Covariant soliton dynamics: Structure of the nucleon

L. S. Celenza, A. Rosenthal,* and C. M. Shakin
Department of Physics and Institute for Nuclear Theory, Brooklyn College of the City University of New York, Brooklyn, New York 11210
(Received 2 April 1984)

We present a model of nucleon structure that is fully covariant. We begin with a Lagrangian that describes quarks coupled to various mesonic fields (σ,π,ρ,ω) and an additional field, χ, which is required to bind the quarks into a nucleon. The couplings of the σ, π, ρ, and ω fields to the quarks are fixed so as to reproduce the empirically determined coupling of these mesons to the *nucleon*. The latter couplings are taken from fits to nucleon-nucleon scattering data made using one-boson-exchange models. Therefore the free parameters of the model are the mass and coupling constant of the χ field and the quark mass, m_q. In order to simplify the problem, the nucleon is assumed to virtually decay into a quark and a diquark. Equations which specify this amplitude are found by using the equations for the quark and meson field operators obtained from our Lagrangian. The equations which we solve are fully covariant and nonlinear and are solved by iteration. In this model the quark dynamics is governed by the mesonic fields whose source is the nucleon itself. The amplitudes for the emission of these fields by the nucleon depend upon the (nucleon)$\rightarrow$(quark + diquark) amplitudes whose structures we are attempting to determine. This model therefore requires a self-consistent solution and leads to the nonlinear equations noted above. At this point we have not calculated mesonic corrections to the nucleon observables such as the magnetic moments, form factors, and g_A, although we have included the effects of all the mesonic fields in the calculation of the nucleon mass. However, the calculation of nucleon observables, using current operators which contain only quark fields, yields a surprisingly good fit to electromagnetic form factors, magnetic moments, g_A, etc. (It is possible that some aspects of these results will be less satisfactory when mesonic corrections are calculated.) The model has the further virtue of generalizing the SU(6) quark model of the nucleon so as to be consistent with the covariance requirements of the theory of special relativity. It is clear that the major limitation of the model, other than the use of the diquark approximation, is the lack of a satisfactory description of the confinement mechanism as the χ field serves to bind the system but does not actually confine the quarks.

I. INTRODUCTION

Since the introduction of the MIT bag model[1] there has been an ongoing interest in developing models of nucleon structure. Much work has been done on chiral bag models[2] and other variants of the MIT model.[3] Recently we have seen models that improve upon the static-cavity approximation that is used in most applications of the bag model. In particular we are here most interested in nontopological models such as that of Friedberg and Lee.[4] This model has the structure of a simple field theory with quarks coupled to a scalar field which confines the quarks to a finite region. The source of the scalar field is the scalar quark density and therefore the equations of this model must be solved in a self-consistent manner. Usually, this is done by introducing a static approximation and solving the classical (c-number) version of the field equations.[5] In an earlier work we demonstrated how one could avoid the static approximation and analyze the Friedberg-Lee model in a manner which maintains the translational invariance of the model throughout.[6] In this work we will proceed along these lines and discuss soliton models in a fully covariant approximation. (The approximations we use will be discussed in detail at a later point.)

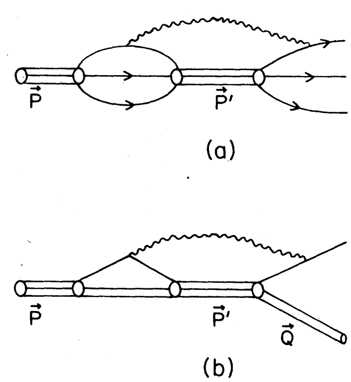

FIG. 1. Schematic description of a covariant self-consistent-field model. (a) A nucleon decays virtually to three quarks, one of which emits a meson. The meson is later absorbed by a quark. (b) A "diquark" approximation. A nucleon of momentum $\vec{P}$ decays virtually to a quark and diquark. A meson is emitted by the quark and is ultimately absorbed after another virtual decay of the nucleon. [See the right-hand side of Eq. (5.5).] In both cases the nucleon form factor for emission of a meson is given in terms of the amplitude for the virtual decay—see Fig. 2.

We should note that the equations of the Friedberg-Lee model have been solved in the static approximation by Goldflam and Wilets[7] and methods for restoring translational invariance to the model have been discussed by Hadjimichael and Fiebig and by Betz and Goldflam among others.[8]

In the Friedberg-Lee model the scalar field which is used to confine the quarks is apparently unrelated to the scalar mesons exchanged between nucleons in the boson-exchange model of nuclear forces. Since we will have to deal with more than one scalar field in this work we will call the scalar field of the Friedberg-Lee model $\chi(x)$. We will use the notation $\sigma(x)$ for a scalar field which might appear as the chiral partner of the pion field or as the scalar field that appears in the boson-exchange model of nuclear forces. These distinctions become important as we now have a new class of soliton models[9,10] in which a bound state (a nucleon) is formed by solving either the equations of the nonlinear or linear σ model of Gell-Mann and Levy.[11] We should also mention the work of Seki and Ohta[12] which combines both a confining field $\chi(x)$ and a chiral σ model. In addition to these models there are also topological models which develop some ideas of Skyrme.[13] In these models a nucleon is obtained from a Lagrangian that contains only boson fields.[14] (The resulting objects are often called Skyrmions.) Models which contain both quarks and meson fields and have interesting topological properties have been discussed by several authors.[15]

Clearly before undertaking studies in this area one must adopt a point of view that will assist in choosing among the rapidly increasing number of models of nucleon structure. We will try to clarify the basis of our work by indicating some of our goals:

(1) We desire to construct a *covariant* soliton model. The preservation of translation invariance in the model is particularly important if one wishes to calculate form factors properly. Calculations of nucleon magnetic moments made using *static* models are subject to large errors, for example.[6] Aside from the importance of a proper treatment of translational invariance for the calculation of dynamic properties such as form factors, static properties such as the nucleon mass are expected to have significant "recoil" or "center-of-mass" corrections.

(2) We desire to discuss the interaction between solitons. This requires a description of solitons that is nonstatic. For example, recent studies of the nucleon-nucleon scattering amplitude have shown that nucleons appear to interact, in part, by the exchange of rather intense (Lorentz) scalar and vector fields.[16] Indeed, that was always true in boson-exchange models. However, as has been discussed recently, if the free-space nucleon-nucleon scattering amplitude is written in terms of Dirac matrices (which act in the space of spinor solutions of the Dirac equation) it is apparent that the scalar and vector terms are very large.[16] This analysis of the nucleon-nucleon scattering amplitude requires only a knowledge of the NN phase shifts and therefore is independent of any model of the interaction. It is clear from these studies that the strong attraction in the NN force is of (Lorentz) scalar character and the repulsion is of vector character. These features of the interaction cannot be obtained by using a *static* soliton model of the nucleon to study the nucleon-nucleon interaction.

(3) It is one of our goals to create a model of a nucleus. Therefore our solitons must be able to move about in the presence of external fields generated by other solitons. Thus with the development of a nonstatic soliton model we might be able to develop a shell model of a nucleus composed of interacting solitons.

(4) The recent successful descriptions of nuclear structure based upon a relativistic many-body theory[17] implies that the boson-exchange model of nuclear forces should be taken seriously as a model for the interaction of nucleons. The essential features of the model are the couplings of the nucleon to fields with the quantum numbers of the σ, π, ρ, and ω mesons.[18] These four fields are the minimum required to describe the nucleon-nucleon interaction. Of these, the σ field plays a particularly important role in the successful description of nuclear dynamics in a relativistic model.[17] We believe our soliton model should incorporate the insight gained from our studies of relativistic nuclear structure physics.

In light of the above comments, the model of Birse and Banerjee[10] is of interest. These authors have studied the linear σ model. (In their work, quarks replace the nucleons which appeared in the original version of the σ model.) This model has a σ field and a pion field coupled to the quark fields. A nucleon is constructed using a "hedgehog" solution to the equations of motion.[19] As Kalbermann and Eisenberg[20] have recently remarked, it appears somewhat unexpected to create "confinement" *via* meson-quark coupling. In particular, the pion is responsible for a large part of the attraction in the Birse-Banerjee model. As we have noted earlier, we believe we should include reference to the σ, π, ρ, and ω fields. Thus one is tempted to add the ρ and ω fields to the Birse-Banerjee model. As we shall see, the interaction of the ρ and ω fields with the quark is repulsive. Indeed the repulsion is sufficiently large to cancel most of the attraction obtained from the σ- and π-quark coupling. Given this observation, one option is to include an additional scalar field, $\chi(x)$, in addition to the mesonic fields. Thus, in this work we consider a model in which we form a soliton solution using a scheme somewhat analogous to that used by Friedberg and Lee[4] and developed by Goldflam and Wilets.[7] We are still interested in including the σ, π, ρ, and ω fields. Thus our Lagrangian has two parts, one of which involves the χ field while the other describes the interaction of the quark field with meson fields $(\sigma,\pi,\rho,\omega,\ldots)$. This model can be seen to have the following unsatisfactory feature. We must assume that only the mesonic modes $(\sigma,\pi,\rho,\omega,\ldots)$ are exchanged between two solitons in interaction and that the scalar field, $\chi(x)$, does not participate in the interaction. This *ad hoc* assumption does have one satisfactory feature, however. It essentially trivializes the nuclear force problem since our solitons interact by exchange of σ, π, ρ, and ω mesons and these exchanges can describe the NN force. This scheme also greatly reduces the number of free parameters to be specified since the coupling of the σ, π, ρ, and ω mesons to the *nucleon* must be given correctly. As we will see, in our model, the coupling constants of these mesons to the *quarks* are related linearly to the meson-nucleon coupling constants which are known from phenomenological studies; therefore there are no free parameters in the quark-meson Lagrangian. (We do have the freedom to include a tensor coupling of the ρ and ω fields to the quarks.[6] The strength of this coupling is a free parameter. It can be seen that a nonzero value for this parameter has the effect of giving the quarks an anomalous magnetic moment, if we use a vector-dominance model to calculate electromagnetic form factors.[6])

We may remark that one could carry out our analysis with a single scalar field which is exchanged between nucleons and also serves to provide sufficient binding to form a nucleon. However, if one were to use a single scalar field one would have to use a somewhat different coupling constant to describe the coupling of this field to the quarks in the calculation of nucleon-nucleon scattering and in the calculation of the structure of the nucleon. While we can provide some arguments to justify such a procedure, we have decided to use two scalar fields in this work. The use of two such fields can be seen as an alternate to introducing two different (effective) coupling constants for a single field. Either of these procedures will lead to the same results in this model.

Before entering upon a discussion of the equations of our model it is useful to indicate, in a schematic fashion, the dynamical basis of our formalism. The mathematical analysis is based upon algebraic manipulations; however, the resulting equations have a useful diagrammatic representation. The interaction for the model we would like to study is shown in Fig. 1(a). Here a nucleon of momentum $\vec{P}$ is assumed to decay (virtually) into three (valence) quarks. One of the quarks emits a meson and the nucleon reforms and propagates with momentum $\vec{P}'$. The meson is then reabsorbed by one of the valence quarks. This defines the interaction to be used in a covariant self-consistent-field description of a (relativistic) bound state. As this problem is too difficult for us at this time, we turn to an approximation discussed in an earlier work.[6] We assume that two quarks form a quasibound state, a "diquark," and consider the dynamical model shown in Fig. 1(b). This is the model we will develop in this work. [More precisely, Fig. 1(b) is a schematic representation of the right-hand side of Eq. (5.5).] It is clear that the field seen by the quark on the far right of the diagram depends upon nucleon form factors which govern the amplitude for the emission of various mesons. In turn, the form factors depend upon the quark "wave functions," that is, upon the nucleon to quark-diquark amplitudes. Thus we see the nature of the self-consistent solution that will be required in our analysis.

The limitations of this model have been discussed previously.[6] In this model all the momentum is carried by quark and mesonic fields. There is no reference to the gluons which appear in the quantum chromodynamics (QCD) Lagrangian and which are thought to carry significant momentum. At this stage of our understanding, however, we are not able to provide a satisfactory model which has gluon degrees of freedom. (We should remark that the quarks of the QCD Lagrangian have only small "current" masses. The quarks in our model have large "constituent" masses. This feature may involve the "dressing" of the quarks by the gluon field. We are not able to further develop such speculations at this time.)

The plan of our work is as follows. In Sec. II we present the Lagrangian of our model and the associated field equations. We also introduce a series of nucleon form factors which play a role in the analysis. In Sec. III we discuss the form of the amplitude for a nucleon transition to a quark and a diquark. In Sec. IV we relate our model to the SU(6) model of nucleon structure by studying our wave functions in the nucleon rest frame. In Sec. V we provide equations which determine various invariant amplitudes which parametrize the (nucleon) to (quark-diquark) amplitudes. These amplitudes are shown to define relativistic quark wave functions which take a simple form in the nucleon rest frame. (Since we are working with a covariant formalism we can discuss the soliton wave function in any Lorentz frame.) In Sec. VI we discuss the calculation of the nucleon form factors introduced in Sec. II. In Sec. VII we discuss how the meson-quark coupling constants may be obtained from the phenomenological meson-nucleon coupling constants and the calculated nucleon form factors. In Secs. VIII and IX we discuss our numerical results. Section X contains a summary and some general remarks.

II. LAGRANGIAN AND FIELD EQUATIONS

We begin by writing the Lagrangian density of our soliton model. This Lagrangian includes the interaction of the quarks with the σ, π, ρ, and ω fields as well as with the field $\chi(x)$. We also include terms which describe the tensor coupling of the ρ and ω fields to the quarks. Here m_q, m_χ, g_χ, and $\tilde{\lambda}$ are parameters of the model. The coupling constants g_σ, g_π, g_ρ, and g_ω are to be fixed by requiring that the empirical values are obtained for the coupling of the mesons to the *nucleon*. Furthermore, $\tilde{\lambda}$ can be fixed by requiring that the *tensor* coupling of the ρ and ω fields to the *nucleon* is given correctly. If $\tilde{\lambda}$ is fixed in this manner, the model has four parameters, m_q, m_χ, g_χ, and a diquark mass parameter, m_d. (In our calculations we have found quite satisfactory results with $\tilde{\lambda}=0$. However, we present the various $\tilde{\lambda}$-dependent terms in the following for completeness.) With the definitions,

$$\vec{F}^{\rho}_{\mu\nu}(x)\equiv\partial_\mu\vec{\rho}_\nu(x)-\partial_\nu\vec{\rho}_\mu(x) \tag{2.1}$$

and

$$F^{\omega}_{\mu\nu}(x)\equiv\partial_\mu\omega_\nu(x)-\partial_\nu\omega_\mu(x)\ , \tag{2.2}$$

we have,

$$\mathscr{L}(x)=\bar{q}(x)\left[i\gamma^\mu\partial_\mu-m_q-g_\sigma\sigma(x)-g_\chi\chi(x)-ig_\pi\gamma_5\vec{\tau}_q\cdot\vec{\Pi}(x)-g_\omega\gamma^\mu\omega_\mu(x)\right.$$
$$\left.+g_\omega\frac{\tilde{\lambda}}{4m_N}\sigma^{\mu\nu}F^{\omega}_{\mu\nu}(x)-g_\rho\gamma^\mu\vec{\tau}_q\cdot\vec{\rho}_\mu(x)+g_\rho\frac{\tilde{\lambda}}{4m_N}\sigma^{\mu\nu}\vec{\tau}_q\cdot\vec{F}^{\rho}_{\mu\nu}(x)\right]q(x)$$

$$+\tfrac{1}{2}[\partial_\mu\chi(x)\partial^\mu\chi(x)-m_\chi^2\chi^2(x)]+\tfrac{1}{2}[\partial_\mu\sigma(x)\partial^\mu\sigma(x)-m_\sigma^2\sigma^2(x)]+\tfrac{1}{2}[\partial_\mu\vec{\Pi}(x)\cdot\partial^\mu\vec{\Pi}(x)-m_\pi^2\vec{\Pi}^{\,2}(x)]$$

$$-\tfrac{1}{4}[\vec{F}^{\rho}_{\mu\nu}(x)\cdot\vec{F}^{\mu\nu}_{\rho}(x)]+\tfrac{1}{2}m_\rho^2\vec{\rho}^{\,\mu}(x)\cdot\vec{\rho}_\mu(x)-\tfrac{1}{4}[F^{\omega}_{\mu\nu}(x)F^{\mu\nu}_{\omega}(x)]+\tfrac{1}{2}m_\omega^2\omega^\mu(x)\omega_\mu(x)\,. \tag{2.3}$$

From this Lagrangian density we obtain the following field equations:

$$(i\gamma^\mu\partial_\mu-m_q)q(x)=g_\sigma q(x)\sigma(x)+g_\chi q(x)\chi(x)+ig_\pi\gamma^5\vec{\tau}_q\cdot q(x)\vec{\Pi}(x)+g_\omega\left[\gamma^\mu q(x)\omega_\mu(x)+\frac{\tilde{\lambda}}{2m_N}\sigma^{\mu\nu}q(x)\partial_\nu\omega_\mu(x)\right]$$

$$+g_\rho\left[\gamma^\mu\vec{\tau}_q q(x)\cdot\vec{\rho}_\mu(x)+\frac{\tilde{\lambda}}{2m_N}\sigma^{\mu\nu}\vec{\tau}_q\cdot q(x)\partial_\nu\vec{\rho}_\mu(x)\right], \tag{2.4}$$

$$(\Box+m_\sigma^2)\sigma(x)=-g_\sigma\bar{q}(x)q(x)\,, \tag{2.5}$$

$$(\Box+m_\chi^2)\chi(x)=-g_\chi\bar{q}(x)q(x)\,, \tag{2.6}$$

$$(\Box+m_\pi^2)\vec{\Pi}(x)=-ig_\pi\bar{q}(x)\gamma^5\vec{\tau}_q q(x)\,, \tag{2.7}$$

$$(\Box+m_\omega^2)\omega^\nu(x)=g_\omega\left\{\bar{q}(x)\gamma^\nu q(x)-\frac{\tilde{\lambda}}{2m_N}\partial_\mu[\bar{q}(x)\sigma^{\mu\nu}q(x)]\right\}, \tag{2.8}$$

$$(\Box+m_\rho^2)\vec{\rho}^{\,\nu}(x)=g_\rho\left\{\bar{q}(x)\gamma^\nu\vec{\tau}_q q(x)-\frac{\tilde{\lambda}}{2m_N}\partial_\mu[\bar{q}(x)\sigma^{\mu\nu}\vec{\tau}_q q(x)]\right\}. \tag{2.9}$$

We will denote the nucleon states as $|\vec{P},s,t)$ and normalize them such that $(\vec{P}',s',t'|\vec{P},s,t)=\delta_{ss'}\delta_{tt'}\delta(\vec{P}'-\vec{P})[E_N(\vec{P})/m_N]$, where $E_N(\vec{P})=(\vec{P}^2+m_N^2)^{1/2}$. Here s and t denote the projections of the nucleon spin and isospin. The mass of the nucleon, m_N, may be calculated using the formalism presented in the Appendix. This quantity also appears in our equations for the nucleon-quark-diquark amplitudes. (See Sec. V.)

The analysis of Eqs. (2.4)—(2.9) proceeds *via* the definition of various form factors which appear in the following equations:

$$(\vec{P}',s',t'|\bar{q}(0)q(0)|\vec{P},s,t)=\bar{u}(\vec{P}',s')u(\vec{P},s)\delta_{tt'}\frac{\rho_s(q^2)}{(2\pi)^3}\,, \tag{2.10}$$

$$(\vec{P}',s',t'|\bar{q}(0)\gamma^5\vec{\tau}_q q(0)|\vec{P},s,t)=\langle t'|\vec{\tau}_N|t\rangle\frac{g_1(q^2)}{(2\pi)^3}\bar{u}(\vec{P}',s')\gamma^5u(\vec{P},s)\,, \tag{2.11}$$

$$\left[\vec{P}',s',t'\left|\bar{q}(0)\left[\gamma^\mu-\tilde{\lambda}\frac{i\sigma^{\mu\nu}}{2m_N}q_\nu\right]q(0)\right|\vec{P},s,t\right]=\frac{\delta_{t't}}{(2\pi)^3}\bar{u}(\vec{P}',s')\left[\gamma^\mu F_{10}(q^2)+\frac{i\sigma^{\mu\nu}}{2m_N}q_\nu F_{20}(q^2)\right]u(\vec{P},s)\,, \tag{2.12}$$

$$\left[\vec{P}',s',t'\left|\bar{q}(0)\left[\gamma^\mu\vec{\tau}_q-\frac{i\tilde{\lambda}\sigma^{\mu\nu}}{2m_N}q_\nu\vec{\tau}_q\right]q(0)\right|\vec{P},s,t\right]=\frac{\langle t'|\vec{\tau}_N|t\rangle}{(2\pi)^3}\bar{u}(\vec{P}',s')\left[\gamma^\mu F_{11}(q^2)+\frac{i\sigma^{\mu\nu}}{2m_N}q_\nu F_{21}(q^2)\right]u(\vec{P},s)\,. \tag{2.13}$$

The calculation of these form factors is discussed in Sec. VI. From these equations we obtain

$$(\vec{P}',s',t'|\sigma(0)|\vec{P},s,t)=\frac{-g_\sigma}{-q^2+m_\sigma^2}\bar{u}(\vec{P}',s')u(\vec{P},s)\frac{\rho_s(q^2)}{(2\pi)^3}\,, \tag{2.14}$$

$$(\vec{P}',s',t'|\chi(0)|\vec{P},s,t)=\frac{-g_\chi}{-q^2+m_\chi^2}\bar{u}(\vec{P}',s')u(\vec{P},s)\frac{\rho_s(q^2)}{(2\pi)^3}\,, \tag{2.15}$$

$$(\vec{P}',s',t'|\vec{\Pi}(0)|\vec{P},s,t)=\langle t'|\vec{\tau}_N|t\rangle\frac{(-ig_\pi)}{-q^2+m_\pi^2}[\bar{u}(\vec{P}',s')\gamma^5u(\vec{P},s)]\frac{g_1(q^2)}{(2\pi)^3}\,, \tag{2.16}$$

$$(\vec{P}',s',t'|\omega^\mu(0)|\vec{P},s,t)=\delta_{tt'}\frac{g_\omega}{-q^2+m_\omega^2}\frac{1}{(2\pi)^3}\bar{u}(\vec{P}',s')\left[\gamma^\mu F_{10}(q^2)+\frac{i\sigma^{\mu\nu}}{2m_N}q_\nu F_{20}(q^2)\right]u(\vec{P},s)\,,$$

$$(\vec{P}',s',t'|\vec{\rho}^{\,\mu}(0)|\vec{P},s,t)=\langle t'|\vec{\tau}_N|t\rangle\frac{g_\rho}{-q^2+m_\rho^2}\frac{1}{(2\pi)^3}\bar{u}(\vec{P}',s')\left[\gamma^\mu F_{11}(q^2)+\frac{i\sigma^{\mu\nu}}{2m_N}q_\nu F_{21}(q^2)\right]u(\vec{P},s)\,. \tag{2.18}$$

Before analyzing our quark field equation, Eq. (2.4), we will introduce certain matrix elements of the quark field operator. These matrix elements are introduced and parametrized in Secs. III and IV. In Sec. V we return to the study of Eq. (2.4) and will there make use of Eqs. (2.14)–(2.18).

III. NUCLEON-DIQUARK AMPLITUDES

The analysis of our field equations follows, in part, the analysis given in Ref. 6. Here we improve upon that work in that we present a manifestly covariant specification of the matrix elements of the quark field operator. We consider the amplitude for a nucleon of momentum $\vec{P}$ to decay into an off-shell quark and an on-shell diquark of momentum $\vec{Q}$. The diquark energy is put equal to $(\vec{Q}^2+m_d)^{1/2}$ with $m_d=2m_q$. The diquark has spin S and helicity λ. The isospin of the diquark is T with projection M_T. Thus we consider the matrix element

$$(\vec{Q}S\lambda TM_T\,|\,q(0)\,|\,\vec{P},s,t)\ .$$

We may consider the simple case, $S=0$, $T=0$ first and write the amplitude in question in terms of scalar invariants, A and B,

$$(\vec{Q}\,|\,q(0)\,|\,\vec{P},s,t)=\frac{1}{(2\pi)^{3/2}}\left[A+\frac{B\not{Q}}{m_d}\right]u(\vec{P},s)\frac{1}{\sqrt{4\pi}}\,|\,t\rangle\ . \tag{3.1}$$

Here $u(\vec{P},s)$ is a Dirac spinor and $|\,t\rangle$ is the isospin wave function of the quark; the quark isospin projection, t_q, is here equal to t. In the nucleon rest frame ($\vec{P}=0$) we have,

$$(\vec{Q}'\,|\,q(0)\,|\,\vec{0},s,t)$$
$$=\frac{1}{(2\pi)^{3/2}}\left[A+\frac{B\not{Q}'}{m_d}\right]u(\vec{0},s)\frac{1}{\sqrt{4\pi}}\,|\,t\rangle\ . \tag{3.2}$$

The scalar invariants A and B may be taken as functions of the Lorentz scalar, $[(P\cdot Q/m_{\rm N})^2-m_d^2]^{1/2}$, which may be identified as the magnitude of the quark (or diquark) momentum in the nucleon rest frame, where $\vec{P}=0$. (In the calculations which follow we will need to consider amplitudes of the form given in Eq. (3.1) where the nucleon has either four-momenta P or P'. In that case we can introduce another Lorentz scalar, $[(\vec{P}'\cdot Q/m_{\rm N})^2-m_d^2]^{1/2}$. Scalar invariants that are functions of $[(P'\cdot Q/m_{\rm N})^2-m_d^2]^{1/2}$ will be denoted as A' and B'.)

We remark that the amplitude given in Eq. (3.1) may be obtained by applying a Lorentz boost to the rest-frame amplitude. We have

$$(\vec{Q}\,|\,q_\alpha(0)\,|\,\vec{P},s,t)=S_{\alpha\beta}(\vec{P})(\vec{Q}'\,|\,q_\beta(0)\,|\,\vec{0},s,t) \tag{3.3}$$

or

$$\frac{1}{(2\pi)^{3/2}}\left[A+\frac{B\not{Q}}{m_d}\right]u(\vec{P},s)\frac{1}{\sqrt{4\pi}}\,|\,t\rangle$$
$$=S(\vec{P})\frac{1}{(2\pi)^{3/2}}\left[A+\frac{B\not{Q}'}{m_d}\right]u(\vec{0},s)\frac{1}{\sqrt{4\pi}}\,|\,t\rangle\ . \tag{3.4}$$

The four-vectors Q_μ and Q'_μ are related by a Lorentz transformation, $S(\vec{P})\not{Q}'S^{-1}(\vec{P})=\not{Q}$. Note that $S(\vec{P})u(\vec{0},s)=u(\vec{P},s)$.

It is instructive to specify the nucleon amplitude in more detail. (We will use the notation $Q=|\vec{Q}|$ and $Q'=|\vec{Q}'|$ in the following.) In the rest frame, $\vec{P}=0$, we have,

$$(\vec{Q}'\,|\,q(0)\,|\,\vec{0},s,t)$$
$$=\frac{1}{(2\pi)^{3/2}}\frac{1}{\sqrt{4\pi}}\begin{bmatrix}R_u(Q')\,|\,s\rangle\\ \vec{\sigma}\cdot\vec{Q}'R_l(Q')\,|\,s\rangle\end{bmatrix}|\,t\rangle\ . \tag{3.5}$$

Here $R_u(Q')$ and $R_l(Q')$ are the Lorentz scalars,

$$R_u(Q')=A+\frac{P\cdot QB}{m_{\rm N}m_d} \tag{3.6}$$

and

$$R_l(Q')=B/m_d\ . \tag{3.7}$$

Note that we may also write,

$$R_u(Q')=A+\frac{E_d(\vec{Q}')B}{m_d}\ , \tag{3.8}$$

which follows upon evaluation of the invariant $[P\cdot Q/(m_{\rm N}m_d)]$ in the frame where $\vec{P}=0$. Furthermore, it is useful to define the function

$$\bar{R}_l(Q')=|\,\vec{Q}'\,|\,R_l(Q')=|\,\vec{Q}'\,|\,B/m_d\ . \tag{3.9}$$

Indeed, at a later point we will provide integral equations for the functions $R_u(Q')$ and $\bar{R}_l(Q')$. We choose the normalization

$$\int_0^\infty\frac{Q^2dQ}{2E_d(\vec{Q})}[R_u^2(Q)+\bar{R}_l^2(Q)]=1\ , \tag{3.10}$$

or defining

$$\hat{R}_u(Q)=R_u(Q)/[2E_d(\vec{Q})]^{1/2}$$

and

$$\hat{R}_l(Q)=\bar{R}_l(Q)/[2E_d(\vec{Q})]^{1/2}$$

we have,

$$\int_0^\infty Q^2dQ[\hat{R}_u^2(Q)+\hat{R}_l^2(Q)]=1\ . \tag{3.11}$$

The description of the (virtual) nucleon decay into a quark and an $S=1$, $T=1$ diquark is somewhat more complicated. In the nucleon rest frame *both* s-wave and d-wave decay is allowed. Therefore there are *four* scalar amplitudes required for a complete specification of the matrix elements of the quark field operator between a nucleon state and a diquark state. To avoid unnecessary complication we have assumed that we can neglect the d-wave transition and therefore we have organized the amplitudes so that they describe an s-wave transition. This amplitude can then be specified in terms of *two* Lorentz scalars, $\tilde{A}$ and $\tilde{B}$. We write,

$$(\vec{Q}S\lambda TM_T \,|\, q(0) \,|\, \vec{P},s,t) = \frac{1}{(2\pi)^{3/2}} \frac{1}{\sqrt{4\pi}} \gamma^5 \left[\frac{\xi_\lambda^* \cdot P}{m_N} A_1 \left[1 + \frac{Q}{m_d} \right] + \xi_\lambda^* \left[\tilde{A} + \frac{\tilde{B} Q}{m_d} \right] \right] u(\vec{P},s) C^{T\ \ 1/2\ 1/2}_{M_T t_q\ \ t} \,|\, t_q \rangle \ . \tag{3.12}$$

Here

$$A_1 \equiv \frac{\tilde{B} - \tilde{A}}{\left[1 + \frac{P \cdot Q}{m_N m_d} \right]} \tag{3.13}$$

and $t_q = t - M_T$. Furthermore, $\xi_\lambda^* \equiv \xi_\lambda^*(\hat{Q})$ is the polarization vector of the $S=1$ diquark.

Again, if we need to consider the amplitudes

$$(\vec{Q}S\lambda TM_T \,|\, q(0) \,|\, \vec{P},s,t)$$

and

$$(\vec{Q}S\lambda TM_T \,|\, q(0) \,|\, \vec{P}',s',t')$$

at the same time we will use the notation, A_1, A, B for functions of the scalar $[(P\cdot Q/m_N)^2 - m_d^2]^{1/2}$ and the notation A_1', A', B' for functions of the scalar $[(P'\cdot Q/m_N)^2 - m_d^2]^{1/2}$.

It is instructive to inspect the wave function of Eq. (3.12) in more detail. For example, when $\vec{P}=0$, we have,

$$(\vec{Q}'S\lambda TM_T \,|\, q(0) \,|\, \vec{0},s,t) = \frac{1}{(2\pi)^{3/2}} \frac{1}{\sqrt{4\pi}} \begin{bmatrix} \left[\tilde{A} + \frac{E_d(\vec{Q}')}{m_d} \tilde{B} \right] \vec{\xi}_\lambda^* \cdot \vec{\sigma} \,|\, s \rangle \\ \vec{\sigma} \cdot \vec{Q}' \frac{\tilde{B}}{m_d} \vec{\xi}_\lambda^* \cdot \vec{\sigma} \,|\, s \rangle \end{bmatrix} C^{T\ \ 1/2\ 1/2}_{M_T t_q\ \ t} \,|\, t_q \rangle \ . \tag{3.14}$$

We use the completeness of the Pauli spinors, and the relation

$$\langle s_q \,|\, \vec{\xi}_\lambda^* \cdot \vec{\sigma} \,|\, s \rangle = -\sqrt{3} C^{1\ 1/2\ 1/2}_{\lambda\ s_q\ \ s} \ , \tag{3.15}$$

which is valid if $\vec{Q}'$ is along the z axis, to write

$$(\vec{Q}'S\lambda TM_T \,|\, q(0) \,|\, \vec{0},s,t) = -\sqrt{3} \frac{1}{(2\pi)^{3/2}} \frac{1}{\sqrt{4\pi}} \begin{bmatrix} \left[\tilde{A} + \frac{E_d(\vec{Q}')\tilde{B}}{m_d} \right] |\, s_q \rangle \\ \sigma \cdot \vec{Q}' \frac{\tilde{B}}{m_d} \,|\, s_q \rangle \end{bmatrix} C^{1\ 1/2\ 1/2}_{\lambda\ s_q\ \ s} C^{T\ \ 1/2\ 1/2}_{M_T t_q\ \ t} \,|\, t_q \rangle \tag{3.16}$$

with $s_q = s - \lambda$.

The motivation for writing the wave function in this form is given in the next section where we also give the result for $\vec{Q}'$ in a general direction. (We might note, however, that in Ref. 6 we considered canonical states for the diquark rather than the helicity states we use in this work. The treatment of the Lorentz boost properties of the $S=1$ states given in Ref. 6 was only approximate while in this work we provide an exact formulation.)

IV. A MODEL BASED UPON SU(6) SYMMETRY

In the general case the relations between the amplitudes A, B, $\tilde{A}$, and $\tilde{B}$ must be determined from dynamical considerations. It is useful, however, to make contact with the standard quark model based on SU(6) symmetry. In part, this was done in Ref. 6 and from that work we can see that the appropriate identification for an SU(6) based model is

$$\tilde{A} = -\frac{1}{\sqrt{3}} A \ , \tag{4.1}$$

$$\widetilde{B}=-\frac{1}{\sqrt{3}}B \; . \tag{4.2}$$

In this case Eq. (3.17) becomes, with $\vec{Q}'=Q'\hat{z}$,

$$(\vec{Q}'S\lambda TM_T \,|\, q(0) \,|\, \vec{0},s,t)=\frac{1}{(2\pi)^{3/2}}\frac{1}{\sqrt{4\pi}}\begin{bmatrix}\left[A+\dfrac{E_d(\vec{Q}')B}{m_d}\right]|s_q\rangle \\ \dfrac{\vec{\sigma}\cdot\vec{Q}'B}{m_d}|s_q\rangle\end{bmatrix} C^{1\;\;1/2\;1/2}_{\lambda\;s_q\;\;s}C^{T\;\;1/2\;1/2}_{M_T\;t_q\;\;t}\,|t_q\rangle \; . \tag{4.3}$$

A more general result with $\vec{Q}'$ in an arbitrary direction may be obtained by using

$$\vec{\xi}^{\,*}_{\lambda}(\hat{Q}')=\sum_{\lambda'}\vec{\xi}^{\,*}_{\lambda'}(\hat{z})D^{(1)}_{\lambda'\lambda}(\hat{Q}') \tag{4.4}$$

and then making use of Eq. (3.16).

It may be seen from Ref. 6, that the use of Eq. (3.1) in conjunction with Eq. (4.3) will reproduce the results of the SU(6) quark model for the ratio μ_p/μ_n, g_A, and other nucleon observables. These results require that the $S=0$, $T=0$ and the $S=1$, $T=1$ amplitudes appear with equal weight in constructing the nucleon wave function.

At this point we should remark that the standard SU(6) quark model is essentially nonrelativistic in character. Attempts to extend this model to provide a representation of the Poincaré group have not been successful. However, as noted earlier, our analysis is fully covariant. We have only used some aspects of the SU(6) model and have maintained covariance of our formulation throughout. Even with a covariant formulation we still obtain the result that $\mu_p/\mu_n=-\frac{3}{2}$ and that $g_A=\frac{5}{3}(1-\frac{4}{3}a)$, where a is the fraction of small component in the quark wave function.

We may use this model based upon SU(6) symmetry to calculate various form factors. For example, we consider the calculation of

$$(\vec{P}',s',t' \,|\, \bar{q}(0)q(0) \,|\, \vec{P},s,t) \; .$$

Here the operator $\bar{q}(0)q(0)$ does not contain the quark isospin operator; therefore, we find,

$$\rho_s(q^2)=\tfrac{3}{2}[\rho_s^{S=0}(q^2)+\rho_s^{S=1}(q^2)] \; . \tag{4.5}$$

The factor of 3 arises since $\rho_s^{S=0}$ and $\rho_s^{S=1}$ are the form factors calculated "per quark." The factor of 2 has its origin in our SU(6) model in which the $S=0$ and $S=1$ contributions are given equal weight.[6] In the case where the operator in question involves the quark isospin, we have, for example, (see Sec. VI),

$$g_1(q^2)=\tfrac{3}{2}[g_1^{S=0}(q^2)-\tfrac{1}{3}g_1^{S=1}(q^2)] \; .$$

Similarly, we have

$$F_{10}(q^2)=\tfrac{3}{2}[F_{10}^{S=0}(q^2)+F_{10}^{S=1}(q^2)] \; , \tag{4.6}$$

$$F_{20}(q^2)=\tfrac{3}{2}[F_{20}^{S=1}(q^2)+F_{20}^{S=1}(q^2)] \; , \tag{4.7}$$

while

$$F_{11}(q^2)=\tfrac{3}{2}[F_{11}^{S=0}(q^2)-\tfrac{1}{3}F_{11}^{S=1}(q^2)] \; , \tag{4.8}$$

$$F_{21}(q^2)=\tfrac{3}{2}[F_{21}^{S=0}(q^2)-\tfrac{1}{3}F_{21}^{S=1}(q^2)] \; . \tag{4.9}$$

The normalization factors have been chosen such that

$$\rho_s^{S=0}(0)=\rho_s^{S=1}(0) \; ,$$

$$F_{10}^{S=0}(0)=F_{10}^{S=1}(0)=1 \; ,$$

and

$$F_{11}^{S=0}(0)=F_{11}^{S=1}(0)=1 \; .$$

It follows that $F_{10}(0)=3$ and $F_{11}(0)=1$. (These normalization factors are discussed in more detail in Sec. VI.) It is further worth remarking that in this model

$$g_1^{S=1}(q^2)=-\tfrac{1}{3}g_1^{S=0}(q^2)$$

so that

$$g_1(q^2)=\tfrac{5}{3}g_1^{S=0}(q^2) \; .$$

We will continue our discussion of form factors in Sec. VI. Further comments on the role of SU(6) symmetry will be presented in the next section.

V. INTEGRAL EQUATIONS FOR INVARIANT AMPLITUDES: QUARK WAVE FUNCTIONS

We are now in possession of a sufficient number of definitions so that we can construct an equation which determines the scalars A and B or the related quantities, $R_u(Q)$ and $R_l(Q)$. One starts with the equation for the quark field operator, Eq. (2.4). We then form matrix elements of this equation by multiplying by nucleon states and diquark states on the right-hand side and left-hand side of Eq. (2.4). We then make use of the fact that

$$(\vec{Q}S\lambda TM_T \,|\, q(x) \,|\, \vec{P},s,t)=e^{i(Q-P)\cdot x}(\vec{Q}S\lambda TM_T \,|\, q(0) \,|\, \vec{P},s,t) \; . \tag{5.1}$$

Furthermore, in carrying out these manipulations one encounters matrix elements of the form $(\vec{Q}S\lambda TM_T \,|\, q(0)\sigma(0) \,|\, \vec{P},s,t)$ which we evaluate by inserting a set of soliton (nucleon) states:[21]

$$(\vec{Q}S\lambda TM_T\,|\,q(0)\sigma(0)\,|\,\vec{P},s,t)=\int\sum_{s't'}(\vec{Q}S\lambda TM_T\,|\,q(0)\,|\,\vec{P}',s',t')\frac{d\vec{P}'m_N}{E_N(\vec{P}')}(\vec{P}',s',t'\,|\,\sigma(0)\,|\,\vec{P},s,t)\,, \tag{5.2}$$

$$=-g_\sigma\sum_{s't'}\int(\vec{Q}S\lambda TM_T\,|\,q(0)\,|\,\vec{P}',s',t')\frac{d\vec{P}'m_N}{E_N(\vec{P}')}\frac{\bar{u}(\vec{P}',s')u(\vec{P},s)}{-q^2+m_\sigma^2}\frac{\rho_s(q^2)}{(2\pi)^3}\delta_{tt'}\,. \tag{5.3}$$

Here we have made use of Eq. (2.14). Finally we use the definition of the nucleon-diquark amplitudes given in terms of A' and B' or $\widetilde{A}'$ and $\widetilde{B}'$ and the relation

$$\left[\frac{P'+m_N}{2m_N}\right]=\sum_{s'}u(\vec{P}',s')\bar{u}(\vec{P}',s')\,. \tag{5.4}$$

Thus, we obtain, for example, an equation for the channel involving the $S=0$, $T=0$ diquark:

$$\begin{aligned}
[\gamma^\mu(P-Q)_\mu-m_q]\left[A+\frac{BQ}{m_d}\right]u(\vec{0},s)=&-g_\chi^2\int\frac{d\vec{P}'}{(2\pi)^3}\frac{m_N}{E_N(\vec{P}')}\left[A'+\frac{B'Q}{m_d}\right]\left[\frac{P'+m_N}{2m_N}\right]u(\vec{0},s)\frac{\rho_s(q^2)}{-q^2+m_\chi^2}\\
&-g_\sigma^2\int\frac{d\vec{P}'}{(2\pi)^3}\frac{m_N}{E_N(\vec{P}')}\left[A'+\frac{B'Q}{m_d}\right]\left[\frac{P'+m_N}{2m_N}\right]u(\vec{0},s)\frac{\rho_s(q^2)}{-q^2+m_\sigma^2}\\
&+g_\pi^2(3)\int\frac{d\vec{P}'}{(2\pi)^3}\frac{m_N}{E_N(\vec{P}')}\gamma^5\left[A'+\frac{B'Q}{m_d}\right]\left[\frac{P'+m_N}{2m_N}\right]\gamma^5u(\vec{0},s)\frac{g_1(q^2)}{-q^2+m_\pi^2}\\
&+g_\omega^2\int\frac{d\vec{P}'}{(2\pi)^3}\frac{m_N}{E_N(\vec{P}')}\left[\gamma^\mu+\frac{\widetilde{\lambda}}{2m_N}i\sigma^{\mu\nu}q_\nu\right]\left[A'+\frac{B'Q}{m_d}\right]\left[\frac{P'+m_N}{2m_N}\right]\\
&\times\frac{1}{-q^2+m_\omega^2}\left[\gamma_\mu F_{10}(q^2)+\frac{i\sigma_{\mu\nu}}{2m_N}q^\nu F_{20}(q^2)\right]u(\vec{0},s)\\
&+g_\rho^2(3)\int\frac{d\vec{P}'}{(2\pi)^3}\frac{m_N}{E_N(\vec{P}')}\left[\gamma^\mu+\frac{\widetilde{\lambda}}{2m_N}i\sigma^{\mu\nu}q_\nu\right]\left[A'+\frac{B'Q}{m_d}\right]\left[\frac{P'+m_N}{2m_N}\right]\\
&\times\frac{1}{-q^2+m_\rho^2}\left[\gamma_\mu F_{11}(q^2)+\frac{i\sigma_{\mu\nu}}{2m_N}q^\nu F_{21}(q^2)\right]u(\vec{0},s)\,.
\end{aligned} \tag{5.5}$$

Here $P=[m_N,\vec{0}]$, $P'=[E_N(\vec{P}'),\vec{P}']$, and $q=P'-P$. Of course, if we were to put $P=[E_N(\vec{P}),\vec{P}]$ the fully covariant nature of this equation would be apparent. This equation has a rather transparent diagrammatic interpretation and may be represented as in Fig. 1(b). Equation (5.5) is a *covariant self-consistent-field* equation in that the form factors are functionals of the Lorentz scalars A, B, A', and B'. It is useful to simplify this equation by obtaining integral equations which do not contain any Dirac matrices. It turns out that it is most convenient to write integral equations for the quantities $R_u(Q)$ and $\bar{R}_l(Q)$. The integral equations for these quantities are best written in a two-dimensional matrix form as this most clearly shows that the kernal of the equation is real and symmetric, that is, we are dealing with a Hermitian interaction. The appropriate variables are the diquark momenta $\vec{Q}$ and $\vec{Q}'$, where $\vec{Q}'$ is the diquark momentum in the frame where the intermediate nucleon (of momentum $\vec{P}'$) is at rest. We make extensive use of the relations[6]

$$\vec{Q}'=\vec{Q}-\vec{P}'\left\{\frac{E_d(\vec{Q})}{m_N}-\frac{\vec{P}'\cdot\vec{Q}}{m_N[E_N(\vec{P}')+m_N]}\right\}, \tag{5.6}$$

$$E_d'\equiv E_d(\vec{Q}')=[E_d(\vec{Q})E_N(\vec{P}')-\vec{P}'\cdot\vec{Q}]/m_N\,, \tag{5.7}$$

$$\frac{\vec{P}'}{\epsilon_N(\vec{P}')}\equiv\frac{\vec{P}'}{[E_N(\vec{P}')+m_N]}=\frac{\vec{Q}-\vec{Q}'}{E_d+E_d'}\,, \tag{5.8}$$

$$\vec{P}'=\frac{(\vec{Q}-\vec{Q}')}{E_d+E_d'}\left[\frac{2m_N}{1-|\vec{Q}-\vec{Q}'|^2/(E_d+E_d')^2}\right]. \tag{5.9}$$

After some manipulations one finds for the $S=0$, $T=0$ channel,

$$\hat{m}_{\rm N}\begin{bmatrix} R_u(Q) \\ \bar{R}_l(Q) \end{bmatrix} = \begin{bmatrix} E_d(\vec{\mathbf{Q}})+m_q & -|\vec{\mathbf{Q}}| \\ -|\vec{\mathbf{Q}}| & E_d(\vec{\mathbf{Q}})-m_q \end{bmatrix}\begin{bmatrix} R_u(Q) \\ \bar{R}_l(Q) \end{bmatrix}$$

$$-g_\chi^2\int\frac{d\vec{\mathbf{P}}'}{(2\pi)^3}\frac{\epsilon_{\rm N}(\vec{\mathbf{P}}')}{2E_{\rm N}(\vec{\mathbf{P}}')}\frac{\rho_s(q^2)}{m_\chi^2-q^2}\begin{bmatrix} 1 & \vec{\mathbf{P}}'\cdot\hat{Q}'/\epsilon_{\rm N}(\vec{\mathbf{P}}') \\ -\vec{\mathbf{P}}'\cdot\hat{Q}/\epsilon_{\rm N}(\vec{\mathbf{P}}') & -\hat{Q}\cdot\hat{Q}' \end{bmatrix}\begin{bmatrix} R_u(Q') \\ \bar{R}_l(Q') \end{bmatrix}$$

$$-g_\sigma^2\int\frac{d\vec{\mathbf{P}}'}{(2\pi)^3}\frac{\epsilon_{\rm N}(\vec{\mathbf{P}}')}{2E_{\rm N}(\vec{\mathbf{P}}')}\frac{\rho_s(q^2)}{m_\sigma^2-q^2}\begin{bmatrix} 1 & \dfrac{\vec{\mathbf{P}}'\cdot\hat{Q}'}{\epsilon_{\rm N}(\vec{\mathbf{P}}')} \\ \dfrac{-\vec{\mathbf{P}}'\cdot\hat{Q}}{\epsilon_{\rm N}(\vec{\mathbf{P}}')} & -\hat{Q}\cdot\hat{Q}' \end{bmatrix}\begin{bmatrix} R_u(Q') \\ \bar{R}_l(Q') \end{bmatrix}$$

$$-3g_\pi^2\int\frac{d\vec{\mathbf{P}}'}{(2\pi)^3}\frac{\epsilon_{\rm N}(\vec{\mathbf{P}}')}{2E_{\rm N}(\vec{\mathbf{P}}')}\frac{g_1(q^2)}{m_\pi^2-q^2}\begin{bmatrix} \vec{\mathbf{P}}'^2/\epsilon_{\rm N}^2(\vec{\mathbf{P}}') & \hat{Q}'\cdot\vec{\mathbf{P}}'/\epsilon_{\rm N}(\vec{\mathbf{P}}') \\ -\vec{\mathbf{P}}'\cdot\hat{Q}/\epsilon_{\rm N}(\vec{\mathbf{P}}') & \dfrac{-[2QQ'-\hat{Q}\cdot\hat{Q}'(Q^2+Q'^2)]}{(E_d+E_d')^2} \end{bmatrix}\begin{bmatrix} R_u(Q') \\ \bar{R}_l(Q') \end{bmatrix}$$

$$+g_\omega^2\int\frac{d\vec{\mathbf{P}}'}{(2\pi)^3}\frac{\epsilon_{\rm N}(\vec{\mathbf{P}}')}{2E_{\rm N}(\vec{\mathbf{P}}')}\frac{1}{m_\omega^2-q^2}\begin{bmatrix} V_{11}^\omega & V_{12}^\omega \\ V_{21}^\omega & V_{22}^\omega \end{bmatrix}\begin{bmatrix} R_u(Q') \\ \bar{R}_l(Q') \end{bmatrix}$$

$$+g_\rho^2(3)\int\frac{d\vec{\mathbf{P}}'}{(2\pi)^3}\frac{\epsilon_{\rm N}(\vec{\mathbf{P}}')}{2E_{\rm N}(\vec{\mathbf{P}}')}\frac{1}{m_\rho^2-q^2}\begin{bmatrix} V_{11}^\rho & V_{12}^\rho \\ V_{21}^\rho & V_{22}^\rho \end{bmatrix}\begin{bmatrix} R_u(Q') \\ \bar{R}_l(Q') \end{bmatrix}, \tag{5.10}$$

where

$$V_{11}^\omega=\left\{\frac{2[2m_{\rm N}-E_{\rm N}(\vec{\mathbf{P}}')]}{\epsilon_{\rm N}(\vec{\mathbf{P}}')}+\frac{3\tilde{\lambda}\vec{\mathbf{P}}'^2}{\epsilon_{\rm N}^2(\vec{\mathbf{P}}')}\right\}[F_{10}(q^2)+F_{20}(q^2)]-F_{20}(q^2)\left[1+\frac{\tilde{\lambda}}{2m_{\rm N}}\frac{\vec{\mathbf{P}}'^2}{\epsilon_{\rm N}(\vec{\mathbf{P}}')}\right], \tag{5.11}$$

$$V_{12}^\omega=\frac{-2\vec{\mathbf{P}}'\cdot\hat{Q}'}{\epsilon_{\rm N}(\vec{\mathbf{P}}')}\left[1-\frac{\tilde{\lambda}}{2}\right][F_{10}(q^2)+F_{20}(q^2)]-F_{20}(q^2)\tilde{\lambda}\left[\frac{\vec{\mathbf{P}}'\cdot\hat{Q}'}{2m_{\rm N}}\right], \tag{5.12}$$

$$V_{21}^\omega=\frac{2\vec{\mathbf{P}}'\cdot\hat{Q}}{\epsilon_{\rm N}(\vec{\mathbf{P}}')}\left[1-\frac{\tilde{\lambda}}{2}\right][F_{10}(q^2)+F_{20}(q^2)]+F_{20}(q^2)\tilde{\lambda}\left[\frac{\vec{\mathbf{P}}'\cdot\hat{Q}}{2m_{\rm N}}\right], \tag{5.13}$$

$$V_{22}^\omega=[F_{10}(q^2)+F_{20}(q^2)]\left\{\frac{2m_{\rm N}}{\epsilon_{\rm N}(\vec{\mathbf{P}}')}\hat{Q}\cdot\hat{Q}'+\frac{2\vec{\mathbf{P}}'\cdot\hat{Q}\vec{\mathbf{P}}'\cdot\hat{Q}'}{\epsilon_{\rm N}^2(\vec{\mathbf{P}}')}+\tilde{\lambda}\left[\frac{2(\vec{\mathbf{P}}'\cdot\hat{Q})(\vec{\mathbf{P}}'\cdot\hat{Q}')}{\epsilon_{\rm N}^2(\vec{\mathbf{P}}')}-\frac{\vec{\mathbf{P}}'^2(\hat{Q}\cdot\hat{Q}')}{\epsilon_{\rm N}^2(\vec{\mathbf{P}}')}\right]\right\}$$

$$-F_{20}(q^2)\left[\hat{Q}\cdot\hat{Q}'\left[1-\frac{\tilde{\lambda}\vec{\mathbf{P}}'^2}{2m_{\rm N}\epsilon_{\rm N}(\vec{\mathbf{P}}')}\right]\right]. \tag{5.14}$$

To obtain V_{11}^ρ, V_{12}^ρ, etc., replace $F_{10}(q^2)$ and $F_{20}(q^2)$ by $F_{11}(q^2)$ and $F_{21}(q^2)$ in Eqs. (5.11)–(5.14).

Further development of this equation requires a change of the variable of integration from $\vec{\mathbf{P}}'$ to $\vec{\mathbf{Q}}'$. These variables are related as in Eqs. (5.8) and (5.9).

The Jacobian of this transformation is given in Ref. 6, where the equation for a quark field coupled to a scalar field is developed in some detail. We found $d\vec{\mathbf{P}}'\to d\vec{\mathbf{Q}}'J$ where,

$$\frac{\epsilon_{\rm N}(\vec{\rm P}\,')}{2E_{\rm N}(\vec{\rm P}\,')}J=\left[\frac{2m_{\rm N}}{E_d+E_d'}\right]^3\left[\frac{1}{1-\left[\frac{\vec{\rm Q}-\vec{\rm Q}\,'}{E_d-E_d'}\right]^2}\right]^4 \times\left[1+\frac{(\vec{\rm Q}-\vec{\rm Q}\,')\cdot\vec{\rm Q}\,'}{E_d'(E_d+E_d')}\right], \tag{5.15}$$

and noted that

$$\left[1+\frac{(\vec{\rm Q}-\vec{\rm Q}\,')\cdot\vec{\rm Q}\,'}{E_d'(E_d+E_d')}\right]=\frac{m_d^2+E_dE_d'+\vec{\rm Q}\cdot\vec{\rm Q}\,'}{E_d'(E_d+E_d')}\,. \tag{5.16}$$

[See Eqs. (B5) and (B8) of Ref. 6.]

Now that we have Eq. (5.10) available we can return to further consideration of the wave function of the nucleon whose structure is based upon an underlying SU(6) model. Thus far we have only presented an equation for the case that the diquark has $S=0$ and $T=0$. A similar equation may be written for the case in which $S=1$ and $T=1$. That equation would involve the amplitudes $\widetilde{A}$, $\widetilde{B}$, $\widetilde{A}\,'$, and $\widetilde{B}\,'$. The $S=0$, $T=0$ equation and the $S=1$, $T=1$ equation would be coupled through the various form factors which appear in these equations.

We do not write the equation for the $S=1$, $T=1$ channel here. However, we can note the difference between that equation and Eq. (5.10). First we note that the equations for the $S=1$, $T=1$ channel contain some small terms proportional to the quantity A_1 defined earlier. In addition, the isospin factors are different. The factors $3g_\pi^2$ and $3g_\rho^2$ in Eq. (5.10) are replaced by $-g_\pi^2$ and $-g_\rho^2$ on passing to the $S=1$, $T=1$ equation. If these isospin factors had been unmodified we could have argued that the use of our SU(6) model would be justified. Since the pion and rho meson contributions are quite different in the two equations we need to present arguments that can be used to justify the SU(6) model. First we can argue that the Δ isobar plays a role in the $S=1$, $T=1$ channel. If the coupling of the delta to the nucleon through the transition $\rm N\to\pi+\Delta$ or $\rm N\to\rho+\Delta$ were considered one could write coupled equations in the $S=1$, $T=1$ channel involving both nucleon-diquark and delta-diquark decay amplitudes. The delta might serve to restore the SU(6) symmetry by giving rise to an *effective* isospin factor of 3 for the pion and rho meson, in the $S=1$, $T=1$ channel. This coupled channel model is under study at this time.

The second argument for the use of the SU(6) model relates to the nature of the binding mechanism. If binding were achieved with the scalar field $\chi(x)$, wave functions could then be constructed that have an SU(6) symmetry. At that point, "turning on" the interactions with the σ, π, ρ, and ω fields could lead to only a relatively small violation of this symmetry since the effects of these mesonic couplings tend to cancel. (The σ and π fields yield attraction while the ρ and ω fields yield repulsion.)

We will assume that the second of these two suggestions is reasonable and defer the study of the coupled equations involving the Δ for another investigation. In the next section we discuss the calculation of the various form factors which appear in Eq. (5.10), and in Sec. VII we specify the coupling constants of the quark-meson interaction.

VI. CALCULATION OF NUCLEON FORM FACTORS

We have defined a series of form factors in Sec. II. In this section we show how these form factors may be calculated in terms of the nucleon-diquark amplitudes introduced earlier. For example, let us recall the definition of the scalar form factor, $\rho_s(q^2)$,

$$(\vec{\rm P}\,',s',t'\,|\,\bar q(0)q(0)\,|\,\vec{\rm P},s,t)=\delta_{tt'}\bar u(\vec{\rm P}\,',s')u(\vec{\rm P},s)\frac{\rho_s(q^2)}{(2\pi)^3}\,. \tag{6.1}$$

We now insert a set of diquark states between the operators $\bar q(0)$ and $q(0)$:

$$(\vec{\rm P}\,',s',t'\,|\,\bar q(0)q(0)\,|\,\vec{\rm P},s,t)=\sum_{S\lambda TM_T}\int(\vec{\rm P}\,',s',t'\,|\,\bar q(0)\,|\,\vec{\rm Q}S\lambda TM_T)\frac{d\vec{\rm Q}}{2E_d(\vec{\rm Q})}(\vec{\rm Q}S\lambda TM_T\,|\,q(0)\,|\,\vec{\rm P},s,t)\,. \tag{6.2}$$

We use the $S=0$ and $S=1$ amplitudes defined earlier and note that with normalization chosen for our SU(6) model, we must insert a factor of $\frac{3}{2}$. We have

$$\begin{aligned}(\vec{\rm P}\,',s',t'\,|\,\bar q(0)q(0)\,|\,\vec{\rm P},s,t)=\frac{3}{2}\Bigg\{&\frac{1}{(2\pi)^3}\delta_{tt'}\frac{1}{4\pi}\int\frac{d\vec{\rm Q}}{2E_d(\vec{\rm Q})}\bar u(\vec{\rm P}\,',s')\left[\frac{\not{Q}B'}{m_d}+A'\right]\left[A+\frac{B\not{Q}}{m_d}\right]u(\vec{\rm P},s)\\ &+\sum_\lambda\frac{1}{(2\pi)^3}\delta_{tt'}\frac{1}{4\pi}\int\frac{d\vec{\rm Q}}{2E_d(\vec{\rm Q})}\bar u(\vec{\rm P}\,',s')\left[\left[\widetilde{A}\,'+\frac{\widetilde{B}\,'\not{Q}}{m_d}\right]\not{\xi}_\lambda+\left[1+\frac{\not{Q}}{m_d}\right]\frac{A_1'P\cdot\xi_\lambda}{m_{\rm N}}\right]\\ &\times(-1)\left[\frac{\xi_\lambda^*\cdot P}{m_{\rm N}}A_1\left[1+\frac{\not{Q}}{m_d}\right]+\not{\xi}_\lambda^*\left[\widetilde{A}+\frac{\widetilde{B}\not{Q}}{m_d}\right]\right]u(\vec{\rm P},s)\Bigg\}\,.\end{aligned} \tag{6.3}$$

Thus the contributions naturally divide into an $S=0$ $T=0$ and $S=1$ $T=1$ part. [Recall that we are using the SU(6) model described earlier where $\widetilde{A}=-(1/\sqrt{3})A$ and $\widetilde{B}=-(1/\sqrt{3})B$.] We write

$$\rho_s(q^2)=\tfrac{3}{2}[\rho_s^{S=0}(q^2)+\rho_s^{S=1}(q^2)] \tag{6.4}$$

and find that,

$$\rho_s^{S=0}(q^2)=\frac{1}{4\pi}\int\frac{d\vec{Q}}{2E_d(\vec{Q})}\left\{(A'A+B'B)+\frac{Q\cdot(P+P')}{(P+P')^2}\frac{2m_{\rm N}}{m_d}(A'B+B'A)\right\}, \tag{6.5}$$

while the general form for $\rho_s^{S=1}(q^2)$ is

$$\begin{aligned}\rho_s^{S=1}(q^2)=\frac{1}{4\pi}\int\frac{d\vec{Q}}{2E_d(\vec{Q})}&\left\{\left[\frac{P\cdot P'}{m_{\rm N}^2}-\frac{(P'\cdot Q)(P\cdot Q)}{m_{\rm N}^2m_d^2}\right]2A_1'A_1\left[1+\frac{2m_{\rm N}}{m_d}\frac{Q\cdot(P+P')}{(P+P')^2}\right]\right.\\&+3\left[\widetilde{A}'\widetilde{A}+\widetilde{B}'\widetilde{B}+\frac{2m_{\rm N}}{m_d}\frac{Q\cdot(P+P')}{(P+P')^2}(\widetilde{A}'\widetilde{B}+\widetilde{A}\widetilde{B}')\right]\\&\left.-2(\widetilde{A}'-\widetilde{B}')(\widetilde{A}-\widetilde{B})\left[1-\left[\frac{2m_{\rm N}}{m_d}\frac{Q\cdot(P+P')}{(P+P')^2}\right]\right]\right\}.\end{aligned} \tag{6.6}$$

These expressions may be evaluated in any frame, but it is useful to take $\vec{P}=0$. Furthermore, the primes on the invariant amplitudes signify the dependence on nucleon rest-frame variables discussed earlier.

It is worth noting that

$$\rho_s(0)=3(1-2a), \tag{6.7}$$

where a is the fraction of lower component in the quark wave function,

$$a=\int Q^2dQ[\hat{R}_l^2(Q)]. \tag{6.8}$$

We note that if one performs the isospin sums explicitly one has

$$g_1(q^2)=\tfrac{3}{2}[g_1^{S=0}(q^2)-\tfrac{1}{3}g_1^{S=1}(q^2)], \tag{6.9}$$

where the $-\frac{1}{3}$ is an isospin factor. We find

$$g_1^{S=1}(q^2)=-\tfrac{1}{3}g_1^{S=0}(q^2)$$

with

$$g_1^{S=0}(q^2)=\frac{1}{4\pi}\int\frac{d\vec{Q}}{2E_d(\vec{Q})}\left[(A'A-B'B)-(A'B-B'A)\times\frac{2m_{\rm N}}{m_d}\frac{Q\cdot(P'-P)}{(P'-P)^2}\right]. \tag{6.10}$$

Thus we have

$$g_1(q^2)=\tfrac{5}{3}g_1^{S=0}(q^2). \tag{6.11}$$

The expressions for the vector form factors are quite lengthy, particularly those for $S=1$, so that we do not present them here.

The procedure for calculating these form factors is quite straightforward. From the solution of Eq. (5.10) one obtains $R_u(Q)$ and $\overline{R}_l(Q)$ from which one can obtain $A(Q)$ and $B(Q)$. We can now refer to Fig. 2 for further insight. We see that if we put $\vec{P}=0$ we can use the amplitudes $A(Q)$ and $B(Q)$ to specify the left-hand vertex since $\vec{Q}$ is the diquark momentum in the nucleon rest frame. For the right-hand vertex, however, the nucleon has momentum $\vec{P}=\vec{q}$. Therefore the argument of the amplitudes A and B is $|\vec{Q}'|$, where $\vec{Q}'$ is the diquark momentum in the rest frame of the nucleon of momentum $\vec{P}'$. The vector $\vec{Q}'$ may be obtained from the knowledge of $\vec{Q}$ and $\vec{P}'$ using Eq. (5.6). As mentioned previously, we use the notation $A'=A(Q')$ and $B'=B(Q')$ where $Q'=|\vec{Q}'|$.

Before leaving this section we show that our choice of normalization yields the correct baryon number for the nucleon. Note that

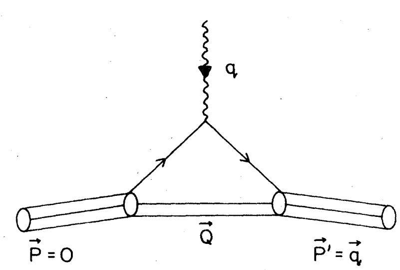

FIG. 2. Calculation of nucleon form factors in the diquark approximation. (This calculation may be carried out in any Lorentz frame but we here specialize to the case $\vec{P}=0$.) The nucleons and the diquark are on the mass shell in this approximation.

$$(\vec{P}',s,t\,|\,B\,|\,\vec{P},s,t)=(\vec{P}',s,t\,|\,\tfrac{1}{3}\int d\vec{x}\,\bar{q}(\vec{x})\gamma^0 q(\vec{x})\,|\,\vec{P},s,t)\,,$$

$$=\tfrac{1}{3}(2\pi)^3\delta(\vec{P}'-\vec{P})(\vec{P},s,t\,|\,\bar{q}(0)\gamma^0 q(0)\,|\,\vec{P},s,t)\,, \tag{6.12}$$

$$=\frac{(2\pi)^3}{3}\delta(\vec{P}'-\vec{P})\bar{u}(\vec{P},s)\gamma^0 u(\vec{P},s)\frac{F_{10}(0)}{(2\pi)^3}\,, \tag{6.13}$$

$$=\delta(\vec{P}'-\vec{P})E_N(\vec{P})/m_N\,, \tag{6.14}$$

since $F_{10}(0)=3$. If we evaluate Eq. (6.12) by inserting $S=0$ and $S=1$ diquark states we would find

$$(\vec{P},s,t\,|\,\bar{q}(0)\gamma^0 q(0)\,|\,\vec{P},s,t)=\frac{3}{(2\pi)^3}\bar{u}(\vec{P},s)\gamma^0\left[\frac{F_{10}^{S=0}(0)+F_{10}^{S=1}(0)}{2}\right]u(\vec{P},s)\,, \tag{6.15}$$

$$=\frac{3}{(2\pi)^3}\frac{E_N(\vec{P})}{m_N}\,, \tag{6.16}$$

which also leads to the correct result. In Eqs. (6.15) and (6.16) the factor of 3 has its origin in that $F_{10}^{S=0}(q^2)$ and $F_{10}^{S=1}(q^2)$ are calculated per quark and are normalized such that $F_{10}^{S=0}(0)=F_{10}^{S=1}(0)=1$ in the SU(6) model. The factor of 2 in Eq. (6.15) also has its origin in our SU(6) model which requires that the $S=0$ and $S=1$ amplitudes appear with equal weight when specifying the nucleon wave function.

As another exercise, let us consider the isospin operator, T_3. As mentioned earlier, we are neglecting the mesonic contributions to the current operators. Therefore, we express T_3 only in terms of the quark fields,

$$T_3=\int d\vec{x}\,\bar{q}(\vec{x})\gamma^0\frac{\tau_3^q}{2}q(\vec{x})\,. \tag{6.17}$$

We have

$$(\vec{P}',s,t\,|\,T_3\,|\,\vec{P},s,t)=(2\pi)^3\delta(\vec{P}'-\vec{P})(\tfrac{1}{2})(\vec{P},s,t\,|\,\bar{q}(0)\gamma^0\tau_q^3 q(0)\,|\,\vec{P},s,t)$$

$$=\frac{(-1)^{(1/2)-t}}{2}\bar{u}(\vec{P},s)\gamma^0 u(\vec{P},s)F_{11}(0)\delta(\vec{P}'-\vec{P})\,, \tag{6.18}$$

$$=\frac{(-1)^{(1/2)-t}}{2}\frac{E_N(\vec{P})}{m_N}\delta(\vec{P}'-\vec{P})\,, \tag{6.19}$$

since $F_{11}(0)=1$. The evaluation in terms of a set of intermediate diquark states yields

$$(\vec{P}',s,t\,|\,T_3\,|\,\vec{P},s,t)=(2\pi)^3\delta(\vec{P}'-\vec{P})(\tfrac{3}{2})[F_{11}^{S=0}(0)-\tfrac{1}{3}F_{11}^{S=1}(0)]\frac{1}{(2\pi)^3}(-1)^{(1/2)-t}\bar{u}(\vec{P},s)\gamma^0 u(\vec{P},s)\,, \tag{6.20}$$

$$=\frac{(-1)^{(1/2)-t}}{2}\frac{E_N(\vec{P})}{m_N}\delta(\vec{P}-\vec{P}')\,, \tag{6.21}$$

where we have used the fact that

$$F_{11}^{S=0}(0)=F_{11}^{S=1}(0)=1$$

in our SU(6) model. Again the factor of 3 in Eq. (6.20) appears because our form factors for specific diquark spins are calculated per quark. The factor of $(\frac{1}{2})$ in Eq. (6.20) again has its origin in the SU(6) based model as noted above.

We have provided a somewhat lengthy discussion of our choice of normalization since this can be a source of some confusion. We also note that in the general case the amplitudes $\tilde{A}$ and $\tilde{B}$ are not linearly related to A and B. These amplitudes must be determined from dynamical considerations through the solution of the $S=0$, $T=0$ and $S=1$, $T=1$ equations. As remarked earlier, these equations are coupled through the appearance of the same form factors in both equations. (The form factors depend upon all the invariants A, B, $\tilde{A}$, and $\tilde{B}$.) A model in which we do not impose SU(6) symmetry at the outset will be presented in a future work as well as calculations of the mesonic field contribution to the electromagnetic form factors and magnetic moments.

VII. DETERMINATION OF COUPLING CONSTANTS

By using Eqs. (2.14)–(2.18) we can relate the meson-quark coupling constants of our model to empirical meson-nucleon coupling constants. For example, we have

$$G_{\sigma NN}=g_\sigma\rho_s(0)\,, \tag{7.1}$$

$$G_{\pi NN}=g_\pi g_1(0)\,, \tag{7.2}$$

$$G_{\rho NN}=g_\rho F_{11}(0)=g_\rho\,, \tag{7.3}$$

$$G_{\omega NN}=g_\omega F_{10}(0)=3g_\omega\,. \tag{7.4}$$

The ratio of tensor to vector coupling for the ρ and ω mesons is given by

$$\left[\frac{G^T_{\rho \mathrm{NN}}}{G^V_{\rho \mathrm{NN}}}\right]=\frac{F_{21}(0)}{F_{11}(0)}=F_{21}(0)\,, \tag{7.5}$$

$$\left[\frac{G^T_{\omega \mathrm{NN}}}{G^V_{\omega \mathrm{NN}}}\right]=\frac{F_{20}(0)}{F_{10}(0)}=\frac{F_{20}(0)}{3}\,. \tag{7.6}$$

Fits to nucleon-nucleon scattering data give values for $G^2_{\sigma \mathrm{NN}}/4\pi$, etc. For example, for the potential of Holinde, Erkelenz, and Alzetta[22] (HEA) one has $G^2_{\sigma \mathrm{NN}}/4\pi=4.63$, $G^2_{\pi \mathrm{NN}}/4\pi=13$, $G^2_{\omega \mathrm{NN}}/4\pi=14$, $G^2_{\rho \mathrm{NN}}/4\pi=1.5$, $(G^T_{\rho \mathrm{NN}}/G^V_{\rho \mathrm{NN}})=3.5$, and $(G^T_{\omega \mathrm{NN}}/G^V_{\omega \mathrm{NN}})=0$. (In this model $m_\pi=138.5$ MeV, $m_\sigma=500$ MeV, $m_\rho=763$ MeV, and $m_\omega=782.8$ MeV.) This one-boson-exchange model of the nucleon-nucleon force also has η, ϕ, and δ mesons which play a relatively minor role in the data fitting. We will not consider these additional mesons at this time. Using the above values we find $G_{\sigma \mathrm{NN}}=7.63$, $G_{\pi \mathrm{NN}}=12.8$, $G_{\omega \mathrm{NN}}=13.3$, and $G_{\rho \mathrm{NN}}=4.34$. Thus $g_\sigma=7.63/\rho_s(0)$; $g_\pi=12.8/g_1(0)$, $g_\omega=4.33$, and $g_\rho=4.34$. (The equality of g_ω and g_ρ obtained in this manner is rather striking.) To obtain a value for $\rho_s(0)$ we note that

$$\rho_s(0)=3(1-2a)\,,$$

where a is the fraction of the lower component in the wave function in an SU(6) model. Furthermore, we may show that $g_A=\frac{5}{3}(1-\frac{4}{3}a)$.[6] Let us assume that we have constructed a model that gives $g_A=1.25$. That would require $a=0.19$ and therefore we could estimate $\rho_s(0)\simeq 1.86$. This in turn would yield $g_\sigma=4.10$ which is surprisingly close to the 4.33 and 4.34 obtained for g_ω and g_ρ. Finally, to obtain a value for g_π we need an estimate for $g_1(0)$. Calculations of this quantity are given in Ref. 6. In that work baglike wave functions were used to parametrize the vertex functions and values of about 2 to 4 were obtained depending on the bag size. In this work we find $g_1(0)=4.78$. (See Tables I and III.)

Ideally, one might iterate our equations by adjusting g_σ and g_π at each stage so that $G_{\sigma \mathrm{NN}}$ and $G_{\pi \mathrm{NN}}$ are given correctly. Alternatively one could fix g_σ and g_π and then calculate values of $G_{\sigma \mathrm{NN}}$ and $G_{\pi \mathrm{NN}}$ based upon the values obtained for $\rho_s(0)$ and $g_1(0)$. We chose a combination of these procedures. Over the first few iterations g_σ and g_π were adjusted so that $G_{\sigma \mathrm{NN}}$ and $G_{\pi \mathrm{NN}}$ were given correctly. Then g_σ and g_π were fixed and the iteration was continued. Because of the iteration scheme chosen, the final value for $G_{\pi \mathrm{NN}}$ was 12.9 instead of the value of 12.8 appropriate to the potential HEA.[22] (Rather rapid convergence of the iteration procedure was obtained, with a stable solution appearing after three or four iterations. Stability was checked by continuing the iteration to about a total of nine iterations.)

VIII. RESULTS OF NUMERICAL CALCULATIONS

Before discussing our results we need to comment on the various aspects of self-consistency in these calculations. The results depend upon the choice of various coupling constants and masses. Indeed, the quark moves in a potential that is quite sensitive to the value of the nucleon mass through its use in various kinematic transformations. In order to keep the scale of the interaction from changing radically, we have used the experimental value of m_N in constructing the Jacobian [Eq. (5.15)] and the various m_N-dependent terms in Eq. (5.10). However, using the techniques of the Appendix we may calculate a value for m_N by constructing the expectation value of the Hamiltonian in the soliton state. The value of m_N so determined is denoted $m_{\mathrm{N}(H)}$. The fact that $m_{\mathrm{N}(H)}$ does not precisely reproduce the experimental value makes the calculation not fully self-consistent. (With an improved model we might be able to achieve self-consistency for this aspect of the calculation.) However, the calculation is self-consistent in what we consider the most important aspect. The form factors of the nucleon are calculated with the invariant amplitudes that are obtained from the solution of Eq. (5.10). Of course, the interaction in this equation is constructed in terms of these form factors. The solutions we present are self-consistent with respect to this feature.

As noted earlier, the model we are discussing has several free parameters, g_χ, m_χ, m_q, and $\tilde{\lambda}$. A full exploration of this parameter space is a large task. Therefore we have put $m_\chi=500$ MeV and $\tilde{\lambda}=0$ and considered variation of m_q and g_χ. The results are sensitive to the variation of these parameters and not all choices lead to stable solutions of the nonlinear equations. The inclusion of the ρ meson in the analysis creates a special problem. Indeed, we find that when the ρ meson is included the soliton tends to collapse in size and a stable solution cannot readily be found. This feature may be traced to the way the ρ is coupled to the nucleon. One notes that $F_{21}(q^2)$ is quite large. From studies of NN scattering using the one-boson-exchange models of the nuclear force one finds that the phenomenological values of

$$F_{21}(0)/F_{11}(0)=F_{21}(0)$$

vary from about 3.50 to about 6.0. (As we will see, it is not difficult to reproduce the lower of these values while at the same time providing a good fit to the nucleon magnetic moments.) The nucleon magnetic moments are given in terms of $F_{20}(0)$ and $F_{21}(0)$ in Ref. 6, Eqs. (8.1) and (8.2). With these large values of $F_{21}(q^2)$ there is a large amplitude for the nucleon to emit a meson that carries a large momentum. [Note that the momentum of the emitted meson is $q=\{-q^0,-\vec{q}\}$ with $q^0=E_\mathrm{N}(\vec{q})-m_\mathrm{N}$.]. We see from Eqs. (5.10)–(5.14) that $V^\rho_{11}(q^2)$ becomes negative for sufficiently large $|\vec{\mathrm{P}}'|=|\vec{q}|$. Thus, inclusion of recoil effects leads to the situation where the ρ meson provides a repulsive potential for small $|\vec{q}|$, and an attractive potential for large $|\vec{q}|$, in $V^\rho_{11}(q^2)$. Furthermore, we see from these equations that since $F_{21}(q^2)$ is large, the term $V^\rho_{12}(q^2)$ is large. Therefore the ρ meson is particularly effective in enhancing the lower components of the wave function, $\overline{R}_l(Q)$. Because of these features which tend to destabilize the calculation, we found it useful to replace g_ρ by $g_\rho(1-q^2/\Lambda^2)^{-2}$ with $\Lambda\simeq 800$ MeV/c. This replacement regulates those high momentum aspects of the model that are due to the coupling of the ρ meson and enables us to obtain reasonable solutions. With this feature in place it was possible to find relatively stable solutions and one further, relatively minor regulation of the high momentum behavior was needed, as will be discussed below.

As noted earlier, the model has several free parameters g_χ, m_χ, m_q, and $\tilde{\lambda}$, and we chose to put $\tilde{\lambda}=0$ and $m_\chi=500$ MeV. With $m_q=600$ MeV and $g_\chi=6.0$ we ob-

TABLE I. Results of calculation using $g_\chi=6.3$, $m_\chi=500$ MeV, and $m_q=600$ MeV. (The meson masses were set at $m_\sigma=500$ MeV, $m_\pi=138.5$ MeV, $m_\omega=782.8$ MeV, and $m_\rho=763$ MeV, which are the values used in constructing the potential HEA of Ref. 22. The values of $G_{\sigma NN}$, $G_{\pi NN}$, $G_{\rho NN}$, and $G_{\omega NN}$ were also taken from the potential HEA—see Sec. VII.)

	Calculated	Experimental
Baryon density rms radius	0.647 fm	
Scalar density rms radius	0.511 fm	
$\epsilon=m_N-\langle E_d\rangle$	181 MeV	
$\langle E_d\rangle$	1314 MeV	
$\hat{m}_N$	1495 MeV	
$\mathcal{E}_\sigma$	205 MeV	
$\mathcal{E}_\pi$	168 MeV	
$\mathcal{E}_\omega$	−133 MeV	
$\mathcal{E}_\rho$	29 MeV	
$\mathcal{E}_\chi$	528 MeV	
$m_{N(H)}$	1342 MeV	938 MeV (1087 MeV)[a]
$\langle r_p^2\rangle_E^{1/2}$	0.867 fm	0.87±0.02 fm[c]
$\langle r_p^2\rangle_M^{1/2}$	0.813 fm	0.80±0.03 fm[e]
$\langle r_n^2\rangle_M^{1/2}$	0.820 fm	0.79±0.15 fm[e]
$-\left[\frac{dG_E^n(q^2)}{dq^2}\right]$	3.65×10^{-2} fm^2	$(1.89+0.04)\times10^{-2}$ fm^2[d]
μ_p	2.76	2.79
μ_n	−1.81	−1.91
μ_p/μ_n	1.50	1.46
g_A	1.27	1.25
$g_\sigma=\frac{G_{\sigma NN}}{\rho_s(0)}$	$\frac{7.63}{1.94}=3.93$	
$g_\pi=\frac{G_{\pi NN}}{g_1(0)}$	$\frac{12.91}{4.78}=2.70$	
$g_\omega=\frac{G_{\omega NN}}{3}$	$\frac{13.3}{3}=4.43$	
$g_\rho=\frac{G_{\rho NN}}{1}$	$\frac{4.34}{1}=4.34$	
$\frac{G_\rho^T}{G_\rho^V}=\frac{F_{21}(0)}{F_{11}(0)}$	3.56	3.50[b]
$\frac{G_\omega^T}{G_\omega^V}=\frac{F_{20}(0)}{F_{10}(0)}$	−0.048	0.0[b]

[a] SU(6) value: $(m_\Delta+m_N)/2$.
[b] HEA: Ref. 22.
[c] Reference 23.
[d] Reference 24.
[e] Reference 25.

tained a solution that converged rapidly to yield an object that had properties quite similar to that of the nucleon. This solution, however, exhibited a very small residual drift upon successive iteration which indicated that the high momentum properties of the model were still not satisfactory. Therefore we implemented a sharp cutoff at $-q^2=40\ \mathrm{fm}^{-2}$ for *all* the mesons. [This was most readily accomplished by putting all the form factors, $\rho_s(q^2)$, $g_1(q^2)$, $F_{10}(q^2)$, etc., equal to zero for $-q^2>40\ \mathrm{fm}^{-2}$. The cutoff for the ρ meson which was implemented by replacing g_ρ by $g_\rho(1-q^2/\Lambda^2)^{-2}$ was maintained, as well.] With this sharp cutoff we found convergent solutions and readily obtained objects that had the properties of the nucleon. (The sharp cutoff limits $|\vec{q}|$ are to be less than about 1.5 GeV. This is a not unreasonable choice as one expects that gluon degrees of freedom should become important for momentum of the order of 1–2 GeV/c.)

Our results are summarized in various tables and figures. In Table I we present values for various quantities obtained when forming the expectation value of the Hamiltonian. (The notation is defined in the Appendix.) As may be seen from the table, the value of the nucleon mass is about 40% too large; however, the values for g_A, μ_p, and μ_n are in very good agreement with the experimental data. We also present values of the rms radius of the baryon and scalar densities obtained from the coordinate space wave functions depicted in Fig. 3. These wave functions are obtained by Fourier transformation of the momentum space wave functions in the nucleon rest frame. In Figs. 4 and 5 we show the calculated values for the proton and neutron form factors and compare these values with the experimental data. Numerical values are given in Table II. The fit to the data is quite good for $|q^2|<0.3\ (\mathrm{GeV}/c)^2$. From the slope of the curve for $G_E(q^2)$ at the origin we can extract a value for the proton electromagnetic radius,

$$\langle r^2\rangle_E^{1/2}=\left[-6\frac{dG_E^{\mathrm{p}}(q^2)}{dq^2}\right]_{q^2=0}^{1/2}. \quad (8.1)$$

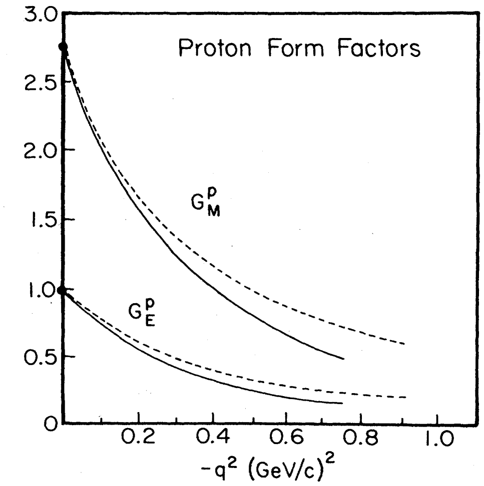

FIG. 4. Proton electric and magnetic form factors. The dashed curves represent the data while the solid line represents the results of our calculations.

The value obtained in this manner is given in Table I. For a strongly coupled system it is not unexpected that this value differs somewhat from the value of the rms radius of the baryon density which is also given in Table I. In Table I we also present values for the neutron and proton magnetic radii defined by

$$\langle r_{\mathrm{p}}^2\rangle_M^{1/2}=\left[-\frac{6}{G_M^{\mathrm{p}}(0)}\frac{dG_M^{\mathrm{p}}(q^2)}{dq^2}\right]_{q^2=0}^{1/2}, \quad (8.2)$$

$$\langle r_{\mathrm{n}}^2\rangle_M^{1/2}=\left[-\frac{6}{G_M^{\mathrm{n}}(0)}\frac{dG_M^{\mathrm{n}}(q^2)}{dq^2}\right]_{q^2=0}. \quad (8.3)$$

Finally, in Table III we present values for $\rho_s(q^2)$, $g_1(q^2)$, $F_{10}(q^2)$, $F_{11}(q^2)$, $F_{20}(q^2)$, and $F_{21}(q^2)$ for various values of q^2. We should again note that we have neglected mesonic corrections to the magnetic moments, electromagnetic form factors, etc. In the calculations report-

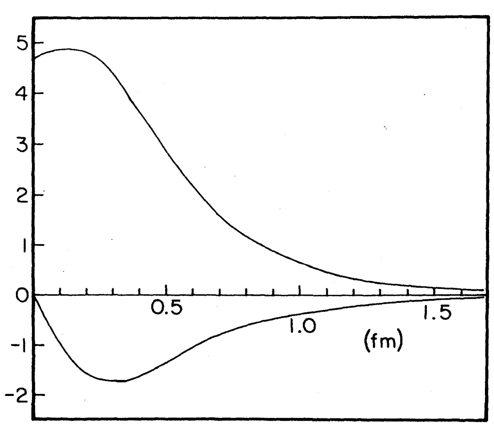

FIG. 3. Coordinate-space quark wave functions obtained by Fourier transformation of the momentum-space amplitude in the nucleon rest frame. The curves represent the upper and lower radial components of the coordinate-space spinor.

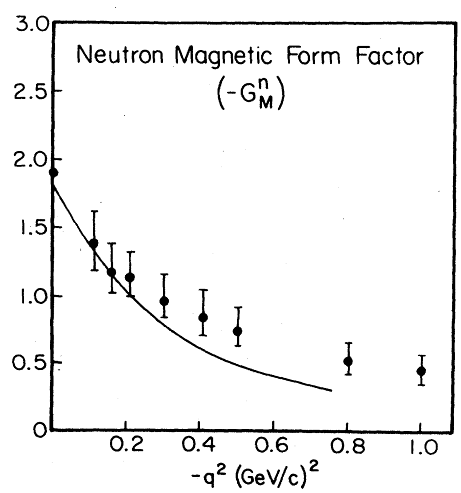

FIG. 5. Neutron magnetic form factor. The solid line represents the results of our calculations.

TABLE II. Calculated electric and magnetic form factors of the proton and neutron.

q^2 (fm^{-2})	$G_E^p(q^2)$	$G_M^p(q^2)$	$G_E^n(q^2)$	$G_M^n(q^2)$
0.0	1.00	2.76	7.23×10^{-6}	-1.81
-4.0	0.61	1.79	4.17×10^{-3}	-1.16
-8.0	0.38	1.21	8.49×10^{-3}	-0.78
-12.0	0.24	0.85	1.16×10^{-2}	-0.54
-16.0	0.15	0.60	1.35×10^{-2}	-0.39
-20.0	0.09	0.44	1.41×10^{-2}	-0.28

ed here we have used currents expressed in terms of quark fields only. The mesonic corrections will form the subject of a future publication.

IX. DISCUSSION OF THE RESULTS OF NUMERICAL CALCULATIONS

In Ref. 6 we considered a model for nucleon form factors which was a simplified version of the model considered here. Indeed, given the simplifications of that reference it was possible to provide analytic expressions for the various form factors of the nucleon. These were given as integrals involving the upper and lower components of the quark wave function in coordinate space. We will use some of the results of that analysis to interpret our numerical results.

A. The axial coupling constant, g_A

The value of g_A was found to be

$$g_A=\tfrac{5}{3}(1-\tfrac{4}{3}a)\,, \tag{9.1}$$

where a was the fractional contribution of the lower component to the normalization integral. A value of $a=0.188$ yields $g_A=1.25$. (The value of a in the MIT bag model is $a=0.26$.) We were able to obtain a solution of our equations with $g_A=1.27$ and very good values for the magnetic moments.

B. The nucleon magnetic moments, μ_p and μ_n

In the MIT bag model the magnetic moments are linearly related to the bag size with a bag size of $R\sim1.4$ fm required to fit the proton moment. In our model, which treats recoil effects properly, one has a more complex situation. The analysis of Ref. 6 provides the result

$$\mu_p=(1-\tfrac{4}{3}a)+\mu_p^C\,, \tag{9.2}$$

$$=\tfrac{3}{5}g_A+\mu_p^C\,, \tag{9.3}$$

where

$$\mu_p^C=-\tfrac{4}{3}m_N(\tfrac{2}{3})\int_0^\infty x^3R_u(x)R_l(x)dx\,. \tag{9.4}$$

The $R_u(r)$ and $R_l(r)$ are Fourier transforms of the momentum-space wave functions[6] and are normalized such that

$$\int_0^\infty [R_u^2(r)+R_l^2(r)]r^2dr=1\,. \tag{9.5}$$

The factor of $(\frac{2}{3})$ in Eq. (9.4) had its origin in our approximating Eq. (5.6) by $\vec{Q}'\simeq\vec{Q}-(\frac{2}{3})\vec{q}$. [This choice corresponds to a "weak coupling" limit where $m_N\sim3m_q$ and $E_d(\vec{Q})\sim2m_q$. In retrospect we feel that this approximation was not appropriate.] In the work reported here we calculate the moments using the relations[6]

$$\mu_p=1+\left[\frac{F_{20}(0)}{6}+\frac{F_{21}(0)}{2}\right], \tag{9.6}$$

$$\mu_n=\left[\frac{F_{20}(0)}{6}-\frac{F_{21}(0)}{2}\right], \tag{9.7}$$

using Eq. (5.6) without making any approximation. However, it is useful to have analytic results such as those obtained in Ref. 6 and reproduced in Eqs. (9.2)—(9.4). From these equations we see that the value obtained for the moments is not simply related to the size of the confinement region. Indeed, we have found solutions that describe objects of the same size that have quite different values of the magnetic moments. For example, if $R_l(r)$ is very small and $a\simeq0$ we have $\mu_p\simeq1.0$. However, such an object can have an rms radius of about 0.7 fm for the baryon density. Since we can find solutions with sizable $R_l(r)$ and $a\simeq0.2$ we can obtain $\mu_p\sim2.7$ with objects of similar size ($\langle r^2\rangle^{1/2}\simeq0.7$ fm) to that described above. Thus the value obtained for a is a sensitive indicator of the quality of the fit to be expected for g_A *and* the magnetic moments.

C. The nucleon form factors

As noted above, we have obtained a good fit to the form factors for $-q^2<0.3$ (GeV/c)^2. The fit deteriorates progressively above this value of $-q^2$. This leads us to believe that we require a more detailed evaluation of the form factor at the larger values of $-q^2$. Indeed as $-q^2$ approaches 1 or 2 (GeV/c)^2 we expect gluon degrees of freedom to become important and other techniques will be required in the evaluation of the form factors. Ultimately, one would hope to move continuously from our calculation to a QCD based analysis, the latter analysis being relatively insensitive (at least in the predicted q^2 dependence) to those details of the wave functions of the nucleon which are probed at low q^2.

TABLE III. Form factors obtained in a self-consistent solution of Eq. (2.4).

q^2 (fm^{-2})	$\rho_s(q^2)$	$g_1(q^2)$	$F_{10}(q^2)$	$F_{11}(q^2)$	$F_{20}(q^2)$	$F_{21}(q^2)$
0.0	1.94	4.78	3.00	1.00	-0.14	3.56
-4.0	1.42	3.16	1.83	0.70	0.05	2.25
-8.0	1.07	2.17	1.17	0.50	0.12	1.47
-12.0	0.81	1.53	0.76	0.36	0.15	1.03
-16.0	0.62	1.11	0.51	0.26	0.15	0.72
-20.0	0.48	0.81	0.34	0.19	0.14	0.52
-24.0	0.37	0.60	0.23	0.14	0.12	0.39

D. Meson-nucleon coupling constants

As we have discussed in some detail, the coupling of the quarks to the mesons has been adjusted to give the empirical values for the coupling of the mesons to the *nucleon.* Some interesting features emerge in this analysis. First, one finds that the quark-meson coupling constants are quite similar in value. Second, the ρ and ω mesons are coupled to the *quarks* in an entirely similar fashion. The results of our calculation using our SU(6) based model yields quite different couplings of these mesons to the *nucleon.* Indeed the model reproduces the strong tensor coupling of the ρ meson to the nucleon quite well and at the same time yields essentially zero tensor coupling of the ω to the nucleon. This feature is also consistent with empirical values obtained for these couplings when use is made of the one-boson-exchange model for the description of nucleon-nucleon scattering. We should also remark that since the photon is coupled to the quarks in a manner similar to the coupling of the ρ and ω, a successful fit to the magnetic moments implies that (in this model) the ρ and ω tensor coupling to the nucleon will be given correctly.

E. The nucleon-nucleon interaction

As discussed in the last subsection, the meson-quark couplings have been adjusted to give the phenomenological meson-nucleon coupling. Therefore if one were to calculate the diagram shown in Fig. 6(a) one would have the correct strength for the various parts of the momentum-space one-boson-exchange potential. [We have adjusted our quark-meson coupling constants in order to reproduce the potential HEA (Ref. 22).] Further iteration of this potential using the techniques described in Ref. 18 would yield the nucleon-nucleon scattering amplitude. The iteration is indicated in a schematic fashion in Fig. 6(b).

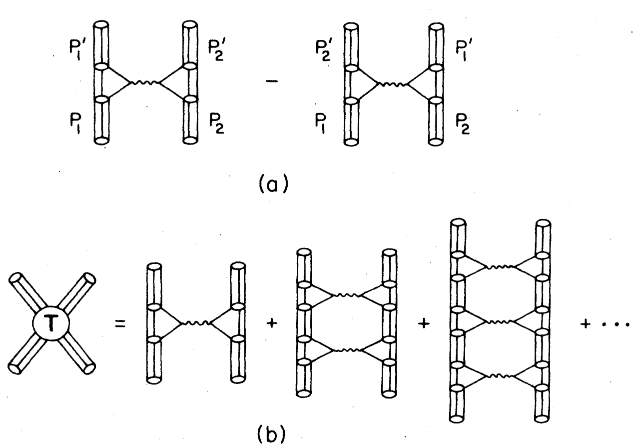

FIG. 6. (a) The nucleon-nucleon potential in a one-boson-exchange model. Both direct and exchange terms are shown. (b) Iteration of the potential constructed in (a) using the techniques of Ref. 18 will yield the nucleon-nucleon scattering amplitude.

F. The size of the nucleon

One of the more interesting aspects of our results is the fact that in a relativistic calculation of a strongly bound system the electromagnetic size of the soliton may differ significantly from the "physical" size. The latter size is obtained by calculating the rms radius of the baryon density, for example. The electromagnetic size is obtained from the slope of the proton electric form factor at $q^2=0$. [See Eq. (8.1).] The different values obtained in the two calculations arise from the way the transformation properties of our vertex functions affect the calculation of form factors. The most relevant equation is Eq. (5.6). From this equation we can infer that if $m_d=2m_q$ is greater than $m_{\rm N}$ we can expect $\langle r_{\rm p}^2\rangle_E^{1/2}$ to be greater than $\langle r^2\rangle^{1/2}$ calculated from the baryon density.

G. The nucleon mass

In our calculations we find $m_{{\rm N}(H)}=1342$ MeV. This is about 43% too large if compared with the nucleon mass. However, one may suggest that in an SU(6) based model it is more appropriate to compare the calculated value with the average of the delta and nucleon mass, a mass of about 1087 MeV. If the latter comparison is made the calculated value is only about 23% too large. In the following paper we present the results of a calculation in which we fit the SU(6) values for the mass rather well; however, the value obtained for g_A in that case is somewhat too small. It is quite possible that a more extensive parameter search will yield good fits for all the nucleon observables simultaneously.

X. CONCLUSIONS AND SUMMARY

In this section it is useful to summarize the various positive and negative features of our model. Of the negative features, the most serious is the absence of a good model for confinement. Indeed the connection of this model to QCD is tenuous at best. Furthermore, the model is not renormalizable; however, since we are dealing with an effective Lagrangian we do not consider that a serious defect. In addition, the diquark approximation should be improved upon and the structure of the meson fields should be addressed. In surveying the results of our calculations, we see that the nucleon mass is too large and the form factors fall off somewhat too fast with increasing values of $-q^2$. Of course, results obtained in a model with several free parameters such as g_χ, m_χ, and m_q are subject to revision.

Having surveyed some of the deficiencies of the model, we now turn to the more positive features. First, we see that we have introduced a new, fully covariant approach to relativistic bound state dynamics. We have obtained good fits to the nucleon magnetic moments and the axial vector coupling constant, g_A. Also there is a good fit to the electric and magnetic form factors of the nucleon, for values of $-q^2$ that are not too large. Another nice feature of this model is that it is consistent with the one-boson-exchange model of nuclear forces.[18] The model is restricted such that $G_{\sigma NN}$, $G_{\pi NN}$, $G_{\rho NN}$, and $G_{\omega NN}$ are given correctly. (This is accomplished by adjusting g_σ and g_π during the first few iterations of our nonlinear equations. Note that g_ρ and g_ω do not require such adjustments as the ρ and ω mesons are coupled to conserved currents.) In addition, the ratio of the tensor to the vector coupling constants of the ρ and ω mesons to the *nucleon* are given correctly by the model. [For example, for the ρ meson this ratio is

$$(G^T_{\rho NN}/G^V_{\rho NN})=F_{21}(0)/F_{11}(0)=3.55\ ,$$

while the phenomenological value obtained for the NN potential HEA (Ref. 22) is 3.50. Correspondingly, the ratio for the ω meson

$$(G^T_{\omega NN}/G^V_{\omega NN})=F_{20}(0)/F_{10}(0)=-0.15/3=-0.05\ ,$$

while the value for the potential HEA is zero.] The fact that it is possible to couple the σ, π, ρ, and ω mesons to the nucleon and still have a viable model appears to us to be quite important in light of the rather complete success found when using a Lagrangian based on the boson-exchange model of the nuclear force for studying the properties of nuclear matter, effective forces in nuclei, and the nuclear optical potential.[17] Other studies of nucleon-nucleus scattering using a relativistic impulse approximation have shown that the NN scattering amplitude is best represented in terms of its Dirac-matrix representation.[16] In this representation it is clear that the Lorentz-scalar part of the amplitude is large and attractive while the Lorentz-vector part is large and repulsive. These features are consistent with an underlying boson-exchange model of the interaction and further support the picture of the NN interaction obtained from application of the boson-exchange model at lower energies.

We may hope that our model will also provide a basis for the construction of a theory of many-body soliton dynamics. One may consider the construction of a shell model for solitons and also investigate the modifications of soliton properties in external fields or inside a nucleus. We explore these matters in the following paper.

ACKNOWLEDGMENTS

We wish to thank Mr. Ramesh Babu Thayyullathil for assistance with various aspects of the calculations reported here. This work was supported in part by the National Science Foundation and the Faculty Research Award Program of the City University of New York.

APPENDIX

In this appendix we calculate the energy of the nucleon by forming the expectation value of the Hamiltonian. Let us consider the states $|\vec{P},s,t\rangle$ which have the normalization

$$\langle\vec{P}',s',t'|\vec{P},s,t\rangle=\delta(\vec{P}-\vec{P}')\ . \tag{A1}$$

We have

$$\langle\vec{P}',s',t'|H|\vec{P},s,t\rangle=\langle\vec{P}',s',t'|\int\mathscr{H}(\vec{x})d\vec{x}\,|\vec{P},s,t\rangle=(2\pi)^3\delta(\vec{P}'-\vec{P})\langle\vec{P},s,t\,|\mathscr{H}(0)|\vec{P},s,t\rangle\delta_{ss'}\delta_{tt'}\ . \tag{A2}$$

Here $\mathscr{H}(\vec{x})$ is the Hamiltonian density. We further define

$$\langle\vec{P},s,t\,|\mathscr{H}(0)|\vec{P},s,t\rangle=E(\vec{P})\ , \tag{A3}$$

where

$$E(\vec{P})=(\vec{P}^2+m^2_{N(H)})^{1/2}\ . \tag{A4}$$

Therefore

$$\langle\vec{P}',s',t'|\mathscr{H}|\vec{0},s,t\rangle=\delta(\vec{P}')m_{N(H)}\ , \tag{A5}$$

where $m_{N(H)}$ is the mass of the soliton. We have included the subscript (H) as an indication of how this quantity was calculated since the notation $\hat{m}_N$ was used for the eigenvalue of Eq. (5.10).

The Hamiltonian density may be obtained from the Lagrangian density given in Eq. (2.3),

$$\mathscr{H}(x)=\bar{q}(x)\left[\frac{1}{i}\vec{\gamma}\cdot\vec{\nabla}+m_q+g_\sigma\sigma(x)+g_\chi\chi(x)+g_\omega\gamma^\mu\omega_\mu(x)+ig_\pi\gamma_5\vec{\tau}_q\cdot\vec{\Pi}(x)+g_\rho\gamma^\mu\vec{\tau}_q\cdot\vec{\rho}_\mu(x)\right.$$
$$\left.-g_\omega\frac{\lambda}{4m_N}\sigma^{\mu\nu}F^\omega_{\mu\nu}(x)-g_\rho\frac{\lambda}{4m_N}\sigma^{\mu\nu}\vec{\tau}_q\cdot\vec{F}^\rho_{\mu\nu}(x)\right]q(x)$$
$$+\tfrac{1}{2}\left[\left[\frac{\partial\chi}{\partial t}\right]^2+|\nabla\chi(x)|^2+m^2_\chi\chi^2(x)\right]+\tfrac{1}{2}\left\{\left[\frac{\partial\sigma(x)}{\partial t}\right]^2+|\vec{\nabla}\sigma|(x)|^2+m^2_\sigma\sigma^2(x)\right\}$$

$$+\tfrac{1}{2}\left\{\left[\frac{\partial\vec{\Pi}(x)}{\partial t}\right]^2+|\nabla\vec{\Pi}(x)|^2+m_\pi^2\vec{\Pi}^{\,2}(x)\right\}+[\tfrac{1}{4}F^{\omega}_{\mu\nu}(x)F^{\mu\nu}_{\omega}(x)-\tfrac{1}{2}m_\omega^2\omega^\mu(x)\omega_\mu(x)]$$
$$+[\tfrac{1}{4}\vec{F}^{\rho}_{\mu\nu}(x)\vec{F}^{\mu\nu}_{\rho}(x)-\tfrac{1}{2}m_\rho^2\vec{\rho}^{\,\mu}(x)\cdot\vec{\rho}_\mu(x)]-\left[\vec{F}^{0\nu}_{\rho}(x)-g_\rho\frac{\lambda}{2m_{\rm N}}\bar{q}(x)\sigma^{0\nu}\vec{\tau}_q q(x)\right]\cdot\frac{\partial\vec{\rho}_\nu}{\partial t}(x)$$
$$-\left[F^{0\nu}_{\omega}(x)-g_\omega\frac{\lambda}{2m_{\rm N}}\bar{q}(x)\sigma^{0\nu}q(x)\right]\frac{\partial\omega_\nu}{\partial t}(x)\,. \tag{A6}$$

We will drop the last two terms of Eq. (A6) when evaluating the energy of the system since these terms are quite small. Using the techniques developed in this work we find

$$m_{{\rm N}(H)}=3(\hat{m}_{\rm N}-\langle E_d\rangle)+\mathscr{E}_\chi+\mathscr{E}_\sigma+\mathscr{E}_\pi+\mathscr{E}_\omega+\mathscr{E}_\rho\,, \tag{A7}$$

where

$$\mathscr{E}_\chi=\frac{g_\chi^2}{2}\int\frac{d\vec{q}}{(2\pi)^3}\left[\frac{\epsilon_{\rm N}(\vec{q})}{2E_{\rm N}(\vec{q})}\right]\left[\frac{2(q^0)^2\rho_s^2(q^2)}{(-q^2+m_\chi^2)^2}+\frac{\rho_s^2(q^2)}{-q^2+m_\chi^2}\right],$$

$$\mathscr{E}_\sigma=\frac{g_\sigma^2}{2}\int\frac{d\vec{q}}{(2\pi)^3}\left[\frac{\epsilon_{\rm N}(\vec{q})}{2E_{\rm N}(\vec{q})}\right]\left[\frac{2(q^0)^2\rho_s^2(q^2)}{(-q^2+m_\sigma^2)^2}+\frac{\rho_s^2(q^2)}{-q^2+m_\sigma^2}\right],$$

$$\mathscr{E}_\pi=\frac{g_\pi^2(3)}{2}\int\frac{d\vec{q}}{(2\pi)^3}\left[\frac{\epsilon_{\rm N}(\vec{q})}{2E_{\rm N}(\vec{q})}\right]\left[\frac{E_{\rm N}(\vec{q})-m_{\rm N}}{E_{\rm N}(\vec{q})+m_{\rm N}}\right]\left[\frac{2(q^0)^2g_1^2(q^2)}{(-q^2+m_\pi^2)^2}+\frac{g_1^2(q^2)}{-q^2+m_\pi^2}\right],$$

$$\mathscr{E}_\omega=-\frac{g_\omega^2}{2}\int\frac{d\vec{q}}{(2\pi)^3}\left[\frac{\epsilon_{\rm N}(\vec{q})}{2E_{\rm N}(\vec{q})}\right]\left[\left\{\left[\frac{2m}{\epsilon_{\rm N}(\vec{q})}\right]^{1/2}[F_{10}(q^2)+F_{20}(q^2)]-\left[\frac{\epsilon_{\rm N}(\vec{q})}{2m_{\rm N}}\right]^{1/2}F_{20}(q^2)\right\}^2\right.$$
$$\left.-\frac{2\vec{q}^{\,2}}{\epsilon_{\rm N}^2(\vec{q})}[F_{10}(q^2)+F_{20}(q^2)]^2\right]\frac{[(q^0)^2+\vec{q}^{\,2}+m_\omega^2]}{[-q^2+m_\omega^2]^2}\,,$$

$$\mathscr{E}_\rho=-\frac{g_\rho^2(3)}{2}\int\frac{d\vec{q}}{(2\pi)^3}\left[\frac{\epsilon_{\rm N}(\vec{q})}{2E_{\rm N}(\vec{q})}\right]\left[\left\{\left[\frac{2m_{\rm N}}{\epsilon_{\rm N}(\vec{q})}\right]^{1/2}[F_{11}(q^2)+F_{21}(q^2)]-\left[\frac{\epsilon_{\rm N}(\vec{q})}{2m_{\rm N}}\right]^{1/2}F_{21}(q^2)\right\}^2\right.$$
$$\left.-\frac{2\vec{q}^{\,2}}{\epsilon_{\rm N}^2(\vec{q})}[F_{11}(q^2)+F_{21}(q^2)]^2\right]\frac{[(q^0)^2+\vec{q}^{\,2}+m_\rho^2]}{(-q^2+m_\rho^2)^2}\,, \tag{A8}$$

and $\hat{m}_{\rm N}$ is the eigenvalue of the equation for the invariant amplitudes, Eq. (5.10). Furthermore,

$$\langle E_d\rangle\equiv\frac{1}{4\pi}\int d\vec{Q}\,E_d(\vec{Q})[\hat{R}_u^2(Q)+\hat{R}_l^2(Q)]\,. \tag{A9}$$

We may put

$$\epsilon\equiv\hat{m}_{\rm N}-\langle E_d\rangle \tag{A10}$$

and

$$\mathscr{E}_m=\mathscr{E}_\chi+\mathscr{E}_\sigma+\mathscr{E}_\pi+\mathscr{E}_\omega+\mathscr{E}_\rho\,, \tag{A11}$$

so that

$$m_{{\rm N}(H)}=3\epsilon+\mathscr{E}_m\,. \tag{A12}$$

*Present address: Department of Physics, Western Michigan University, Kalamazoo, MI 49001.

[1]See, for example, T. DeGrand, R. L. Jaffe, K. Johnson, and J. Kiskis, Phys. Rev. D **12**, 2060 (1975); K. Johnson, Acta Phys. Pol. **B6**, 865 (1975). More recent work is described in C. E. Carlson, T. H. Hansson, and C. Peterson, Phys. Rev. D **27**, 1556 (1983).

[2]G. E. Brown and M. Rho, Phys. Lett. **82B**, 177 (1979); V. Vento, M. Rho, and G. E. Brown, *ibid.* **103B**, 285 (1981); V. Vento, W. K. Cheng, and A. Halprin, Phys. Rev. D **20**, 727 (1979); V. Vento, J. G. Jun, E. M. Nyman, M. Rho, and G. E. Brown, Nucl. Phys. **A345**, 413 (1980); F. Myhrer, G. E. Brown, and Z. Xu, *ibid.* **A362**, 317 (1981).

[3]G. A. Miller, A. W. Thomas, and S. Theberge, Phys. Lett. **B91**, 192 (1980); A. W. Thomas, S. Theberge, and G. A. Miller, Phys. Rev. D **24**, 216 (1981).

[4]R. Friedberg and T. D. Lee, Phys. Rev. D **15**, 1694 (1977); **16**, 1096 (1977); **18**, 2623 (1978). See also T. D. Lee, *ibid.* **19**, 1802 (1979).

[5]For a general review, see R. Rajaraman, *Solitons and Instantons* (North-Holland, Amsterdam, 1982); R. Jackiw, Rev. Mod. Phys. **49**, 681 (1977).

[6]L. S. Celenza and C. M. Shakin, Phys. Rev. C **28**, 2042 (1983).

[7]R. Goldflam and L. Wilets, Phys. Rev. D **25**, 1951 (1982).

[8]E. Hadjimichael and R. Fiebig, Phys. Rev. D **30**, 181 (1984); **30**, 195 (1984); D. M. Betz and R. Goldflam, *ibid.* **28**, 2848 (1983).

[9]S. Kahana, G. Ripka, and V. Soni, Nucl. Phys. **A415**, 351 (1984).

[10]M. C. Birse and M. K. Banerjee, Phys. Lett. **136B**, 284 (1984).

[11]M. Gell-Mann and M. Levy, Nuovo Cimento **16**, 705 (1960).

[12]R. Seki and S. Ohta, California Institute of Technology Report MAP-52, 1983.

[13]T. M. R. Skyrme, Proc. R. Soc. London **A260**, 127 (1961); Nucl. Phys. **31**, 556 (1962).

[14]E. Witten, Nucl. Phys. **B223**, 433 (1983); **B223**, 422 (1984); G. S. Adkins, C. R. Nappi, and E. Witten, Nucl. Phys. **B228**, 552 (1983); G. S. Adkins and C. R. Nappi, *ibid.* **B233**, 109 (1984); Phys. Lett. **137B**, 251 (1984); A. D. Jackson and M. Rho, Phys. Rev. Lett. **51**, 751 (1983).

[15]M. Rho, A. S. Goldhaber, and G. E. Brown, Phys. Rev. Lett. **51**, 747 (1983); M. Rho, Proceedings of the International School of Nuclear Physics, Erice, Italy, 1983 (unpublished).

[16]J. A. McNeil, J. R. Shepard, and S. J. Wallace, Phys. Rev. Lett. **50**, 1439 (1983); J. A. McNeil, L. Ray, and S. J. Wallace, Phys. Rev. C **27**, 2123 (1983).

[17]M. A. Anastasio, L. S. Celenza, W. S. Pong, and C. M. Shakin, Phys. Rep. **100**, 327 (1983).

[18]See, for example, K. Erkelenz, Phys. Rep. **C13**, 191 (1974).

[19]A. Chodos and C. B. Thorn, Phys. Rev. D **12**, 2733 (1975).

[20]G. Kalbermann and J. M. Eisenberg, Tel Aviv University Report TAUP 1182-83, 1983 (unpublished).

[21]A similar technique has been used for nonrelativistic problems: A. K. Kerman and A. Klein, Phys. Lett. **1**, 185 (1962); Phys. Rev. **132**, 1326 (1963); **138**, B1323 (1965). This method has been applied in the study of soliton properties by J. Goldstone and R. Jackiw, Phys. Rev. D **11**, 1486 (1975). See also A. Klein, Phys. Rev. D **14**, 558 (1976); A. Klein and F. Krejs, *ibid.* **12**, 3112 (1975); A. Klein and H. A. Welton, *ibid.* **17**, 1009 (1978), and references therein.

[22]K. Holinde, K. Erkelenz, and R. Alzetta, Nucl. Phys. **A198**, 598 (1972).

[23]F. Borkowski, G. G. Simon, V. H. Walther, and R. D. Wending, Z. Phys. A **275**, 29 (1975).

[24]V. E. Krohn and C. E. Ringo, Phys. Rev. D **8**, 1305 (1973).

[25]L. N. Hand *et al.*, Rev. Mod. Phys. **35**, 335 (1963).