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Interpretation of the angular distribution of the ^{4}He$(\pi^-,n)^3$H reaction based on the Δ-hole model

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The angular distribution and the magnitude of the cross section of the ^{4}He$(\pi^-,n)^3$H reaction for pion kinetic energies from 100 to 250 MeV are reasonably well reproduced in the framework of the Δ-hole model with an improved overlap function $\langle\psi_{^3\mathrm{H}}|\psi_{^4\mathrm{He}}\rangle$. It is pointed out that the Δ-hole formation dominates the reaction in the forward region while the one nucleon mechanism becomes more important at larger angles.

The purpose of the present Rapid Communication is to give an interpretation for the angular distribution of the ^{4}He$(\pi^-,n)^3$H reaction through an interplay between reaction mechanisms and nuclear structure effects. There have been several theoretical calculations[1-4] of this reaction, but the calculated angular distributions have minima too deep and tend to fall off too rapidly compared with the observed ones.[5] Two reaction mechanisms are known to contribute to the reaction, i.e., the one-nucleon mechanism (ONM) and the two-nucleon mechanism (TNM). In the energy region we are interested in, i.e., $T_\pi = 100$–250 MeV, the momentum transfer involved in the reaction is more than 2 fm^{-1} even at forward angles. Therefore, the TNM is usually considered to be more favorable than the ONM since two nucleons can share the large momentum transfer in the TNM while it must be absorbed by one nucleon in the ONM. This is not the case for the ^{4}He$(\pi^-,n)^3$H reaction. The wave function for ^{4}He is accurately known[6-8] and the overlap function $\langle\psi_{^3\mathrm{H}}|\psi_{^4\mathrm{He}}\rangle$ which determines the ONM part has large momentum components. Thus the two mechanisms can compete in the present case and one has to include both in a proper way.

In our previous calculation,[4] we used a formalism based on the Δ-hole model[9-12] which enabled us to treat both mechanisms in a unified and consistent way. However, a simple Gaussian function was used for the overlap function and therefore the ONM contribution was very small. The calculated angular distribution did not show the second maximum observed in the experimental one and fell off too rapidly. In the present calculation, we use an improved overlap function and try to explain the observed angular distribution by a superposition of the contributions from the two mechanisms.

The delta degree of freedom is treated dynamically in our formalism. At present, the one Slater determinant approximation for the ground state of ^{4}He is inevitable for the actual calculation based on the above dynamical treatment. In order to determine the single particle wave function to be used in the Slater determinant, we use the argument by Gaudin *et al.*[13] They pointed out that given a density matrix calculated from a wave function with Jastrow-type correlations, one can construct a Slater determinant from the eigenfunctions of the density matrix with the eigenvalue close to one, which gives a density very similar to the original one, and also gives, in general, a good approximation to the expectation value of any one-body operator. We thus choose the single particle state in the Salter determinant so as to fit the charge form factor of ^{4}He calculated in Ref. 7 with the three-body force. The single particle wave function is expressed by harmonic oscillator wave functions as follows:

$$\psi_s(r) = \sum_{i=1}^{4} a_i R_{i-1\,s}(\nu r^2) \ , \tag{1}$$

where $R_{i-1\,s}(\nu r^2)$ is the s-wave harmonic oscillator wave function with the node number $i-1$, and $a_1=0.9391$, $a_2=-0.3060$, $a_3=0.1284$, $a_4=-0.0890$, and $\nu=0.734$ fm^{-2}. We approximate the overlap function $\langle{}^3\mathrm{H}|{}^4\mathrm{He}\rangle$ by this single particle wave function. The momentum distribution of the overlap function thus obtained is shown in Fig. 1.

The formulation of the Δ-hole model for the $(\pi,$N$)$ reaction is given in Refs. 4 and 12. The reaction amplitude consists of the background term T_{bg} and the Δ-hole term $T_{\Delta\mathrm{h}}$, i.e.,

$$T = T_{bg} + T_{\Delta\mathrm{h}} \ . \tag{2}$$

The background term T_{bg} represents the contribution of all the processes in which Δ is not explicitly involved, and is assumed to be given by the distorted wave approximation for the ONM, where the $P_{33}(\Delta)$ contribution is excluded from the pion optical potential:

$$\langle \mathbf{p}, \nu_\mathrm{n}^{-1} | T_{bg} | \mathbf{k}, 0\rangle = \langle \chi^{(-)}_{\mathrm{N},\mathbf{p}} | F_{\pi\mathrm{NN}} | \psi_{\nu_\mathrm{h}}, \chi^{(+)}_{\pi,\mathbf{k}}\rangle \ , \tag{3}$$

where $\mathbf{k}$ and $\mathbf{p}$ are the incident pion and outgoing nucleon momenta, respectively, and ν_h specifies a state of the residual nucleus with one hole in an orbit described by the wave function ψ_{ν_h}, which we identify with ψ_s given by Eq. (1). χ_N and χ_π are the nucleon and pion distorted waves, respectively. $F_{\pi NN}$ is the πNN vertex and is given by

$$F_{\pi NN} = \frac{f_{\pi NN}}{\mu}\tau_\alpha\left(\sigma\cdot\nabla_\pi - \frac{E}{M}\nabla_N\right)\frac{\Lambda_{\pi NN}^2}{\Lambda_{\pi NN}^2 - \nabla_\pi^2} , \quad (4)$$

where $f_{\pi NN}$ is the πNN coupling constant, μ is the pion mass, τ and σ are the isospin and spin matrices for the nucleon, respectively, and α denotes the charge state of the pion. E is the pion energy, M is the nucleon mass, $\Lambda_{\pi NN}$ is the cutoff mass for the vertex, and ∇_π should be operated only on the pion distorted wave. The Δ-hole term $T_{\Delta h}$ represents the contribution of the formation of Δ-hole states followed by the emission of a nucleon leading to a one-hole state.

$$\langle \mathbf{p},\nu_n^{-1}|T_{\Delta h}|\mathbf{k},0\rangle = \sum_{\substack{\nu_\Delta'\nu_h' \\ \nu_\Delta''\nu_h''}} \langle \chi_{N,\mathbf{p}}^{(-)}\psi_{\nu_h'}|t_{NN,N\Delta}|\psi_{\nu_\Delta'}\psi_{\nu_h}\rangle\langle\nu_\Delta'\nu_h'^{-1}|G_{\Delta h}|\nu_\Delta''\nu_h''^{-1}\rangle\langle\psi_{\nu_\Delta''}|F_{\pi N\Delta}|\psi_{\nu_h''},\chi_{\pi,\mathbf{k}}^{(+)}\rangle , \quad (5)$$

where $|\nu_\Delta'\nu_h'^{-1}\rangle$ and $|\nu_\Delta''\nu_h''^{-1}\rangle$ are the intermediate Δ-hole states and ψ_{ν_Δ} is the wave function for a Δ in the orbit ν_Δ. The $\pi N\Delta$ vertex $F_{\pi N\Delta}$ and the Green's function $G_{\Delta h}$ describe the formation and the propagation of the Δ-hole states and are determined by the analysis of elastic scattering. The transition interaction $t_{NN,N\Delta}$ is a two-body operator and is assumed to consist of the one pion exchange part $t_{NN,N\Delta}^{(\pi)}$ and a short range part $t_{NN,N\Delta}^{(s)}$, i.e.,

$$t_{NN,N\Delta}(1,2) = t_{NN,N\Delta}^{(\pi)}(1,2) + t_{NN,N\Delta}^{(s)}(1,2) , \quad (6)$$

$$t_{NN,N\Delta}^{(\pi)}(1,2) = F_{\pi NN}(1)G_\pi(1,2)F_{\pi N\Delta}^\dagger(2) , \quad (7)$$

$$t_{NN,N\Delta}^{(s)}(1,2) = (\tau_1\cdot T_2^\dagger)[a_c(\sigma_1\cdot \mathbf{S}_2^\dagger)(\nabla_1\cdot\nabla_2) + a_T\{(\sigma_1\cdot\nabla_2)(\mathbf{S}_2^\dagger\cdot\nabla_2) - \tfrac{1}{3}(\sigma_1\cdot\mathbf{S}_2^\dagger)(\nabla_1\cdot\nabla_2)\}]e^{-\Lambda_s|\mathbf{r}_1-\mathbf{r}_2|} , \quad (8)$$

where G_π is the pion Green's function, $T(S)$ is the transition isospin (spin), $a_c(a_T)$ is the complex strength for the central (tensor) term, and Λ_s is the range parameter. The transition interactioin $t_{NN,N\Delta}$ is first applied to the $^2H(\pi^+,p)p$ reaction. With the vertex parameters $\Lambda_{\pi NN}=800$ MeV and $\Lambda_{\pi N\Delta}=350$ MeV, the free Green's function for G_π and the range parameter $\Lambda_s = 800$ MeV, the strength parameters a_c and a_T are determined so as to reproduce the observed differential cross sections and the polarizations for this reaction. The obtained values of a_c and a_T were given in Ref. 14. The parameters a_c and a_T at the pion kinetic energies 100, 150, 200, and 250 MeV are obtained by a simple interpolation. The nucleon distorted wave χ_N is calculated with the optical potential of Ref. 15.

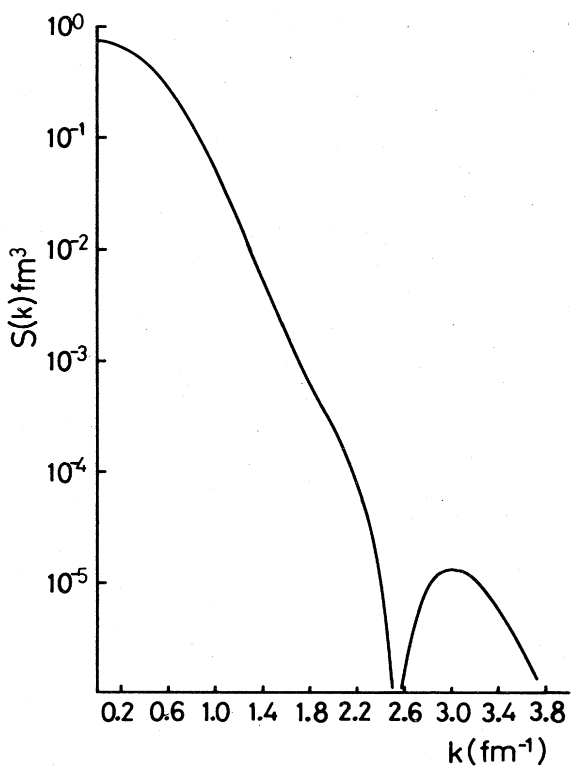

FIG. 1. The momentum distribution of the overlap function $\langle\psi_{^3H}|\psi_{^4He}\rangle$ $\left[S(k) = \left|\int d\mathbf{r}\langle\psi_{^3H}|\psi_{^4He}\rangle e^{i\mathbf{k}\mathbf{r}}\right|^2\right]$ is expressed as the function of the momentum k.

Let us proceed to the analysis of the $^4He(\pi^-,n)^3H$ reaction. The calculated results are shown in Fig. 2. The results of calculation including only the background term T_{bg} and only the Δ-hole term are shown by the dotted lines and the dashed lines, respectively. The results with the contributions of both terms are shown by the solid lines. The Δ-hole term which allows sharing the large momentum transfer between two nucleons dominates the reaction as usual up to 50°, 45°, 40°, and 32° corresponding to the pion kinetic energies 100, 150, 200, and 250 MeV. On the other hand the background term becomes more important over these angles because the second maximum in the Fourier component of the overlap function ψ_s, as shown in Fig. 1, plays a very important role in this region of momentum transfer (~ 3.5 fm^{-1}). We can roughly say that the angular distribution is determined mainly by the Δ-hole term (TNM) in the forward region up to the dip or the inflection point while the background term (ONM) dominates the reaction in the region of the second maximum or the shoulder. Although the present calculation does not perfectly reproduce the observed angular distributions, we see a great improvement over the previous calculation, which is due to a large contribution from the ONM through the high momentum component of the overlap function. This clearly indicates an important interplay between reaction mechanisms and nuclear structure effects in the present reaction. For further improvements, we have to go beyond the one Slater determinant approximation using the accurately calculated wave functions for ^4He and ^3He. The transition interaction $t_{NN,N\Delta}^{(s)}$ should also be refined in such a calculation.

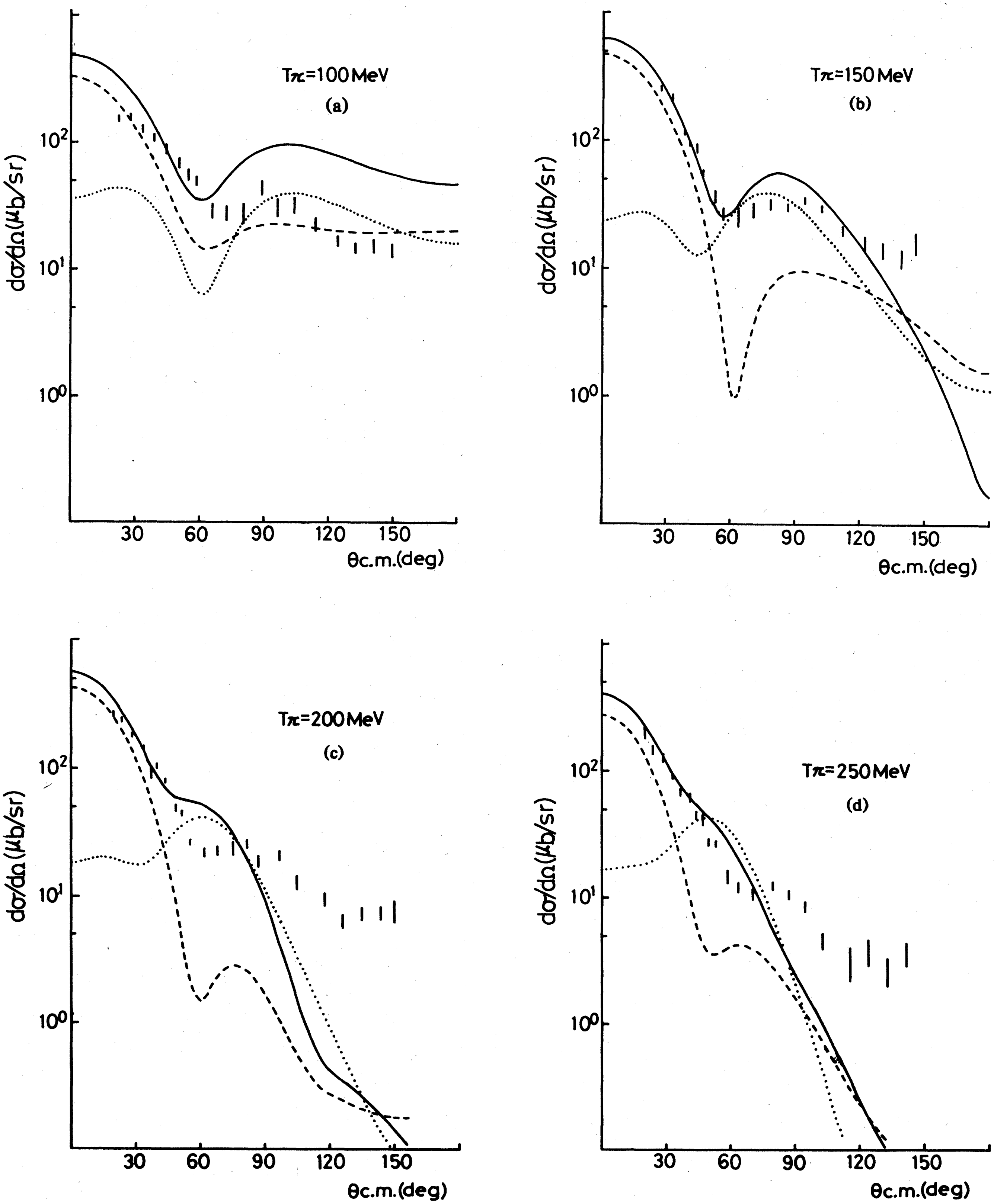

FIG. 2. The differential cross section for the reaction ${}^4He(\pi^-,n){}^3H$ at $T_\pi = 100$, 150, 200, and 250 MeV. The data are taken from Ref. 5. The solid lines represent the results of the full calculation with the Δ-hole and background terms. The dashed lines show the results with the Δ-hole term only, while the dotted lines show the results with the background term only.

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