PHYSICAL REVIEW C VOLUME 31, NUMBER 6 JUNE 1985

Transfer in the light Hg isotopes and the U(6/12) models

M. Vergnes, S. Grafeuille, G. Rotbard, G. Berrier-Ronsin, J. Vernotte, J. M. Maison, and S. Fortier
Institut de Physique Nucléaire, 91406 Orsay, France

R. Tamisier
Institut de Physique Nucléaire, 91406 Orsay, France
and Institut de Physique de Nantes, 44072 Nantes, France

P. Van Isacker and J. Jolie
Instituut voor Nukleaire Wetenschappen, B-9000 Gent, Belgium

(Received 21 February 1985)

It was suggested recently that the level schemes of the odd Hg isotopes with $193 \leq A \leq 199$ could be described in the framework of the U(5) limit of the U(6/12) supersymmetry scheme. This suggestion has been further tested using the reactions ^{200,198,196}Hg(p,d)^{199,197,195}Hg. The comparison of the experimental spectroscopic factors with the ones computed using the U(6/12) model shows that among the three nuclei studied, ^{195}Hg and ^{197}Hg can be considered as reasonably described. The agreement in this case with U(6/12) is better in the U(5) limit than in the O(6) limit.

The Hg isotopes, at the edge of the transitional region and only two protons away from the Pb closed shell, have been tentatively described in the past by many models.[1] Among the most recent attempts was a description[2] of the level schemes of the odd-A isotopes, between $A = 193$ and 199, using the U(6/12) model[3] with a U(5) core. Although our preliminary (p,t) results[4] did not show clear evidence for such a U(5) nature of the even-even isotopes, it was found interesting enough to further test the U(6/12) description of the Hg isotopes with single particle transfer reactions populating the levels of the odd nuclei. The results discussed in the present paper concern the pickup reactions on even-even targets.

In the U(6/12) model, where the odd particle can occupy orbits with $j = \frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$, two chains of subgroups can be constructed, each corresponding to a U(5) symmetry for the even-even core:[5–7]

$$
\begin{aligned}
&\mathrm{U}(6/12) \supset \mathrm{U}^B(6) \times \mathrm{U}^F(12) \supset \mathrm{U}^B(6) \times \mathrm{U}^F(6) \times \mathrm{SU}^F(2)\,,\\
&\qquad \supset \left\{ \begin{matrix} (I),\ \mathrm{U}^B(5) \times \mathrm{U}^F(5) \times \mathrm{SU}^F(2) \\ (I'),\ \mathrm{U}^{B+F}(6) \times \mathrm{SU}^F(2) \end{matrix} \right\} \supset \mathrm{U}^{B+F}(5) \times \mathrm{SU}^F(2)\,,\\
&\qquad \supset \mathrm{O}^{B+F}(5) \times \mathrm{SU}^F(2) \supset \mathrm{O}^{B+F}(3) \times \mathrm{SU}^F(2) \supset \mathrm{Spin}(3)\,,\\
&\qquad \supset \mathrm{Spin}(2)\,.
\end{aligned}
\tag{1}
$$

In Ref. 2 a description of the odd-mass $^{193-199}$Hg isotopes was proposed, using a Hamiltonian corresponding to the limit (I') of Eq. (1). However, the comparison with the experimental data was carried out only for the energies. Since it is known[7] that the most general spectrum of limit I' coincides with a particular spectrum of limit I, a further test of the U(6/12) description of the Hg isotopes preferably should also deal with the question: Which one of the two limits is appropriate for these nuclei. In Ref. 7 a detailed account is given of the properties of limits I and I'. In particular it is shown that the single-particle structure of the wave functions in the two limits is different. One-nucleon transfer reactions are therefore most

TABLE I. One-nucleon transfer intensities in the U(5) limits I and I' of U(6/12).

Intensity	Limit I	Limit I'
$I(0_1^+ \to \frac{1}{2}_1)$	$2\xi_{1/2}^2$	$2\xi_{1/2}^2$
$I(0_1^+ \to \frac{3}{2}_1)$	$4\xi_{3/2}^2$	$\frac{4N}{N+1}\xi_{3/2}^2$
$I(0_1^+ \to \frac{3}{2}_2)$	0	$\frac{4}{N+1}\xi_{3/2}^2$
$I(0_1^+ \to \frac{5}{2}_1)$	$6\xi_{5/2}^2$	$\frac{6N}{N+1}\xi_{5/2}^2$
$I(0_1^+ \to \frac{5}{2}_2)$	0	$\frac{6}{N+1}\xi_{5/2}^2$

TABLE II. Isotopic analysis of the Hg targets (as given by ORNL, supplier of the isotopic material).

Target A	204	202	201	(at %) 200	199	198	196	Thickness (μg/cm^2)
Natural Hg	6.8	29.7	13.2	23.1	16.9	10.1	0.15	30
200	0.2	1.02	1.82	95.74	1.04	0.18	$<$0.02	340
198	0.14	0.72	0.59	2.00	11.2	85.3	0.05	280
196	1.20	7.45	4.23	11.08	13.15	15.06	47.83	30

useful to probe these differences. The transfer intensity is defined as

$$I(i\rightarrow f)=|\langle f||P_j||i\rangle|^2\,, \tag{2}$$

where P_j is the transfer operator. For the pickup reaction from an even-even nucleus with N bosons to an odd-A nucleus with N bosons (the reaction which is considered in this paper) the transfer operator is[8] $P_j=\xi_j a_j^\dagger$ and the intensities $I(i\rightarrow f)$ in limits I and I' of U(6/12) are summarized in Table I. The ξ_j are parameters.

The (p,d) experiments were performed using a 25 MeV proton beam from the Orsay MP tandem accelerator and detecting the outgoing deuterons with a 50 cm long position sensitive detector[9] in the focal plane of the split pole spectrometer. The targets, already used to study the (p,t) reaction,[4] consisted of HgS evaporated onto a thin carbon foil. The enrichments and thicknesses are given in Table II, for the ^{200}Hg, ^{198}Hg, and ^{196}Hg targets. A natural HgS target was also used to help in normalizing the results for the different isotopes. A spectrum, corresponding to the ^{200}Hg(p,d)^{199}Hg reaction, is shown in Fig. 1. The energy resolution, mainly due to the target thickness, varies between 13 and 20 keV (full width at half maximum). The spectroscopic factors have been extracted in the usual way,[10] using the DWBA method with the optical potentials already used[10] to analyze our Pt(p,d) results at 26 MeV. In the case of close doublets (or multiplets) of levels, either an automatic peak fitting code (permitting, if the separation in energy of the levels is larger than 1.5 times the full width at half maximum, to extract the different components of the complex peak), or an analysis of the angular distribution of the total complex peak using mixtures of pure experimental shapes corresponding to the different known angular momentum transfer, were used in order to get the spectroscopic factors of the different levels.

Our present aim being mainly to test the U(6/12) description, we will only discuss here the population of the lowest-lying levels of the final odd Hg isotopes: ^{199}Hg, ^{197}Hg, and ^{195}Hg. A more complete account of the experimental results concerning all the isotopes between ^{203}Hg and ^{195}Hg will be given elsewhere.

In the U(5) limits of U(6/12) it is predicted that all the $p_{1/2}$, $p_{3/2}$, and $f_{5/2}$ strength should be found on the five low-lying $\frac{1}{2}^-_1$, $\frac{3}{2}^-_1$, $\frac{5}{2}^-_1$, $\frac{3}{2}^-_2$, and $\frac{5}{2}^-_2$ levels. It can be seen in Fig. 1 that this is at least qualitatively true for ^{199}Hg. The systematic behavior of these lowest-lying levels for the odd Hg isotopes has been studied by Wood[11]

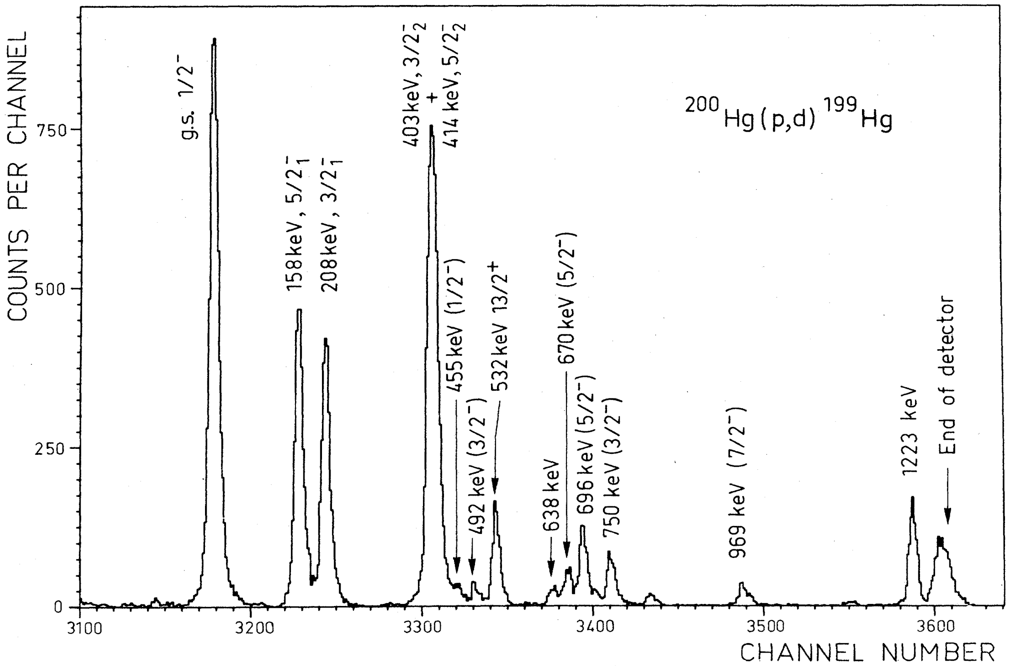

FIG. 1. Energy spectrum of emitted deuterons from the ^{200}Hg(p,d)^{199}Hg reaction at $\theta_{\rm lab}=5°$. Peaks are labeled by excitation energy in keV and J^π values of levels in the final nucleus.

for the nuclei between ^{189}Hg and ^{199}Hg. We will adopt in our comparison with the U(6/12) model the energies and J^{π} assignments proposed in Fig. 6 of Ref. 11. The experimental results for the five lowest levels are given in Table III for the three reactions discussed. The corresponding angular distributions are shown in Fig. 2 for the reaction ^{200}Hg(p,d)^{199}Hg. A direct comparison of the five experimental spectroscopic factors available for each nucleus with the predictions of Table I [for an even-even target the spectroscopic factor S_{if} is equal to the transfer intensity $I(i \rightarrow f)$] would not be very meaningful because the theoretical results depend on three parameters $\xi_{1/2}^2$, $\xi_{3/2}^2$, and $\xi_{5/2}^2$. These parameters are in principle different for each target and equal to the occupation probabilities of the corresponding single particle orbitals. However, an interesting prediction of the U(6/12) model is that the ratios

$$R_{3/2}=\frac{S_{3/2_1^-}}{S_{3/2_2^-}} \text{ and } R_{5/2}=\frac{S_{5/2_1^-}}{S_{5/2_2^-}}$$

are parameter-free. They are indeed both predicted to be infinite in limit I and to be equal to N in limit I', and can be used as a first test of the model.

The most striking feature of the experimental spectroscopic factors shown in Table III is the very rapid change, when going from ^{199}Hg to ^{195}Hg, of the distribution of the $p_{3/2}$ and $f_{5/2}$ strengths among the levels discussed. In ^{199}Hg the strength for the $\frac{3}{2}{}_2^-$ (or $\frac{5}{2}{}_2^-$) level is about equal to the strength for the $\frac{3}{2}{}_1^-$ (or $\frac{5}{2}{}_1^-$) level; in ^{197}Hg only about 15% of the strength goes to the $\frac{3}{2}{}_2^-$ and $\frac{5}{2}{}_2^-$ levels and, in ^{195}Hg, this fraction is vanishingly small.

A comparison of the experimental ratios $R_{3/2}$ and $R_{5/2}$ with the supersymmetry predictions (Table IV) shows that the relation $R_{3/2}=R_{5/2}$ is approximately verified for the three Hg nuclei. Furthermore, for ^{195}Hg the experimental ratios are very large, consistent with limit I of U(6/12), and, for ^{197}Hg, the ratios are reasonably close to the predictions in limit I' of U(6/12). The data for ^{199}Hg, however, are completely at variance with U(6/12).

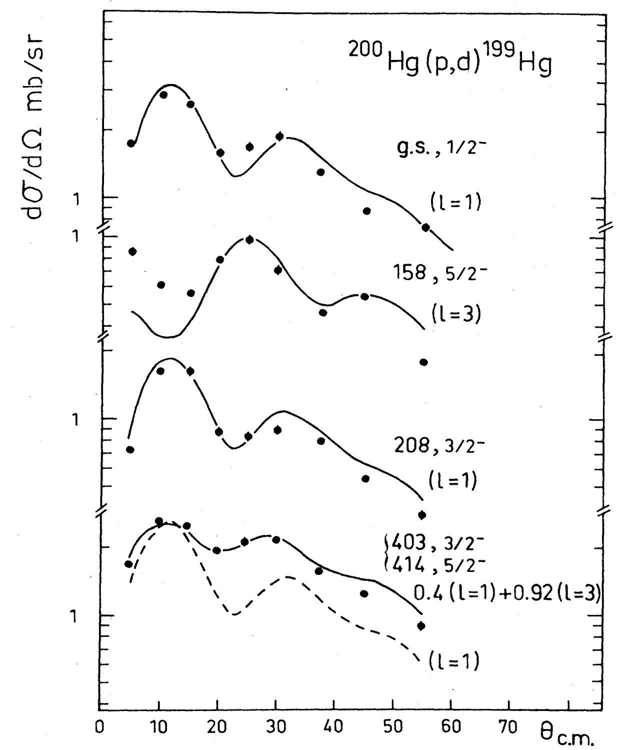

FIG. 2. Angular distributions of the lowest lying levels of ^{199}Hg in the ^{200}Hg(p,d)^{199}Hg reaction. The continuous curves are DWBA fits for the three lowest levels. The angular distribution corresponding to the peak "403+414 keV" of Fig. 1 cannot be fitted by any pure angular distribution, either $l=1$ or $l=3$ (the pure $l=1$ is shown as a dashed curve). A fit by a mixture of $l=1$ (coefficient α) and $l=3$ (coefficient β) is shown as a continuous curve. In view of the poor DWBA fit of the forward angles for the $l=3$ transfer, another independent fit was made using—instead of DWBA curves—empirical "average" curves deduced from the experimental shapes for pure transitions. The resulting α and β coefficients are the same.

We can even go a little further in the case of ^{195}Hg and ^{197}Hg, without playing too much with parameters. We make the very strong arbitrary simplification of assuming that the occupation probabilities of the three orbitals are the same: $\xi_{1/2}^2=\xi_{3/2}^2=\xi_{5/2}^2=\xi_{\rm eff}^2$. We then obtain a reasonable description of the absolute spectroscopic factors (Table V), with one parameter for each nucleus, fixed

TABLE III. Spectroscopic factors for the lowest $\frac{1}{2}^-$, $\frac{3}{2}^-$, and $\frac{5}{2}^-$ levels of ^{199}Hg, ^{197}Hg, and ^{195}Hg.

	^{199}Hg		^{197}Hg		^{195}Hg	
J^{π}	$E_{\rm exc}^{\rm a,b}$ (keV)	$S^{\rm c}$	$E_{\rm exc}^{\rm a,d}$ (keV)	$S^{\rm c}$	$E_{\rm exc}^{\rm a,e}$ (keV)	$S^{\rm c}$
$\frac{1}{2}{}_1^-$	0	1.10	0	0.84	0	0.74
$\frac{3}{2}{}_1^-$	208	0.56	152	1.20 (±0.3)	37	1.41
$\frac{3}{2}{}_2^-$	403	0.84 (±0.10)	308.6	0.14 (±0.07)	279	≤0.05
$\frac{5}{2}{}_1^-$	158	1.6	134	2.7 (±0.5)	53	2.5
$\frac{5}{2}{}_2^-$	414	1.5 (±0.2)	307.8	0.70 (±0.3)	300	≤0.09

[a] Reference 11.
[b] Reference 12.
[c] Present work. Only the errors due to uncertainties in the decomposition of close doublets are indicated, when >6%.
[d] Reference 13.
[e] Reference 14.

TABLE IV. Comparison of the experimental ratios $R_{3/2}$ and $R_{5/2}$ (see the text) with the predictions of the U(5) limits of U(6/12).

	^{199}Hg	^{197}Hg	^{195}Hg
$R_{3/2}$	0.67 ± 0.07	8.6 ± 4.8	$\geqq 30$
$R_{5/2}$	1.06 ± 0.12	3.8 ± 1.7	$\geqq 30$
$R_{\text{th}}(I)$	∞	∞	∞
$R_{\text{th}}(I')$	4	5	6

TABLE V. One parameter fit of the ^{197}Hg and ^{195}Hg spectroscopic factors (see the text).

	^{197}Hg		^{195}Hg	
J_i^π	Expt.	Limit I'	Expt.	Limit I
$\frac{1}{2}{}_1^-$	0.84	0.93	0.74	0.78
$\frac{3}{2}{}_1^-$	1.2 (±0.3)	1.55	1.41	1.55
$\frac{3}{2}{}_2^-$	0.14 (±0.07)	0.31	$\leqq 0.05$	0
$\frac{5}{2}{}_1^-$	2.7 (±0.5)	2.325	2.5	2.33
$\frac{5}{2}{}_2^-$	0.70 (±0.3)	0.465	$\leqq 0.09$	0
Σ	5.58	5.58	$\geqq 4.65$ $\leqq 4.79$	4.66

at $\xi^2_{\text{eff}}=0.465$ for ^{197}Hg and $\xi^2_{\text{eff}}=0.39$ for ^{195}Hg. The breaking as defined in Ref. 15, is 21.9% for ^{197}Hg and at most 10.2% for ^{195}Hg.

Although it appears possible to describe the lowest levels of $^{195-197}$Hg in the U(5) limit of U(6/12), it is interesting to look also to the results predicted in the O(6) limit. The transfer intensities, corresponding to the pickup spectroscopic factors measured in the present study, have been worked out:

$$I(0_1^+\rightarrow\tfrac{1}{2}{}_1^-)=\frac{N+4}{N+2}\xi^2_{1/2}\,, \tag{3a}$$

$$I(0_1^+\rightarrow\tfrac{3}{2}{}_1^-)=\frac{16N(N+4)}{5(N+1)(N+3)}\xi^2_{3/2}\,, \tag{3b}$$

$$I(0_1^+\rightarrow\tfrac{3}{2}{}_2^-)=\frac{2(N+4)(N+5)}{5(N+1)(N+2)}\xi^2_{3/2}\,, \tag{3c}$$

$$I(0_1^+\rightarrow\tfrac{5}{2}{}_1^-)=\frac{24N(N+4)}{5(N+1)(N+3)}\xi^2_{5/2}\,, \tag{3d}$$

$$I(0_1^+\rightarrow\tfrac{5}{2}{}_2^-)=\frac{3(N+4)(N+5)}{5(N+1)(N+2)}\xi^2_{5/2}\,, \tag{3e}$$

and the parameter-free ratios are

$$R_{3/2}=R_{5/2}=\frac{8N(N+2)}{(N+3)(N+5)}\;. \tag{4}$$

In the O(6) limit of U(6/12), also two group chains are possible:[6] one where boson and fermion degrees of freedom are coupled on the level of O(6), the other one where they are coupled on the level of U(6). In contrast to the U(5) limit of U(6/12), however, the wave functions of the lowest states in the two O(6) limits are the same and, in particular, the predictions (3) for the spectroscopic intensities are valid in the two limits.

Applying these formulas, it appears that the theoretical ratios $R=3.05$ for ^{199}Hg ($N=4$), $R=3.50$ for ^{197}Hg ($N=5$), and $R=3.88$ for ^{195}Hg ($N=6$), are not in agreement with the experimental ratios shown in Table IV and that for $^{195-197}$Hg, no single parameter fit of the five experimental spectroscopic factors can give an agreement similar to the one shown in Table V.

To summarize and conclude, we have shown a very striking variation, between ^{199}Hg and ^{195}Hg, of the population of the $\frac{3}{2}{}_2^-$ and $\frac{5}{2}{}_2^-$ levels, as compared to the population of the $\frac{3}{2}{}_1^-$ and $\frac{5}{2}{}_1^-$ levels. This strong variation appears as somewhat contradictory with the smooth energy behavior shown in Fig. 6 of Ref. 11. The two experimental ratios $R_{3/2}$ and $R_{5/2}$ are of similar magnitude, as predicted by the U(6/12) model, in each of the three Hg isotopes studied, although they both vary strongly from one isotope to the next. It seems possible to describe reasonably well the lowest-lying levels of ^{197}Hg and ^{195}Hg within the framework of U(6/12). However, due to the remarkable differences between the two nuclei, one must assume a *different* U(5) limit of U(6/12) to adequately describe each of them. The possible physical meaning of that result is not yet clear. Also, the O(6) limit of U(6/12) does not appear to give an acceptable description of the spectroscopic data of any of the $^{195-199}$Hg isotopes.

Although the present results do not contradict—at least for $^{195-197}$Hg—the suggestion of Ref. 2 that the light odd Hg isotopes can be described by the U(6/12) model, the rapid experimental variation of the ratio R suggests the possibility that the agreement shown for $^{195-197}$Hg could be somewhat fortuitous. In this connection, it is fair to remark that it has always been claimed that dynamic symmetries can only occur in very special and limited cases and that, indeed, most of the previous similar experimental tests of the U(6/4) and U(6/12) models have concluded to a good agreement in only very limited regions of the chart of the nuclides.[15,16]

---

[1] See for example: D. Kolb and C. Y. Wong, Nucl. Phys. **A245**, 205 (1975); I. Morrison, A. Faessler, and C. Lima, *ibid.* **A372**, 13 (1981); Th. J. Köppel, A. Faessler, and U. Götz, *ibid.* **A403**, 263 (1983).

[2] H. Z. Sun, A. Frank, and P. Van Isacker, Phys. Lett. **124B**, 275 (1983).

[3] A. B. Balentekin, I. Bars, R. Bijker, and F. Iachello, Phys. Rev. C **27**, 1761 (1983).

[4] M. Vergnes, G. Berrier-Ronsin, G. Rotbard, J. Vernotte, S. Fortier, J. M. Maison, R. Tamisier, P. D. Duval, and E. R. Flynn, in *Proceedings of the International Conference on Nuclear Physics, Florence, 1983,* edited by P. Blasi and R. A. Ricci (Tipographia Compositori, Bologna, 1983), Vol. 1, p. 185; M. Vergnes, G. Berrier-Ronsin, G. Rotbard, J. Vernotte, S. Fortier, J. M. Maison, R. Tamisier, S. Grafeuille, P. D. Duval, and E. R. Flynn, Institut de Physique Nucléaire Inter-

nal Report, Orsay, IPNO-DRE 83-24 (unpublished).

[5] R. Bijker and V. K. B. Kota, Ann. Phys. (N.Y.) **156**, 110 (1984).

[6] P. Van Isacker, A. Frank, and H. Z. Sun, Ann. Phys. (N.Y.) **157**, 183 (1984).

[7] J. Vervier, P. Van Isacker, J. Jolie, V. K. B. Kota, and R. Bijker (unpublished).

[8] O. Scholten, Ph.D. thesis, University of Groningen, The Netherlands, 1980. O. Scholten and A. E. L. Dieperink, in *Interacting Bose-Fermi Systems in Nuclei,* edited by F. Iachello (Plenum, New York, 1981), p. 343.

[9] R. G. Markham and R. G. H. Robertson, Nucl. Instrum. Methods **129**, 131 (1975); E. Bartol, S. Fortier, J. M. Maison, A. Richard, and G. Rotbard, Annuaire Institut de Physique Nucléaire, Orsay, Report IPNO 1981, 1981 (unpublished), p. 77.

[10] G. Berrier-Ronsin, M. Vergnes, G. Rotbard, J. Vernotte, J. Kalifa, R. Seltz, and H. L. Sharma, Phys. Rev. C **17**, 529 (1978).

[11] John L. Wood, in *Proceedings of the International Symposium on Future Directions in Studies of Nuclei Far from Stability, Nashville, 1979,* edited by J. H. Hamilton *et al.* (North-Holland, Amsterdam, 1980) p. 37; and private communication.

[12] J. Halperin, Nucl. Data Sheets **B24**, 57 (1978); R. F. Casten, D. D. Warner, G. M. Gowdy, N. Rofail, and K. P. Lieb, Phys. Rev. C **27**, 1310 (1983).

[13] B. Harmatz, Nucl. Data Sheets, B **20**, 73 (1977); R. A. Braga, J. L. Wood, G. M. Gowdy, and R. W. Fink, Phys. Rev. C **19**, 2305 (1979).

[14] M. J. Martin, Nucl. Data Sheets **B8**, 431 (1972); G. M. Gowdy, J. L. Wood, and R. W. Fink, Nucl. Phys. **A312**, 56 (1978).

[15] M. Vergnes, G. Berrier-Ronsin, and R. Bijker, Phys. Rev. C **28**, 360 (1983).

[16] J. A. Cizewski, in *Interacting Bose-Fermi Systems in Nuclei,* edited by F. Iachello (Plenum, New York, 1981), p. 389.