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Three-nucleon bound-state collapse with Tabakin potentials

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The collapse of the three-nucleon ground state with separable two-body interactions of a Tabakin type is investigated. A simple scaling property of this phenomenon is derived. Moreover, an interpretation of the collapse is given in terms of the local equivalent of the Tabakin potential.

I. INTRODUCTION

Searching for interactions that can describe satisfactorily the nucleon-nucleon scattering data and at the same time allow a relatively simple evaluation of three-nucleon observables, one is led to use separable potentials, preferably of low rank. In Ref. 1 Tabakin constructed a separable potential of rank one with both attraction and repulsion, which is capable of reproducing accurately the S-wave NN phase shifts up to moderately high energies. In particular, the sign change of the phases, in the $^3S_1$ as well as the $^1S_0$ partial waves, can be accounted for with no need to introduce higher-rank interactions. Shortly after Tabakin's proposal, however, application to the 3N system by different authors[2,3] produced an unphysically large value for the triton binding energy, in spite of a proper description of the two-body data. In Ref. 2 Beam ascribed this phenomenon, without explaining it, to the existence of a zero-width resonance, alias continuum bound state (CBS), in the Tabakin potential. In a series of papers, about a decade later, Sofianos *et al.*[4] and Pantis *et al.*[5] studied in great detail this dramatic overbinding of the triton (and of ^{4}He), calling it few-body bound-state collapse (BSC). They used nonlocal potentials of higher (2) rank, either purely separable or with local parts, to investigate the connection between the occurrence and location of a CBS or a genuine resonance in the two-particle T matrix. To that end the potential parameters were varied, keeping the deuteron properties fixed as much as possible. Their observations implied a generalization of Beam's conclusion, namely that the BSC can occur (but not inevitably) when the two-body resonance pole lies close enough to the real axis. Furthermore, the extent of collapse surprisingly turned out to grow for the CBS (or resonance) moving to higher energies. Nevertheless, neither of these phenomena could be explained satisfactorily, although abundantly many numerical results gave some insight into what was going on.

In this paper we study certain aspects of the BSC for the Tabakin potential to shed more light on the character of the collapse phenomenon. The interaction has a slightly different form than in Ref. 1, allowing for a relativistically covariant extension, which is used to study relativistic effects in the three-nucleon system within the framework of the Bethe-Salpeter equation.[6] In Sec. II the parameters of the form factors are given together with the results for the trinucleon binding energies. Using simple scaling arguments we show in Sec. III that the three-body binding energy grows when the CBS moves to higher energies. Another way to look at the collapse phenomenon is to introduce an effective local two-body potential. In terms of such a local interaction we find in the region where the collapse takes place a qualitatively very different behavior. This is discussed in Sec. IV.

II. TRITON BOUND STATES FROM THE TABAKIN POTENTIAL

Throughout this paper we use a separable potential of the form

$$V(p,p')=\lambda g(p)g(p') , \tag{1}$$

with

$$g(p)=\sqrt{m_{\rm N}/E_p}\,\frac{p_c^2-p^2}{(p^2+\beta^2)^2}\,\frac{p^2+\alpha^2}{p^2+\gamma^2},\quad E_p=\sqrt{p^2+m_{\rm N}^2}\ , \tag{2}$$

where λ, α, β, γ, and p_c are parameters, to be adjusted separately to the $^3S_1$ and $^1S_0$ proton-neutron scattering data. The factor $\sqrt{m_{\rm N}/E_p}$ is included in order to have the same form factor as used in Ref. 6, where the square root in the nonrelativistic case makes the Lippmann-Schwinger (LS) equation equivalent to the Blankenbeckler-Sugar (BSLT) (Ref. 7) equation. The effect of this factor is very small, both in the two-body and three-body calculations. The original form factor of Tabakin reads

$$g(p)=\frac{p_c^2-p^2}{p^4+\beta^4}\,\frac{p^2+\alpha^2}{p^2+\gamma^2}\ . \tag{3}$$

With the above potential the equal-mass LS equation for the T matrix can be solved in closed form, yielding

$$T(p,p';E)=\lambda g(p)\tau(E)g(p') , \tag{4}$$

with

$$\tau^{-1}(E)=1+\frac{\lambda}{8\pi^2 m_N}\int_0^\infty dk\frac{k^2g^2(k)}{E_k(k^2-p^2-i\epsilon)} ,$$
$$p=\sqrt{m_N E} , \tag{5}$$
$$E_k=\sqrt{k^2+m_N^2} .$$

The integral in Eq. (5), as for its principal value, is evaluated numerically with high precision, after a suitable subtraction. The parameters are not fitted to the experimental phase shifts, but fixed by the S-wave scattering lengths a, effective ranges r_0, zeros of phases, and the deuteron binding energy E_d ($^3S_1$) (see Ref. 6 for the resulting phase shifts). The triplet and singlet parameters are given in Table I. With this two-body input we can solve the triton binding energy E_t and wave function from coupled homogeneous Faddeev equations, restricting ourselves to the $^3S_1$ and $^1S_0$ channels. For the case of three identical fermions interacting through pairwise, separable S-wave potentials of rank one, they read, after partial-wave decomposition,

TABLE I. Tabakin potential parameters and low-energy neutron-proton scattering quantities.

	$^3S_1$	$^1S_0$
λ	-1835.6 GeV^4	-1539.3 GeV^4
β	1.0152 GeV	1.0 GeV
α	0.16576 GeV	0.22525 GeV
γ	0.14868 GeV	0.17887 GeV
p_c	0.41843 GeV	0.33479 GeV
a	5.424 fm	-23.748 fm
r_0	1.759 fm	2.75 fm
E_d	2.2246 MeV	

$$\phi^i(q,q'';E_t)=\sum_{j=1}^{2}\int_0^\infty \frac{q'^2dq'}{2\pi^2}V^{ij}(q,q';E_t)\tau^j\left[-E_t-\frac{3}{4}\frac{q'^2}{m_N}\right]\phi^j(q',q'';E_t) , \tag{6}$$

with the driving term

$$V^{ij}(q,q';E_t)=C^{ij}\int_{-1}^{1}dx\frac{\lambda g_i(p)g_j(p')}{-E_t-(q^2+q'^2+qq'x)/m_N},\quad x=\frac{\mathbf{q}\cdot\mathbf{q}'}{qq'} , \tag{7}$$

where the

$$C^{ij}=\begin{bmatrix}\frac{1}{4} & -\frac{3}{4}\\ -\frac{3}{4} & \frac{1}{4}\end{bmatrix}$$

are recoupling coefficients for spin and isospin, and where the relative momenta p and p' are given by $\mathbf{p}=-\mathbf{q}'-(\mathbf{q}/2)$ and $\mathbf{p}'=\mathbf{q}+(\mathbf{q}'/2)$. Equation (6) is a homogeneous integral equation in one continuous variable. As a result it can be solved with high accuracy in the bound-state region by straightforward discretization procedures. With our Tabakin interaction the model triton supports two bound states with energies $E_t=858$ MeV and $E_{t'}=5.83$ MeV, to be compared with the experimental triton bound-state energy of 8.48 MeV. With the

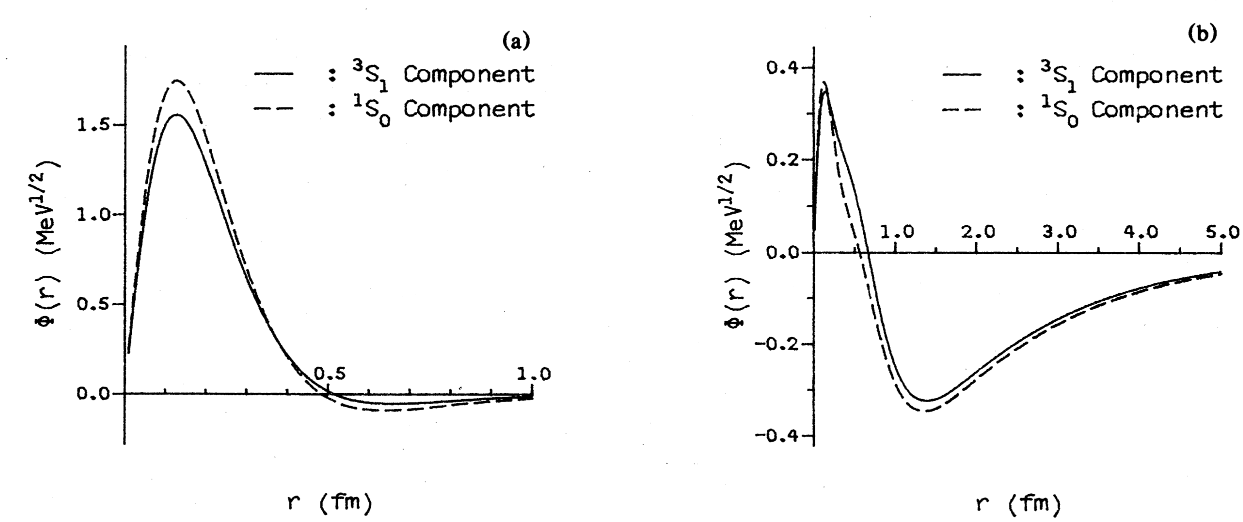

FIG. 1. Triton wave function with Tabakin potential. (a) Collapsed ground state; (b) first excited state.

original Tabakin potential we obtain the values $E_t=333$ MeV and $E_{t'}=3.03$ MeV, which is in rough agreement with the old results.[2,3] The latter numbers are considerably different from ours, which is partly due to the fact that Tabakin's original triplet interaction has its CBS at a lower energy, partly to the somewhat different β dependence. Our ensuing triton wave functions are depicted in Fig. 1. The ground state clearly manifests a collapsed condition, its spatial extension being about one order of magnitude too small. The excited state has its primary peak at roughly the same place as the ground state, but on the other hand exhibits a huge secondary bump, making it a much more extended object. Interpreting this radial excitation as the "physical" triton, as sometimes suggested, is rather arbitrary, since the deep-lying ground state is in no way spurious. It is a proper solution of the Faddeev equation and does not violate any bound (see also below). In the next two sections we study, on the basis of a slightly simplified, three-boson system, the occurrence and behavior of a collapsed triton, and try to give an interpretation.

III. SCALING OF TRITON BINDING ENERGY

Henceforth we will consider, for the sake of simplicity, a system of three identical bosons interacting pairwise through the $^3S_1$ Tabakin potential given in the preceding section. The corresponding Faddeev equation is equivalent to Eq. (6), but without the summation and the recoupling coefficients. For the fixed parameters above we again obtain two bound states with energies $E_t=860$ MeV and $E_{t'}=13.1$ MeV. Let us now examine how E_t changes if we vary the parameter β, while at the same time we keep the deuteron binding energy fixed and do not allow the CBS to become a genuine resonance. Thereto, the coupling λ and the zero of the phases p_c have to be adjusted accordingly. Fulfilling these constraints, E_t and p_c are calculated as a function of β and plotted in Fig. 2. Also, the lower bound on E_t from Ref. 8,

$$E_{\rm LB}=2E_{\rm d}(\tfrac{3}{2}\lambda)\ , \tag{8}$$

is shown.

From Fig. 2 we see that E_t decreases roughly quadratically and p_c linearly with β. In other words, E_t grows when the CBS moves to higher energies. This is precisely the phenomenon which was observed in Ref. 5, but could not be explained. In our simple rank-one model it turns out to be an inherent scaling property of the two- and three-particle equations. First we look at the two-body T matrix of Eqs. (4) and (5). Let us forget for the moment the factor $\sqrt{m_{\rm N}/E_k}$. The condition on λ to get the deuteron pole reads

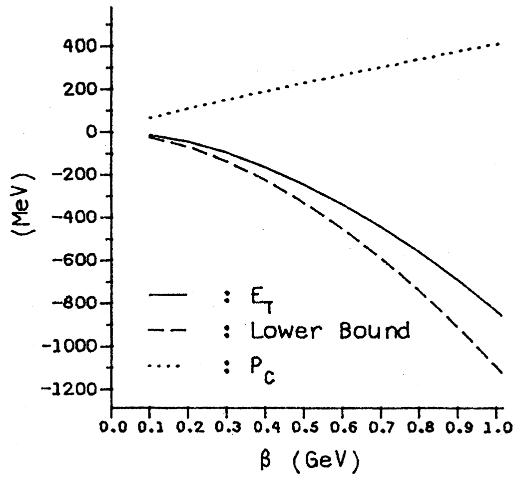

FIG. 2. Scaling in β of three-boson triton binding energy for $^3S_1$ Tabakin potential with CBS.

$$\frac{1}{\lambda}=-\frac{1}{8\pi^2 m_{\rm N}}\int_0^\infty \frac{k^2dk}{k^2+m_{\rm N}E_{\rm d}}\frac{(p_c^2-k^2)^2}{(k^2+\beta^2)^4}\frac{(k^2+\alpha^2)^2}{(k^2+\gamma^2)^2}\ . \tag{9}$$

For $\beta\gg\alpha,\ \gamma,\ \sqrt{m_{\rm N}E_{\rm d}}$ we can write approximately

$$\frac{1}{\lambda}\approx-\frac{1}{8\pi^2 m_{\rm N}}\int_0^\infty dk\frac{(p_c^2-k^2)^2}{(k^2+\beta^2)^4}\ . \tag{10}$$

If we change β to β' and simultaneously p_c to $p_c'=(\beta'/\beta)p_c$, the structure of the integrand in Eq. (10) does not alter. After the change of variables $k\to(\beta'/\beta)k$, we get for the new coupling λ',

$$\frac{1}{\lambda'}\approx-\frac{1}{8\pi^2 m_{\rm N}}\frac{\beta^3}{\beta'^3}\int_0^\infty dk\frac{(p_c^2-k^2)^2}{(k^2+\beta^2)^4}\ . \tag{11}$$

So if we take $\lambda'\approx(\beta'/\beta)^3\lambda$ and $p_c'\approx(\beta'/\beta)p_c$ the deuteron binding energy can be kept fixed. Moreover, analogously it follows that with this choice for λ' and p_c' the condition for a CBS at an energy $E=p_c^2/m_{\rm N}$,

$$\frac{1}{\lambda}\approx-\frac{1}{8\pi^2 m_{\rm N}}\int_0^\infty dk\,k^2\frac{k^2-p_c^2}{(k^2+\beta^2)^4}\ , \tag{12}$$

does not change. Taking into account the factor $\sqrt{m_{\rm N}/E_k}$ will not influence the scaling of p_c. As for λ, significant deviations only occur for very large values of β, as has been checked numerically. Turning to the three-body system we have the integral equation

$$\phi(q,q'';E_t)=\frac{\lambda}{2\pi^2}\int_0^\infty dq'\int_{-1}^{1}dx\frac{q'^2g(p)g(p')}{-E_t-(q^2+q'^2+qq'x)/m_{\rm N}}\tau\left[-E_t-\frac{3}{4}\frac{q'^2}{m_{\rm N}}\right]\phi(q',q'';E_t)\ , \tag{13}$$

with $\mathbf{p}=-\mathbf{q}'-(\mathbf{q}/2)$ and $\mathbf{p}'=\mathbf{q}+(\mathbf{q}'/2)$. If we now change β to β', p_c to $(\beta'/\beta)p_c$, and the variables q and q' to $(\beta/\beta')q$ and $(\beta/\beta')q'$, respectively, the different factors in the integrand alter as follows:

$$g(p)g(p')\to\left[\frac{\beta}{\beta'}\right]^4 g(p)g(p'),\quad \lambda\to\left[\frac{\beta'}{\beta}\right]^3\lambda\,,$$
$$\frac{dq'q'^2}{-E_t-(q^2+q'^2+qq'x)/m_N}\to\frac{\beta'}{\beta}\,\frac{dq'q'^2}{-(\beta/\beta')^2E_t'-(q^2+q'^2+qq'x)/m_N}\,. \tag{14}$$

As τ does not change and all the factors β'/β drop out, we regain the original equation provided that $E_t'=(\beta'/\beta)^2E_t$, and indeed we see that E_t grows quadratically with β.

IV. BOUND-STATE COLLAPSE OF THE TRITON

An alternative way to study the BSC is allowing the CBS to become a resonance. This can be accomplished by varying β and adjusting only λ to the deuteron binding energy. Figure 3 shows how this resonance pole moves into the lower half of the complex k plane ($k=\sqrt{m_N E}$), together with its counterpart at $-k^*$. The effect on E_t is depicted in Fig. 4, being clearly much more dramatic than in the scaling case. We see that E_t increases by almost a factor of 20, while β changes only 10%, which can indeed be called a collapse. This qualification becomes even more appropriate if we see what happens with the triton wave function, which is plotted in Fig. 5 for three values of β. $\beta=1.0$ GeV obviously corresponds to a collapsed state, $\beta=0.6$ GeV to a noncollapsed one, whereas for $\beta=0.8$ GeV we are apparently in an intermediate situation.

From Figs. 3–5 it seems safe to conclude that the three-body collapse, at least for the interaction we use, occurs when the two-body resonance pole is close enough to the real axis, and will be raised if the pole moves farther into the complex plane. The latter conjecture, however, gets invalidated if we go to higher values of β. With a β of 2.0 GeV, for example, E_t becomes as big as 4250 MeV, although in that case the pole has a larger imaginary part than in the noncollapsed one of $\beta=0.7$ GeV ($E_t=17.8$ MeV). In order to get at least an idea of what might give rise to the collapse after all, we will try to make the (nonlocal) Tabakin potential more transparent. A way to do this is by constructing a local potential in coordinate space, that, at some fixed energy E, produces exactly the same two-body wave function. For an S-wave bound state (deuteron), this potential is given by

$$V(r)=\frac{d^2\phi(r)/dr^2}{m_N\phi(r)}-E_d,\quad E_d>0\,. \tag{15}$$

$\phi(r)$ in Eq. (15) is just the Fourier transform of $g(p)[E_d+(p^2/m_N)]^{-1}$, where $g(p)$ is the Tabakin form factor. For calculational reasons we again drop the factor $\sqrt{m_N/E_p}$ from g. This does not change the qualitative properties of V and ϕ. Taking for E_d the deuteron binding energy, we compute $V(r)$ and $\phi(r)$ for the same values of β as above. The results for V are plotted in Fig. 6, the deuteron wave function in Fig. 7. For comparison these quantities are also calculated for the Graz-II (Ref. 9) potential, which is a realistic separable potential of rank

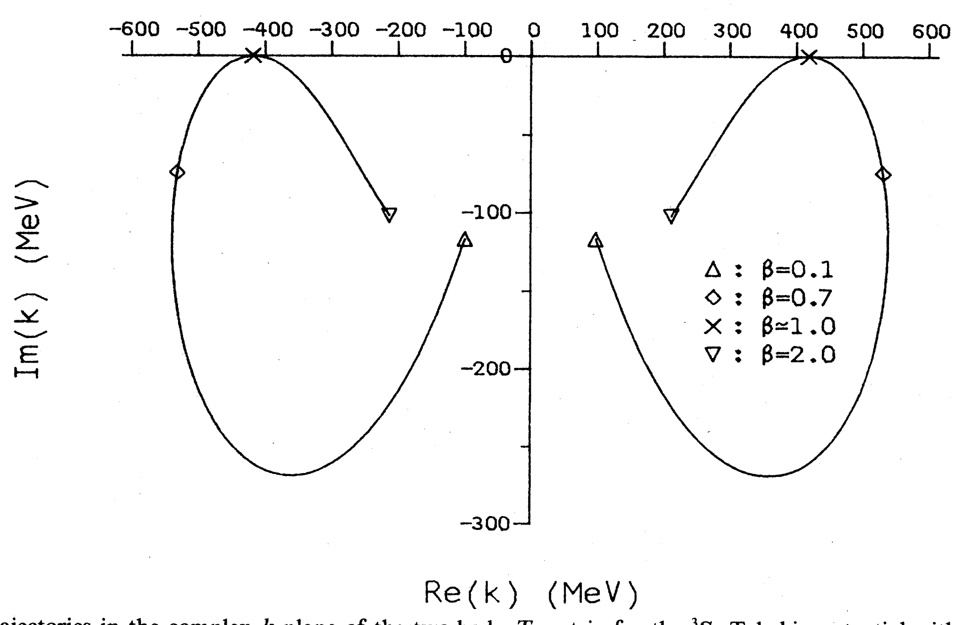

FIG. 3. Pole trajectories in the complex k plane of the two-body T matrix for the $^3S_1$ Tabakin potential with fixed α,γ,p_c, as a function of β.

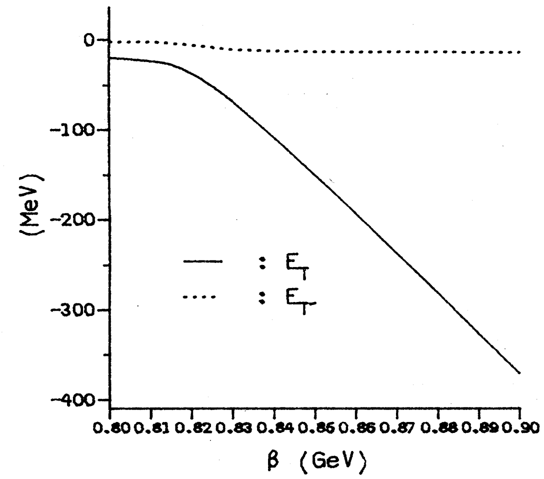

FIG. 4. Collapse of the three-boson triton binding energy for the $^3S_1$ Tabakin potential with fixed α,γ,p_c, as a function of β.

three ($^3S_1$) with reasonable off-shell properties. From Fig. 6 we see that at intermediate and long distances the Tabakin potential with $\beta=0.6$ GeV qualitatively behaves in the same way as Graz-II, due to the location of the zeros of $\phi(r)$ and $\phi''(r)$. In that region both interactions exhibit a repulsive core and an attractive tail. At short distances ($r<0.3$ fm for Tabakin and $r<0.1$ fm for Graz-II) there is a deep negative part with a local maximum, whereby Graz-II has an additional structure for $r<0.05$ fm. Nevertheless, the core repulsion is apparently strong enough to largely screen this inner region. Close to $\beta=0.8$ GeV, however, a crossover of these zeros takes place, thereby changing drastically the properties of the equivalent local potential. For $\beta \gtrsim 0.8$ GeV, V suddenly becomes attractive everywhere, in two more or less separated regions, however. Remarkably close to this crossover point the triton binding energy starts to rise steeply. For values of β larger than 1 GeV the potential

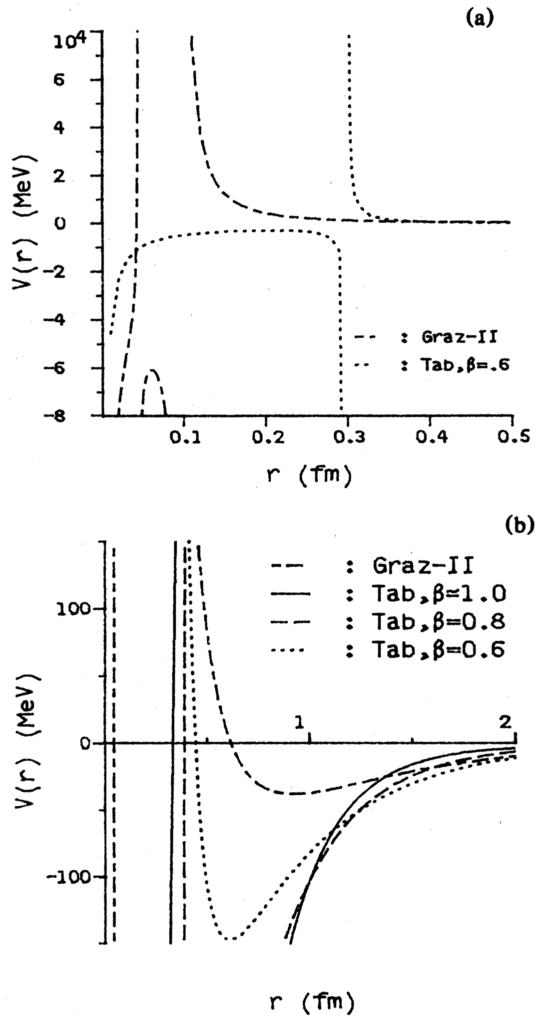

FIG. 6. Equivalent local potentials of $^3S_1$ Tabakin (fixed α,γ,p_c) and Graz-II interactions at deuteron binding energy. (a) Inner region; (b) intermediate and outer region.

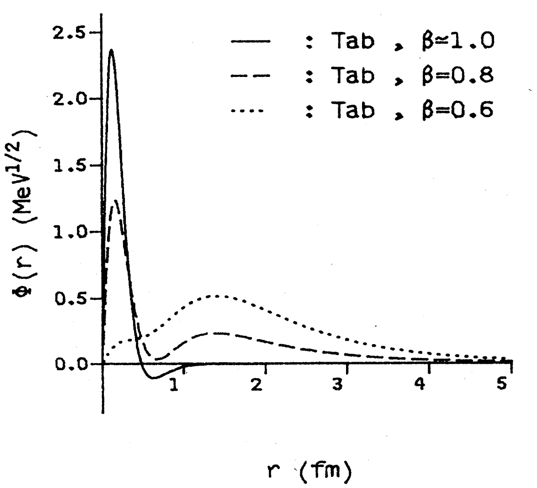

FIG. 5. Triton three-boson wave function with the $^3S_1$ Tabakin potential for varying β and fixed α,γ,p_c.

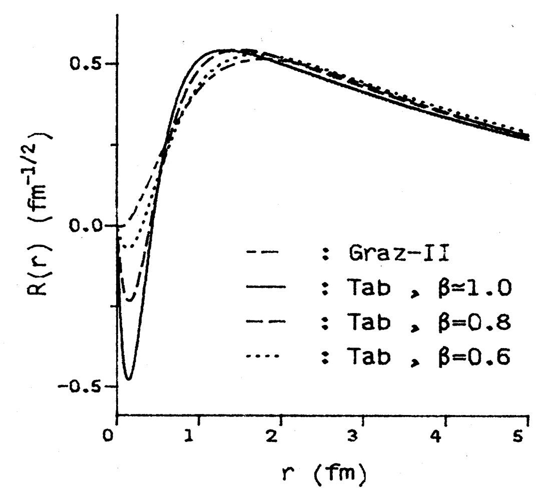

FIG. 7. Radial S-wave deuteron wave functions for $^3S_1$ Tabakin (fixed α,γ,p_c) and Graz-II potentials.

does not change its character anymore, there is no second crossover. This might indicate why in this region the collapse steadily proceeds. If the BCS is interpreted in this way, also the systematically large triton binding energy in the case with a CBS becomes plausible, since then there is no crossover at all, the local potential being attractive always. On the level of the deuteron wave function the effect of the crossover can be seen in Fig. 7. For $\beta=1.0$ GeV there is an unrealistic, huge negative peak at short distances. This bump, however, shrinks rapidly for decreasing β, a manifestation of the mentioned screening effect.

Of course one should beware of drawing too rigorous conclusions from such an incomplete picture. After all, the local equivalent constructed above merely provides a single-energy snapshot of the Tabakin potential. On the other hand, it may emphasize once more the importance of getting the deuteron properties right when looking for a realistic description of the triton.

V. CONCLUDING REMARKS

The study of the Tabakin potential in this paper has been motivated by the wish to understand how such a simple separable potential can exhibit so totally different aspects when incorporated in two- or three-body calculations. In the nucleon-nucleon system it allows a surprisingly accurate reproduction of the *S*-wave scattering data up to moderately high energies. When applied to the triton, however, with unaltered parameters, it produces a dramatic overbinding. From simple scaling considerations the puzzling dependence of the three-body binding energy on the position of the continuum bound state can be explained. Moreover, in terms of the effective local two-body potential, the Tabakin interaction leads to a strong attraction at short distances, which, in combination with the absence of a hard-core repulsion at intermediate distances, allows the collapse of the three-body wave function to take place.

It is true this collapse can be circumvented by modifying the parameters, but not without spoiling the nice two-particle results. Hammel *et al.*[10] have used Tabakin-type potentials in relativistic three-nucleon calculations based on a quasipotential approach. Their chosen triplet interaction, however, gave a rather poor description of the NN phase shifts, even at low energies. So, although it is interesting to see that the three-body BSC can be avoided by changing the potential parameters in only one of the channels, in actual three-nucleon model studies more conclusive results may be obtained by replacing in that channel the Tabakin potential by one that at least properly fits the low-energy scattering data, like, for instance, the Yamaguchi interaction. This is actually what is done in Ref. 6, where the effect of relativity in the triton is investigated employing the full three-body Bethe-Salpeter equation.

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