Pion scattering on aligned ^{165}Ho

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We evaluate the angular distributions and the orientation asymmetry variable $A_S(\theta)$ in the description of pion scattering from oriented, deformed nuclei. The results show a strong, selective sensitivity of $A_S(\theta)$ to the neutron and proton deformation β_2^N and β_2^C. We also evaluate the energy dependence of $A_S(\theta)$ and conclude that single charge exchange from ^{165}Ho at $T_\pi=160$ MeV would be an interesting case to study experimentally. Results of deformed Hartree-Fock calculations indicate $A_S(\theta)=6\%$ for this nucleus.

I. INTRODUCTION

In a previous paper[1] we found that pion charge exchange scattering is sensitive to a poorly known quantity characterizing deformed nuclei, namely the deformation of the excess neutron density, β_2^N. The important quantity for determining β_2^N is the orientation asymmetry variable $A_s(\theta)$,

$$A_S(\theta)=\frac{d\sigma^{\perp}/d\Omega-d\sigma^{\parallel}/d\Omega}{d\sigma^{\perp}/d\Omega+d\sigma^{\parallel}/d\omega}\ , \tag{1}$$

where $d\sigma^{\perp}/d\Omega$ and $d\sigma^{\parallel}/d\Omega$ are the cross sections for scattering from deformed nuclei oriented perpendicular and parallel, respectively, to the direction of the incident pions. The main result in Ref. 1 was that a -10% to $+10\%$ variation of β_2^N/β_2^C about the value $B_2^N=\beta_2^C$ would give rise to a change of $A_s(0)$ from roughly 0 to 0.3 in a measurement of single or double charge exchange. Here β_2^C is the quadrupole deformation of the protons. In this paper we want to explore in greater detail the sensitivity of the predictions to various assumptions of the model.

As in our previous work, we use the semiclassical eikonal theory to describe the scattering. In a sequence of theoretical[2] and experimental[3] investigations, the eikonal approach has been shown to reflect the sensitivity of more exact solutions of the scattering equations and also to reproduce the trends of charge-exchange experimental data. We therefore continue to use this theory to anticipate features of the experimental results which we expect to be forthcoming.[4] We are hopeful that these measurements can lead to an empirical verification of predictions of models such as the deformed Hartree-Fock (DHF) theory[5] or macroscopic-microscopic models.[6] It has been difficult to obtain unambiguous experimental information about the shape of the neutron distribution in highly deformed nuclei from other measurements.[1]

II. NUCLEAR MODEL

We are interested in the specific case of ^{165}Ho ($N=98$, $Z=67$). This nucleus is known to be highly deformed. The spin quantum number of the ground state of ^{165}Ho is $I=\frac{7}{2}=K$, where I is the total angular momentum and K is its projection on the body-fixed axis. Atomic ^{165}Ho has a net magnetic moment and can be aligned in a magnetic field; this alignment has been accomplished in experiments at LAMPF for the purpose of measuring pion total cross sections.[8] Studies with μ^--atom techniques have determined the parameters of the charge density in the ground state of ^{165}Ho.[7] A Woods-Saxon shape for the charge distribution was used in the analysis

$$\rho_C(r,R)=\frac{\rho_0}{1+e^{(r-R)/a}} \tag{2}$$

with the half-density radius R mapped onto an ellipsoidal surface

$$R=R_0[1+\beta_2^C Y_{20}(\theta)]\ , \tag{3}$$

whose major axis lies along the z' axis of the body-fixed coordinate system. The results of the analysis give $\beta_2^C=0.32$ with

$$R_0=6.15,\ \ a=0.49\ . \tag{4}$$

A general representation of the density for nonspherical nuclei may be made in terms of the components $\rho(l,r)$ of an expansion in spherical harmonics

$$\rho(l,r)=\left[\frac{2l+1}{4\pi}\right]^{1/2}\int d\Omega\,\rho(r,\theta)Y_{l0}(\Omega)\ , \tag{5}$$

and each $\rho(l,r)$ is characterized by its multipole moment Q_l,

$$Q_l=\frac{8\pi}{2l+1}\int_0^\infty \rho(l,r)r^{l+2}dr\ . \tag{6}$$

Assuming that β_2 is small, a Taylor expansion of Eq. (2) permits an approximate identification of the first two multipole components for ^{165}Ho,

$$\rho(r,R_0,\beta)=\rho(r,R_0)+R_0\beta_2 Y_{20}(\theta)\rho'(r,R_0)\ . \tag{7}$$

The DHF densities $\rho(l,r)$ for $l=0$ and $l=2$ have shapes very similar to the corresponding terms in Eq. (7), and we therefore adopt this form for neutron and proton densities with $\rho(r,R_0)$ the Woods-Saxon shape in Eqs. (2) and (3). Choosing the parameters of Eq. (4) for the protons, we obtain the results in the upper part of Table I for the root

TABLE I. Values of root-mean-square radii ($r_{\rm rms}$) and quadrupole moment Q_2.

R_0 (fm)	a (fm)	β_2	$r_{\rm rms}$ (fm)	Q_2 (fm^2)
Protons				
6.15	0.49	0.32	5.10	744
6.25	0.49	0.366	5.19	765
Expt.			5.19	744
Neutrons				
6.15	0.49	0.32	5.10	949
6.50	0.49	0.336	5.35	1106

mean square radius $r_{\rm rms}$ and moment Q_2. The first row corresponds to the parameters given in Ref. 5. The $r_{\rm rms}$ is slightly smaller than the experimental value, presumably occurring as a result of the expansion in going from Eqs. (2) and (3) to Eq. (7). In the second row the values of R_0 and β_L have been adjusted to give the $r_{\rm rms}$ and Q_2 obtained from solving the DHF equations of Ref. 5. We show the same quantities for the neutron distribution in the lower part of Table I. The first row is calculated with the same parameters that describe the protons in Ref. 7. These values of R_0 and a give smaller $r_{\rm rms}$ that the DHF calculations; the second row shows an adjustment of R_0 and β_2 which gives the theoretical values of $r_{\rm rms}$ and β_2.

For highly-deformed nuclei there exist a large number of low-lying rotational levels that are easily excited by a medium-energy projectile. These states must be summed in both the intermediate states of the amplitude and the final states for the evaluation of the cross section for a low resolution measurement. To accomplish this we use the closure approximation, which implies that the orientation of the nucleus is not changed during the scattering process. The scattering amplitude depends, therefore, on the orientation Ω of the body-fixed system in the laboratory. We characterize this orientation by the Euler angles[9] $(\alpha,\beta,\gamma)\equiv\Omega$.

In order to calculate the cross section we must know the wave function of the nuclear ground state. We assume that this is the product of the intrinsic wave function in the body-fixed system and the eigenstate $\psi_{IMK}(\Omega)$ of the collective variable Ω describing the orientation of the nucleus. For the state ψ we make the usual assumption; that is, ψ is an eigenstate of the rigid rotator Hamiltonian. These wave functions are just the rotation matrices[9] $D^I_{MK}(\alpha,\beta,\gamma)$. Two specific cases will be useful later. These are as follows: (1) The nucleus is polarized along the z axis. In this case the projection M of the total angular momentum on the laboratory z axis is $M=I$ and

$$\psi^{(\parallel)}_{IIK}(\Omega)=\left[\frac{2I+1}{8\pi^2}\right]^{1/2}D^I_{IK}(\phi,\theta,0)\ . \qquad (8)$$

(2) The nucleus is polarized along the x axis. In this case we rotate Eq. (8) by $\pi/2$ about the y axis and find

$$\psi^{(\perp)}_{IIK}(\Omega)=\left[\frac{2I+1}{8\pi^2}\right]^{1/2}\sum_M D^I_{IM}\left[0,\frac{\pi}{2},0\right]D^I_{MK}(\phi,\theta,0)\ . \qquad (9)$$

Suppose we now have a scattering from state I,M,K to I',M',K'. The desired amplitude is

$$\langle I'M'K'\,|\,F(q,\Omega)\,|\,IMK\rangle=\int d\Omega\,\psi^*_{I'M'K'}(\Omega)\times F(q,\Omega)\psi_{IMK}(q,\Omega)\ , \qquad (10)$$

where $F(q,\Omega)$ is the scattering from the intrinsic state whose orientation is Ω relative to the laboratory coordinate system. If the scattering to levels I',M',K' of the final nucleus is not resolved, then the cross section from the initial state IMK is

$$\frac{d\sigma}{d\Omega}=\sum_{\substack{I'M'\\K'}}|\langle I'M'K'\,|\,F(q,\Omega)\,|\,IMK\rangle|^2 =\langle IMK\,|\,[F(q,\Omega)]^2\,|\,IMK\rangle\ , \qquad (11)$$

where closure has been used. We have found that for the small β_N and β_C that we use, the off-diagonal elements are small and that

$$\frac{d\sigma}{d\Omega}=\langle IMK\,|\,[F(q,\Omega)]^2\,|\,IMK\rangle \approx|\langle IMK\,|\,F(q,\Omega)\,|\,IMK\rangle|^2\ . \qquad (12)$$

So we use the following expressions,

$$F^{\parallel}(q)=\frac{2I+1}{2}\int_0^\pi\sin\theta\,d\theta\,|d^I_{IK}(\theta)|^2\int F(q,\Omega)\frac{d\phi}{2\pi} \qquad (13)$$

and

$$F^{\perp}(q)=\frac{2I+1}{2}\sum_M\left|d^I_{IM}\left[\frac{\pi}{2}\right]\right|^2\int_0^\pi\sin\theta\,d\theta\,|d^I_{MK}(\theta)|^2\times\int F(q,\Omega)\frac{d\phi}{2\pi}\ . \qquad (14)$$

III. SCATTERING THEORY

The semiclassical results are an approximation to the solution of the Klein-Gordon equation with an optical potential U,

$$U=u_0+u_1(\boldsymbol{\phi}\cdot\mathbf{T}_N)+u_2(\boldsymbol{\phi}\cdot\mathbf{T}_N)^2\ , \qquad (15)$$

where u_0, u_1, and u_2 are referred to, respectively, as the isoscalar, isovector, and isotensor components of the optical potential. Here $\boldsymbol{\phi}$ is the pion and $\mathbf{T}_N$ the nuclear isotopic spin operator. In the case of interest, U is the optical potential for the nucleus in its intrinsic state and depends on the orientation Ω of the body-fixed axis relative to the laboratory system.

Our procedure will be to calculate first the amplitude F_τ for elastic and inelastic scattering from the initial state to a given final state, which is a member of the rotational band built on the ground state. We generate the charge-exchange amplitude to the analogs of the final state from this result by applying isospin invariance. The elastic and inelastic scattering is evaluated in channels of total isospin τ,

$$\tau=\boldsymbol{\phi}+\mathbf{T}_N \ . \tag{16}$$

These channels are not those measured in the laboratory (which are eigenstates of ϕ^2, ϕ_z, T_N^2, and T_{N_z}) but are of advantage in the theory because U and the scattering T matrix are diagonal in this basis. The relationship between the amplitude F_τ and the physical amplitude $F^{(ij)}$ for elastic scattering $[(ij)=(++)$ and $(--)$ for π^+ and π^-, respectively], single charge exchange (SCX) $[(i,j)=(0,+)]$, and double charge exchange (DCX) $[(i,j)=(-,+)]$ may be expressed formally as

$$F^{(ij)}=\sum_\tau A^{(ij)}(\tau)F_\tau \ , \tag{17}$$

where $\tau=T$, $T+1$, and $T-1$ and where the set $A^{(ij)}(\tau)$ are Clebsch-Gordan coefficients and are given explicitly in Ref. 2. The cross section for a transition from a given initial to a given final state is then given by Eqs. (10)—(14) with the F there identified with the $F^{(ij)}$.

We will calculate F_τ in the eikonal theory. The result is

$$F_\tau(q,\theta)=ik\int_0^\infty b\,db\,J_0(qb)[1-G_\tau(b,\theta)] \tag{18}$$

and

$$G_\tau(b\theta)=\frac{1}{2\pi}\int_0^{2\pi}d\phi' e^{-\chi_\tau(b,\theta,\phi')} \ ,$$

where

$$\chi_\tau(b,\Omega)=\frac{1}{2k}\int_{-\infty}^{\infty}dz\,U_\tau(b,z,\Omega) \ , \tag{19}$$

and U_τ is the projection of U onto the state of total isospin τM_τ,

$$U_\tau(\mathbf{b},z,\Omega)=\langle\tau\,|\,u_0+u_1\boldsymbol{\phi}\cdot\mathbf{T}_N+u_2(\boldsymbol{\phi}\cdot\mathbf{T}_N)^2\,|\,\tau\rangle \ . \tag{20}$$

We use a local form for the optical potential, so that

$$U_\tau=k^2\xi_\tau+\tfrac{1}{2}\nabla^2\xi_\tau \ , \tag{21}$$

where ξ_τ depends on the nuclear densities $\rho(r)$ and $\Delta\rho(r)$, which are the core and excess neutron densities. If we ignore the effect of the Coulomb interaction on nuclear structure, then the excess neutron density is the difference between the neutron density ρ_n and proton density ρ_p,

$$\rho(r)=N\rho_n(r)+Z\rho_p(r) \ , \tag{22}$$

$$\Delta\rho(r)=N\rho_n(r)-Z\rho_p(r) \ . \tag{23}$$

The quantities ξ_τ also depend on the free pion-nucleon scattering amplitude through an isoscalar $\lambda_0^{(1)}$ and an isovector $\lambda_1^{(1)}$ coefficient.[2] We may write

$$\xi_\tau(r,\Omega)=\lambda_0^{(1)}\rho(r,\Omega)+\gamma^{(1)}(\tau)\lambda_i^{(1)}\Delta\rho(r,\Omega) \ , \tag{24}$$

where $\gamma^{(1)}(\tau)$ are fixed by the isospin geometry (see Ref. 2) and where we have now displayed the explicit dependences of ρ and $\Delta\rho$ on the nuclear orientation Ω. The Laplacian term in Eq. (21) arises from the p-wave character of the pion-nucleon interaction.[2] We omit all other partial waves. They are small because of the proximity to the (3,3) resonance.

The parameters $\lambda_i^{(1)}$ are given explicitly in terms of the free pion-nucleon scattering amplitude in Ref. 2. However, in the region of the Δ_{33} resonance these parameters are renormalized by higher order terms in U. The relationship between these renormalizations and the explicit forms of the higher-order optical potential was studied in Ref. 2. In the present work we take $\lambda_0^{(1)}$ and $\lambda_1^{(1)}$ from the free pion-nucleon scattering amplitude in the usual way, without an energy shift. The cross sections for charge exchange evaluated in this fashion are generally too small and require a substantial renormalization. However, as we shall verify in Sec. IV, the *asymmetry* $A_S(\theta)$ is insensitive to these corrections.

That $A_S(\theta)$ is a useful measure of β_2^N can easily be seen in a schematic model of diffractive scattering for which the charge-exchange scattering occurs only at the edge of the nucleus and the amplitude is proportional to the circumference of the excess neutron distribution. Assuming that this distribution is an ellipsoid with semimajor axis b and semiminor axis a, then the cross section for parallel orientation is

$$\frac{d\sigma^{\parallel}}{d\Omega}\propto a^2 \ , \tag{25}$$

and for perpendicular orientation

$$\frac{d\sigma^{\perp}}{d\Omega}\propto\frac{a^2+b^2}{2} \ . \tag{26}$$

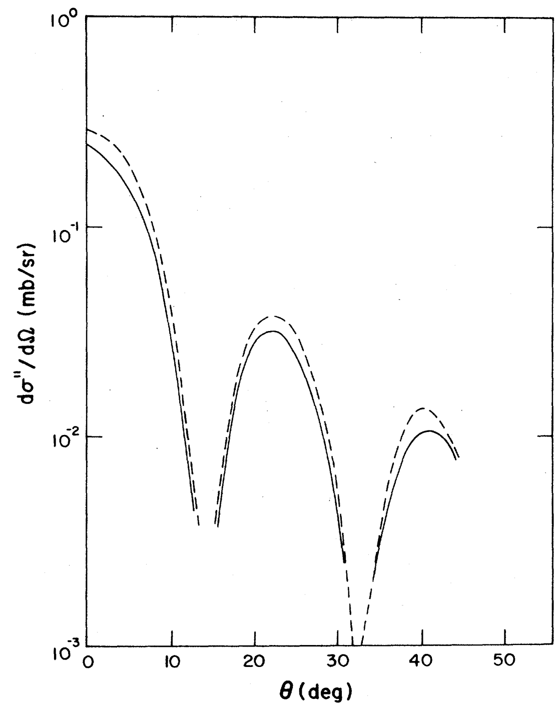

FIG. 1. Importance of virtual excitations of rotational levels in calculating the cross section $d\sigma^{\parallel}/d\Omega$ for the reactions ^{165}Ho$(\pi^+,\pi^0)^{165}$Er at 200 MeV. The solid curve includes virtual excitations, the dashed curve assumes that the nucleus remains in its ground state. We assume $\beta_2^N=\beta_2^C=0.32$, $R_0=6.15$ fm.

If we now change a and b by an amount δa and δb, we must hold the volume fixed at V_0,

$$V_0=\tfrac{4}{3}\pi a^2 b \tag{27}$$

so that the ratio $\delta b/\delta a$ remains fixed at

$$\delta b/\delta a=-\frac{2b}{a} \ . \tag{28}$$

We then easily find, for $b \approx a$,

$$\delta\frac{d\sigma^{\parallel}}{d\Omega}\propto a\,\delta a \tag{29}$$

and

$$\delta\frac{d\sigma^{\perp}}{d\Omega}\propto -\frac{a\,\delta a}{2} \ , \tag{30}$$

so that *the result of a change* δa *is opposite in the parallel and perpendicular orientations.* This is just the sensitivity we need to make $A_S(\theta)$ an efficient measure of the deformation of the neutrons,

$$\delta A_S(0^\circ)\propto \delta a \ . \tag{31}$$

The coefficient of proportionality depends on details, and one of the objectives for investigations is to determine it carefully.

There is a limiting form of the theory in which the coefficient can be calculated analytically. Let us assume that the nucleus remains in its ground state during all intermediate scatterings in F. We shall call this the coherent approximation. Such an assumption was made, for example, by Hoodboy in Ref. 10 for calculation of total pion cross sections from ^{165}Ho. The result is obtained by replacing χ in Eq. (19) by $\langle\psi|\chi|\psi\rangle$, which is equivalent to replacing β_2 by β_2^L,

$$\beta_2^L\equiv\frac{[3M^2-I(I+1)][3K^2-I(I+1)]}{I(2I-1)(I+1)(2I+3)}\beta_2 \ . \tag{32}$$

We then make an analytical approximation following Ref. 2. Making a Taylor-series expansion of F about some radius $\bar{R}$ and keeping terms up to second derivatives, one finds

$$F^{(0+)}=\frac{1}{\sqrt{T}}\,\frac{1}{2T+1}\,\frac{1}{T+1}S^{(0+)}\frac{dF}{d\bar{R}}(\theta,\bar{R}) \ , \tag{33}$$

where

$$S^{(0+)}=(2T^2-1)(R_T-R_{T-1})+T(2R_{T+1}-R_T-R_{T-1}) \ , \tag{34}$$

$$\bar{R}=\tfrac{1}{2}(R_T+R_{T-1})-\frac{T}{2T^2-1}\,\frac{[R_T-R_{T+1}(R_{T+1}-R_{T-1})]}{R_T-R_{T-1}} \ , \tag{35}$$

TABLE II. Sensitivity of forward cross section and asymmetry to model.

Effect of changing β. $R_0^C=6.15$ fm. $\lambda_0^{(1)}=0.75+8.89i$

Reaction	β_2^C	β_2^N	$d\sigma^{\parallel}/d\Omega$ (mb/sr)	$d\sigma^{\perp}/d\Omega$ (mb/sr)	A_S (%)
	0.32	0.32	0.207	0.262	11.5
(π^+,π^0)	0.32	0.32	0.121	0.212	27
	0.32	0.29	0.309	0.307	−0.3
	0.32	0.32	0.89×10^{-4}	0.11×10^{-3}	10.1
(π^+,π^-)	0.32	0.35	0.69×10^{-4}	0.11×10^{-3}	22
	0.32	0.29	0.122×10^{-3}	0.12×10^{-3}	−1.4

Effect of changing R_0^N. $\beta_2^C=\beta_2^N=0.32$. $\lambda^{(1)}$ as in upper part of Table II.

Reaction	R_0^C (fm)	R_0^N (fm)	$d\sigma^{\parallel}/d\Omega$ (mb/sr)	$d\sigma^{\perp}/d\Omega$ (mb/sr)	A_S (%)
(π^+,π^0)	6.15	6.15	0.207	0.262	11.5
	6.15	6.75	2.05	2.52	10.5
(π^+,π^-)	6.15	6.15	0.89×10^{-4}	0.11×10^{-3}	10.1
	6.15	6.75	0.29×10^{-2}	0.34×10^{-2}	7.2

Effect of changing $\lambda^{(1)}$. $R_0^C=R_0^N=6.15$ fm. $\beta_2^C=\beta_2^N=0.32$.

Reaction	$\lambda_0^{(1)}$ (fm^3)	$\lambda_1^{(1)}$ (fm^3)	$d\sigma^{\parallel}/d\Omega$ (mb/sr)	$d\sigma^{\perp}/d\Omega$ (mb/sr)	A_S (%)
	$0.75+8.9i$	$1.2+8.9i$	0.207	0.262	11.5
(π^+,π^0)	$0.64+7.6i$	$1.33+10.0i$	0.45	0.56	11.4
	$0.86+10.0i$	$0.98+7.5i$	0.100	0.13	11.9
	$0.75+8.9i$	$1.2+8.9i$	0.89×10^{-4}	0.11×10^{-3}	10.1
(π^+,π^-)	$0.64+7.6i$	$1.3+10.0i$	0.35×10^{-3}	0.44×10^{-3}	10.9
	$0.86+10.0i$	$0.98+7.5i$	7.23×10^{-4}	0.28×10^{-4}	9.8

and R_J are the black-disk radii in channels of total isospin τ. These radii can be calculated analytically in terms of U as in Ref. 2. We then find

$$A_S(0) \cong \frac{3}{2}\left[\frac{5}{16\pi}\right]^{1/2} \beta_2^N(I=M) , \tag{36}$$

where we have assumed $\beta_2^N=\beta_2^C$. One sees explicitly by following the derivation that the geometrical factors that fix the size of the individual cross sections, as well as the dynamical factors $\lambda_i^{(1)}$, drop out of $A_S(\theta)$, making it uniquely sensitive to β_2^N. If we take as an example ^{165}Ho, then

$$A_S(0) \simeq 7\% . \tag{37}$$

This result is confirmed by explicit evaluation of the integrals of the eikonal approximation in the coherent approximation.

However, based on the studies of inelastic proton scattering,[11] virtual excitations of the nucleus are expected to be important. These excitations may be included in the closure approximation, which fixes the orientation Ω of the nucleus and averages over Ω only after the scattering has occurred. In this case we cannot easily make analytical approximations, but it is possible to do the integral over the ϕ direction in Eq. (18). Making the dependence on ϕ explicit in Eq. (19),

$$\chi_\tau(b,\theta,\phi)=\chi_\tau^{(1)}(b,\theta)+\chi_\tau^{(2)}(b,\theta)\cos^2\phi , \tag{38}$$

we find

$$G(b,\theta) \equiv \frac{1}{2\pi}\int_0^{2\pi} d\phi\, e^{-\chi(b,\theta,\phi)}$$
$$=\{\exp[-\chi^{(1)}(b,\theta)-\tfrac{1}{2}\chi^{(2)}(b,\theta)]\}\{I_0[\tfrac{1}{2}\chi^{(2)}(b,\theta)]\}, \tag{39}$$

where $I_0(\chi)$ is a Bessel function of imaginary argument of order zero.

A comparison of the closure and coherent approximations is made in Fig. 1 for $\beta_N=\beta_C=0.32$. The size of the correction is consistent with the findings of Ref. 11. The asymmetry increases from 7% to 11%, which implies a difference of 22% in the cross sections for the parallel and perpendicular alignments.

We conclude that it is important to retain the virtual inelastic excitations in order to achieve numerical accuracy. In obtaining the results in the next section we therefore use the closure approximation.

IV. RESULTS

In this section we show results that utilize the full eikonal model in Eqs. (17)—(20). We show in Table II the results of calculations for different values of R_0, β_2, and $\lambda^{(1)}$ to demonstrate the sensitivity of the cross sections $d\sigma^\perp/d\Omega$ and $d\sigma^\parallel/d\Omega$ and the asymmetry A_S to these quantities. We give results for SCX and DCX at $T_\pi=180$ MeV.

In the upper part of Table II we show the effect of changing the deformation of neutrons and protons. We see that the cross sections and especially $A_S(0)$ are sensitive to changes in β_2^N/β_2^C. This is true for both SCX and DCX and is in accord with the results of Ref. 1. In the middle part of Table II we change R_0^N keeping all other parameters of the model fixed. The $A_S(0°)$ is remarkably insensitive to R_0^N, even for changes that give rise to more than an order of magnitude variation in the individual cross sections. In the lower part of Table II we show effects of 10% changes in the values of $\lambda^{(1)}$. Again we see that the results for $A_S(0°)$ are much more stable than the cross sections. The results of Table II are in accord with the expectations based on our derivation of Eq. (36) and demonstrate the selective sensitivity to β_2, which measures the relative strength of the $l=0$ and $l=1$ components of the density. The main result of our work is this strong, selective sensitivity to β_2^C and β_2^N.

Figure 2 displays the cross sections and orientation asymmetry for SCX plotted as a function of angle at $T_\pi=180$ MeV. The most favorable angle for observing a large asymmetry is 0°, because the cross section is largest here. For this calculation we have used the values of R_0, and a in Eq. (4), but if we fix the parameters according to the DHF solution the zero-degree cross sections become $d\sigma^\parallel/d\Omega(0°)=0.88$ mb/sr and $d\sigma^\perp/d\Omega=0.98$ mb/sr. This demonstrates, once again, the sensitivity of SCX cross sections to the neutron and proton density. We ex-

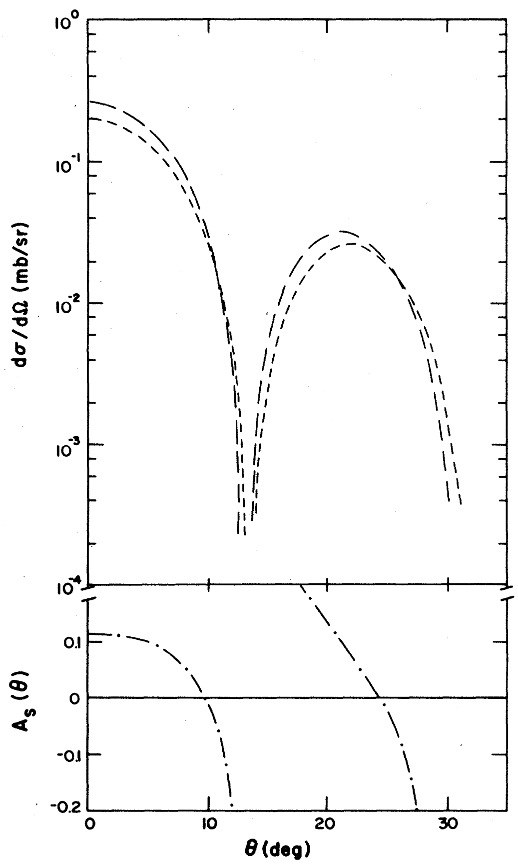

FIG. 2. Cross sections and asymmetry for ^{165}Ho$(\pi^+,\pi^0)^{165}$Er at 180 MeV for $\beta_2^N=\beta_2^C=0.32$ and $R_0=6.15$ fm. The long dashes are $d\sigma^\perp/d\Omega$ and the short dashes $d\sigma^\parallel/d\Omega$.

pect these numbers to be smaller than the data because of our omission of second order terms in U. The magnitudes of the cross sections may be inferred from previous experiments, which show that zero-degree cross sections behave as $g(N-Z)/A^{1.35}$. Results of Ref. 3 give $g=49$ for $T_\pi=165$ MeV, so for ^{165}Ho cross sections should be roughly

$$\langle d\sigma/d\Omega(0°)\rangle = 1.5 \text{ mb/sr} .$$

To get a feeling for the dependence of $d\sigma^{\parallel}/d\Omega$ for SCX on β_2^N, we show in Fig. 3 the effect of varying β_2^N by $\pm 10\%$. The same calculation for DCX is shown in Fig. 4. The omitted second order terms in U may have a large effect on the shape of the DCX cross section, but for SCX they affect mostly the overall magnitude of the cross section. In any case, the shape of the cross sections shows only a slight dependence on β_2^N.

Finally, we give in Fig. 5 the energy dependence for SCX of $A_S(0)$. The energy dependence for SCX is rather flat with some tendency to be largest at 160 MeV. For DCX the largest asymmetry occurs at a slightly higher energy. However, the double isobaric analog state has been observed in heavy nuclei only at much higher energies ($T_\pi=292$ MeV), where the asymmetry appears to be getting smaller. We thus conclude that the SCX reaction at $T_\pi \simeq 160$ MeV would be most favorable for observing the effects of deformation. The fact that $A_S(0°)$ is slowly varying with energy for SCX also favors this reaction.

What is the expected value of $A_S(0)$? Because there have been no measurements of β_2^N in ^{165}Ho, we cannot be sure of its value. However, if $\beta_2^C=\beta_2^N=0.32$, the empiri-

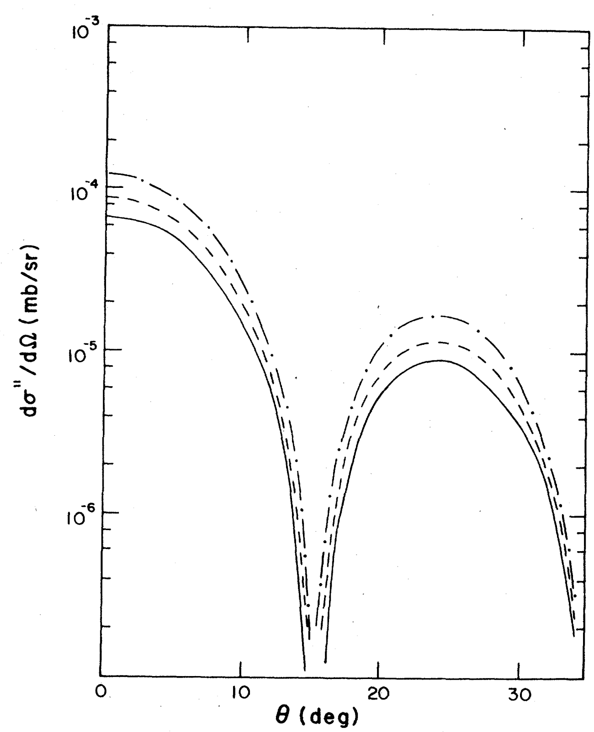

FIG. 4. The effect on $d\sigma^{\parallel}/d\Omega$ of varying β_2^N, for the DCX reaction ^{165}Ho$(\pi^+,\pi^-)^{165}$Tm at 180 MeV. Legend is the same as in Fig. 3.

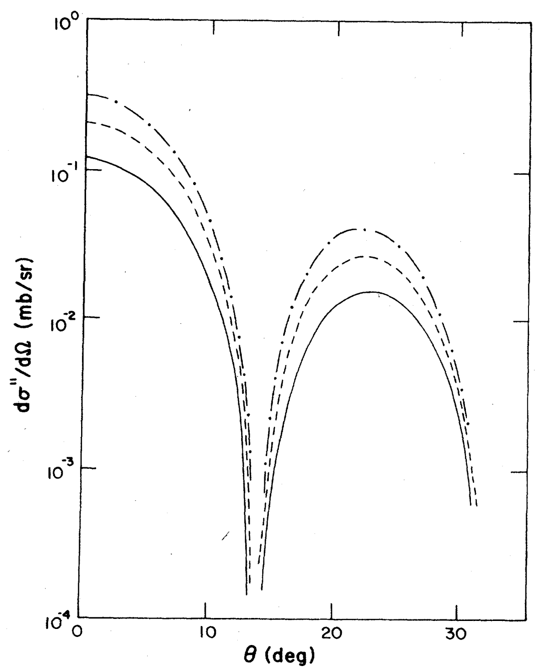

FIG. 3. The effect on $d\sigma^{\perp}/d\Omega$ of varying β_2^N, for the SCX reaction ^{165}Ho$(\pi^+,\pi^0)^{165}$Er at 180 MeV. We fix $\beta_2^C=0.32$. Solid curve, $\beta_2^N=1.1\beta_2^C$; dashed curve, $\beta_2^N=\beta_2^C$; dot-dashed curve, $\beta_2^N=0.9\beta_2^C$.

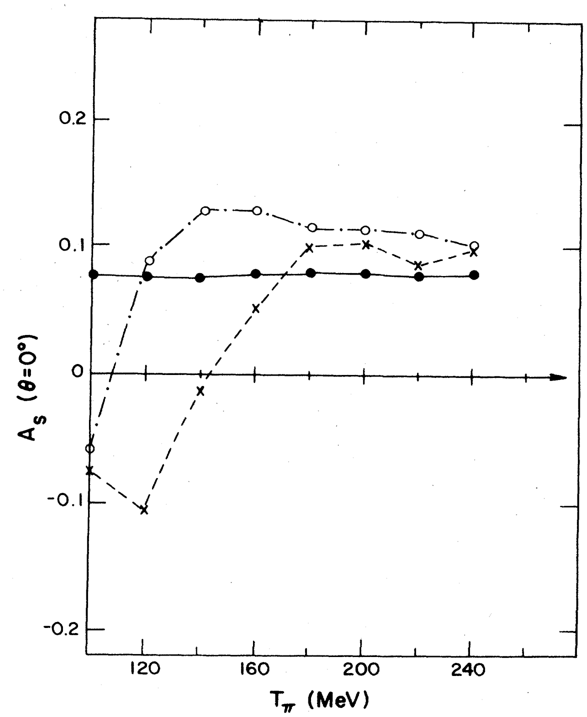

FIG. 5. Energy dependence of $A_s(\theta=0)$ for $\beta_2^N=\beta_2^C=0.32$. Solid points, π^+ elastic; open points, SCX; and crosses, DCX. The curves are drawn to guide the eye.

cal value[7] for the proton deformation, we calculate $A_S(0°) \simeq 11\%$, or a 22% difference in cross sections. If we take the values in Table I corresponding to the DHF (Ref. 5) calculation, we find $A_S(0°) = 6\%$.

V. CONCLUSION

The results of this investigation substantiate the conclusion of Ref. 1, that the asymmetry $A_S(\theta)$ for single charge exchange from oriented, deformed nuclei is sensitive to the deformation of the excess neutrons.[13] We have further demonstrated that $A_S(0)$ is insensitive to other less well-known aspects of the problem, such as higher order terms in U, which can affect the size of $d\sigma/d\Omega$.

Extensions of these calculations into several directions would be useful. We have assumed a special form for the neutron and proton densities [see Eq. (7)], in which $\rho(l,r)$ for $l=2$ is proportional to the derivative of the $l=0$ term, which is taken to be a Woods-Saxon shape. Although the DHF theory resembles this, there are differences in detail which must be carefully taken into account when comparing theory to experiment. The multipoles $l>2$ also should be included. As a second extension, we believe that it would be useful to solve numerically the coupled channel equations including second order effects explicitly. Parameters of the second order optical potential have recently[12] been obtained empirically for $T_\pi = 165$ MeV. This extension would permit a reliable theoretical prediction of the magnitudes and shapes of the individual cross sections $d\sigma^{\parallel}/d\Omega$ and $d\sigma^{\perp}/d\Omega$.

Although both SCX and DCX show a strong sensitivity to β_2^N, we would prefer measurements of SCX, because the cross sections are larger than those of DCX and because they depend on the second order optical potential in a more straightforward way. We have further seen that the SCX cross sections are more slowly varying with energy, which makes the connection between $A_S(0)$ and the densities more stable against effects which are not included in our calculation, especially dynamical modifications of Δ_{33} propagation in the nucleus.

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