Alpha transfer mechanism in heavy-ion reactions

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In some heavy-ion reactions, such as $^{24}Mg(^{16}O,^{12}C)^{28}Si$ oscillatory structure often appears in the whole angle region in the differential cross section. This anomalous phenomenon, unexpected from ordinary distorted-wave Born approximation theory, is explained by an alpha transfer process between two unidentical nuclear cores. Nuclear molecular-orbit theory has been formulated, and the coupled system of wave equations describing both the elastic scattering and transfer reaction channels are obtained. An independent α particle model is assumed in the evaluation of nuclear wave functions. The differential cross sections of both channels have been calculated simultaneously for two cases: $^{24}Mg(^{16}O,^{16}O)^{24}Mg$ and $^{24}Mg(^{16}O,^{12}C)^{28}Si$ as well as $^{28}Si(^{16}O,^{16}O)^{28}Si$ and $^{28}Si(^{16}O,^{12}C)^{32}S$. Agreement with the experimental data can be reached in the whole angle region for both of these cases.

I. INTRODUCTION

In the elastic scattering between $4N$ nuclei, such as the $^{24}Mg(^{16}O,^{16}O)^{24}Mg$ case, an uprising oscillatory structure often appears in the differential cross section at the backward angle region as shown in Fig. 1. The conventional optical model calculation generally could not describe this oscillatory structure as the dashed curve shown in the figure.[1]

This anomalous phenomenon has been studied by an α-transfer mechanism.[2] ^{24}Mg was treated as a system with two α particles revolving about a nuclear core ^{16}O. In the scattering process of $^{16}O+^{24}Mg$ the two valence α particles will move around the two identical ^{16}O cores in some molecular orbits. The nuclear molecular-orbit theory or the linear combination of nuclear orbits (LCNO) (Ref. 3) was used for this α transfer process. The oscillatory structure did appear and agreement with the experimental data was reached naturally for the whole angle region as shown by the solid curve in Fig. 1.

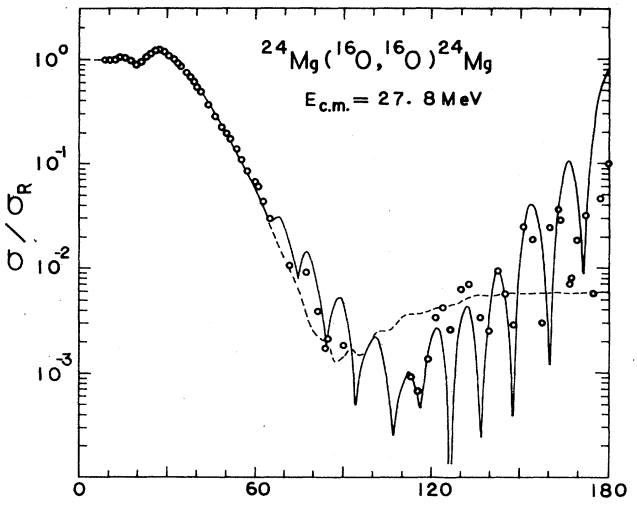

FIG. 1. Differential cross section of the elastic scattering between ^{16}O and ^{24}Mg (Ref. 1). A conventional optical model calculation cannot interpret the uprising oscillatory structure at the large angle region. The solid curve is the result of the LCNO theory calculation based on the α transfer mechanism.

However, in the collision between ^{16}O and ^{24}Mg, beside the elastic scattering, the nuclear reaction channel $^{24}Mg(^{16}O,^{12}C)^{28}Si$ also plays a significant role, as shown in Fig. 2.[1] In this reaction oscillatory structure appears in the differential cross section throughout the whole angle region. Paul and his collaborators have made a calculation using the distorted wave Born approximation (DWBA). Wide discrepancy with the experimental data appeared in the backward angle region as shown by the dashed curve in the lower part of Fig. 2.[1]

It seems worthwhile to study both the elastic scattering and reaction channels simultaneously for this case with the α transfer mechanism. So far in the treatment of elastic transfer scattering only the cases with the transferred nucleon or alpha moving around two identical nuclear cores are considered.[3] In the heavy ion collision process the α particle may also move around two unidentical cores in molecular orbits. In this particular case of the $^{24}Mg+^{16}O$ system ^{16}O may be considered as a system composed of one α particle revolving around a ^{12}C core. In the collision process this valence α particle moves around these two unidentical cores ^{12}C and ^{24}Mg.

A nuclear molecular orbital theory can be formulated for these two-channel α transfer mechanisms. A channel coupling wave equation system has been obtained and applied to two cases: $^{16}O+^{24}Mg$ and $^{16}O+^{28}Si$. The theoretical calculations agree with the experimental data well for both cases as shown by the solid curves in Figs. 2 and 4.

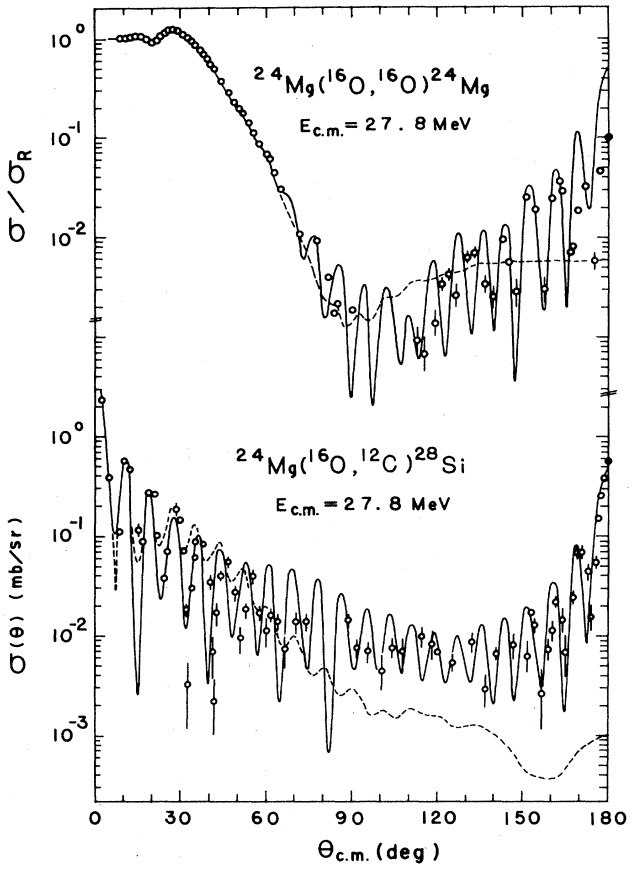

FIG. 2. Differential cross sections for the ^{24}Mg(^{16}O,^{16}O)^{24}Mg scattering channel and the ^{24}Mg(^{16}O,^{12}C)^{28}Si reaction channel (Refs. 1 and 7). The dotted curves are the conventional optical model and DWBA calculations, respectively. The solid curves are the results of the LCNO coupled channel theory calculations.

II. LCNO THEORY FOR THE α TRANSFER MECHANISM BETWEEN UNIDENTICAL CORES

Let us consider a system with an alpha cluster α revolving about two unidentical cores as shown in Fig. 3.

$$(a+C_1)+C_2 \rightarrow C_2+(a+C_1) \quad (\alpha \text{ channel}) ,$$

$$(a+C_1)+C_2 \rightarrow (C_2+a)+C_1 \quad (\beta \text{ channel}) ,$$

where C_1 and C_2 are two unidentical nuclear cores with zero spin and a is the α cluster transferred between these two cores.

Originally the α cluster is attached to core C_1 to form nucleus A ($A=C_1+a$). During the collision process either a may stay with core C_1, and the elastic scattering channel takes place, or a may be transferred to C_2 to form nucleus B ($B=C_2+a$) and the system takes a nuclear reaction channel.

The total Hamiltonian of the system for the scattering channel is

$$H_\alpha = T_\alpha(\bar{R}_1)+V_{\text{opt}}(R_1)+h_\alpha(\bar{R}_1,\bar{r}_2) , \tag{1}$$

where the first two terms are the relative kinetic energy

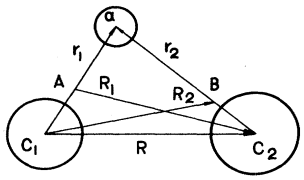

FIG. 3. Coordinate system for the molecular state with a valence alpha particle α revolving around two unidentical core nuclei C_1 and C_2.

and optical potential between nucleus A and core C_2. The last term is the residual Hamiltonian for the transferred α cluster.[4]

$$h_\alpha(\bar{R}_1,\bar{r}_2)=t(\bar{r}_1)+v_{aC_1}(\bar{r}_1)+v_{aC_2}(\bar{r}_2) . \tag{2}$$

The total Hamiltonian of the system may also be written as

$$H_\beta = T_\beta(\bar{R}_2)+V_{\text{opt}}(R_2)+h_\beta(\bar{R}_2,\bar{r}_1) , \tag{3}$$

where

$$h_\beta(\bar{R}_2,\bar{r}_1)=t(\bar{r}_2)+v_{aC_2}(\bar{r}_2)+v_{aC_1}(\bar{r}_1) . \tag{4}$$

According to the LCNO theory the total wave function for the system can be expressed as

$$\Psi=\sum_m [C_m^\alpha \psi_m^\alpha(\bar{R}_1)\phi_m^\alpha(\bar{r}_1)+C_m^\beta \psi_m^\beta(\bar{R}_2)\phi_m^\beta(\bar{r}_2)] , \tag{5}$$

where $\phi_m^\alpha(\bar{r}_1)$ and $\phi_m^\beta(\bar{r}_2)$ are the wave functions for the valence particle α bound to nuclear cores C_1 and C_2, respectively.

If the recoil effect of the cores is neglected, R_1 and R_2 can be replaced by R. All of this of course means that the Born-Oppenheimer (BO) approximation is adopted. It has been shown that even for a small mass ratio of core to the valence particle, the BO approximation still holds.[5]

We also assume that the probability of the valence particle staying in an excited state is negligible. Only contributions of the ground states are considered. The system wave function then becomes

$$\Psi=C^\alpha \psi^\alpha(\bar{R})\phi^\alpha(\bar{r}_1)+C^\beta \psi^\beta(\bar{R})\phi^\beta(\bar{r}_2) . \tag{6}$$

Now we perform the following operations:

$$\begin{aligned} &\langle \phi^\alpha(\bar{r}_1) \mid H_\alpha - E \mid \Psi \rangle = 0 , \\ &\langle \phi^\beta(\bar{r}_1) \mid H_\beta - E \mid \Psi \rangle = 0 , \end{aligned} \tag{7}$$

and consider $\phi^\alpha(\bar{r}_1)$ and $\phi^\beta(\bar{r}_2)$ to be orthogonal to each other. As is known, the overlapping integral

$$\langle \phi^\alpha(\bar{r}_1) \mid \phi^\beta(\bar{r}_2) \rangle = \delta$$

is a small quantity.[4] It gives only higher order modification. A system of differential equations is obtained from Eq. (7) describing the coupling between α and β channels

$$[T_\alpha(\bar{R})+\bar{V}_1(R)+K_1(R)+\epsilon_1-E]\psi^\alpha(\bar{R})+C_1 J_1(R)\psi^\beta(\bar{R}) = 0 , \tag{8}$$

$$[T_\beta(\bar{R})+\bar{V}_1(R)+K_2(R)+\epsilon_1-E]\psi^\beta(\bar{R})+C_2J_2(R)\psi^\alpha(\bar{R})=0\ ,$$

where

$$V_i(R)=V_i^{\rm opt}(R)+V_1^C(R),\quad i=1,2\ . \tag{9}$$

The Wood-Saxon potential is used for the nuclear potential

$$V^{\rm opt}=-V\{1+\exp[(R-R_V)/a_v]\}^{-1} -iW\{1+\exp[(R-R_W)/a_W]\}^{-1} \tag{10}$$

and the Coulomb potential $V^C(R)$ is taken to be

$$V^C(R)=\begin{cases}Z_1Z_2e^2(3-R^2/R_C^2)/2R_C, & R\le R_C\\ Z_1Z_2e^2/R, & R>R_C\end{cases}\ , \tag{11}$$

$$K_1(R)=\langle\phi^\alpha(\bar{r}_1)\,|\,v(r_2)\,|\,\phi^\alpha(\bar{r}_1)\rangle\ , \tag{12}$$

$$K_2(R)=\langle\phi^\beta(\bar{r}_2)\,|\,v(r_1)\,|\,\phi^\beta(\bar{r}_2)\rangle\ , \tag{13}$$

$$\epsilon_1=\langle\phi^\alpha(\bar{r}_1)\,|\,h(\bar{r}_1)\,|\,\phi^\alpha(\bar{r}_1)\rangle\ ,\quad h(\bar{r}_1)=t(\bar{r}_1)+v(r_1)\ , \tag{14}$$

$$\epsilon_2=\langle\phi^\beta(\bar{r}_2)\,|\,h(\bar{r}_2)\,|\,\phi^\beta(\bar{r}_2)\rangle\ ,\quad h(\bar{r}_2)=t(\bar{r}_2)+v(r_2)\ , \tag{15}$$

$$J_1(R)=\langle\phi^\alpha(\bar{r}_1)\,|\,v(r_2)\,|\,\phi^\beta(\bar{r}_2)\rangle\ , \tag{16}$$

$$J_2(R)=\langle\phi^\beta(\bar{r}_2)\,|\,v(r_1)\,|\,\phi^\alpha(\bar{r}_1)\rangle\ . \tag{17}$$

The interaction between the α particle and the nuclear core v in the above integrals is taken to be the Gaussian potential

$$v_{C_ia}(r_i)=V_{R_1}\exp(-r_i^2/a_{R_i}^2)+V_{A_i}\exp(-r_i^2/a_{A_i}^2),\quad (i=1,2)\ , \tag{18}$$

which contains a repulsive term with $V_{R_i}>0$ and an attractive term with $V_{A_i}<0$. For the bound state wave function of the valence particle ϕ, an independent particle model has been used.[6] In this model the wave function for the valence α particle can be expressed simply as a combination of harmonic oscillator functions,

$$\phi(\bar{r}_i)=\frac{1}{\sqrt{8}}(\pi a_i^2)^{-3/4}[(2-\sqrt{6})+\sqrt{8/3}(r_i/a_i)^2]\times\exp(-r_i^2/2a_i^2),\quad i=1,2\ , \tag{19}$$

where a_i is the oscillator length parameter.

The LCNO matrix elements of Eqs. (12), (13), (16), and (17) can then be easily integrated in closed forms and the results are listed in the Appendix. The values of the integrals of Eqs. (14) and (15) are taken directly to be the experimental energies of the ground states.

Finally the coupled differential equation system Eq. (8) is solved numerically and the phase shifts determined. From the approaching behavior of the wave functions one gets the scattering amplitudes for the two channels as follows:

$$f_{\alpha\alpha}=f_{\alpha\alpha}^C+\frac{1}{2ik_\alpha}\sum_l\sqrt{4\pi(2l+1)}\times(e^{2i\delta_l^\alpha}-1)e^{2i\delta_l^{\alpha C}}Y_{l0}(\theta)\ , \tag{20}$$

$$f_{\beta\alpha}=\frac{A^\beta}{2ik_\beta}\sum_l\sqrt{4\pi(2l+1)}e^{2i(\delta_l^\beta+\delta_l^{\beta C})}Y_{l0}(\theta)\ , \tag{21}$$

where

$$f_{\alpha\alpha}^C=-\frac{\eta_\alpha}{2k_\alpha\sin^2\theta/2}e^{2i(\delta_0^C-\eta_\alpha\ln\sin\theta/2)} \tag{22}$$

is the Coulomb scattering amplitude. $\delta_l^{\alpha C}$ and $\delta_l^{\beta C}$ are Coulomb phase shifts. The differential cross sections for scattering and reaction channels will be

$$d\sigma_{\alpha\alpha}/d\Omega=|f_{\alpha\alpha}/f_{\alpha\alpha}^C|^2\ , \tag{23}$$

$$d\sigma_{\beta\alpha}/d\Omega=|f_{\beta\alpha}|^2,\quad \beta\neq\alpha\ . \tag{24}$$

III. CHOICE OF PARAMETER VALUES AND RESULTS

The formulation of the LCNO theory for the two-channel transfer mechanism has been applied to calculate the angular distribution of two cases: ^{24}Mg(^{16}O,^{16}O)^{24}Mg and ^{24}Mg(^{16}O,^{12}C)^{28}Si; ^{28}Si(^{16}O,^{16}O)^{28}Si and ^{28}Si(^{16}O,^{12}C)^{32}S. Experimental data of these cases are available.[1,7–9] The optical model parameters in Eqs. (10) and (11) are kept the same as those used by previous authors.[1,10] (See Table I.) The parameters of the Gaussian potential between the valence α particle and the nuclear core in Eqs. (18) and (19) are listed in Table II.

TABLE I. Wood-Saxon optical potential and Coulomb potential parameters taken from Refs. 1 and 10 without adjustment.

System	V (MeV)	R_{0V}[a] (fm)	a_V (fm)	W (MeV)	R_{0W}[a] (fm)	a_W (fm)	R_{0C}[a] (fm)
^{16}O+^{24}Mg	37.0	1.37	0.394	78.0	1.32	0.208	1.2
^{16}O+^{28}Si	10.0	1.35	0.618	23.4	1.23	0.552	1.0

[a]This is the value of the root mean square radius with $R_i=R_{0i}(A_1^{1/3}+A_2^{1/3})$, $i=V,W,C$.

TABLE II. Gaussian potential parameters in Eq. (18) for the interaction between α and nuclear cores and the harmonic oscillator length parameters in the wave function for the α particle in Eq. (19).

Core	V_R (MeV)	a_R (fm)	V_A (MeV)	a_A (fm)	a_{HO} (fm)
^{12}C	55.0	0.95	−30.2	1.7	1.2[a]
^{24}Mg	310.0	0.93	−285.0	2.5	1.55
^{28}Si	250.0	1.0	−212.0	2.1	1.5

[a]This value is determined from spectroscopic data (Ref. 2).

The harmonic oscillator length for ^{12}C is taken to be $a_{HO}=1.2$ as determined from spectroscopic data.[2] The rest of the parameters in Table II are considered to be adjustable. Their values may give some information about how an α particle interacts with some $4N$ nuclei of different mass numbers.

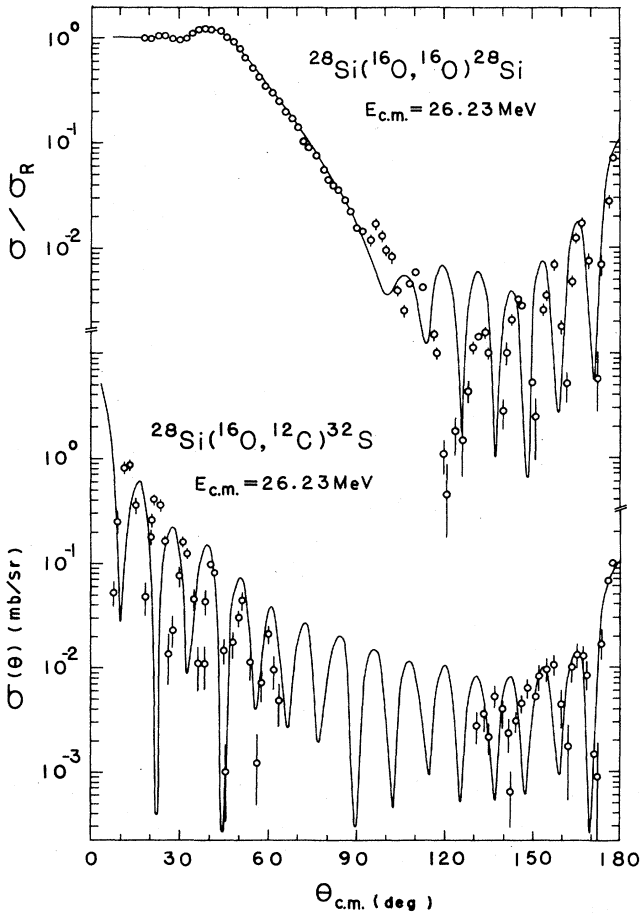

FIG. 4. Differential cross sections for the $^{28}Si(^{16}O,^{16}O)^{28}Si$ scattering channel and the $^{28}Si(^{16}O,^{12}C)^{32}S$ reaction channel (Ref. 9). The solid curves are the results of the LCNO coupled channel theory calculations.

The calculated results of the angular distributions for both scattering channel $^{24}Mg(^{16}O,^{16}O)^{24}Mg$ and reaction channel $^{24}Mg(^{16}O,^{12}C)^{28}Si$ are plotted as the solid curves shown in Fig. 2. The experimental points are the measurements made by Paul *et al.*[1,7] While for the elastic scattering channel the oscillatory structure appears only in the large angle region, the oscillatory structure covers the whole angle region for the reaction channel. The dotted curves are the results of their calculation based on conventional optical model and DWBA theory, and oscillatory structures do not show up in the backward angle region. These oscillatory structures do appear in the α transfer LCNO theory calculations. The number of peaks and their magnitudes agree with the experimental data except in the 90° region.

For the other example, similar oscillatory structures also appear in the angular distribution of both scattering channel $^{28}Si(^{16}O,^{16}O)^{28}Si$ and the reaction channel $^{28}Si(^{16}O,^{12}C)^{32}S$ as shown in Fig. 4.[9] Agreements can also be reached for the LCNO theory calculations as the solid curves show in the figure. It is interesting to note that for both cases the positions of crest and valley in the scattering channel nearly coincide with those in the reaction channel.

IV. DISCUSSION

The nucleon or a cluster of nucleons transfer mechanism, which has been used so successfully in interpreting the elastic transfer scattering between two identical nuclear cores, can be extended to apply to some heavy ion nuclear reactions between two unidentical cores. This way the appearance of oscillatory structure in the differential cross section in certain heavy ion reactions between $4N$ nuclei may be also interpreted. An α particle is supposed to move in some molecular orbits around the two unidentical cores. The LCNO theory works well to describe these α transfer mechanisms in coupled scattering and reaction channels.

It seems to be profitable to apply the independent α particle model to get simple expressions for the bound state wave function of the α particle. This way one can integrate out the LCNO matrix elements and get the exchange potential in closed form. This makes it easier to analyze the behavior of these exchange potentials which play the major role in causing the uprising oscillatory structure in the backward angle region of the differential cross section.

Both the scattering and reaction channel experimental data have been fitted simultaneously with a common set of parameters. If two sets of adjustable parameters are used separately, better agreement may be obtained. The recoil effect of the cores has been neglected and improvement may be gained by taking it into consideration.

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APPENDIX

The LCNO matrix elements, Eqs. (16) and (17), may be written as

$$v(r_i)=v_{Ri}\exp(-\lambda_{Ri}r_i^2)+v_{Ai}\exp(-\lambda_{Ai}r_i^2),\quad i=1,2$$

and

$$\phi(r_i)=(a_i+b_i r_i^2)\exp(-\tau_i r_i^2)\ ,$$

where

$$a_i=-0.067\,3460\,a_{\mathrm{HO}i}^{-3/2},\quad b_i=0.244\,688\,a_{\mathrm{HO}i}^{-7/2},\quad \tau_i=0.5\,a_{\mathrm{HO}i}^{-2}\ .$$

(1) $$K_1(R)=\langle\phi(\bar r_1)\,|\,v(r_2)\,|\,\phi(\bar r_1)\rangle$$
$$=F_1(v_{R2},\lambda_{R2},a_1,b_1,\tau_1,R)+F_1(v_{A2},\lambda_{A2},a_1,b_1,\tau_1,R),$$

where

$$F_1(v,\lambda,a_1,b_1,\tau_1,R)=\frac{2\pi vAE_0}{\lambda R}[C_1+(C_3+C_5Z^2a^{-1})Z^2a^{-1}]Za^{-1}\ ,$$

$$C_1=a_1^2+3a_1b_1/a+15b_1^2/4a^2,\quad C_3=2a_1b_1+5b_1^2/a\ ,$$

$$C_5=b_1^2,\quad a=2\tau_1+\lambda,\quad Z=\lambda R(2\tau_1+\lambda)^{-1/2}\ ,$$

$$A=\exp(-\lambda R^2+Z^2),\quad E_0=\tfrac{1}{2}\sqrt{\pi}[1+\mathrm{erf}(Z)]\ .$$

(2) $$K_2(R)=F_1(v_{R1},\lambda_{R1},a_2,b_2,\tau_2,R)+F_1(v_{A1},\lambda_{A1},a_2,b_2,\tau_2,R)\ .$$

(3) $$J_1(R)=\langle\phi(\bar r_1)\,|\,v(r_2)\,|\,\phi(\bar r_2)\rangle$$
$$=F_2(v_{R2},\lambda_{R2},a_1,b_1,\tau_1,a_2,b_2,\tau_2,R)+F_2(v_{A2},\lambda_{A2},a_1,b_1,\tau_1,a_2,b_2,\tau_2,R)\ ,$$

where

$$F_2(v,\lambda,a_1,b_1,\tau_1,a_2,b_2,\tau_2,R)$$
$$=\frac{2\pi vAE_0}{(\lambda+\tau_2)R}[D_0+(D_1+\{D_2+[D_3+(D_4+D_5Za^{-1/2})Za^{-1/2}]Za^{-1/2}\}Za^{-1/2})Za^{-1/2}]a^{-1/2}\ ,$$

$$D_0=-a_1b_2R/a-3b_1b_2R/2a^2\ ,$$

$$D_1=a_1[a_2+b_2[R^2+(\lambda+\tau_2)^{-1}]+\tfrac{3}{2}(a_1b_2+b_1\{a_2+b_2[R^2+(\lambda+\tau_2)^{-1}]+5b_2/2a\})]a^{-1}\ ,$$

$$D_2=-2(a_1+3b_1/a)b_2R,\quad D_3=a_1b_2+b_1\{a_2+b_2[R^2+(\lambda+\tau_2)^{-1}+5a^{-1}]\}\ ,$$

$$D_4=-2b_1b_2R,\quad D_5=b_1b_2,\quad a=\lambda+\tau_1+\tau_2\ ,$$

$$Z=(\lambda+\tau_2)R(\lambda+\tau_1+\tau_2)^{-1/2},\quad A=\exp[-(\lambda+\tau_2)R^2+Z^2]\ .$$

(4) $$J_2(R)=\langle\phi(\bar r_2)\,|\,v(r_1)\,|\,\phi(\bar r_1)\rangle$$
$$=F_2(v_{R1},\lambda_{R1},a_2,b_2,\tau_2,a_1,b_1,\tau_1,R)+F_2(v_{A1},\lambda_{A1},a_2,b_2,\tau_2,a_1,b_1,\tau_1,R)\ .$$

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