Calculation of the $^{48}Ca(\beta^-)^{48}Sc$ decay rate

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A calculation is made of the partial half-life for the β^- decay of ^{48}Ca to the 5^+ state of ^{48}Sc at 130.94 ± 0.04 keV (an energy release of 147.1 ± 5.0 keV). The estimated partial half-life is 760×10^{18} yr. This value is competitive with the most recent estimate of the partial half-life for the 2ν mode of double beta decay.

I. INTRODUCTION

Of the naturally occurring nuclei for which double β^- decay is energetically allowed, only ^{48}Ca and ^{96}Zr can also decay by single β^- emission. None of the possible decay modes of ^{48}Ca have been observed; however, experimental limits have been set on the partial half-life (inverse decay rate) for the 0ν and 2ν modes of double-beta decay. These limits[1] are $t\,(\equiv t_{1/2})>2\times10^{21}$ and $t>36\times10^{18}$ yr, respectively. The present limit[2] on single β^- decay is only $t>2\times10^{16}$ yr, but at least one experiment is in progress that should improve this limit.[3]

Because of renewed interest in double-beta decay—especially the possible 0ν mode—several calculations of the $^{48}Ca(2\beta^-)^{48}Ti$ partial lifetimes have been made. These have recently been reviewed by Haxton and Stephenson.[4] For both the 0ν and 2ν decay modes the matrix element is found to be quite weak. Estimates of the partial half-life for the dominating 2ν mode vary in the range ~30–300×10^{18} yr.[4] A central ingredient of these calculations is the closure approximation. Recently, this approximation was examined critically by Tsuboi, Muto, and Horie,[5] who show it to be a potential source of large errors in the case of ^{48}Ca $2\beta2\nu$ decay. Subsequently, Brown[6] has considered the closure approximation together with the effects of using a limited shell-model space. He has also considered the influence of some of the quenching effects appropriate to Gamow-Teller β decay. He finds a $2\beta2\nu$ half-life of $t(2\beta2\nu)\sim90\times10^{18}$ yr with the possibility of a value considerably greater than this.

In the past, crude estimates (see, e.g., Ref. 1) of the $^{48}Ca(\beta^-)^{48}Sc$ decay rate indicated a value considerably less than that which was then currently estimated for the $2\beta2\nu$ decay mode. However, with the possible retardation of the $2\beta2\nu$ mode, the single β^- mode may well dominate, or at least strongly influence, the total ^{48}Ca decay rate. Thus, a careful estimate of the $^{48}Ca(\beta^-)^{48}Sc$ decay rate is of interest. Such a calculation is described in Sec. II.

II. UNIQUE FOURTH-FORBIDDEN β^- DECAY OF ^{48}Ca

A. Introduction

A schematic of ^{48}Ca decay is shown in Fig. 1. It is seen that three ^{48}Sc states are energetically accessible for ^{48}Ca β^- decay.[7] The decays to the 4^+ state at 252 keV and the 6^+ ground state are nonunique fourth and sixth forbidden, respectively, while that to the 5^+ state at 131 keV is unique fourth forbidden. Of these three decays it is straightforwardly found—following Behrens and Jänecke[8]—that the decay to the 5^+ state will strongly dominate. Briefly, the decays to the 4^+ and 5^+ states are of the same order but the energy releases differ markedly. Thus, phase space considerations favor decay to the 5^+ state by a factor of 10^5. The decay to the 6^+ state is two orders more forbidden and the estimated reduction relative to decay to the 5^+ state is a factor of $\sim10^9$. In summary, decays to the 4^+ and 6^+ states will be negligible compared to that to the 5^+ state.

B. The comparative half-life of the fourth forbidden unique branch

We use the definition of the unique n-forbidden nuclear matrix element $\langle G_n\rangle^2$ and the comparative half-life, f_nt, of Warburton, Garvey, and Towner,[9]

$$\langle G_n\rangle^2=(\ln2)(2\pi^3/g^2C_A^2)\left\{\frac{[(2n+1)!!]^2}{(2n+1)}\right\}\lambda\!\!\!^{-}{}_{ce}^{2n}(f_nt)^{-1}\ ,\quad (1)$$

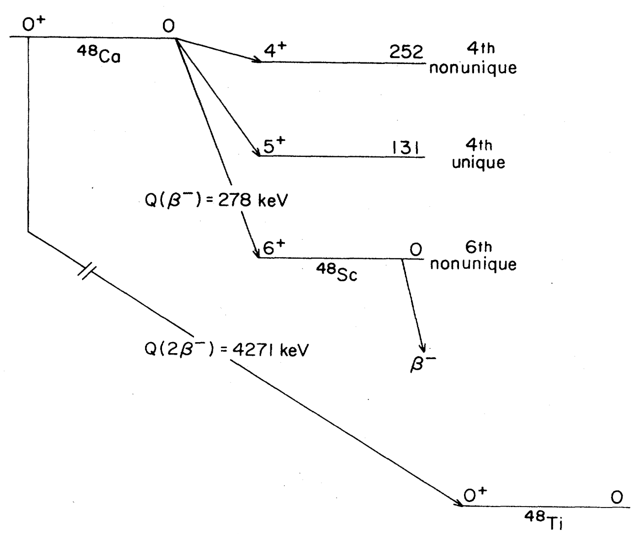

FIG. 1. The possible single and double β^- decays of ^{48}Ca.

where $2\pi\lambda\!\!\!^{-}_{ce}$ is the Compton wavelength of the electron. We shall use[10]

$$(\ln 2)(2\pi^3/g^2C_A^2)=3.883\times 10^3 \text{ sec}$$

and $\lambda\!\!\!^{-}_{ce}=386.145$ fm with the result

$$\langle G_4\rangle^2=6035\times 10^{18}(f_4t)^{-1}\text{ fm}^8\ , \qquad (2)$$

where, as throughout this paper, the half-life t is in years.

The integrated Fermi function is expressed as[9]

$$f_n=\int_1^{W_0} a_n(Z,W)F(Z,W)pq^2W\,dW\ , \qquad (3)$$

where W is the β energy and W_0 is the disintegration energy (including the rest mass), p and q are the β and ν momenta, and $F(Z,W)$ is the Fermi function.[8] The units of energy and momentum are m_0c^2 and m_0c, respectively. With the normalization of Eq. (1), the shape factor $a_n(Z,W)$ is given by

$$a_n(Z,W)=(2L-1)!\sum_{\nu=1}^{L}\frac{p^{2(\nu-1)}q^{2(L-\nu)}}{(2\nu-1)!(2L-2\nu+1)!}\lambda_\nu\ , \qquad (4)$$

where $L=n+1$.[11] The λ_ν are defined by Behrens and Jänecke.[8] The shape factor for fourth forbidden unique beta decay is

$$a_n(Z,W)=q^8+12q^6p^2\lambda_2+(\tfrac{125}{6})q^4p^4\lambda_3 + 12q^2p^6\lambda_4+p^8\lambda_5\ . \qquad (5)$$

Equation (3) was evaluated by numerical integration following previous procedures.[9] Because of the low energy release and high degree of forbiddance, the screening correction was found to be relatively large. Other corrections—finite size, finite de Broglie wave length, radiative—were routinely made but were negligible. The screening correction was made in the modified Wentzel-Kramers-Brillouin (WKB) approximation[12] in which W is replaced by W-V^s in expressions for p and W. Here V^s, the screening potential, is usually taken for light elements as $1.45Z^{4/3}\alpha^2$. We refer to this approximation as V_0. The WKB prescription is verified and accurate for allowed transitions, but according to Behrens and Bühring,[13] its accuracy has not been investigated for the $p^{2\nu-2}\lambda_\nu$ which appear in the shape factor. Behrens and Jänecke[8] tabulate "exact" screening corrections for λ_2 so that it is a simple matter to check the approximation for $\nu=2$ at least. We find that the screening results tabulated by Behrens and Jänecke for λ_2 at $Z=21$ are reproduced to better than 0.3% by $V^s=0.91V_0$. We assume that the WKB approximation is adequate for $\nu=3$, 4, and 5, but note that this assumption needs verification. However, the integrated Fermi function is insensitive to the behavior of the λ_ν at the low values of electron momentum where the WKB approximation is suspect.

The effect of screening and of Coulomb effects on the shape factor and hence on f_4 is summarized in Table I. It is seen that the screening effect is more important than the other Z-related effects which cause the λ_ν to deviate from unity.

The uncertainty in f_4 is dominated by the uncertainty[7] of ± 5.0 keV in $Q(\beta^-)$. The final result is given in Table II.

TABLE I. Coulomb effects on the fourth-forbidden unique integral f_4 for an energy release of 147.1 keV.

Condition	f_4 (units of 10^{-4})
$V_s=0$	1.097
$V_s=0$, $\lambda_\nu=1.0$ (all ν)	1.066
$V_s=V_0$	0.976
$V_s=0.91V_0$	0.987

C. Calculation of $\langle G_4\rangle^2$

Calculations were made assuming a closed $2s,1d$ shell at ^{40}Ca. Harmonic oscillator radial wave functions were used with the size parameter from

$$b^2=41.467/(45A^{-1/3}-25A^{-2/3})\text{ fm}^2\ ,$$

which gives $b=1.988$ fm. The calculations assumed a closed $f_{7/2}$-neutron subshell for ^{48}Ca and, for ^{48}Sc, a $f_{7/2}$-neutron hole and a proton in the full $1f,2p$ shell. Schematically,

$$^{48}\text{Ca}[\nu(f_{7/2})^8]_{0^+}\xrightarrow{\beta^-}{}^{48}\text{Sc}[\nu(f_{7/2})^7\pi(f,p)]_{5^+}\ . \qquad (6)$$

For each of the three possible transitions, the contribution to $\langle G_n\rangle$ is obtained by combining Eqs. (12) and (44) of Ref. 9.

If the proton is confined to the $f_{7/2}$ subshell, the result is independent of the assumed nucleon-nucleon interaction and is $\langle G_4\rangle=580$ fm^4 ($f_{7/2}$ subshell only). The calculation for the space of Eq. (6) was performed with the computer code[14] OXBASH using both the Utrecht interaction[15] (designated FPV) and a modification[16,17] (FPV2) of this interaction. The results of these calculations are $\langle G_4\rangle=405$ fm^4 and 409 fm^4 for FPV and FPV2, respectively.

How realistic are these calculations? Fortunately, recent (p,n) experiments have been performed[17] at energies where the interaction is dominated by the spin-dependent operators analogous to those inducing beta decay. These experiments can act as calibrators of the shell-model calculations. Thus, it was found,[17] for example, that the calculation with FPV overestimates, by a factor of 2.05, the cross section for ^{48}Ca(p,n) proceeding to the unresolved 4^+, 5^+, and 6^+ levels of ^{48}Sc. Since the calculation also indicates that the 5^+ level dominates, the same factor applies to the beta decay and thus, we adopt

$$\langle G_4\rangle^2=0.49(405)^2\text{ fm}^8=8.04\times 10^4\text{ fm}^8\ .$$

TABLE II. The integrated Fermi function f_4 as a function of $Q(\beta^-)$.

$Q(\beta^-)$ (keV)	f_4 ($\times 10^{-4}$)
147.1	0.986
142.1	0.744
152.1	1.296

The quenching of the (p,n) cross section and $\langle G_4 \rangle^2$ is presumably due to inadequacies in the shell-model space, such as the restricted f,p space, to $2\hbar\omega$ excitations which will cause further inhibitions, and to quenching effects analogous to those encountered in allowed Gamow-Teller decay.

III. THE LIFETIME OF ^{48}Ca

Combining the derived values of f_4 and $\langle G_4 \rangle^2$ into Eq. (2) gives a half-life of 760×10^{18} yr for $^{48}Ca(\beta^-)^{48}Sc$. The estimated uncertainty is $\sim 70\%$. The result is considerably longer than Brown's estimate[6] of $\sim 90 \times 10^{18}$ yr for the 2ν mode of double beta decay. However, as stated earlier, it is quite possible that $t(2\beta 2\nu)$ is underestimated. In this case, the β^- mode could contribute significantly to the decay of ^{48}Ca.

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[1] R. K. Bardin, P. J. Gollon, J. D. Ullman, and C. S. Wu, Nucl. Phys. **A158**, 337 (1970).

[2] J. W. Jones and T. P. Kohman, Phys. Rev. **85**, 941 (1952).

[3] D. E. Alburger and J. B. Cumming (unpublished).

[4] W. C. Haxton and G. J. Stephenson, Jr., Prog. Part. Nucl. Phys. (to be published).

[5] T. Tsuboi, K. Muto, and H. Horie, Phys. Lett. **143B**, 293 (1984).

[6] B. A. Brown, in Proceedings of the International Symposium on Nuclear Shell Models, 1984, Drexel University (to be published); and private communication.

[7] J. R. Beene, Nucl. Data Sheets **23**, 1 (1978); D. E. Alburger, Nucl. Data Sheets (to be published).

[8] H. Behrens and J. Jänecke, *Numerical Tables for Beta Decay and Electron Capture, Landolt-Bornstein, New Series, Vol. I/4* (Springer, Berlin, 1969).

[9] E. K. Warburton, G. T. Garvey, and I. S. Towner, Ann. Phys. (N.Y.) **57**, 174 (1970).

[10] D. H. Wilkinson, in *Nuclear Physics with Heavy Ions and Mesons,* edited by R. Balian, M. Rho, and G. Ripka (North-Holland, Amsterdam, 1978), Vol. 2, p. 877.

[11] Equation (4) follows from a rearrangement of Eq. (44) of Behrens and Jänecke (Ref. 8). It is identical to Eq. (4) of Ref. 9 with $C_{n,\nu}(Z,W) \equiv C_n(Z,W) \equiv \lambda_{n+1}$.

[12] M. E. Rose, Phys. Rev. **49**, 727 (1936).

[13] H. Behrens and W. Buhring, *Electron Radial Wave Functions and Nuclear Beta Decay* (Clarendon, Oxford, 1982).

[14] W. D. M. Rae, A. Etchegoyen, N. S. Godwin, and B. A. Brown (unpublished).

[15] A. G. M. van Hees and P. W. M. Glaudemans, Z. Phys. A **303**, 267 (1981).

[16] For FPV2 the $f_{5/2}$ single-particle energy was increased by 2 MeV. This gave better agreement with the 1^+ particle-hole spectrum of ^{48}Ca [B. A. Brown (private communication)].

[17] B. D. Anderson *et al.*, Phys. Rev. C **31**, 1147 (1985); **31**, 1161 (1985).