Mass dispersion through particle emission from a vibrating nucleus

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(Received 24 July 1984)

We investigate numerically the predictions of a formula, proposed recently by Balian and Vénéroni, for evaluating fluctuations of one-body observables in a mean-field framework.

In a recent article,[1] Balian and Vénéroni (BV) have proposed a prescription for calculating fluctuations of single-particle observables in a mean-field framework. For a nucleus described by a one-body density matrix ρ such that $\rho^2=\rho$ at time $t=t_0$, the fluctuation in a one-body observable Q at a later time t_1 is[1]

$$\Delta Q^2|_{t_1}=\frac{1}{2}\lim_{\epsilon\to 0}\frac{1}{\epsilon^2}\mathrm{Tr}[\rho(t_0)-\sigma(t_0,\epsilon)]^2 \ , \quad (1)$$

where $\sigma(t,\epsilon)$ is the solution of the time-dependent Hartree-Fock (TDHF) equation

$$i\hbar\dot{\rho}(t)=[w(t),\rho(t)] \ , \quad (2)$$

satisfying the boundary condition

$$\sigma(t_1,\epsilon)=\exp(i\epsilon Q)\rho(t_1)\exp(-i\epsilon Q) \ . \quad (3)$$

The prescription defined by Eqs. (1)–(3) clearly differs from the conventional TDHF expression

$$\Delta Q^2|_{t_1}=\mathrm{Tr}[Q^2\rho(t_1)]-[\mathrm{Tr}Q\rho(t_1)]^2 \ , \quad (4)$$

which is known to lead to too small values for the widths of mass distributions in TDHF calculations.[2] The purpose of the present paper is to investigate numerically the predictions of Eq. (1) in the framework of a simple model and to compare these predictions with those of Eq. (4). [An attractive feature of Eq. (1) is that it requires only the use of existing TDHF codes in contrast with earlier attempts[3] to improve Eq. (4).]

The investigation we have performed is based on a TDHF calculation of monopole vibrations in calcium 40 described in Ref. 4. At time $t=t_0$, the density matrix $\rho(t_0)$ of the nucleus is constructed from a constrained Hartree-Fock calculation in the presence of an external field λr^2, which compresses the nucleus. At later times, the nucleus experiences monopole oscillations. Because of the coupling to continuum states present in the nonlinear TDHF equations, an emission of particles occurs which leads to a damping of collective oscillations. Eventually, the density matrix of the nucleus $\tilde{\rho}(R)$ restricted to a bounded region of space $r\leq R$ will become a solution of the static Hartree-Fock equation

$$[W,\tilde{\rho}(R)]=0 \ . \quad (5)$$

However, $\tilde{\rho}(R)$ does not correspond to a Slater determinant since, because of the emitted particles, it does not satisfy $\tilde{\rho}^2=\tilde{\rho}$.

In the present work, the single-particle observable Q which has been considered, is the number of particles inside a bounded region of space

$$\hat{N}(R)=\int\psi^*(\vec{r})\psi(\vec{r})\theta(R-r)d\vec{r} \ , \quad (6)$$

where $\theta(x)=1$, if $x>0$ and zero otherwise. The fluctuation in the operator $\hat{N}$ gives a measure of the mass dispersion of the nucleus after particle emission has taken place. In our numerical calculations, we have used the same ingredients as in Ref. 4. The TDHF equation (2) has been solved with a zero-range density dependent force on a 0.2 fm mesh inside a sphere of radius $R_0=20$ fm. The initial compression of the nucleus was obtained by applying an external field λr^2 with $\lambda=10$ MeV/fm^2 which reduces the radius from 3.19 to 2.64 fm. The time t_1 at which fluctuations were calculated from Eqs. (1) and (4) was chosen to be $t_1=2\times10^{-22}$ sec with a nuclear radius of 6.36 fm, i.e., after particle emission and before particles reflected from the boundary at $R_0=20$ fm reach the nucleus. The value of the radius R in Eq. (6) has been taken to be 8 fm. At time $t=t_1$, the expectation value of $\hat{N}(R)$ has been found to be 33.10, which corresponds to an emission of 7 particles. The

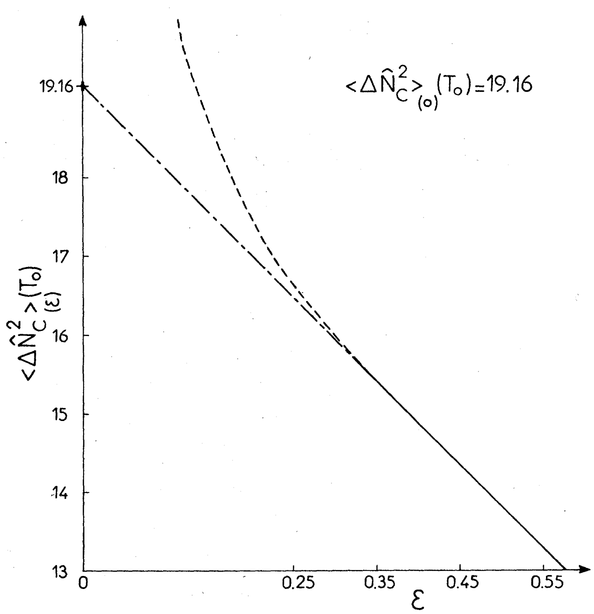

FIG. 1. Fluctuation in the number of particles after particle emission from the calcium-40 nucleus, calculated for various values of ϵ in Eq. (1).

dispersion in $\hat{N}$ calculated from the conventional TDHF expression (4) was found to be

$$\Delta N^2_{\rm TDHF} = 5.27 \ . \tag{7}$$

In order to evaluate the dispersion ΔN^2 from Eq. (1), we have calculated the right-hand side of this equation for various values of ϵ. The result displayed in Fig. 1 shows that large truncation errors occur for values of ϵ below 0.3. This is because the unitary transformation defined by Eq. (3) is too small in this case to produce a meaningful change in the wave function. However, it can be seen that the results obtained in Fig. 1 for $\epsilon > 0.3$ can be safely extrapolated up to $\epsilon = 0$. Using a linear extrapolation, as in Fig. 1, one obtains the following value:

$$\Delta N^2_{\rm BV} = 19.16 \ , \tag{8}$$

which is about four times the TDHF value (7). The prescription of Balian and Vénéroni thus appears to produce results which are qualitatively larger than those of the conventional TDHF expression. This difference is encouraging, since it leaves some hope to improve mass dispersions in heavy-ion collisions in a mean-field framework. Recent TDHF calculations of the collision between two oxygen nuclei appear, in fact, to support this conclusion.[5,6]

We would like to thank M. Vénéroni for helpful discussions.

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