Consequences of Pauli effects in ^{6}Li and the reaction ^{6}Li(e,e′d)^{4}He

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The reaction ^{6}Li(e,e′d)^{4}He, carried out with quasifree kinematics, is ideally suited for mapping out the $^6\text{Li}\rightarrow\alpha+\text{d}$ momentum distribution in the region where Pauli effects from the underlying $S_{1/2}$ αN interaction are manifest. Data for the $^6\text{Li}\rightarrow\alpha+\text{d}$ momentum distribution in the range of alpha-particle recoil momenta of 0.0 to 1.4 fm^{-1} are important to understanding three-body models of ^{6}Li in general and the $S_{1/2}$ αN interaction in particular. In anticipation of ^{6}Li(e,e′d)^{4}He coincidence measurements, the cross section is calculated with three-body models of ^{6}Li under the assumptions of one-photon exchange and deuteron-pole dominance. The required kinematical conditions that make the theory applicable are emphasized.

Recently, Parke and Lehman (PL)[1] concluded qualitatively that the ^{6}Li(e,e′d)^{4}He reaction should provide the best method for extracting the $^6\text{Li}\rightarrow\alpha+\text{d}$ momentum distribution and thereby testing detailed predictions about the distribution as derived from three-body (αNN) models of ^{6}Li. Specifically, it is clear from the work of PL, plus earlier work of Lehman and Rajan (LR),[2] that the location of the minimum and the recovery to the secondary maximum of the momentum distribution is mainly controlled by the $S_{1/2}$ partial wave of the αN interaction. Therefore, availability of experimental data concerning this momentum distribution for relative αd momentum, q, in the range $0.0\leqslant q\lesssim 1.4$ fm^{-1} should constitute a demanding test of the three-body model of ^{6}Li and the underlying dynamical input. The objective of this Brief Report is to quantify the qualitative statements of PL by calculating the ^{6}Li(e,e′d)^{4}He cross section and to give explicitly the required kinematical conditions that make the underlying theory viable. Such a calculation should be of value to those who contemplate such experiments.

There is ample evidence that the $A=6$ system can be treated as a three-body problem (αNN) for excitation energies $\lesssim 20$ MeV.[1–3] The basic two-body interactions, NN and αN, are represented by separable potentials fit to experimentally determined phase shifts and other low-energy observables. In particular, the $S_{1/2}$ αN interaction leads to phase shifts that reflect the composite nature of the α particle and thereby embodies the main effects of the Pauli exclusion principle. Simply put, the $1s$ nuclear shell can accommodate only four nucleons (the α) and any nucleon that scatters on the α particle experiences a repulsion at close distances due to the Pauli effect. The Pauli principle can be incorporated into the αN interaction through either a repulsive or an attractive (with an excluded-bound state) potential.[4] Both are consistent with present αN scattering data.

Previous investigations by LR on the alpha-deuteron (αd) structure of ^{6}Li demonstrate that, in three-body models, the $S_{1/2}$ αN interaction Pauli effect is manifest in the nodal structure of the effective αd coordinate-space wave function or, equivalently, through the presence of a diffraction minimum in the $^6\text{Li}\rightarrow\alpha+\text{d}$ momentum distribution.[2] LR and PL calculated the allowed $l=0$ and $l=2$ partial-wave components of the $^6\text{Li}\rightarrow\alpha+\text{d}$ momentum-distribution amplitude, denoted by them as $f_l(q)$. Furthermore, PL compared the momentum distributions from the two different models of the $S_{1/2}$ αN interaction, i.e., repulsive or attractive-projected $S_{1/2}$ αN interaction and all other two-body interactions unchanged, with the distribution extracted from the best available ^{6}Li(p,pd)^{4}He data. Both distributions are in good agreement with the data in the region where the pole-dominance assumption is justified ($0.0\leqslant q\lesssim 0.3$ fm^{-1}). Outside this region ($0.3\lesssim q\lesssim 0.7$ fm^{-1}), both theoretical curves fall below the experimental distribution, the attractive-projected model being the lower of the two. The discrepancy in the region $0.3\lesssim q\lesssim 0.7$ fm^{-1} can be attributed to failure of the pole-dominance assumption (theory) or, more likely, due to the incorrect values of the p-d cross section used in extracting the experimental distribution.[2] Nevertheless, the data stop just before the predicted diffraction minimum whose location is set primarily by the nature of the $S_{1/2}$ αN interaction in the three-body model. Thus, it would be an excellent test of the three-body model and the handling of Pauli effects in the underlying $S_{1/2}$ αN interaction if data were available for the $^6\text{Li}\rightarrow\alpha+\text{d}$ momentum distribution for $0.0\leqslant q\lesssim 1.4$ fm^{-1}. Moreover, the experimental demonstration of a diffraction minimum in the $^6\text{Li}\rightarrow\alpha+\text{d}$ momentum distribution is in itself of importance since it is a clear signal of Pauli effects. This aim motivates our present calculation of the ^{6}Li(e,e′d)^{4}He cross section. We believe this reaction will be the most direct means of extracting the $^6\text{Li}\rightarrow\alpha+\text{d}$ momentum distribution.

The electron is preferable to the proton as a probe, e.g., the ^{6}Li(e,e′d)^{4}He reaction as opposed to the ^{6}Li(p,pd)^{4}He reaction, for two reasons: (a) difficulty in obtaining sufficient energy resolution for the recoil α particle in (p,pd); and (b) absence of distortion effects in electron scattering at sufficiently high energies, i.e., rescattering between either the incoming or outgoing electron and the target or outgoing nuclear products, respectively, is negligible. In contrast, distortion effects usually play a role for proton projectiles since they interact strongly.

Under the two assumptions of one-photon exchange and deuteron-pole dominance between the virtual photon and $^6\text{Li}\rightarrow\alpha+\text{d}$ vertices, the ^{6}Li(e,e′d)^{4}He coincidence cross section can be written as

$$\frac{d^3\sigma}{d\Omega_e d\Omega_d dE_e}\cong(\text{k.f.})\times\left(\frac{d\sigma}{d\tilde{\Omega}_{ed}}\right)_{\text{elastic}}\times[f_0(q)]^2\ , \qquad (1)$$

where k.f. represents the kinematical factor given by

$$\text{k.f.} = \frac{\tilde{k}_1^e(\tilde{E}_1^e + \tilde{E}_1^d)^2 k_2^e (k_2^d)^2 E_2^\alpha}{\tilde{k}_2^e \tilde{E}_1^d E_2^d k_1^e \,|(k_2^d/E_2^d)[E_2^\alpha + E_2^d] - \hat{k}_2^d \cdot (\vec{k}_1^e - \vec{k}_2^e)|} . \quad (2)$$

In the kinematical factor, Eq. (2), $E(k)$ refers to the energy (momentum) of the particle indicated by a superscript and the subscripts 1 and 2 refer to incoming and outgoing, respectively. The tilde ($\sim$) above quantities in Eqs. (1) or (2) means that they are to be evaluated in the center-of-momentum (c.m.) frame of the electron and deuteron; otherwise the quantities in Eq. (2) are to be evaluated in the laboratory frame of the ^{6}Li(e,e′d)^{4}He reaction. Lastly, the approximately equals sign in Eq. (1) indicates that the $l=2$ partial-wave component of the $^6\mathrm{Li} \to \alpha + \mathrm{d}$ momentum-distribution amplitude has been neglected. Omission of $f_2(q)$ is justified by its smallness of magnitude compared with $f_0(q)$. The main contribution from $f_2(q)$ appears around the diffraction minimum ($q \sim 0.7$ fm^{-1}) of the momentum distribution where it fills the minimum slightly. The absence of $f_2(q)$ leads to the factorized form of Eq. (1).

The relatively simple expression for the coincidence cross section as given by Eq. (1) holds only if one assumes deuteron-pole dominance, i.e., the electron interacts with only the ejected deuteron and no final-state rescattering of the αd pair occurs. When rescattering happens, factorization is impossible, obscuring the function of interest—$[f_0(q)]^2$. Deuteron-pole dominance, or equivalently, quasi-free scattering of the electron from the deuteron, can be ensured by choosing kinematical regions for the reaction where the deuteron is ejected with considerable kinetic energy ($\gtrsim 80$ MeV) and the relative energy of the αd pair is large ($\gtrsim 70$ MeV).[5] This assures that the detected deuteron, rather than the α particle, did indeed interact with the incident electron, and it should minimize, if not eliminate, any significant αd rescattering effects.

Of course, $[f_0(q)]^2$ would be difficult to extract from experiment if the kinematical factor and/or e-d cross section were rapidly varying in the kinematical region of interest. We shall see below, however, that the kinematical factor and the e-d elastic scattering cross section are slowly varying functions of q under the chosen kinematical conditions. Thus, experimental determination of the coincidence cross section can be used to reveal the form of $[f_0(q)]^2$ through Eq. (1), which, in turn, will provide information as to the nature of the $S_{1/2}$ αN interaction.

In order to evaluate the coincidence cross section given by Eq. (1), we need, besides the kinematical factor given by Eq. (2), the c.m. elastic e-d cross section and $[f_0(q)]^2$. Even though the exchanged deuteron is off shell, it is adequate for our purposes to use the on-shell versions of both the kinematical factor and the e-d cross section. To achieve this, we adopt the convention of computing the incident electron energy for on-shell elastic scattering from the final-state kinematics. The cross section in the laboratory frame is calculated from

$$\frac{d\sigma}{d\Omega_{\mathrm{ed}}} = \frac{\alpha^2\cos^2(\theta/2)}{4E_1^2\sin^4(\theta/2)} \frac{A(q_e^2) + \tan^2(\theta/2)B(q_e^2)}{[1+(2E_1/M_\mathrm{d})\sin^2(\theta/2)]} , \quad (3)$$

where E_1 is the incident electron energy, θ is the scattered-electron angle in the laboratory, q_e^μ is the four-momentum transfer of the electron, M_d is the deuteron mass, and $A(q_e^2)$ and $B(q_e^2)$ are the elastic structure functions of the deuteron. $A(q_e^2)$ and $B(q_e^2)$ are obtained from the graphical data in Elias *et al.*[6] and Rand *et al.*,[7] while the c.m. e-d cross section is obtained from Eq. (3) after multiplication by $d(\cos\theta)/d(\cos\tilde{\theta})$. Finally, the s-wave momentum distribution amplitude, $f_0(q)$, is that of LR for the repulsive $S_{1/2}$ αN interaction and that of PL for the attractive-projected $S_{1/2}$ αN interaction, both with an NN interaction that produces a 4% D state in the deuteron.

After an extensive kinematical search, we find that the reaction ^{6}Li(e,e′d)^{4}He, constrained to have relative energy of the outgoing αd pair $E_{\alpha\mathrm{d}} \gtrsim 70$ MeV, requires incident electron energies $E_1 \gtrsim 400$ MeV in order to cover the required range of q, $0 \leqslant q \lesssim 1.4$ fm^{-1}, for reasonable θ. The key element is $E_{\alpha\mathrm{d}}$ which increases with θ for a given q. For example, with the scattered-electron and outgoing-deuteron angles coplanar on opposite sides of the incident beam direction, and $E_1 = 400$ MeV, then $E_{\alpha\mathrm{d}}(q=0.1) = 49$ MeV and $E_{\alpha\mathrm{d}}(q=1.4) = 130$ MeV when $\theta = 85°$ and $\theta_\mathrm{d} = 42.2°$, whereas $E_{\alpha\mathrm{d}}(q=0.1) = 63$ MeV and $E_{\alpha\mathrm{d}}(q=1.4) = 149$ MeV when $\theta = 105°$ and $\theta_\mathrm{d} = 32.4°$. The value for the deuteron angle represents the minimum allowed value at $q = 0.0$ for the given E_1 and θ.

For $E_1 = 400$ MeV and the angles specified in the previous paragraph, we plot the kinematical factor, the e-d cross section, and the coincidence cross section in Figs. 1–3, respectively. The first thing that we note is that the kinematical factor and the e-d cross section are slowly and monotonically varying functions of q. As a result, the coincidence cross section varies primarily as $[f_0(q)]^2$ and thus the extraction of $f_0(q)$ from a ^{6}Li(e,e′d)^{4}He experiment is a straightforward matter under the proper conditions.

Since the key element is to satisfy the quasifree (pole-dominance) scattering conditions, i.e., large enough $E_{\alpha\mathrm{d}}$ to make Eq. (1) valid, it appears that at $E_1 = 400$ MeV part of the experiment should be carried out at $\theta = 105°$ to maximize $E_{\alpha\mathrm{d}}$ for low q, say $q \lesssim 0.7$ fm^{-1}, and at $\theta = 85°$ for $q \gtrsim 0.5$ fm^{-1} where the larger coincidence cross section is also important. The low q region is important for checking the overall normalization with the existing high energy ^{6}Li(p,pd)^{4}He data and theory. The suggested overlap region for the two kinematics, $0.5 \lesssim q \lesssim 0.7$ fm^{-1}, would serve to test the effect of rescattering on the reaction. If the deuteron-pole graph is dominant, changing the kinemat-

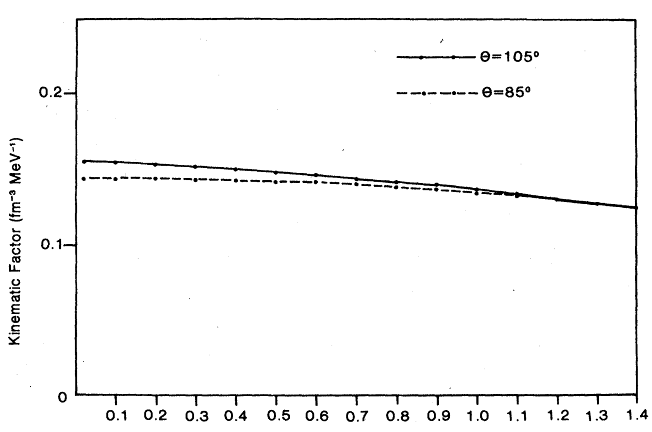

FIG. 1. Kinematic factor for ^{6}Li(e,e′d)^{4}He scattering cross section at $E_1^e = 400$ MeV as a function of q.

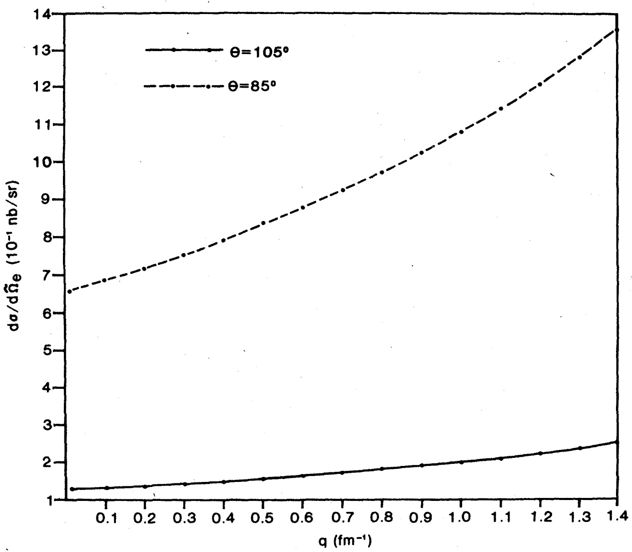

FIG. 2. Elastic scattering cross section $d\sigma/d\tilde{\Omega}_e$ in the c.m. of the electron-deuteron system.

ics should lead to the same value for $f_0(q)$ at fixed q. Finally, the most important region is $q \gtrsim 0.7\ \mathrm{fm}^{-1}$—the location of the minimum in $f_0(q)$ and the magnitude of $f_0(q)$ at the secondary maximum. In this latter region, the predictions of the three-body models should be stringently tested.

Clearly, the predicted cross section in the region of the minimum and the secondary maximum is small (see Fig. 3) and measurements to determine its value in this range of q are likely to be difficult to carry out. However, this situation is typical of most coincidence measurements at present. Despite its difficulty, we believe the experiment would be a worthwhile effort. The results will be critical to refining our understanding of the ^{6}Li three-body model in general and the $S_{1/2}$ component of the αN interaction, in particular.[8]

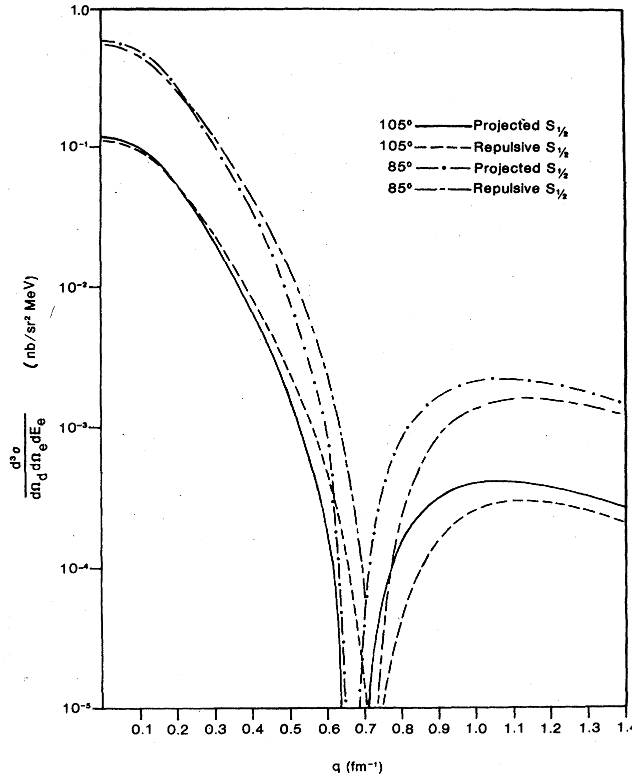

FIG. 3. Coincidence cross section for the ^{6}Li(e, e′d)^{4}He reaction as a function of q, derived from both a repulsive $S_{1/2}$ αN interaction and an attractive projected $S_{1/2}$ interaction. The angles indicated are those of the scattered electron.

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