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Evidence for an underlying SU(3) structure near neutron number $N = 104$

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It is shown that an underlying SU(3) structure of the interacting boson approximation is exhibited by nuclei near neutron number 104 in the rare earth region. This is particularly evident in Yb and Hf and to a lesser extent in the heavy Er and light W nuclei. Although the structure and mixing effects are complex, this region seems to represent the closest approach to the SU(3) limit yet observed in heavy nuclei.

One of the important concepts in nuclear structure is that of symmetries and one of the interesting aspects of the interacting boson approximation (IBA) model[1] is that its inherent group structure leads to the appearance of three dynamical symmetries or limiting coupling schemes evolving from the parent group U(6). These symmetries are usually labeled by their group notation, that is, U(5), SU(3), and O(6). The U(5) limit represents an anharmonic vibrator; SU(3) is a special case of the deformed symmetric rotor and the O(6) limit is an axially asymmetric, γ-unstable[2] (γ-independent) rotor. When the IBA was first proposed, it was thought that many examples of the U(5) and SU(3) symmetries were well known, and soon thereafter the O(6) limit was also discovered.[2] However, closer inspection of the detailed structure of the SU(3) limit shows that it exhibits several very particular features and that until now, no nuclei are known that adequately display all of them. For example, in the strict SU(3) limit, states of the same spin in the β and γ vibrational bands are degenerate and $E2$ transitions from either of these bands to the ground state are forbidden by the SU(3) selection rules. Although ^{156}Gd has often been cited[3] as a typical SU(3) nucleus on the basis of the former feature, the relatively strong $\gamma \rightarrow g$ band $E2$ transitions require deviations from the SU(3) limit that are comparable to those of most other deformed nuclei from Gd–W. Indeed, if the wave functions for typical deformed nuclei, such as ^{168}Er, are expanded in terms of SU(3) basis states, it is found[4] that they contain admixtures of "minor" amplitudes that typically range from 0.4 to 0.6, signaling a significant departure from those of the limiting symmetry.

Given the importance of symmetries both in themselves as facilitating the interpretation of a given nucleus, and as benchmarks for the simple treatment of neighboring nuclei, it is clearly of interest to search for nuclei that display more closely the SU(3) limit of the IBA. Interesting in this regard, therefore, is a recent study[5] of ^{178}Hf which disclosed relatively close lying β and γ bands as well as particular $\gamma \rightarrow g$ $E2$ branching ratios approaching those predicted by the SU(3) symmetry. This has prompted a more detailed inspection of this mass region and has led to the present Rapid Communication whose purpose is to show that nuclei near neutron number $N = 104$, most particularly the Yb and Hf isotopes, indeed represent just such an SU(3)-like region. At the same time it will be pointed out that there are clear deviations from SU(3) even here and that complex mixing effects with two quasiparticle excitations undoubtedly take place.

It is useful at this point to outline the various empirical quantities which can serve as signatures of the SU(3) limit. Figure 1 displays a highly schematic level scheme for a typical deformed nucleus involving a ground band, a γ-vibrational band, and a β-vibrational band, along with γ-ray transitions between them. The most characteristic and easily observable identifiers of the SU(3) limit are listed in a box in the lower right part of the figure. The γ and β bands form a separate representation from the ground state band. Thus, the SU(3) $E2$ selection rule forbidding changes of representation implies that both $\gamma \rightarrow g$ and $\beta \rightarrow g$ $B(E2)$ values should vanish. As noted above, in most deformed nuclei, which are not SU(3) nuclei, these selection rules are violated and $\gamma \rightarrow g$ transitions, in particular, are collective. [Typical $B(E2{:}2_\gamma^+ \rightarrow 0_g^+)$ values are several single particle units.] Therefore, a characteristic signature of the onset of SU(3) nuclei will be a sharp decrease in $\gamma \rightarrow g$

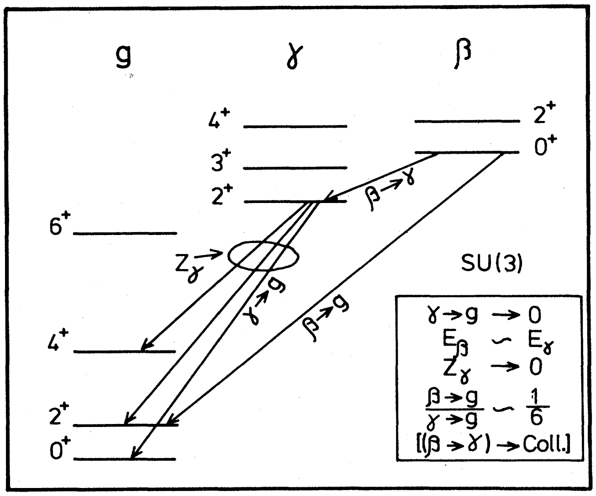

FIG. 1. Schematic level scheme for a deformed nucleus indicating the most important and observable signatures of an approach towards SU(3).

$E2$ transition rates.

Secondly, as noted above, in the strict SU(3) limit, states of equal spin in the β and γ bands should be degenerate. Each of these two signatures is suggestive of, but not sufficient to, establish an SU(3) region since, on the one hand, β-band energies fluctuate widely and might be accidentally degenerate with the γ band and, on the other, it is not easy to distinguish the forbiddeness of $\gamma \rightarrow g$ (or $\beta \rightarrow g$) transitions from a simple decreasing collectivity that can occur when these vibrations lie high in energy near the region of two quasiparticle states.

Since the γ and ground bands belong to different representations in the SU(3) limit, $\gamma \rightarrow g$ $B(E2)$ values, though weak,, should nevertheless exhibit branching ratios that approach for large boson numbers those of the Alaga rules. Empirically, most deformed nuclei show substantial deviations from these rules. It is traditional to describe[6] such deviations quantitatively in terms of a parameter Z_γ which can be thought of, either in the framework of geometrical models or the IBA, as characterizing the amount of mixing between γ and ground bands. A third signature of SU(3), then, is that empirical Z_γ values, deduced from the $\gamma \rightarrow g$ $B(E2)$ values, should therefore approach zero for nuclei close to this limit.

A fourth signature stems from a rather interesting relation between $\beta \rightarrow g$ and $\gamma \rightarrow g$ $E2$ transitions. Although both approach 0 in the SU(3) limit, it has been shown, both numerically[7] and by use of the coherent state formalism,[8] that the $B(E2)$ ratio

$$B(E2{:}2_\beta^+ \rightarrow 0_g^+)/B(E2{:}2_\gamma^+ \rightarrow 0_g^+)$$

actually approaches a finite limiting value (for large boson numbers) of approximately $\frac{1}{6}$.

Finally, a characteristic feature of the SU(3) limit is that $\beta \rightarrow \gamma$ transitions, which do not change representation, are allowed and remain collective. Unfortunately, this last criterion, which is true even in broken SU(3) calculations characterizing actual deformed nuclei, is rather useless in practice for the identification of SU(3) nuclei since, in precisely such nuclei, these levels are particularly close in energy and the low $\beta \rightarrow \gamma$ transition energies would lead to negligible transition strengths even for $B(E2)$ values of collective magnitude.

To summarize, there are at least four characteristic criteria which can be used to search for and identify a region of SU(3) symmetry. At the same time, of course, it must be realized that, given the specific nature of some of these signatures, namely, vanishing transition strengths, vanishing mixing effects, and a specific ratio of very weak $B(E2)$ values, one cannot expect precise adherence to the SU(3) selection rules. Nevertheless, in the remainder of this Rapid Communication each of these four criteria will be investigated in turn and it will be shown that the combined evidence from all of them suggests an underlying SU(3) symmetry near $N = 104$. This evidence is displayed in Figs. 2–4.[9–16]

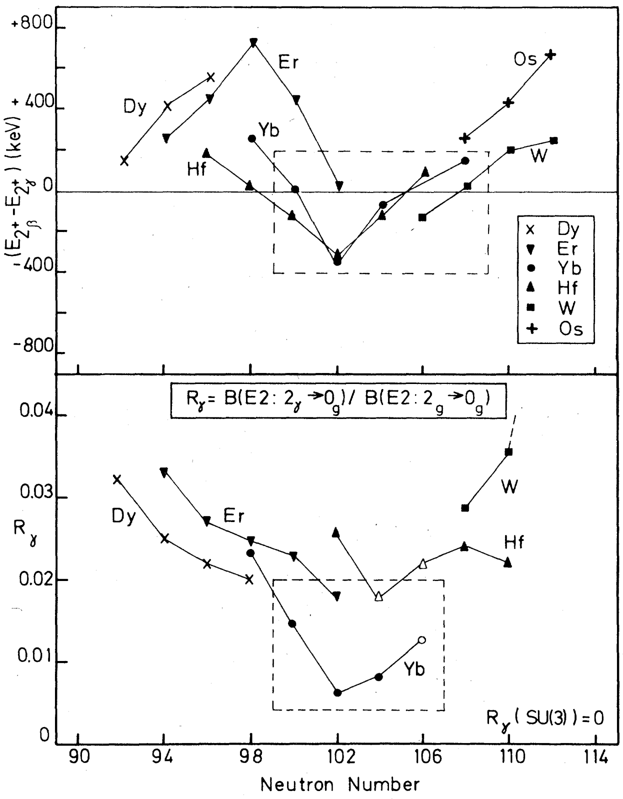

FIG. 2. Top: empirical systematics of the energy difference $E_{2^+_\beta} - E_{2^+_\gamma}$. The SU(3) value is zero. The β band is taken as the lowest excited 0^+ band. In a few cases where the 2_{β^+} level is not assigned, its energy has been estimated by adding a typical rotational energy for that nucleus to $E_{0^+_\beta}$: on the scale of the figure, this is a negligible approximation. Bottom: systematics of R_γ (defined in the figure). R_γ vanishes in the SU(3) limit. Open data points denote average values for nuclei where various measurements differ. The dashed boxes here, and in Figs. 3 and 4, are meant to highlight those nuclei where the characteristic SU(3) values are approached. Data from Refs. 5 and 9–16.

Figure 2 (bottom) shows the first of these, namely, the systematics of $\gamma \rightarrow g$ $E2$ strength in the rare earth nuclei. Although the $B(E2)$ ratio (denoted R_γ) shown is remarkably constant around 0.03 in most of this region, it is evident that, near $N = 104$, it drops towards zero, especially in ^{172}Yb and ^{174}Yb. Likewise, ^{176}Hf suggests a tendency in the same direction. The heavy Er and Dy nuclei and the light W nuclei also display systematics pointing toward similar minima near $N = 104$. Of course, this might be interpreted simply as reflecting a loss of collectivity. Since, indeed, the energies of the vibrational excitations near $N = 104$ do rise substantially, it is all the more important therefore to inspect the other signatures of SU(3) given above. The upper part of Fig. 2 shows one of these, the energy difference of the 2_β^+ and 2_γ^+ levels. It is remarkable that, in almost precisely the same region, this difference crosses zero. One must assess these results carefully, however. In the nuclei in this region there are several known excited 0^+ bands lying just above the lowest one, and complicated mixing can be expected. This is particularly true for $N = 102$ where, indeed, a five band mixing calculation[12] for ^{172}Yb has been reasonably successful. Although the β and γ bands in ^{172}Yb and ^{174}Hf can hardly be described as degenerate and, although there are rapid changes in β- and γ-band energies with neutron number in the region, it is likewise difficult to dismiss the unique clustering of close lying β and γ bands that occurs here as an accident.

Turning to the third criterion discussed above, namely, the approach of Z_γ values to 0, Fig. 3 shows that it is rather well fulfilled. In particular, at $N=102$ and $N=104$, Z_γ values for Yb, Hf, and even Os are all nearly consistent with the SU(3) limit. The Z_γ values for Dy and Er, as well as W, also drop very sharply in the same mass region. Again, caution is required in assessing this result. Many of the Z_γ values near the minimum are obtained solely from the decay the 2_γ^+ level since, even for the 3_γ^+ level, mixing effects with noncollective two quasiparticle excitations, particularly in ^{172}Yb, preclude[16] even the unambiguous determination of Z_γ. Nevertheless, the Z_γ effect is sufficiently strong that it is worth analyzing a bit further. In typical deformed nuclei, the effect of SU(3) breaking is to admix different SU(3) basis states in the actual wave functions.[4] The dominant mixing is $\Delta K=0$. If the Z_γ values of Fig. 3 are used to specify the one parameter (χ) that is needed[17] to determine the structure of calculated IBA wave functions, it is found[18] that the admixtures of the SU(3) β band in the calculated ground band, or of the $\beta\gamma$ SU(3) excitation in the calculated γ band are ≤ 0.08 for nuclei near the minimum in Z_γ. Clearly, this is an overestimate of the SU(3) purity: a similar calculation based on the R_γ values of Fig. 2 gives impurity amplitudes of ~ 0.2. Nevertheless, the approach to SU(3) is unmistakable.

Finally, Fig. 4 shows a ratio of $\beta \rightarrow g$ and $\gamma \rightarrow g$ $B(E2)$ values. As pointed out above, both of these $B(E2)$ values vanish in the SU(3) limit, but they should approach a finite ratio, namely, $\frac{1}{6}$. Once again, in the mass region near $N=102$–106, the empirical ratios for the Yb, Hf, and W nuclei all cross a value of $\frac{1}{6}$ and the Er nuclei seem to be approaching this value. As before, one cannot expect exact agreement for a ratio involving such small $B(E2)$ values. Nevertheless, $B(E2)$ ratios near the predicted limiting ratio in fact do characterize this mass region. It is also worth noting that, in the lighter nuclei near $N=98$, this $B(E2)$ ratio shows extremely large fluctuations and values that are orders of magnitude different from $\frac{1}{6}$. Near $N=94$ there is another crossing of the SU(3) value in Er. Here, however, this cannot be taken as a signature of SU(3) since the other characteristic SU(3) features do not appear. This, as well as the caveats cited in the discussion of each of these SU(3) signatures, highlights the importance of looking at a confluence of evidence that, in association, more than individually, can disclose an underlying symmetry even in a region of complex structure.

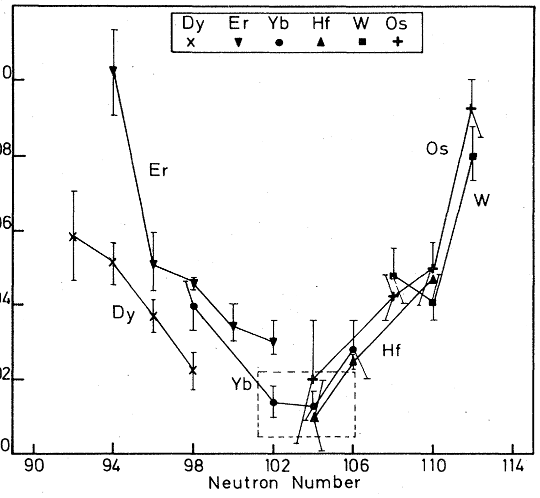

FIG. 3. Z_γ values. (See text for discussion of this quantity which describes the mixing of γ and ground bands.) The SU(3) limiting value is zero. From Refs. 5 and 9–16.

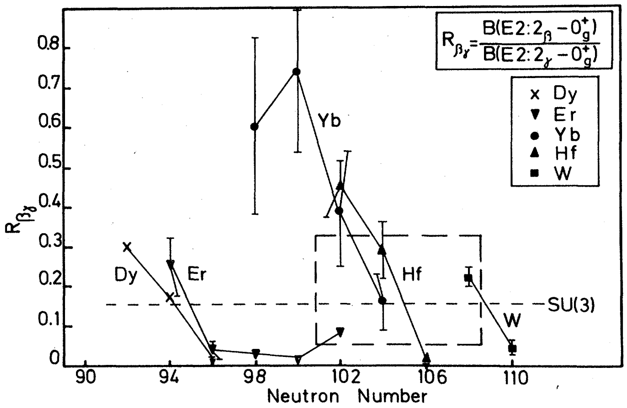

FIG. 4. Systematics of $R_{\beta\gamma}$ (defined in the figure). The SU(3) value is shown as a dashed line. From Refs. 9–16.

To summarize, the nuclei near $N=102$–106, especially the Yb and Hf isotopes, display a number of empirical features which, together, suggest an approach toward the SU(3) limit of the IBA. While sufficient data do not exist for these neutron numbers in Er, Dy, W, and Os, their systematics also point toward a similar structural evolution. While no single nucleus in this region fully displays all the features of the SU(3) limit (although ^{174}Yb and ^{176}Hf nearly do) and the undoubtedly complex mixing with two-quasiparticle states lying just above the β and γ bands has also been emphasized, this region on a whole nevertheless exhibits the closest approach to SU(3) of any known to date. An apt description might be that there is an underlying SU(3) symmetry, partially broken and partially obscured, but whose outlines can nevertheless be discerned. Whether or not its presence can be exploited in detailed IBA calculations is of course in doubt, since, already at the first intrinsic excitations, there is significant breaking of the SU(3) symmetry by interaction with noncollective levels that are outside the basis of the IBA. Nevertheless, these results are of use in providing a simple starting point for the interpretation of an extremely complex region and a benchmark for the treatment of neighboring even and odd mass nuclei, either in terms of supersymmetry ideas[19] or via numerical calculations. Moreover, with the evidence presented here, one now has in hand examples of nuclei resembling all three of the IBA symmetries which evolve from the parent U(6) group.

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