Influence of neck formation on heavy ion subbarrier fusion

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We calculate the effect of the neck degree of freedom on subbarrier fusion reaction cross sections. The potential energy and mass tensor are calculated on the basis of a two-dimensional representation of the liquid drop model. The WKB and Hill-Wheeler approximations are used to calculate the tunneling amplitudes for motion along the direction of the neck size degree of freedom. The resulting cross section for collisions between two identical Ni isotopes show a marked enhancement at low energies compared with the standard one-dimensional radial penetration calculations.

In the last few years it was observed that cross sections for subbarrier fusion of heavy systems are strongly enhanced compared to standard one-dimensional barrier penetration calculations.[1] The inadequacy of the conventional approach has been taken as evidence of the important role played by additional degrees of freedom on the fusion process. The effect of static deformation,[2] slow collective vibrations,[1,3] nucleon transfer,[4] and coupling, in general, to other reaction channels[5,6] have been considered to explain some of the existing data. Indeed, as was pointed out in Ref. 7, the inclusion of virtually any additional degree of freedom is bound to lead to an increase in the subbarrier fusion cross section.

Recently a macroscopic theory of nucleus-nucleus reactions[8] has been applied to the study of the fusion of two heavy ions at energies above the Coulomb barrier.[9] In this theory three degress of freedom, namely, the radial center-of-mass distance, the asymmetry of the system, and the size of the neck connecting the two pieces, are introduced to describe the dynamics of the collision. The success of this theory for the qualitative understanding of heavy-ion fusion cross sections at moderate energies[3,10] leads us to consider how such a reaction model could be extended to describe subbarrier fusion.

The full dynamical problem, which is essentially that of tunneling under a three-dimensional potential barrier, is certainly very complicated, but we can expect to gain much in physical insight by separately exploring the role of these additional degrees of freedom.

In the present paper we study the effect of neck formation on subbarrier fusion cross sections. Although there have been several suggestions that this degree of freedom could play an important role in low energy fusion reactions,[7,11-13] no quantitative calculation treating it as an independent variable has been presented. One should remark, though, that in Ref. 7 a schematic representation of neck formation was discussed in terms of an arbitrarily adjustable harmonic oscillator degree of freedom linearly coupled to the radial motion. Also, in Ref. 13 a shape parametrization in which a neck appears between the two nuclei is considered, but since there is a one-to-one correspondence between the radial distance and the dinuclear shape in this case, the calculations do not explore the role of the neck variable as a separate degree of freedom.

We will consider the shapes shown in Fig. 1. For separated shapes, each fragment is a sphere modified by a smoothly fitted portion of a hyperboloid of two sheets. For an undivided shape the two spheres are joined by a smoothly fitted portion of a single hyperbolic of ellipsoidal surface of revolution.[14] Since we are interested here in assessing the effect of neck formation, we have restricted ourselves to symmetrical configurations, thus suppressing the asymmetry as an additional degree of freedom.

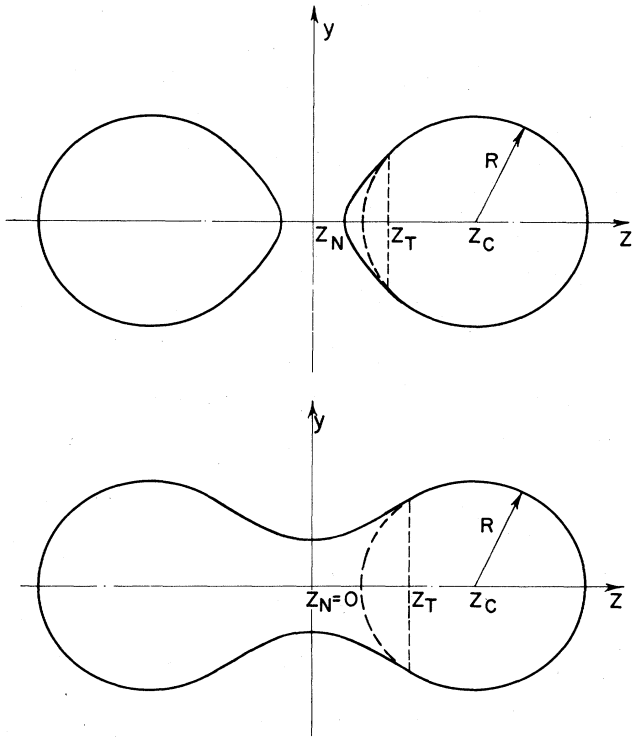

FIG. 1. Examples of separated and undivided shapes described by our parametrization together with the coordinate system in which it is defined. See text for additional details.

Taking the coordinate system indicated in Fig. 1, the shapes are given by

$$y^2 = a + bz^2, \quad z_N \leq z \leq z_T \ , \qquad (1)$$
$$y^2 = R^2 - (z - z_C)^2, \quad z_T \leq z \leq z_C + R \ ,$$

and its reflection with respect to the plane $z = 0$.

In Eq. (1) z_C is the coordinate of the center of the right sphere, z_T that of the circular tangency between it and its quadratic extension, and $z_N = (-a/b)^{1/2}$ the point where this last surface cuts through the z axis, provided the system is separated, or alternatively the origin of the z axis in the case of an undivided system. The three parameters a,b,R determining this shape reduce to two degrees of freedom by the volume conservation requirement. As variables describing our system we then choose the dimensionless quantities[14]

$$\rho = \frac{z_c}{R} \ , \qquad (2)$$

$$\lambda = \frac{l}{R} \ , \qquad (3)$$

where $l = R - (z_C - z_T)$ is the thickness of the missing spherical tip.

The first variable is a measure of the distance between nuclear centers while the second one measures the neck size. Depending on its value we have, for a given ρ, two separated spherical (if $\lambda = 0$), or deformed (if $0 < \lambda < 1 - 1/\rho$) nuclei, or two nuclei joined by a concave (if $1 - 1/\rho < \lambda < 1$) or convex (if $1 < \lambda \leq 2$) neck.

Having selected the parameters ρ, λ characterizing the configuration of our system we may now proceed to study its dynamical behavior. The kinetic energy T,

$$T = \tfrac{1}{2} M_{\rho\rho} \dot{\rho}^2 + M_{\rho\lambda} \dot{\rho} \dot{\lambda} + \tfrac{1}{2} M_{\lambda\lambda} \dot{\lambda}^2 \qquad (4)$$

is calculated using the Werner-Wheeler simplified description of an incompressible, irrotational flow.[15] This leads to analytical expressions for the mass tensor elements $M_{\rho\rho}, M_{\rho\lambda}, M_{\lambda\lambda}$.

For the potential energy V, we have calculated separately its Coulomb, nuclear, and centrifugal contributions,

$$V = V_{\text{Coul}}(\rho,\lambda) + V_{\text{nucl}}(\rho,\lambda) + V_{\text{cent}}(\rho,\lambda) \ . \qquad (5)$$

The Coulomb energy is calculated numerically by dividing the system into disks perpendicular to the z axis and evaluating their interaction and self-energies.

The nuclear energy is calculated as the sum of two terms. One is the usual surface energy,

$$V_{\text{surf}} = \gamma \cdot \text{area}(\rho,\lambda) \ , \qquad (6)$$

where γ is the surface tension coefficient and the area is given by a simple analytical expression in terms of ρ and λ. The diffuseness of the nuclear surface is taken into account by adding a proximity energy V_{prox} obtained by integrating the proximity interaction[16] between each surface element and its neighboring one.

The centrifugal energy is given by

$$V_{\text{cent}} = \frac{\hbar^2 L(L+1)}{2I(\rho,\lambda)} \ , \qquad (7)$$

where L is the orbital angular momentum quantum number and $I(\rho,\lambda)$ the moment of inertia of the system with respect to an axis through the center of mass perpendicular to the symmetry axis. This quantity can also be written as a simple analytical function of ρ and λ.[17]

We have assumed uniform charge and mass densities inside the volume limited by the sharp surface given by Eq. (1). The actual size of the system for a given number of nucleons A is obtained from the volume of a sphere with radius constant r_0, $R_0 = r_0 A^{1/3}$ plus the constant volume condition already mentioned. For the parameter r_0 the equivalent sharp surface value $r_0 = 1.18$ fm was taken for the calculation of Coulomb and surface energies as well as for moment of inertia and mass tensor.

For the proximity energy the relevant quantity is the central surface, which lies slightly inside the sharp surface mentioned above. In the spherical case the radius of this central surface, C, is related to the equivalent sharp radius R_0 through $C = R_0(1 - d^2/R_0^2)$, with $d = 0.99$ fm.[16] To calculate the proximity interaction in the general case we have constructed a central surface as follows. The spherical portions of the sharp surface of radius R define central surface radii C in the same way as before

$$C = R(1 - d^2/R^2) \ . \qquad (8)$$

The central radial parameter ρ_c is therefore related to the

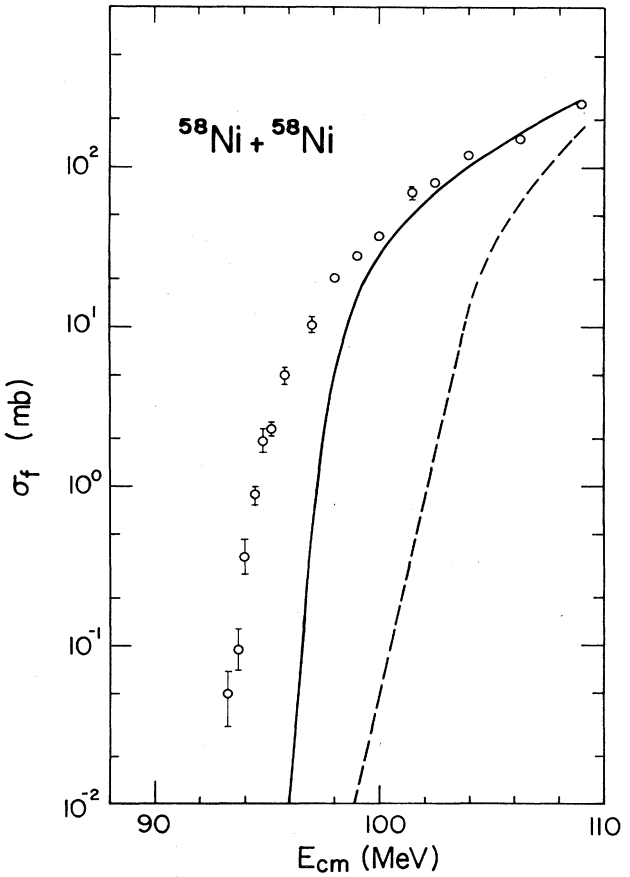

FIG. 2. The fusion cross sections of identical nickel isotopes calculated for barrier penetration along the direction of the neck size degree of freedom (solid curves) and in the radial direction (dashed lines). The data are from Ref. 18.

sharp ρ through

$$\rho_c = \rho R / C \ , \tag{9}$$

and we have assumed the neck size parameter λ to be the same for the central and sharp surfaces. This prescription yields the correct results for the case of one or two spheres and quite reasonably looking shapes for other situations.[17]

With the potential energy so defined we have calculated $T_L(E)$, the transmission coefficients as function of the center of mass energy E, and the orbital angular momentum quantum number L, using the WKB approximation,

$$T_L(E) = \left[\exp\left(\int d\lambda \sqrt{2M_{\lambda\lambda}[E - V(\rho_0, \lambda)]}/\hbar\right) + 1\right]^{-2} \ , \tag{10}$$

where ρ_0 is chosen to be equal to the classical turning point for motion restricted to the radial separation variable and the integration ranges from the entrance ($\lambda = 0$) to the exit point of the tunneling process. At energies above the barrier we have calculated $T_L(E)$ in the standard Hill-Wheeler approximation, using the potential of Eq. (5) with $\lambda = 0$. The fusion cross section is then calculated using

$$\sigma_F(E) = \frac{\pi}{k^2} \sum_L (2L+1) T_L(E) \ , \tag{11}$$

where k is the wave number corresponding to the energy E and the reduced mass of the system.

The results of these calculations for the case of symmetric collision between nickel isotopes are presented in Fig. 2 (full lines), where they are compared with the data of Beckerman *et al.*[18] and with the calculations considering tunneling in the radial direction alone (dashed curves).

These results demonstrate how unfreezing of the neck degree of freedom leads to a marked enhancement in the subbarrier fusion cross section. The lower potential barrier and smaller mass tensor elements in the neck degree of freedom direction make these results easy to understand qualitatively. It should be noted that although the data could be adjusted to obtain a better fit by slightly modifying the parameters r_0 and d, and this could perhaps be justified with respect to known structural differences between different nuclear species, such adjustments have not been attempted here. In our view a better way of performing more realistic calculations is through a careful treatment of the dynamics inside the two-dimensional barrier.

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[1]W. Reisdorf *et al.*, Phys. Rev. Lett. **49**, 1811 (1982), and references contained therein.

[2]R. G. Stokstad and E. E. Gross, Phys. Rev. C **23**, 381 (1983).

[3]H. Esbensen, Nucl. Phys. **A352**, 147 (1981).

[4]R. A. Broglia *et al.*, Phys. Rev. C **27**, 2433 (1983).

[5]C. H. Dasso, S. Landowne, and A. Winther, Nucl. Phys. **A405**, 381 (1983); *ibid.* **A407**, 221 (1983).

[6]M. J. Rhoades-Brown and P. Braun-Munzinger, Phys. Lett. **138B**, 19 (1984).

[7]H. J. Krappe *et al.*, Z. Phys. A **314**, 23 (1983).

[8]W. J. Swiatecki, Phys. Scr. **24**, 113 (1981).

[9]W. J. Swiatecki, Nucl. Phys. **A376**, 275 (1982).

[10]S. Bjørnholm and W. J. Swiatecki, Nucl. Phys. **A391**, 471 (1982).

[11]L. C. Vaz, J. M. Alexander, and G. L. Stachler, Phys. Rep. **69**, 373 (1981).

[12]V. Jahnke *et al.*, Phys. Rev. Lett. **48**, 17 (1982).

[13]G. Royer and B. Remaud, J. Phys. G **9**, 1103 (1983).

[14]J. Blocki and W. J. Swiatecki, Lawrence Berkeley Laboratory Report No. 12811, 1982 (unpublished).

[15]I. Kelson, Phys. Rev. **136**, B1667 (1964).

[16]J. Blocki *et al.*, Ann. Phys. (N.Y.) **105**, 427 (1977).

[17]C. E. Aguiar, L. F. Canto, and R. Donangelo (unpublished).

[18]M. Beckerman *et al.*, Phys. Rev. Lett. **45**, 1472 (1980); Phys. Rev. C **23**, 1581 (1981).