RAPID COMMUNICATIONS

PHYSICAL REVIEW C VOLUME 31, NUMBER 1 JANUARY 1985

Precompound limits of linear momentum transfer in heavy ion reactions

Marshall Blann
Lawrence Livermore National Laboratory, Livermore, California 94550
(Received 14 September 1984)

The Boltzmann master equation is used to predict linear momentum transfer for reactions of ^{60}Ni with ^{16}O at lab energies of 8.8 to 100 MeV/nucleon. Results are in excellent agreement with experimental measurements at ^{16}O energies up to 19.6 MeV/nucleon.

A great effort is being exerted in investigating heavy ion reactions involving projectiles with energies in excess of 10 MeV per nucleon. Interpretation of results requires assumptions as to the time evolution of the reaction, in particular, the removal of excitation energy, momentum, and angular momentum versus time during the fusion and equilibration periods. Whether or not new phenomena are being observed depends on whether the observed phenomena exceed the precompound, phase space "background."

The Boltzmann master equation code due to Harp, Miller, and Berne,[1] provides a method of following the time-dependent characteristics of deexcitation and relaxation of a Fermi gas. Modified forms of this code are applicable to heavy ion reactions.[2,3] Quantities are calculated relativistically in this code, so that it may be applied to reactions over a very broad range of excitation. The output of the code has been modified to calculate some interesting reaction characteristics versus time, e.g., the number of escaped neutrons (protons), the energy removed due to neutron (proton) emission, total energy removed, average kinetic energy removed, and momentum loss. This Rapid Communication concerns application of the Boltzmann master equation model to estimating limits of linear momentum transfer in heavy ion reactions. The system $^{16}O+{}^{60}Ni$ was selected for investigation, at incident ^{16}O energies of 8.8 to 100 MeV/nucleon.

The time dependent master equation code, as adopted for heavy ion reactions has been described in detail in Ref. 3; we refer to this work for general details. Calculations are performed in reaction time increments of 2×10^{-23} sec, which is short with respect to the nucleon-nucleon collision times relevant to the excitation energies considered in this work. Estimation of the momentum transfer is an addition to the calculation described in Ref. 3 which involves some additional assumptions requiring description and discussion. The manner of energy partition assumed for the fusion process is as in Ref. 3, but bears repetition.

Results to be presented herein were calculated assuming that the total available excitation energy ($E_{\text{c.m.}}+Q$ value for compound nucleus formation) was partitioned with equal *a priori* probability between some arbitrary number of degrees of freedom. The rationalization for this is that each projectile nucleon has both a center of mass motion, and a Fermi motion which generally makes a broad range of final nucleon energies accessible. All final exciton energies which would leave particles in occupied orbitals must be excluded. These conditions, including energy conservation, are satisfied using the exciton distribution function[4,5]

$$N_n(E)=\frac{g(gE)^{n-1}}{p!h!(n-1)!}\ , \tag{1}$$

which for the 1 MeV bins used in the master equation code, is better written as evaluated for the number of excitons per unit of energy per unit time

$$N_n(U)\Delta U=\Delta A_t[U^{n-1}-(U-\Delta U)^{n-1}]/E^{n-1}\ , \tag{2}$$

where $N_n(U)\Delta U$ is the number of excitons injected per unit time at an energy between U and $U+\Delta U$. ΔA_t is the number of nucleons entering the target nucleus from the projectile per unit time (see Ref. 3), and $n-1$ is the number of degress of freedom assumed for the energy partition. The ΔA_t values are calculated assuming a constant velocity of fusion based on c.m. projectile energy decremented by the Coulomb barrier.

For the ^{16}O induced reactions to be considered in this work, we have assumed a value of $n=19$. The basis for this is that it gives good agreement for the inclusive proton spectra from the reaction $^{16}O+{}^{197}Au$ at 140, 215, and 310 MeV as reported by Awes *et al.*,[6] for proton energies up to approximately 40 MeV.

For proton energies >40 MeV the experimental proton spectra decreased more rapidly[6] than the master equation prediction with $n=19$ in Eq. (2). This is reasonable as the exciton distribution function (1) does not properly restrict the maximum initial exciton energies to the upper limit due to coupling Fermi motion with center of mass motion. To this end, increasing bombarding energies would require higher initial exciton numbers to reproduce the highest emission energies, as indeed was found by Awes *et al.*[6] Nonetheless, the major part of the nucleon emission cross section is in the range where a constant initial exciton number is valid,[6] so we have used $n=19$ for results to be presented. A broader discussion of this parameter will appear in a more extensive discussion of this work.

The spectra of neutrons and protons emitted are calculated for each time step in the calculation. The initial excitons will be forward peaked due to momentum conservation; the higher the energy the more strongly forward peaked they must be. The case of continuum precompound decay for nucleon induced reactions has been discussed by many authors.[7-10] An early work on this subject by Mantzouranis, Weidenmuller, and Agassi[8] pointed out a diffraction limit for the angular localization

$$R\Delta\theta\geqslant 2\pi/k\ , \tag{3}$$

where R is the radius and k the nucleon wave number. We have used the result of (3) to estimate (crudely) a mean emission angle versus nucleon energy, where R is evaluated for a nucleus of the composite target plus projectile mass and the sharp radius value of Myers[11] was assumed. The linear momentum loss due to nucleon emission was assumed to be $\sqrt{2\epsilon m}\cos(\Delta\theta)$, where ϵ is the nucleon kinetic energy and m the nucleon mass; if $\Delta\theta$ exceeded 90°, a value of 90° was assumed. This means that lower energy nucleons contribute relatively little or nothing at all to the linear momentum loss, whereas high energy nucleons will be mainly responsible for the loss in linear momentum.

An additional contribution has been assumed to the linear momentum loss, i.e., an estimate has been made for d, t, and α emission. This estimate was based on the measurements of Bertrand, Peelle, and Kalbach-Cline[12] and of Wu, Chang, and Holmgren[13] for reactions induced on a range of targets by 39, 62, and 90 MeV protons. Whatever the mechanism of emission, cluster emission cross sections at energies above the evaporation region are roughly in a constant ratio to proton emission for different incident proton energies and target mass (within a factor of 2). The ratio $p:d:t:{}^3\text{He}:\alpha$ is roughly 10:1:0.02:0.02:0.5. We have assumed that a similar ratio will result from the nucleon cascade of the master equation, with average cluster energies equal to half the nucleon energies. This means that the linear momentum removed by nucleons was increased by $\sim 8\%$ as an estimate of cluster emission resulting from the nucleon cascade. If the heavy ions have an intrinsic cluster structure leading to enhancement of cluster emission over that generated by the nucleon cascade, larger momentum decrements would result. The data of Awes *et al.*[6] do show considerably higher cluster emission probabilities than result from proton induced reactions. The latter results were for inclusive reactions and may have large contributions from peripheral breakup reactions, which would not be relevant to the more central collisions of interest in this work. Recent results of Bisplinghoff suggest, however, that high angular momenta should enhance precompound cluster decay.[14] Measurements of cluster emission cross sections coincident with production of evaporation-residue-like products will be valuable in making this aspect of the master equation calculation more quantitatively correct.

In Fig. 1 we present calculated linear momentum transfer results for $^{16}O+{}^{60}Ni$ at ^{16}O lab energies of 8.8, 13, 19, 50, and 100 MeV/nucleon. We compare these results with measured values at 13 and 19 MeV/nucleon, and for 8.8 MeV/nucleon ^{16}O on ^{40}Ca (since experimental results at this energy have not yet been reported on ^{60}Ni, to our knowledge). One problem in extracting the model prediction is deciding the time step at which the system has equilibrated. We have taken two sets of values: The open circles of Fig. 1 use the time step for which the change in neutron kinetic energy per unit time versus the equilibration time becomes reasonably constant. The open squares represent momentum loss at the time at which the rate of momentum loss per unit time becomes reasonably constant and minimal. At the lower bombarding energies, the latter point is not plotted, because it does not differ from the result based on neutron kinetic energy.

It may be seen in Fig. 1 that the Boltzmann master equation, a phase space approach, agrees with the data presently available to within the experimental uncertainties.[15] The explanation for the momentum decrement in heavy (and light) ion induced reactions may therefore have a trivial precompound decay interpretation, treatable by existing tools. Towards this end, measurements of nucleon spectra and cluster multiplicities in coincidence with evaporation residuelike fragments would be valuable. The former determines how realistic the original [Eq. (2)] exciton distribution function is, imposing a strict limitation on both the shape, absolute cross sections, and nucleon multiplicities predicted by the master equation approach. The cluster data provide an important change to the momentum loss calculation which may be dependent on the projectile where there is an intrinsic clustering probability (as, e.g., would be expected for $^{6,7}Li$, ^{12}C, etc.), and perhaps with an important angular momentum dependence as well.[14] The model itself should be improved by using a distribution function which includes the limit of exciton energies due to coupling the Fermi and center of mass motion, which is not done in Eq. (2); while this shortcoming does not enter nucleon induced precompound calculations, it does become relevant to heavy ion systems. Nonetheless, Eq. (2) should give a good result for the major part of the cross section if one is guided by the results of Awes *et al.*[6] Results of other reaction characteristics predicted by the master equation approach for this and other reaction systems, including a more detailed discussion of the parameter space, will be forthcoming.

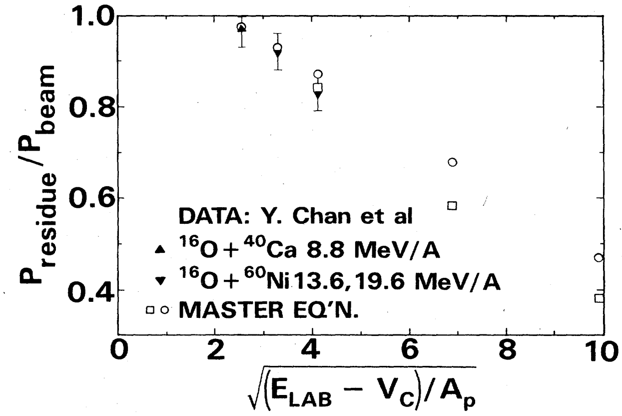

FIG. 1. Fraction of projectile momentum transferred to heavy residues for reactions of 8.8 to 100 MeV/nucleon (lab) ^{16}O with ^{60}Ni. The fractional momentum transfer is given on the ordinate. The abscissa is the square root of [laboratory energy minus the Coulomb barrier (41 MeV) divided by 16]. The two sets of calculated results (open circles and open squares) are discussed in the text. Experimental results are due to Chan *et al.*, Ref. 15.

This work was performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore National Laboratory under Contract No. W-7405-ENG-48.

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