^{2}H(d,γ)^{4}He reaction at low energies

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The branching ratio Γ_γ/Γ_p for the d+d reaction has been measured for deuteron bombarding energies ranging from 50 to 150 keV. The branching ratio is found to be roughly constant over this energy range and the best value is $(1.2\pm0.3)\times10^{-7}$. This result, together with previous branching ratio measurements at higher energies, is compared with direct capture calculations.

I. INTRODUCTION

Nuclear reactions among light ions at low energies have long been recognized as being of fundamental importance to many areas of basic and applied physics. Perhaps the two most notable areas to which these reactions find application are fusion energy and astrophysics. Recent studies of these reactions include precision measurements of the ^{2}H(t,n)^{4}He, ^{2}H(d,p)^{3}H, and ^{2}H(d,n)^{3}He (Ref. 1) reactions at low energy. We have been studying the corresponding radiative capture reactions among these isotopes of hydrogen. Our measurement of the ^{2}H(t,γ)^{5}He reaction at center of mass energies down to 25 keV has been reported.[2] We have completed an analogous study of the ^{2}H(d,γ)^{4}He for center of mass energies of 25 to 80 keV. One motivation for this study lies in the ongoing problem of high temperature deuterium plasma diagnostics.[3] This reaction may likewise contribute to the primordial abundance of ^{4}He.[4] These possible applications of a knowledge of the cross section of the ^{2}H(d,γ)^{4}He reaction dictate the center of mass energies at which the cross section must be known. For a Maxwellian plasma, most of the nuclear reactions occur at energies near the Gamow peak. The value of this Gamow peak is given, nonrelativistically, by[5]

$$E_G=[(\tfrac{1}{2})^{1/2}\pi\alpha Z_1Z_2\sqrt{\mu c^2}kT]^{2/3} ,$$

where α is the fine structure constant, the Z's are the nuclear charges of the reactants, μ is their reduced mass, and T is the plasma temperature. For a deuterium plasma, therefore,

$$E_G\ (\text{MeV})=0.63(kT)^{2/3} ,$$

where kT is in MeV. For plasma temperatures in the range of 10^8—10^9 K, that is, for temperature ranging from the hottest plasmas currently being achieved in the controlled thermonuclear reactor (CTR) effort[6] to the primordial conditions characteristic of the maximum deuterium concentration,[4] the Gamow peak energy increases from 26 to 120 keV. Previous measurements of the ^{2}H(d,γ)^{4}He reaction have been made down to center of mass energies of 400 keV (Ref. 7) and are, as a result, not directly applicable to the above temperature range. Our present measurements, down to a center of mass energy of 25 keV, will allow the yield of the ^{2}H(d,γ)^{4}He reaction to be predicted for deuterium plasmas at temperatures down to 10^8 K.

II. EXPERIMENTAL PROCEDURE

A deuterium target was made by deuterating the back wall of a semicircular scattering chamber with a well-collimated deuterium beam. The charged particle beam was produced by the Cockroft-Walton accelerator of the physics department at the Colorado School of Mines. Thick target yield measurements were made for the ^{2}H(d,p)^{3}H reaction and they verified that the target was well saturated since the deduced total cross sections were approximately in agreement with values of the total cross section found in the literature.[1,8] Deuterating the wall of the scattering chamber enabled us to increase the solid angle subtended by the front face of the gamma detector to nearly 2π steradians by bringing the detector to within a few millimeters of the target.

The gamma detector, which was fabricated by the Harshaw Chemical Company, consisted of a 10.2 cm$\times$10.2 cm NaI(Tl) "plug" detector surrounded by a 30.5 cm (diam)$\times$20.3 cm axially split NaI(Tl) annulus. This annular assembly was used as an anticoincidence shield in which a pulse from the plug detector would not be processed if a signal greater than 600 keV was obtained from either half of the split annulus. Figure 1 displays the setup.

As shown in Fig. 1 a silicon surface barrier detector was mounted at 135° from the forward beam direction.

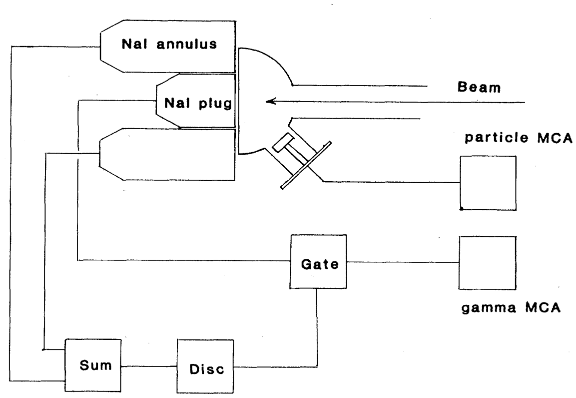

FIG. 1. Schematic layout of experimental setup. The target is located at the back wall of the semicircular scattering chamber and is viewed at 135 deg by the silicon particle detector. The gate on the gamma signal is normally open and is closed by a signal from the annulus.

This arrangement allowed us to simultaneously measure the proton yield from the $^2H(d,p)^3H$ branch of the d + d reaction while the gammas from the $^2H(d,\gamma)^4He$ branch were being measured by the gamma detector. It likewise allowed our use of the gamma and charged particle branches of the $^{11}B(p,\gamma)^{12}C$ and $^{11}B(p,\alpha)^8Be$ resonant reactions, where $E_p=163$ keV (Ref. 9) to determine the absolute detection efficiencies of the system. The 4.4, 11.7, and 16.11 MeV gammas of the $^{11}B(p,\gamma)^{12}C$ reaction were used along with the 19.8 MeV gamma of the $^3H(p,\gamma)^4He$ reaction to obtain an energy calibration and peak shape for the 23.84 MeV gamma ray of the $^2H(d,\gamma)^4He$ reaction.

The yield of gammas detected is determined by the thick target reaction rate for the gamma ray producing reaction. The detected gamma ray yield per incident deuteron of energy E_0 is given by

$$Y_{dd\gamma}=\epsilon(E_\gamma)\int_{E_0}^{0}\frac{\Gamma_\gamma}{\Gamma_{tot}}\frac{\sigma_{tot}(E)}{\frac{dE(E)}{dn}}f(E)dE . \quad (1)$$

Similarly for the charged particle branch, the detected number of protons is given by

$$Y_{ddp}=\epsilon(E_p)\int_{E_0}^{0}\frac{\Gamma_p}{\Gamma_{tot}}\frac{\sigma_{tot}(E)}{\frac{dE(E)}{dn}}f(E)dE . \quad (2)$$

In these two equations $\epsilon_\gamma(E_\gamma)$ and $\epsilon_p(E_p)$ are the detection efficiencies including solid angle, $\Gamma_\gamma/\Gamma_{tot}$ and Γ_p/Γ_{tot} are the gamma to total and proton to total branching ratios, respectively, $\sigma_{tot}(E)$ is the total cross section for the d + d reaction, $dE(E)/dn$ is the stopping power, and $f(E)$ is the fractional density of deuterium atoms in the target at an incident deuteron energy depth E. If the ratio of Eq. (1) and Eq. (2) is taken and it is assumed that the branching ratios $\Gamma_\gamma/\Gamma_{tot}$ and Γ_p/Γ_{tot} are independent of energy, then the ratio Γ_γ/Γ_p can be written in terms of the gamma and proton yields as follows:

$$\frac{\Gamma_\gamma}{\Gamma_p}=\frac{Y_{dd\gamma}}{Y_{ddp}}\frac{\epsilon_p(E_p)}{\epsilon_\gamma(E_\gamma)} . \quad (3)$$

Since the branching ratios $\Gamma_\gamma/\Gamma_{tot}$ and $\Gamma_\alpha/\Gamma_{tot}$ for the $^{11}B+p$ reaction are known at the resonance energy of 163 keV, analogous expressions obtain for the $^{11}B+p$, 11.67 MeV gamma ray and 5.8 MeV alpha particle yields. These yields can then be used to determine the detection efficiency of the NaI(Tl) system. The detection efficiency for the 23.84 MeV gamma can be thus determined with the detection assembly in the same geometry; it is expressed as

$$\epsilon_\gamma(23.84)=\frac{Y_{Bp\gamma}}{Y_{Bp\alpha}}\frac{\Gamma_{Bp\alpha}}{\Gamma_{Bp\gamma}}\epsilon_\alpha(E_\alpha) \times\frac{\int_{\Delta\Omega_\gamma}[W_\gamma(\theta)A(\theta)]_{dd\gamma}d\Omega}{\int_{\Delta\Omega_\gamma}[W_\gamma(\theta)A(\theta)]_{Bp\gamma}d\Omega} . \quad (4)$$

Here $W_\gamma(\theta)$ are the normalized angular distributions relative to the forward beam direction for the 11.67 and 23.8 MeV gammas in the laboratory frame, and the $A(\theta)$'s correct for the attenuation of gammas in the NaI(Tl) detector, which was positioned at $\theta=0°$. The gamma ray angular distribution for the 11.67 MeV gamma of the $^{11}B(p,\gamma)^{12}C$ reaction was determined by Grant *et al.*[10] to be

$$W_{11.67}(\theta)=1+0.23\cos^2\theta . \quad (5)$$

The 11.67 MeV gamma of the $^{11}B(p,\gamma)^{12}C$ reaction was used here instead of the 16.11 MeV gamma since the 11.67 MeV gamma has a smaller error in its reported fractional decay width.[9]

The angular distribution $W_\gamma(\theta)$ for the dd reaction has been measured at higher energies[7] and found to be proportional to $\sin^2\theta\cos^2\theta$ as expected for the electric quadrupole multipolarity of the transition.[11] There are no measurements of the angular distribution at low energies. Consequently, we calculated two values of the efficiency ϵ_γ assuming: (1) a $\sin^2\theta\cos^2\theta$ distribution and (2) an isotropic distribution.

By virtue of the proximity of the detector to the target, the two values of the efficiency differed by about 15% (in the limit of a very large detector subtending 2π steradians, the two efficiencies would have been identical). This uncertainty in our efficiency is carried though the analysis and is reflected in the final quoted value of the gamma ray branching ratio.

Since a surface barrier silicon detector was used, the absolute efficiency of the charged particle detector is equal to one. The efficiency used in Eq. (2) includes the detector's solid angle and must therefore be corrected for the anisotropic scattering of the protons[1] and the alphas[12] in the laboratory frame. Combining these corrections with Eqs. (3) and (4), the branching ratio Γ_γ/Γ_p for the d + d reaction is obtained and written as

$$\frac{\Gamma_\gamma}{\Gamma_p}=\frac{Y_{dd\gamma}}{Y_{ddp}}\frac{Y_{Bp\alpha}}{Y_{Bp\gamma}}\frac{\Gamma_{Bp\gamma}}{\Gamma_{Bp\alpha}}\left[\frac{\sigma_{lab}}{\sigma_{c.m.}}\right]_{ddp}\left[\frac{\sigma_{c.m.}}{\sigma_{lab}}\right]_{Bp\alpha} \times\frac{\int_{\Delta\Omega_\gamma}[W_\gamma(\theta)A(\theta)]_{dd\gamma}d\Omega}{\int_{\Delta\Omega_\gamma}[W_\gamma(\theta)A(\theta)]_{Bp\gamma}d\Omega} . \quad (6)$$

To obtain Eq. (6) we assumed that the branching ratios $\Gamma_\gamma/\Gamma_{tot}$ and Γ_p/Γ_{tot} are independent of energy. These assumptions are verified by measuring the Γ_γ/Γ_p branching ratio at several energies. As a result of this assumption, it is possible to determine the branching ratio without knowing any details of the reaction cross section, the stopping powers, or the concentration of the deuterium atoms in the target. This procedure is also convenient since the branching ratio can be determined without knowing the total number of reactions.

III. RESULTS AND DISCUSSION

Figure 2 compares an $^{11}B(p,\gamma)^{12}C$ spectra with and without the anticoincidence condition. This figure demonstrates the extent to which the annular detection assembly was able to suppress cosmic ray background in the plug detector. For gamma ray energies between 15 and 25 MeV, the annulus eliminates roughly 99% of the cosmic ray events. In Fig. 3 the gamma spectrum of the $^2H(d,\gamma)^4He$ reaction which was taken at the deuteron bombarding energy of 120 keV is shown. The actual

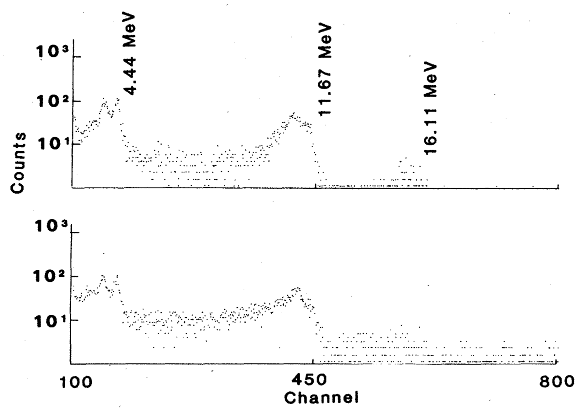

FIG. 2. Gamma ray spectra from the resonant reaction $^{11}B(p,\gamma)^{12}C$ with (top spectrum) and without (bottom spectrum) the annulus anticoincidence condition.

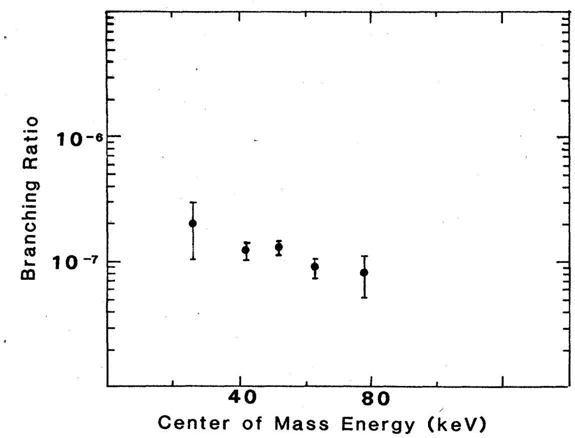

FIG. 4. Measured values of the thick target branching ratios for d-d center of mass energies between 25 and 80 keV.

number of gammas ($Y_{dd\gamma}$) in the peak of interest, centered at about channel 430, was determined by subtracting the background due to cosmic rays and the background which resulted from prompt neutron-induced reactions in the NaI(Tl), from the total number of peak counts. The background below the peak of interest was primarily due to prompt neutron-induced reactions in the NaI(Tl). This background was fitted to an exponential function, and any background counts which contributed to the total number of counts in the peak were subtracted out.

The branching ratios which we determined, using the techniques described in Sec. II, for the $^2H(d,\gamma)^4He$ reaction are shown in Fig. 4. The figure shows that as assumed, the branching ratio is roughly constant over the range of bombarding energies being considered here. The best value of Γ_γ/Γ_p for these five data points, calculated by weighted average, is

$$\Gamma_\gamma/\Gamma_p = 1.2 \pm 0.3 \times 10^{-7} .$$

Although the lowest energy point has relatively large error bars, it does suggest a slight energy dependence of the branching ratio. If such a dependence is allowed, the branching ratio, extrapolated to zero energy, assumes a value of

$$(1.9 \pm 0.3) \times 10^{-7} .$$

The error in the branching ratios shown in Fig. 4 corresponds to the statistical error in the gamma ray yield. A 20% systematic error due to the uncertainty in the branching ratios for the $^{11}B + p$ reaction is also included in the quoted error.

Our measured values of the thick target branching ratio Γ_γ/Γ_p are compared in Fig. 5 to values of the branching ratio determined from earlier measurements of the cross section for the $^2H(d,\gamma)^4He$ reactions at center of mass energies in excess of 400 keV.[7] Except for the lack of measurements between center of mass energies of 80–400 keV, the branching ratio appears to vary in a fairly uniform manner from energies of 25 keV to about 10 MeV. The measured values of the branching ratio given in Fig.

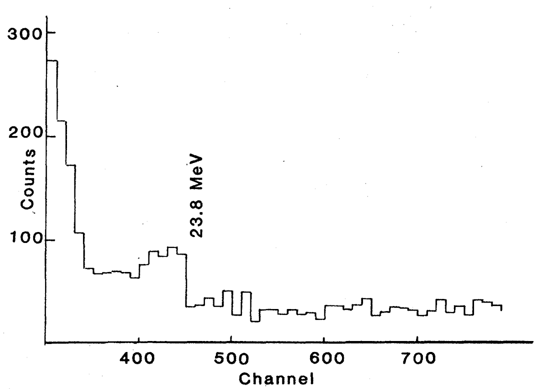

FIG. 3. Gamma ray spectrum from the reaction $^2H(d,\gamma)^4He$ measured at a deuteron bombarding energy of 120 keV. The energy scale in this spectrum is reduced relative to the spectra in Fig. 2.

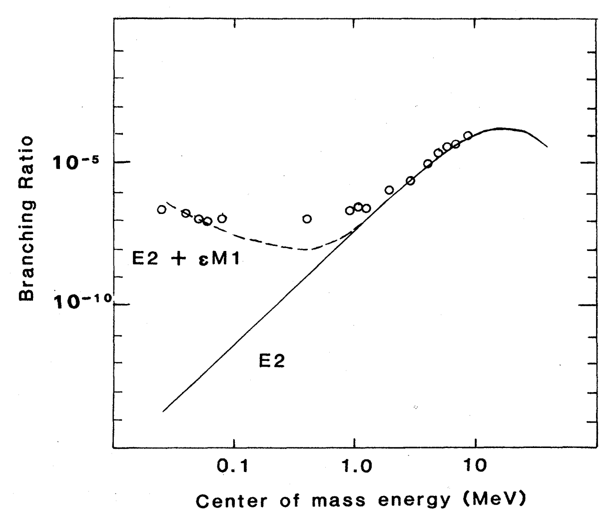

FIG. 5. Comparison of measured branching ratios for the $^2H(d,\gamma)^4He$ reaction to calculated branching ratios. Present measurements are for center of mass energies below 100 keV. The solid curve corresponds to a purely electric quadrupole capture ($E2$) process, while the dashed curve includes an admixture of magnetic dipole capture ($E2+\epsilon M1$).

5 may be compared to values of the branching ratio determined from direct capture reaction calculations of the cross section for the $^2H(d,\gamma)^4He$ reaction and well-measured values of the cross section for the $^2H(d,p)^3H$ reaction.[1,8] The direct capture reaction calculations were based upon alpha particle photodisintegration calculations by Flowers and Mandl.[11]

Specifically, we calculated

$$\frac{\Gamma_\gamma}{\Gamma_p}=\frac{\sigma_{^2H(d,\gamma)^4He}}{\sigma_{^2H(d,p)^3H}}$$

$$=\frac{\sigma_{^4He(\gamma,d)^2H}}{\sigma_{^2H(d,p)^3H}}\left[\frac{(E_{c.m.}+23.8)^2}{4220\times E_{c.m.}}\right]P_l\ ,$$

where $\sigma_{^2H(d,p)^3H}$ are the measured values of the charged particle branch of the dd reaction and $\sigma_{^4He(\gamma,d)^2H}$ is the photodisintegration cross section calculated by Flowers and Mandl. The bracketed quantity relates the photodisintegration to the capture cross section at a center of mass energy $E_{c.m.}$ in MeV, assuming the principle of detailed balance. The factor P_l is the barrier penetration factor and is given in the Wentzel-Kramers-Brillouin (WKB) approximation by[13]

$$\exp\left\{-\int_{R_{min}}^{R_{max}}\sqrt{2\mu c^2/\hbar c}\left[\frac{e^2}{r}+\frac{l(l+1)(\hbar c)^2}{2\mu c^2 r^2}-E_{c.m.}\right]^{1/2}dr\right\},$$

where $E_{c.m.}$ is the deuteron-deuteron center of mass energy.

The branching ratio thus calculated, using alpha and deuteron spatial distribution parameters (K and λ in the notation of Flowers and Mandl) determined from recent tabulations of root mean square radii, and using a minimum barrier radius of 4 fm, is plotted as a function of energy in Fig. 5. The agreement with the measured cross sections is fairly good at energies in excess of about 1 MeV. The agreement with our data (at energies below 100 keV) is however, atrocious, with the measured branching ratio exceeding the predicted ratio by more than five orders of magnitude. Naturally, the predicted branching ratio is dependent upon the particular set of input parameters; however, no choice of parameters which give a reasonable fit at high energies would improve the character of fit at low energies.

The source of this disagreement is not clear. The source of the predicted energy dependence of the branching ratio is, however, not difficult to understand. The $^2H(d,\gamma)^4He$ cross section is dominated, at low energies, by the barrier penetration probability. For $l=2$ and for $\mu c^2=938$ (MeV) the integrand in the expression for P_l assumes the value (with r in fm and $E_{c.m.}$ in MeV)

$$\left[\frac{1.44}{r}+\frac{124}{r^2}-E_{c.m.}\right]^{1/2}.$$

The $^2H(d,p)^3H$ cross section, on the other hand, is a predominantly S wave transition and consequently at low energies is determined by the factor P_0, whose integrand is

$$\left[\frac{1.44}{r}-E_{c.m.}\right]^{1/2}.$$

The $^2H(d,\gamma)^4He$ penetration probability is thus dominated, for radii out to 20 fm, by the centrifugal barrier, whereas the much weaker Coulomb barrier is the sole component of the $^2H(d,p)^3H$ penetration probability. Indeed, the ratio of the penetration probabilities for $R_{min}=2$ and $E_{c.m.}=0.025$ MeV is easily calculated to be

$$\frac{P_2}{P_0}=3.7\times10^{-7}\ .$$

A possible explanation of the disagreement of low energies is therefore a small admixture of an $M1$ component to the capture process.

Such an $M1$ component is forbidden under the assumptions of a totally symmetric, two boson deuteron-deuteron initial continuum state and an S-state alpha particle ground state wave function. An $M1$ transition is, however, allowed by coupling the recently discovered[14] D state component of the alpha particle ground state to the allowed[11] quintet D state in the deuteron-deuteron continuum wave function. The capture cross section will be proportional to the square of the alpha particle D state component and to the square of the sum of the neutron and proton magnetic moments. ($\mu_n=-1.91$ and $\mu_p=2.79$ in nuclear magneton units.)

Indeed a plane wave estimate of the $M1$ contribution to the d-d capture cross section at low energies may be borrowed from the textbook calculation of the n-p capture cross section.

$$\sigma_{M1}(d+d\to\alpha+\gamma)\simeq\pi\left[\frac{2B}{E_{lab}}\right]^{1/2}\left[\frac{B}{MC^2}\right]\left[\frac{\hbar c}{MC^2}\right]^2\times(\mu_n+\mu_p)^2P_0(E_{c.m.})D_\alpha^2\ ,$$

where M is the deuteron rest mass, E_{lab} is the laboratory energy of the deuteron, B is the binding energy of a deuteron to an alpha particle, and D_α is the amplitude of the D state component in the alpha particle ground state.

The branching ratio calculated with $D_\alpha=0.05$ (Ref. 14) and assuming no interference between the $M1$ and $E2$ contributions is shown in Fig. 5 and is labeled ($E2+\epsilon M1$).

The agreement of the mixed $E2$ $M1$ calculation with the low energy data is embarrassingly good given the rough approximation of the $M1$ calculation. We would only point out that an inhibited $M1$ contribution, as permitted by the alpha particle ground state D component, appears to offer one possible solution to the enormous discrepancy between the measured branching ratio and the ratio calculated assuming a pure $E2$ capture process.

Another possible solution to this discrepancy might lie in the feasibility of a two-step radiative capture process. For example, one could imagine the processes $^2H(d,n)^3He(n,\gamma)^4He$ or $^2H(d,p)^3H(p,\gamma)^4He$ as prime candidates for such two-step reactions. While multistep reactions have been shown to be extremely crucial to our understanding of purely hadronic nuclear transfer reactions,[15] there have been few discussions of the possible importance of such contributions to radiative processes.[16]

Given the extent of the discrepancy between the measured branching ratio at low energies and the straightforward calculations of Flowers and Mandl, a serious theoretical effort, perhaps in the directions which we have indicated, aimed at understanding the low energy behavior of this reaction, is warranted.

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[1]Nelson Jarmie, R. E. Brown, and R. A. Hardekopf, Phys. Rev. C **29**, 2031 (1984); Nelson Jarmie and Ronald E. Brown, Los Alamos National Laboratory Report LA-UR-84-3349, 1984.

[2]F. E. Cecil and F. J. Wilkinson, Phys. Rev. Lett. **53**, 767 (1984).

[3]F. E. Cecil and David E. Newman, Nucl. Instrum. Methods **221**, 449 (1984); S. S. Medley and Hans Hendel, Bull. Am. Phys. Soc. **26**, 980 (1982); and Princeton Plasma Physics Laboratory Report No. PPPL-1950, 1982.

[4]J. Yang, M. S. Turner, G. Steigman, D. N. Schramm, and K. Olive, Astrophys. J. **281**, 493 (1984); Robert, V. Wagoner, William A. Fowler, and F. Hoyle, *ibid.* **148**, 3 (1967).

[5]C. Rolfs and H. P. Trautvetter, Annu. Rev. Nucl. Part. Sci. **28**, 122 (1978).

[6]S. L. Davis *et al.*, Plasma Phys. **25**, 189 (1983).

[7]R. W. Zurmuhle, W. E. Stephens, and H. H. Staub, Phys. Rev. **132**, 751 (1963); W. E. Meyerhof, W. Feldman, S. Gilbert, and W. O'Connel, Nucl. Phys. **A131**, 489 (1969); J. M. Poutissou and W. Del Bianco, *ibid.* **A199**, 517 (1973).

[8]G. H. Miley, H. Towner, and N. Ivich, University of Illinois Report No. C00-2218-17, 1974.

[9]F. Ajzenberg-Selove and C. L. Bush, Nucl. Phys. **A336**, 1 (1980).

[10]P. J. Grant, F. C. Flack, J. G. Rutherglen, and W. M. Deuchars, Proc. Phys. Soc. London, Sect. A **67**, 751 (1954).

[11]B. H. Flowers and F. Mandl, Proc. R. Soc. London, Ser. A **206**, 131 (1951).

[12]B. D. Anderson, M. R. Dwarakanath, J. S. Schweitzer, and A. V. Nero, Nucl. Phys. **A233**, 286 (1974).

[13]E. J. Konopinski and E. Teller, Phys. Rev. **73**, 822 (1948).

[14]H. R. Weller, P. Colby, N. R. Roberson, and D. R. Tilley, Phys. Rev. Lett. **53**, 1325 (1984); B. C. Karp, E. J. Ludwig, W. J. Thompson, and F. D. Santos, *ibid.* **53**, 1619 (1984); J. A. Tostevin, J. M. Nelson, O. Karban, A. K. Basak, and S. Roman, Phys. Lett. **149B**, 9 (1984).

[15]P. D. Kunz, in *Proceedings of the International Conference on Light Ion Reaction Mechanism, Osaka, 1983,* edited by H. Ogata, S. Katayama, and T. Kammuri (RCNP, Osaka University, Osaka, 1983), p. 304.

[16]T. A. Tombrello and G. C. Phillips, Bull. Am. Phys. Soc. **6**, 295 (1961); T. A. Tombrello, Ph.D. thesis, Rice University, 1961 (unpublished).