Isoscalar giant monopole in a macroscopic-microscopic approach

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We present an approach to study the interplay between the surface and volume vibrations of nuclei with application to the isoscalar giant monopole resonance. The main ingredients are the well-known properties of the mean field and the curvatures of the liquid-drop energy surface. No specific form of a two-body force is assumed. The single-particle continuum is treated exactly. First results, obtained using a local Woods-Saxon potential, are discussed.

Within the last decade a significant amount of work has been devoted to the study of the nuclear response function in the region of 10—40 MeV of excitation energy.[1,2] The random phase approximation (RPA) with effective two-body interactions is quite successful in reproducing the experimental results,[3] providing a better understanding of properties of nuclei. In particular, the discovery of the isoscalar giant monopole resonance (ISGMR) in medium and heavy nuclei[4] provided useful information regarding nuclear incompressibility. Using the RPA and considering various forms of effective two-body interactions, it has been concluded[5,6] that the experimental results imply a compressibility coefficient $K_\infty=200$ MeV for the nuclear matter. It has been pointed out, however, that this conclusion may be due to the limited forms of the effective two-body interactions used in the calculation.[7,8] Since the nuclear surface plays an important role in determining the properties of nuclei, a study of the interplay between the volume and surface vibrations can provide a better insight on the relationship between the properties of the ISGMR and K_∞.

In this work we describe a macroscopic-microscopic approach to study the interplay between the volume and surface vibrations in a way directly related to the curvatures of the liquid drop (LD) energy surface, i.e., the volume and surface compressibility coefficients. No specific form of the two-body interaction is assumed. In fact, one can vary the volume and surface compressibility coefficients independently. This is equivalent to considering various forms of two-body interactions which generate the same mean field. We also obtain simple expressions for the transition density and transition potential of the states. We note that a classical model[9] using a specific form for the effective two-body interaction was employed recently to study the effect of the surface vibration on the ISGMR.

The present method is based on the collective coupling approximation of Bohr and Mottelson,[10] extended in the collective transport theory of Hofmann and Siemens[11,12] to include several collective degrees of freedom. The main elements of the theory are the following. We first identify the relevant collective degrees of freedom $\{Q_\mu\}$ and approximate the Hamiltonian of the system in terms of the mean field $H_{\rm MF}(\mathbf{r},\mathbf{p},\{Q_\mu\})$, assumed to depend on $\{Q_\mu\}$. We make use of the Strutinsky renormalization procedure[13] to ensure that the approximated Hamiltonian reproduces the constrained energy surface $E(Q)$. We therefore have

$$H(Q)=\sum_i H_{\rm MF}(\mathbf{r}_i,\mathbf{p}_i,\{Q_\mu\})+\delta E_{\rm LD}(Q)\ , \tag{1}$$

with

$$\delta E_{\rm LD}(Q)=E_{\rm LD}(Q)-\left\langle \sum_i H_{\rm MF}(\mathbf{r}_i,\mathbf{p}_i,\{Q_\mu\})\right\rangle_{\rm smooth}\ , \tag{2}$$

so that

$$E(Q)=\langle H(Q)\rangle=E_{\rm LD}(Q)+E_{\rm shell}(Q)\ . \tag{3}$$

Here $E_{\rm LD}(Q)$ is the LD energy and $E_{\rm shell}(Q)$ is the Strutinsky shell correction term. We require that the mean field give a reasonable single particle spectrum and a one-body density matrix. We then use the linear response theory to derive equations of motion for the collective degrees of freedom, obtaining the RPA equations[12]

$$\chi^{\rm RPA}_{\mu\nu}(\omega)=\sum_{\sigma\delta} k_{\mu\sigma}[k-\chi^{\rm sp}(\omega)]^{-1}_{\sigma\delta}\chi^{\rm sp}_{\delta\nu}(\omega)\ , \tag{4}$$

where

$$\chi^{\rm sp}_{\mu\nu}(\omega)={\rm Tr}(F_\mu G^0(\omega)F_\nu) \tag{5}$$

is the free particle-hole Green's function in the space of collective coordinates. The form factor F_ν and the coupling constant $k_{\mu\nu}$ are given by

$$F_\nu=\left.\frac{\partial H}{\partial Q_\nu}\right|_{\{Q^0_\mu\}}\ , \tag{6}$$

$$k_{\mu\nu}=\left\langle\frac{\partial^2 H}{\partial Q_\mu\partial Q_\nu}\right\rangle_0=\frac{\partial^2\langle H\rangle_0}{\partial Q^0_\mu\partial Q^0_\nu}-\chi^{\rm sp}_{\mu\nu}(\omega=0)\ . \tag{7}$$

We point out that the RPA equations (4) can be obtained in the usual way (avoiding collective coordinates) assuming a separable two-body interaction of the form

$$v(1,2)=\sum_{\mu\nu}F_\mu(1)k^{-1}_{\mu\nu}F_\nu(2)\ . \tag{8}$$

It is clear from Eqs. (1)—(7) that the input in the present approach is the mean field and the curvatures

$\partial^2 E(Q)/\partial Q_\mu \partial Q_\nu$ of the LD energy surface. Following Shlomo and Bertsch,[14] the single particle continuum is treated exactly using the complete representation

$$g(r_1,r_2,\omega)=(H_{\rm MF}-E)^{-1}=-\frac{2m}{\hbar^2}u(r_<)v(r_>)/W \quad (9)$$

for the single particle propagator. Here u and v are the regular and irregular solutions of the Schrödinger equation and W is the Wronskian. The free p-h Green's function G^0 is then evaluated from

$$G^0(r_1,r_2,\omega)=-\sum_h \phi_h^*(r_1)[g(r_1,r_2,\epsilon_h+\omega)+g(r_1,r_2,\epsilon_h-\omega)]\phi_h(r_2)\,, \quad (10)$$

where $\phi_h(r)$ is the wave function of the occupied orbit h. Using Eqs. (9), (10), and (6) we find $\chi^{\rm sp}(\omega)$ from Eq. (5) and substitute the result in Eq. (4). The RPA response function of the system to the scattering operator F_μ is given by[3]

$$S_\mu(\omega)=\frac{1}{\pi}\,{\rm Im}\chi_{\mu\mu}(\omega) = \sum_n |\langle 0|F_\mu|n\rangle|^2\delta(E_n-E_0-\omega)\,. \quad (11)$$

Using Eq. (8), the transition potential F_{n0} to the collective state $|n\rangle$ is found as a linear combination of F_μ,

$$F_{n0}=\sum_\mu\left[\sum_\nu k_{\mu\nu}^{-1}\,{\rm Tr}(F_\nu\rho_{n0})\right]F_\mu=\sum_\mu \alpha_\mu F_\mu\,, \quad (12)$$

where ${\rm Tr}(F_\nu\rho_{n0})$ is easily obtained from $\chi^{\rm RPA}$ in the vicinity of the resonance. The corresponding transition density $\rho_{n0}(r)$ is then given by $\Sigma\alpha_\mu\rho_\mu$, in obvious notation.

We now simply apply the method to study the interplay between volume and surface vibrations in the ISGMR. For the purpose of demonstration we assume a local mean field of Woods-Saxon form which includes central, spin-orbit, symmetry, and Coulomb potential:

$$V_{\rm MF}(r)=\left[1-0.65\tau_z\frac{N-Z}{A}\right]\times\left[V(r)+V_{ls}r_0^2\sigma\cdot 1\frac{1}{r}\frac{df}{dr}\right]+\tfrac{1}{2}(1-\tau_z)V_{\rm Coul}\,, \quad (13)$$

where

$$V(r)=V_0f(r)=V_0/[1+e^{(r-R)/a_v}]\,, \quad (14)$$

with $R=r_0A^{1/3}$, $r_0=1.2$ fm, $V_0=-57$ MeV, $a_v=0.65$ fm, and $V_{ls}=8.5$ MeV. The Coulomb potential is that of a uniform charge distribution with the empirical rms radius. Clearly R, a_v, and V_0 provide a natural choice for the collective coordinates describing the oscillations of the mean field. We define the renormalized collective coordinates $\{Q_i\}$ by writing

$$V(r,\{Q_i\})=V[r;Q_1R,Q_2a_v,Q_3V_0]\,,$$

so that the ground state values are $Q_i^0=1$. The corresponding form factors

$$F_i=\left.\frac{\partial V}{\partial Q_i}\right|_{Q=1}$$

are

$$F_1=-R\frac{\partial V}{\partial r},\quad F_2=(R-r)\frac{\partial V}{\partial r},\quad F_3=V(r)\,. \quad (15)$$

It is possible to investigate the properties of the ISGMR treating the quantities $\partial^2\langle H\rangle/\partial Q_i\partial Q_j$ as parameters. However, these six parameters have no simple physical meaning. It is more transparent to relate them to the volume and surface vibrations of the *matter density*.

We relate the mean field vibrations to those of the matter density

$$\rho(r)=\sum \phi_h^*(r)\phi_h(r)$$

by fitting the adiabatic density calculated in the mean field $\rho(r)$ to a Fermi distribution ρ_F,

$$\rho_F[r,q_1c,q_2a,q_3\rho_0]=q_3\rho_0/\{1+\exp[(r-q_1c)/(q_2a)]\}\,. \quad (16)$$

The values of $\{q_i\}$ are obtained from the conditions

$$\langle r^n\rangle=\int r^n\rho(r)d\mathbf{r}=\int r^n\rho_F(r)d\mathbf{r}\,, \quad (17)$$

for $n=0$, 2, and 4. The ground state values $q_i^0=1$ correspond to $Q_j^0=1$. Due to the constraint $\langle r^0\rangle=A$, we have only two independent coordinates, taken to be q_2 (surface) and q_3 (volume). Considering small oscillations, we evaluate

$$D_{ij}=\left.\frac{\partial q_i}{\partial Q_j}\right|_{Q=1}$$

by varying Q_j and solve the linear equations $(q-1)=D(Q-1)$ to find Q_i in terms of q_2 and q_3. We therefore have, in our model calculations, the two form factors

$$F_a=\frac{\partial V}{\partial q_2}=\sum C_i^aF_i,\quad F_\rho=\frac{\partial V}{\partial q_3}=\sum_i C_i^\rho F_i\,, \quad (18)$$

where F_i are given in Eq. (15), and C_i^a and C_i^ρ are obtained numerically. The corresponding transition densities are of the form

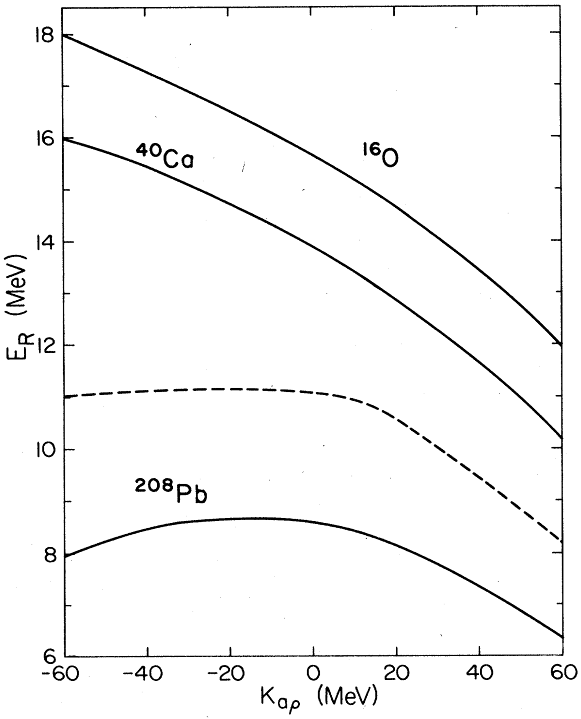

FIG. 1. Energy E_R of the isoscalar giant monopole resonance as a function of $K_{a\rho}$, using $K_{aa}=150$ MeV and $K_{\rho\rho}=60$ MeV (solid line) and 120 MeV (dashed line).

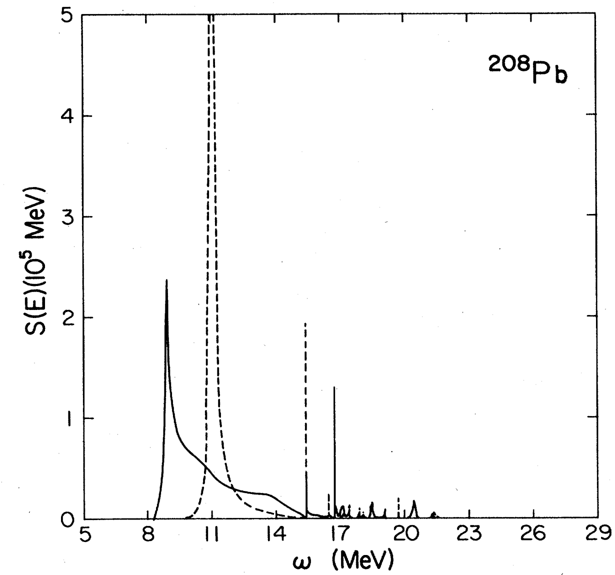

FIG. 2. Response function of ^{208}Pb to F_ρ using $K_{aa}=150$ MeV, $K_{\rho\rho}=120$ MeV, and $K_{a\rho}=40$ MeV (solid line) and $K_{a\rho}=-40$ MeV (dashed line).

$$\rho_a=\left[\frac{3c}{x}-r\right]\frac{\partial\rho_F}{\partial r},\ \ \rho_\rho=\frac{1}{x}\left[x\rho_F+c\frac{\partial\rho_F}{\partial r}\right], \qquad (19)$$

where $x=(3+y)/(1+y)$ with $y=(\pi a/c)^2$. The surface, K_{aa}, volume, $K_{\rho\rho}$, and coupling, $K_{a\rho}$, compressibility coefficients used in the present evaluation of Eq. (4) are defined by

$$K_{aa}=\frac{9}{A}a^2\frac{\partial^2 E}{\partial a^2}\ ,$$
$$K_{\rho\rho}=\frac{9}{A}\rho_0^2\frac{\partial^2 E}{\partial \rho_0^2}\ , \qquad (20)$$
$$K_{a\rho}=\frac{9}{A}a\rho_0\frac{\partial^2 E}{\partial a\,\partial\rho_0}\ .$$

We study the ISGMR by varying these parameters.

Figure 1 shows a plot of the resonance energy E_R of the ISGMR, for ^{16}O, ^{40}Ca, and ^{208}Pb, as a function of the coupling $K_{a\rho}$. In Fig. 2 the RPA response function of ^{208}Pb for F_ρ is shown for $K_{aa}=150$ MeV, $K_{\rho\rho}=120$ MeV, and $K_{a\rho}=-40$ MeV (dashed line) and $K_{a\rho}=40$ MeV (solid line). For ^{208}Pb, the ground state density parameters Eq. (16) are the following: $c=6.48$ fm, $a=0.50$ fm, and $\rho_0=0.17$ fm^{-3}. The form factors (18) are $F_a=-0.30F_1+1.18F_2-1.30F_3$ and $F_\rho=-0.41F_1-0.10F_2+0.22F_3$. From this preliminary investigation we arrive at the following conclusions: (i) E_R is quite sensitive to $K_{a\rho}$, the coupling between volume and surface vibrations. In the range $|K_{a\rho}|\leq 60$ MeV, E_R is changed by 2–4 MeV, leading to a significant change in K_∞. (ii) The width of the ISGMR increases by more than a factor of 2 compared to the uncoupled case (with only ρ_0). (iii) Varying K_{aa}, $K_{a\rho}$, and $K_{\rho\rho}$ within a reasonable range,[9] we find that the surface mode is not very collective. This may be due to the fact that at high excitation energy the particle decay width is very large. (iv) The form factor has a large component of F_ρ ($\alpha_\rho/\alpha_a>2$). (v) Even though there are two collective degrees of freedom, there are many eigenmodes, only one of which shows collective structure (contrast Ref. 9 and other methods cited therein).

In summary, we have demonstrated that the present method, based on the collective coordinates RPA, for studying the ISGMR provides new insight into nuclear vibration. This is due to the fact that the volume and surface compressibility coefficients can be varied independently. The method can be easily applied to studying giant resonances of other multipolarities,[15] and it is numerically fast. However, in order to arrive at a firm conclusion concerning the compressibility of nuclear matter, the following improvements are needed: (i) To reproduce the empirical values of E_R, the nucleon effective mass should be included. (ii) To extract the LD compressibility coefficients, the contribution of the shell correction term (K^{shell}) should be evaluated. This may be particularly important in K_{aa}. (iii) Investigate the sensitivity of the results to the parameters of the mean field and the explicit form for the fitted LD mass density distribution. (iv) Carry out a systematic study for $A>40$. Work on these points is now in progress and the results will be published elsewhere.

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