

# PHYSICAL REVIEW C

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### Specific distortion effects in the five-nucleon system

P. N. Shen and Y. C. Tang

*School of Physics, University of Minnesota, Minneapolis, Minnesota 55455*

Y. Fujiwara

*Department of Physics, University of Michigan, Ann Arbor, Michigan 48109*

H. Kanada

*Department of Physics, Niigata University, Niigata 950-21, Japan*

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The effects of specific distortion in the channel spin  $\frac{3}{2}$  state of the  $d + {}^3\text{He}$  system are considered by including in the resonating-group formulation inelastic channels involving deuteron pseudo-states. The result shows that, because of the high compressibility of the deuteron cluster, these effects are quite important in the  $l=0$  configuration. With specific distortion taken into account, it is found that a satisfactory explanation of the experimental data on  $d + {}^3\text{He}$  resonance state and scattering angular distributions can be achieved.

#### I. INTRODUCTION

In a previous publication,<sup>1</sup> hereafter referred to as CTT, specific distortion effects in the channel spin  $s = \frac{1}{2}$  state of the  $d + {}^3\text{He}$  (or  $d + {}^3\text{H}$ ) system were examined by performing a coupled-channel calculation involving both  $d + {}^3\text{He}$  and  $p + \alpha$  cluster configurations. The results showed that, because of the high compressibility of the deuteron cluster, these effects are quite important. In particular, it was found that the  $d + {}^3\text{He}$  phase shifts obtained in the coupled-channel study differ appreciably from those obtained in a single-configuration calculation.<sup>2</sup>

There exists, however, an undesirable feature in the CTT investigation. For the purpose of simplifying the calculation, specific distortion effects in the channel spin  $s = \frac{3}{2}$  state have not been explicitly considered. As a result, it became necessary to perform the calculation by adopting an effective nucleon-nucleon potential in the  $s = \frac{3}{2}$  state which is substantially stronger than that in the  $s = \frac{1}{2}$  state. This is clearly an unsatisfactory procedure, considering the fact that the deuteron cluster has been shown to be easily distortable.<sup>1,3</sup> In this investigation, our purpose is to correct this defect by taking specific distortion effects in the  $s = \frac{3}{2}$  state explicitly into account. This will be achieved by introducing inelastic channels involving cluster pseudo-states,<sup>4</sup> as has been done in recent studies of the  ${}^3\text{He} + \alpha$  and  $d + \alpha$  systems.<sup>5,6</sup>

In the following section, a brief discussion of the

$d + {}^3\text{He}$  formulation in the  $s = \frac{3}{2}$  state will be given. Results for the phase shift, transmission coefficient, and differential scattering cross section are presented in Sec. III. Finally, in Sec. IV, concluding remarks are made.

#### II. BRIEF DISCUSSION OF THE FORMULATION

The coupled-channel resonating-group formulation in the  $s = \frac{1}{2}$  state with  $d + {}^3\text{He}$  and  $p + \alpha$  cluster configurations is described in CTT and, hence, will not be further discussed here. In the  $s = \frac{3}{2}$  state, the trial wave function is chosen to be

$$\Psi_4 = \sum_{i=1}^3 \mathcal{A}[\phi_3 \phi_d^i F_i(\mathbf{R}) \xi_4 Z(\mathbf{R}_{\text{c.m.}})], \quad (1)$$

where  $\mathcal{A}$  denotes the antisymmetrization operator,  $\xi_4$  denotes the spin-isospin function appropriate in our present case with channel-spin multiplicity  $\lambda=4$ , and  $Z(\mathbf{R}_{\text{c.m.}})$  is any normalizable function describing the motion of the total center of mass. The function  $\phi_3$  describes the spatial behavior of the three-nucleon cluster and is given in CTT. For the choice of the deuteron ground-state function  $\phi_d^1$  and excited pseudo-state functions  $\phi_d^2$  and  $\phi_d^3$ , we use the procedure of first adopting a set of three basic functions

$$\chi_j = \exp \left[ \frac{1}{2} \alpha_j \sum_{k=1}^2 (\mathbf{r}_k - \mathbf{R}_d)^2 \right], \quad j=1-3 \quad (2)$$

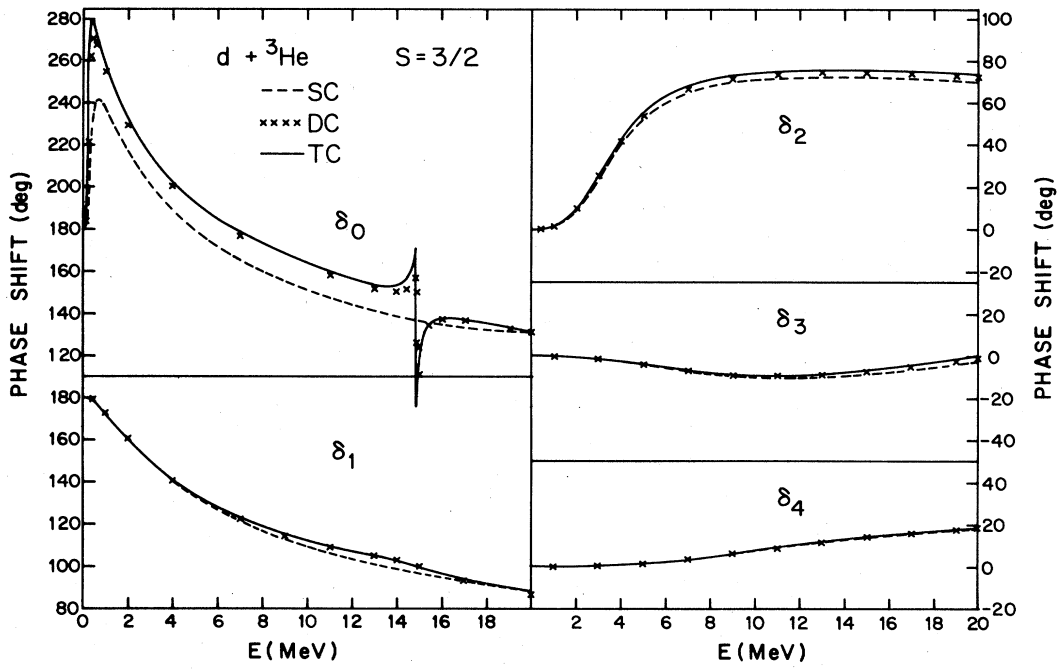


FIG. 1. Phase shifts in the  $s = \frac{3}{2}$  state of the  $d + {}^3\text{He}$  system. Dashed curves, crosses, and solid curves represent results obtained with the SC, DC, and TC calculations, respectively.

and then performing an orthonormalization process to minimize the expectation values  $E_i$  of the deuteron Hamiltonian. The results for the nonlinear variational parameters  $\alpha_j$  turn out to be

$$\begin{aligned} \alpha_1 &= 0.07284 \text{ fm}^{-2}, \\ \alpha_2 &= 0.3657 \text{ fm}^{-2}, \\ \alpha_3 &= 1.4696 \text{ fm}^{-2}, \end{aligned} \tag{3}$$

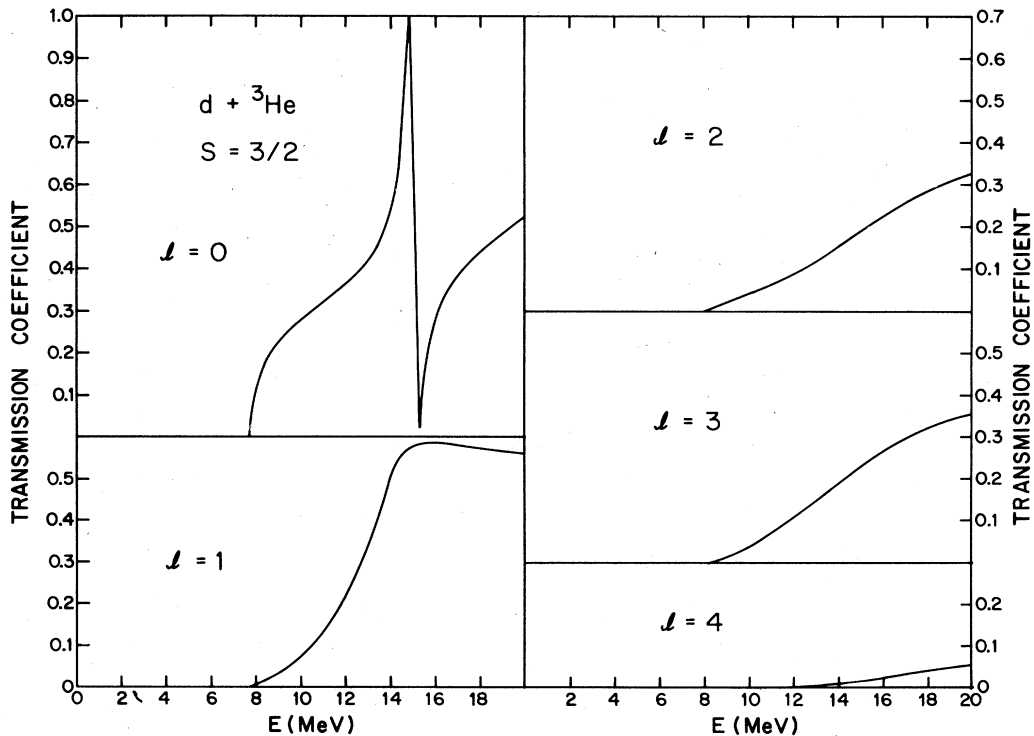


FIG. 2. Transmission coefficients obtained with the TC calculation in the  $s = \frac{3}{2}$  state of the  $d + {}^3\text{He}$  system.

TABLE I. Variational results for deuteron configurations.

Cluster wave function	$A_j^i$			$E_i$ (MeV)	$\bar{R}_i$ (fm)
	$j=1$	$j=2$	$j=3$		
$\phi_d^1$	0.04385	0.1592	0.2520	-2.200	1.899
$\phi_d^2$	0.1214	-0.3194	0.1199	5.486	3.470
$\phi_d^3$	0.05044	-0.5290	1.4174	45.410	1.739

and the deuteron spatial wave functions are

$$\phi_d^i = \sum_{j=1}^3 A_j^i \chi_j. \quad (4)$$

The coefficients  $A_j^i$ , together with the corresponding energies  $E_i$  and rms matter radii  $\bar{R}_i$ , are listed in Table I. As

can be noted from this table, the deuteron in its ground state has a binding energy of 2.20 MeV and a rms matter radius of 1.899 fm. These values are very close to those determined with the ground-state eigenfunction.<sup>7</sup>

The microscopic five-nucleon Hamiltonian is given by

$$H = -\frac{\hbar^2}{2M} \sum_{i=1}^5 \nabla_i^2 + \sum_{i>j=1}^5 V_{ij} - T_{c.m.}, \quad (5)$$

with  $T_{c.m.}$  being the kinetic-energy operator of the total center of mass and  $\hbar^2/2M = 20.735$  MeV fm<sup>2</sup>. The nucleon-nucleon potential  $V_{ij}$  is given in CTT. It contains an exchange-mixture parameter  $u$  which, in the single-channel  $s = \frac{3}{2}$  calculation performed in CTT, was taken to be 1.10 in order to fit the experimental resonance energies of the  $l=0$  states in <sup>5</sup>He and <sup>5</sup>Li. In our present study, it should especially be noted that, since specific distortion is taken into account by considering pseudo-inelastic effects, the  $u$  value of 0.95, determined in CTT by fitting empirical  $n + \alpha$ ,  $p$ -wave phase shifts ( $s = \frac{1}{2}$ ), should now be adopted.

To gain a clear understanding of the effects of specific distortion, we carry out the investigation in several steps. These steps include a single-configuration (SC;  $d + {}^3\text{He}$ )

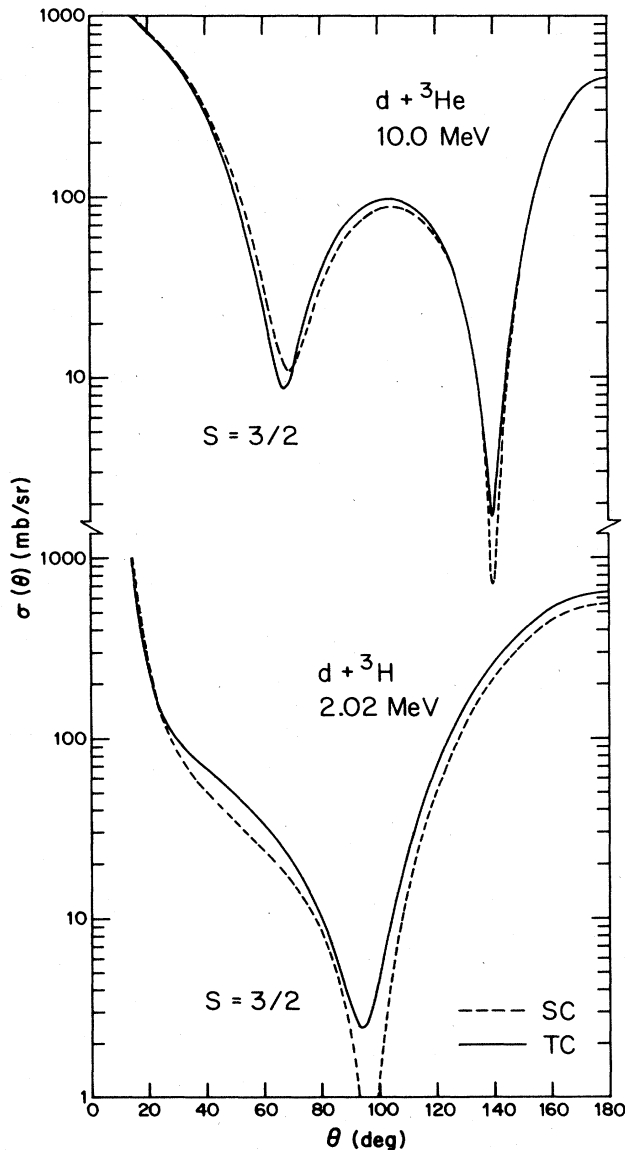


FIG. 3. Comparison of  $s = \frac{3}{2}$  differential scattering cross sections obtained with the SC and TC calculations.

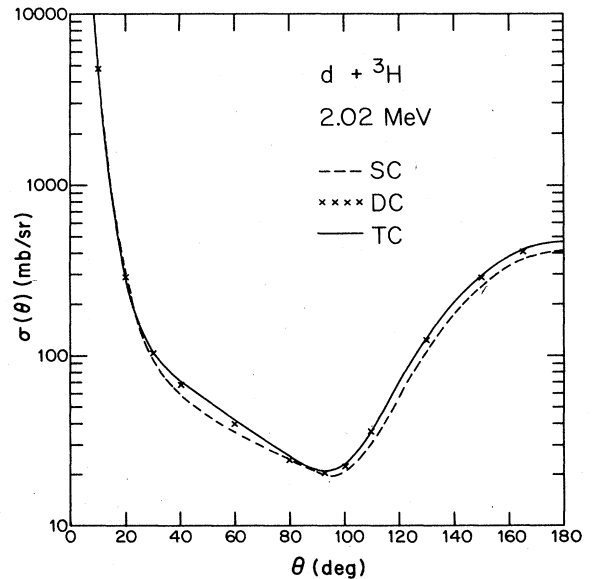


FIG. 4. Comparison of  $d + {}^3\text{H}$  differential scattering cross sections at 2.02 MeV, obtained with the SC, DC, and TC calculations. Contributions from both channel-spin states are included.

calculation with  $F_2(\mathbf{R})$  and  $F_3(\mathbf{R})$  set as zero, a double-configuration (DC;  $d + {}^3\text{He}$  and  $d^* + {}^3\text{He}$ ) calculation with  $F_3(\mathbf{R})$  set as zero, and a triple-configuration (TC;  $d + {}^3\text{He}$ ,  $d^* + {}^3\text{He}$ , and  $d^{**} + {}^3\text{He}$ ) calculation involving all three channels.

The coupled integrodifferential equations satisfied by the variational amplitudes  $F_i(\mathbf{R})$  are solved by a variational technique discussed by Kamimura.<sup>8</sup> From the results, we extract the values of the  $S$ -matrix elements. In the following, we shall discuss, in the energy region below 20 MeV, the behavior of the  $d + {}^3\text{He}$  phase shift  $\delta_l$  and the transmission coefficient<sup>9</sup>  $\eta_l$  for inelastic scattering to the  $d^* + {}^3\text{He}$  pseudo-channel.

### III. RESULTS

#### A. Phase shift and transmission coefficient

The  $d + {}^3\text{He}$  phase shifts  $\delta_l$  and transmission coefficients  $\eta_l$  are shown in Figs. 1 and 2 for energies up to 20 MeV. For the phase shifts, the values obtained in the SC (dashed curves), DC (crosses), and TC (solid curves) cases are shown, while for the transmission coefficients only the values obtained in the TC case are depicted.

By examining Figs. 1 and 2, one notes the following interesting features:

(i) Phase-shift and transmission-coefficient curves in  $l=0$  and 1 states show that dispersionlike resonances are present in DC and TC cases. These resonances arise from the coupling between the  $d + {}^3\text{He}$  configuration and the pseudo-inelastic  $d^* + {}^3\text{He}$  configuration and are not likely to have much physical significance.

(ii) Results from DC and TC calculations are quite similar in all orbital angular-momentum states. This indicates that, because of high energy threshold, the  $d^{**} + {}^3\text{He}$  configuration has little influence in the considered energy region.

(iii) Specific distortion seems to be appreciable mainly in the  $l=0$  state. This is quite clearly a consequence of the centrifugal-barrier effect.

Using the phase-shift values calculated in the very low-energy region, one can determine the resonance energies of the  $(l,s)=(0, \frac{3}{2})$  states in  ${}^5\text{He}$  and  ${}^5\text{Li}$ . The results are given in Table II. Here one finds again that specific-distortion effects are important in the  $l=0$  state and that the  $d^* + {}^3\text{He}$  ( $d^* + {}^3\text{H}$ ) configuration is more significant than the  $d^{**} + {}^3\text{He}$  ( $d^{**} + {}^3\text{H}$ ) configuration. Experimentally,<sup>10</sup> these resonance states are found at 0.06 and 0.27 MeV in  ${}^5\text{He}$  and  ${}^5\text{Li}$ , respectively, which compare quite well with the calculated values in the TC case.

#### B. Differential scattering cross section

The influence of specific distortion is further examined by comparing, in Fig. 3, the  $s = \frac{3}{2}$  differential scattering cross sections calculated in the SC and TC cases for  $d + {}^3\text{H}$  scattering at 2.02 MeV and  $d + {}^3\text{He}$  scattering at 10 MeV. Here one sees that, because specific distortion effects are significant only in low orbital angular-momentum states, the SC and TC results are in fact only moderately different, especially at the higher energy.

Differential cross sections for  $d + {}^3\text{H}$  scattering at 2.02 MeV, calculated by combining the  $s = \frac{1}{2}$  result obtained in CTT and the  $s = \frac{3}{2}$  result obtained here, are shown in Fig. 4. From this figure, it is noted that the inclusion of specific distortion at this energy increases the cross-section values by about 20% in the angular region around  $50^\circ$  and by about 15% in the backward angular region.

#### C. Comparison with experiment

Although virtual breakup effects of the deuteron cluster are considered in our calculation through the inclusion of the  $d^* + {}^3\text{He}$  cluster configuration, we must emphasize that these effects are not sufficiently taken into account because the  $d^* + {}^3\text{He}$  channel has a rather high energy threshold equal to 7.69 MeV. Thus, for a comparison with experimental data, we must again introduce phenomenological imaginary potentials into the resonating-group formulation in order to account approximately for the various reaction channels not explicitly considered. In the present study, we adopt in the  $d + {}^3\text{He}$  channel the same absorptive potential  $W_4(R)$  as used in CTT. The single depth parameter  $W_{04}$  is then adjusted at each energy to yield a best overall fit to the experimental differential scattering cross-section result.<sup>11</sup> The resultant values for this parameter and the corresponding total reaction cross sections turn out to be rather similar to those given in Table I of CTT.

A comparison between calculated and experimental<sup>12</sup> differential cross-section results for  $d + {}^3\text{H}$  scattering at 2.02, 4.19, and 6.6 MeV is shown in Fig. 5. As is seen from this figure, there is very satisfactory agreement. This indicates that, with specific distortion effects taken into account in the  $s = \frac{3}{2}$  state, one can now adopt the same effective nucleon-nucleon potential in both channel-spin states, a feature which is obviously very desirable from the viewpoint of a microscopic calculation.

The situation is somewhat different at higher energies. In Fig. 6, calculated and experimental<sup>13</sup> results for  $d + {}^3\text{He}$  scattering at 8.64, 10.0, and 13.84 MeV are compared. Here one notes that, while the calculation correctly yields the essential features of the experimental data, the fit does become progressively less satisfactory as the energy increases. At present, we are not certain as to the exact cause for this type of discrepancy, although we suspect that it may result from the presence of a rather broad compound-nucleus resonance level which is not well described by the wave functions of this calculation or that the calculated  $F$ - and  $G$ -wave phase shifts may not be sufficiently accurate in this energy region.

### IV. CONCLUSION

Specific distortion effects in the channel spin  $s = \frac{3}{2}$  state of the  $d + {}^3\text{He}$  system are studied by including in the resonating-group formulation inelastic channels involving deuteron pseudo-states. The result shows that, because the deuteron cluster is easily compressible, these effects are quite important, but mainly only in the  $l=0$  configuration. With specific distortion taken into account, it is found that, in contrast to the calculation reported in CTT,

TABLE II. Resonance energies of  $(l,s)=(0,\frac{3}{2})$  states in  ${}^5\text{He}$  and  ${}^5\text{Li}$ .

Nucleus	Resonance energy (MeV)		
	SC	DC	TC
${}^5\text{He}$	0.216	0.044	0.023
${}^5\text{Li}$	0.596	0.286	0.236

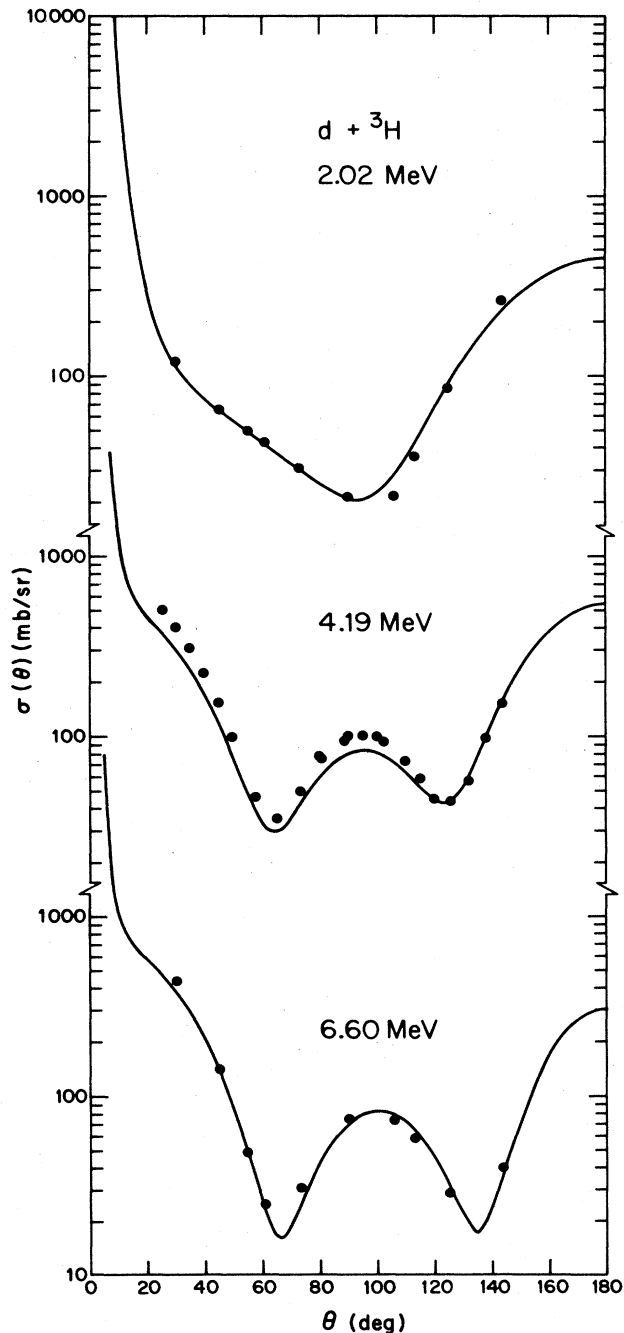


FIG. 5. Comparison of calculated differential cross sections for  $d + {}^3\text{H}$  scattering at 2.02, 4.19, and 6.6 MeV with experimental data.

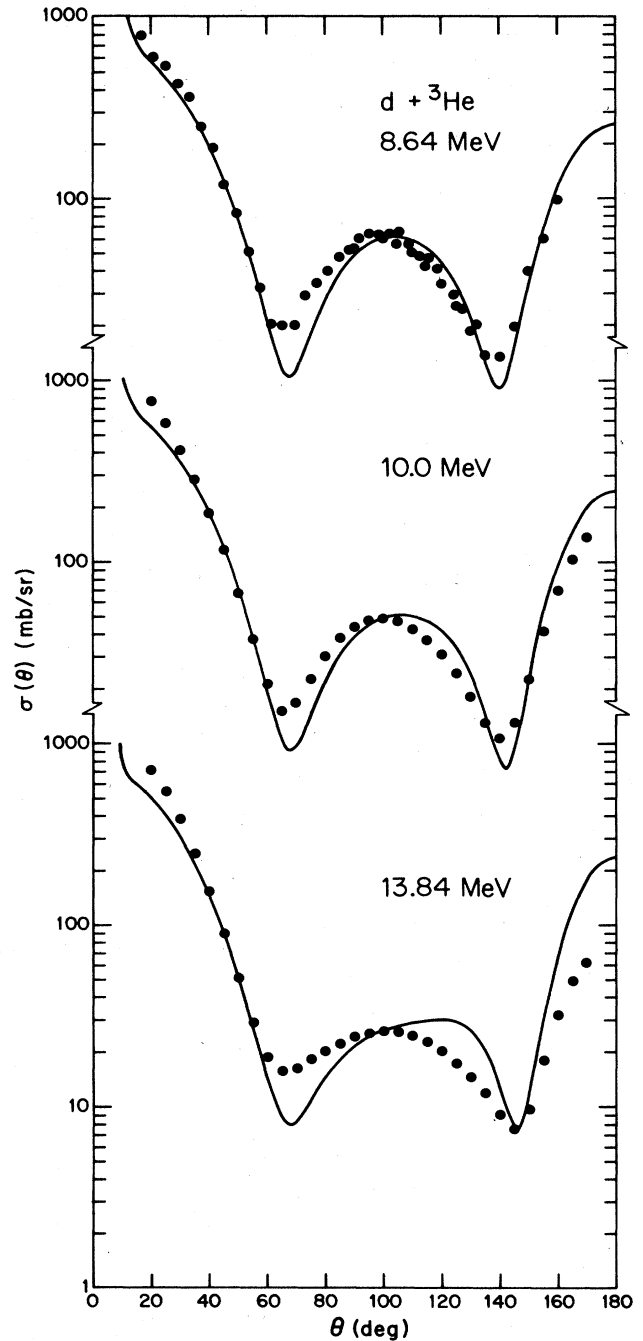


FIG. 6. Comparison of calculated differential cross sections for  $d + {}^3\text{He}$  scattering at 8.64, 10.0, and 13.84 MeV with experimental data.

a satisfactory explanation of the experimental data can now be achieved by employing the same effective nucleon-nucleon potential in both channel-spin states.

The present calculation demonstrates that the pseudo-state method is useful in treating specific-distortion effects. We should emphasize, however, that this method cannot be indiscriminately applied to any nuclear system of interest without first making a careful qualitative examination of the nature of the system. In the  $s = \frac{1}{2}$  state

of the  $d + {}^3\text{He}$  system, for example, a similar study employing  $d + {}^3\text{He}$ ,  $d^* + {}^3\text{He}$ , and  $d^{**} + {}^3\text{He}$  structures, but omitting the  $p + \alpha$  cluster configuration, yielded unsatisfactory results. The reason for this is, in fact, rather simple. For a satisfactory description of the behavior of the system, it has been noted recently<sup>14</sup> that the energetically most-favored cluster configuration must always be included in the resonating-group formulation. Thus, the failure of the above-mentioned  $s = \frac{1}{2}$  calculation is attributable to the fact that the favored  $p + \alpha$  configuration cannot be well represented by a linear superposition of  $d^* + {}^3\text{He}$  and  $d^{**} + {}^3\text{He}$  wave functions.

Based on the results obtained from present and previ-

ous<sup>6</sup> investigations, we can conclude that, if a nuclear system contains easily distortable clusters such as deuteron cluster,  ${}^6\text{Li}$  cluster, and so on, specific-distortion effects must be properly considered. From the computational viewpoint, this is somewhat unfortunate since multiconfiguration resonating-group calculations for heavier systems are tedious to perform, especially in situations where the pseudo-state method is not readily applicable and rearrangement channels have to be taken into account.

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<sup>1</sup>F. S. Chwieroth, Y. C. Tang, and D. R. Thompson, *Phys. Rev. C* **9**, 56 (1974).

<sup>2</sup>F. S. Chwieroth, R. E. Brown, Y. C. Tang, and D. R. Thompson, *Phys. Rev. C* **8**, 938 (1973).

<sup>3</sup>H. Jacobs, K. Wildermuth, and E. Wurster, *Phys. Lett.* **29B**, 455 (1969); D. R. Thompson and Y. C. Tang, *Phys. Rev. C* **8**, 1649 (1973).

<sup>4</sup>Z. C. Kuruoglu and F. S. Levin, *Phys. Rev. Lett.* **48**, 899 (1982).

<sup>5</sup>H. Kanada, T. Kaneko, and Y. C. Tang, *Nucl. Phys.* **A380**, 87 (1982).

<sup>6</sup>H. Kanada, T. Kaneko, and Y. C. Tang, *Nucl. Phys.* **A389**, 285 (1982); H. Kanada, T. Kaneko, M. Nomoto, and Y. C. Tang, *Prog. Theor. Phys.* **72**, 369 (1984).

<sup>7</sup>F. S. Chwieroth, Y. C. Tang, and D. R. Thompson, *Nucl. Phys.* **A189**, 1 (1972).

<sup>8</sup>M. Kamimura, *Prog. Theor. Phys. Suppl.* **62**, 236 (1977).

<sup>9</sup>The transmission coefficient defined here was called the absorption coefficient in CTT.

<sup>10</sup>F. Ajzenberg-Selove, *Nucl. Phys.* **A413**, 1 (1984).

<sup>11</sup>The only exception is for  $d + {}^3\text{H}$  scattering at 2.02 MeV, where the parameter  $W_{04}$  is adjusted to yield the measured total reaction cross section [J. E. Brolley, Jr., T. M. Putnam, L. Rosen, and L. Stewart, *Phys. Rev.* **117**, 1307 (1960)].

<sup>12</sup>M. Ivanovich, P. G. Young, and G. G. Ohlsen, *Nucl. Phys.* **A110**, 441 (1968).

<sup>13</sup>T. R. King and R. Smythe, *Nucl. Phys.* **A183**, 657 (1972).

<sup>14</sup>Y. Fujiwara and Y. C. Tang, *Phys. Rev. C* **29**, 2025 (1984).