Dynamical model for pion photoproduction in the Δ region

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A phenomenological model for pion photoproduction is constructed incorporating the dynamics of pions, nucleons, and Δ's. The importance of nonresonant background interactions for the elastic πN scattering and pion photoproduction is emphasized. The photoproduction amplitudes calculated in our model satisfy two-body unitarity, so that the requirement imposed by Watson's theorem is automatically fulfilled. By fitting the amplitudes to data, $M1$ and $E2$ $\gamma N\to\Delta$ transition amplitudes are estimated, eliminating background contributions. The results are $A_{1/2}(M1)=(-84\pm5)\times10^{-3}$ GeV$^{-1/2}$ and $E2/M1=(3.7\pm0.4)\%$. Our $M1$ amplitude disentangled from the πN rescattering term is in good agreement with the quark model predictions. The sign and magnitude of our $E2$ amplitude are, however, incompatible with the existing quark models.

I. INTRODUCTION

In the early study of the quark model, Becchi and Morpurgo[1] showed that the decay $\Delta\to N\gamma$ is pure $M1$. The recent development of the quark model has led to a small violation of this rule.[2–5] At present the origin of the nonvanishing $E2$ amplitude is attributed to the admixture of D state components into baryon wave functions brought about by a tensor-type quark-quark potential. The experimental situation on the $E2$ amplitude is, however, not clear. The difficulty stems from the smallness of the resonant $E2$ amplitude. The analyses of the experimental data either based on the dispersion relation[6] or the isobar model[7] assume the decomposition of the total amplitude into resonant and background parts. We have to be careful in regard to this decomposition because in general simple addition of two unitary amplitudes will not produce a unitary amplitude.

As Olsson[8] discussed, to retain the unitarity of the total amplitude, a modification of the resonant amplitude is necessary. The physical origin of this modification is the rescattering term which is generated by producing the pion through the background interaction and then having πN scatter via the Δ. The addition of this term to the unitary resonant amplitude results in its renormalization by a complex factor $\gamma\exp(i\delta_p)$. Since the resonant part of the $E2$ amplitude is small compared with the nonresonant background, the effect of the renormalization is expected to be quite important.

Following the procedure used by Olsson, Blomqvist and Laget[9] tried to modify the resonant $M1$ amplitude. However, as Wittman, Davidson, and Mukhopadhyay[10] pointed out, their modification is not complete and an improvement is necessary to make the total amplitude unitary. Apart from this point, the approach of Blomqvist and Laget is not convenient for its application to the nuclear many body calculation. In order to compute the photonuclear reaction amplitudes, off-shell transition matrix elements for elementary processes are required. In the actual calculation they replace the off-shell matrix elements by its on-shell ones. Thus the Δ is not treated dynamically. Koch and Moniz[11] also employed the procedure of Olsson in the Δ-hole approach. They incorporated the Δ dynamics properly and their elementary $M1$ photoproduction amplitude satisfies Watson's theorem.[12] If we want to extend their approach to the $E2$ amplitude, however, a more detailed description of the process is needed because determination of the small piece will subtly depend on the utilized model.

The purpose of this paper is twofold. The first purpose is to construct a dynamical model of the interactions between photon, pion, nucleon, and Δ by a phenomenological fit to the pion-nucleon scattering and pion photoproduction data. It will be a starting point for the study of the photonuclear reaction processes. The unitarity of the photoproduction amplitude in our model is guaranteed by restricting the model space within the two-body space. Since the assumptions of Watson's theorem are two-body unitarity and time reversal invariance, its constraint is inevitably satisfied by our model. The second purpose is to estimate the $M1$ and $E2$ transition amplitudes for the bare Δ. In our model the nonresonant background is unambiguously defined so that it is easy to eliminate the background contributions from the total amplitude. This will make it possible to extract empirical values for the $\gamma N\Delta$ coupling constants which can be compared to the quark model predictions.

In Sec. II we derive basic equations and unitarity relations for the pion photoproduction process. The phenomenological Hamiltonian is constructed in Sec. III. $M1$ and $E2$ transition amplitudes are calculated in Sec. IV.

II. BASIS EQUATIONS AND UNITARITY RELATIONS

Our consideration here is devoted only to the πN P_{33} channel and $\gamma N\to\pi N$ $M_{1+}(\frac{3}{2})$ and $E_{1+}(\frac{3}{2})$ multipoles in which the Δ resonance participates. Furthermore, in these channels we restrict the Hilbert space of the states into $\Delta\oplus\pi N\oplus\gamma N$, neglecting other possible states, $\pi\pi N$, $\pi\Delta$, $\gamma\Delta$, etc. Therefore the amplitude calculated here satisfies the two-body unitarity. This restriction may be justified if the energy is sufficiently below the region where two pion production is important. Experimentally the inelasticity of the P_{33} wave is essentially zero below the total energy $W \cong 1400$ MeV.[13] From $W \cong 1300$ MeV the reaction $\gamma N \to \pi\pi N$ begins to be significant. However it is dominated, at least up to $W \cong 1400$ MeV, by $\pi\Delta$ in a relative S wave which cannot couple to the P_{33} channel.[14] Therefore we compare our calculation with data from threshold to this energy.

The total Hamiltonian can be written as

$$H = H_0 + V , \tag{2.1}$$

where H_0 is the Hamiltonian for free photon, pion, nucleon, and Δ, and V is the interaction Hamiltonian. We now introduce projection operators P_Δ, $P_{\pi N}$, and $P_{\gamma N}$ for Δ, πN, and γN states, respectively. The interaction Hamiltonian is decomposed into the following channel coupling interactions (Fig. 1):

$$v_\pi = P_\Delta V P_{\pi N} , \tag{2.2}$$

$$v_\gamma = P_\Delta V P_{\gamma N} , \tag{2.3}$$

$$v^B_{\pi\pi} = P_{\pi N} V P_{\pi N} , \tag{2.4}$$

$$v^B_{\pi\gamma} = P_{\pi N} V P_{\gamma N} . \tag{2.5}$$

We consider the electromagnetic interaction up to order e, so that $v_{\gamma\gamma} = P_{\gamma N} V P_{\gamma N}$ is neglected.

The transition matrix T is the solution of the integral equation

$$T = V + V G_0 T , \tag{2.6}$$

where G_0 is the free propagator. The transition amplitude for the πN scattering $t_{\pi\pi} = P_{\pi N} T P_{\pi N}$ satisfies the integral equation

$$t_{\pi\pi} = v_{\pi\pi} + v_{\pi\pi} G_0 t_{\pi\pi} \tag{2.7}$$

generated by the driving term

$$v_{\pi\pi} = v^B_{\pi\pi} + v^\dagger_\pi G_0 v_\pi . \tag{2.8}$$

$t_{\pi\pi}$ can be split into two parts

$$t_{\pi\pi} = t^B_{\pi\pi} + \tilde{v}^\dagger_\pi G_\Delta \tilde{v}_\pi , \tag{2.9}$$

where the first term is the background πN amplitude

$$t^B_{\pi\pi} = v^B_{\pi\pi} + v^B_{\pi\pi} G_0 t^B_{\pi\pi} . \tag{2.10}$$

$\tilde{v}^\dagger_\pi$ and $\tilde{v}_\pi$ in Eq. (2.9) are the $\pi N \Delta$ vertex functions modified by the background

$$\tilde{v}^\dagger_\pi = v^\dagger_\pi + t^B_{\pi\pi} G_0 v^\dagger_\pi , \tag{2.11}$$

$$\tilde{v}_\pi = v_\pi + v_\pi G_0 t^B_{\pi\pi} . \tag{2.12}$$

G_Δ is the dressed Δ propagator defined by

$$G_\Delta^{-1} = G_0^{-1} - \Sigma_\Delta , \tag{2.13}$$

where Σ_Δ is the Δ self-energy operator

$$\Sigma_\Delta = v_\pi G_0 v^\dagger_\pi + v_\pi G_0 t^B_{\pi\pi} G_0 v^\dagger_\pi . \tag{2.14}$$

The above equations are shown diagramatically in Figs. 2(a)—(e).

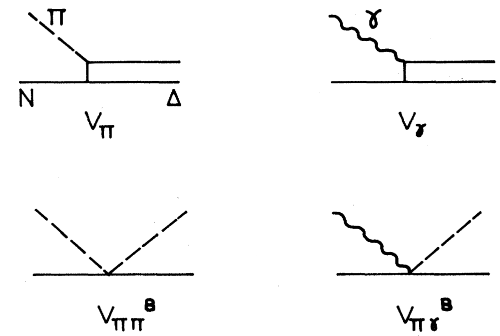

FIG. 1. Elementary interactions for the single pion photoproduction.

The photoproduction amplitude $t_{\pi\gamma} = P_{\pi N} T P_{\gamma N}$ is written in terms of $t_{\pi\pi}$ as

$$t_{\pi\gamma} = v_{\pi\gamma} + t_{\pi\pi} G_0 v_{\pi\gamma} . \tag{2.15}$$

The reaction is started by the interaction

$$v_{\pi\gamma} = v^B_{\pi\gamma} + v^\dagger_\pi G_0 v_\gamma . \tag{2.16}$$

We again split $t_{\pi\gamma}$ into the background amplitude $t^B_{\pi\gamma}$ and the resonant one $t^R_{\pi\gamma}$,

$$t_{\pi\gamma} = t^B_{\pi\gamma} + t^R_{\pi\gamma} . \tag{2.17}$$

$t^B_{\pi\gamma}$ is calculated from $t^B_{\pi\pi}$

$$t^B_{\pi\gamma} = v^B_{\pi\gamma} + t^B_{\pi\pi} G_0 v^B_{\pi\gamma} , \tag{2.18}$$

while the resonant part is made up of two portions

$$t^R_{\pi\gamma} = \tilde{v}^\dagger_\pi G_\Delta v_\gamma + \tilde{v}^\dagger_\pi G_\Delta v_\pi G_0 t^B_{\pi\gamma} . \tag{2.19}$$

The first term is the unitary resonant amplitude

$$t^\Delta_{\pi\gamma} = \tilde{v}^\dagger_\pi G_\Delta v_\gamma \tag{2.20}$$

and the second is the rescattering term. The diagrammatic representations of Eqs. (2.17)—(2.20) are given in Figs. 3(a)—(c).

The photoproduction amplitude can be rewritten into a

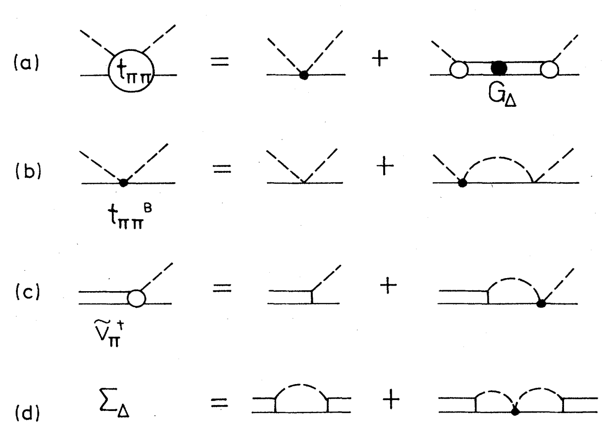

FIG. 2. Classification of the digrams for the πN P_{33} scattering amplitude.

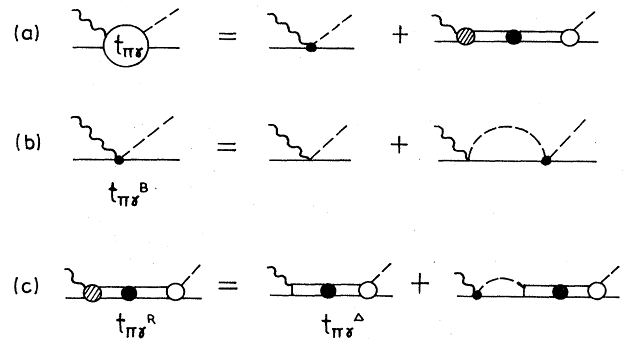

FIG. 3. Classification of the diagrams for the pion photoproduction amplitude.

more familiar form by using unitarity relations. Since both $t_{\pi\pi}$ and $t^B_{\pi\pi}$ are unitary amplitudes, their one-shell values are parametrized in terms of phase shifts δ and δ_e, respectively,

$$t_{\pi\pi}=-\frac{1}{\rho}\exp(i\delta)\sin\delta\ , \tag{2.21}$$

$$t^B_{\pi\pi}=-\frac{1}{\rho}\exp(i\delta_e)\sin\delta_e\ , \tag{2.22}$$

where ρ is the phase space factor for πN states. Watson's theorem requires the on-shell values of $t_{\pi\gamma}$ and $t^B_{\pi\gamma}$ to be

$$t_{\pi\gamma}=|\,t_{\pi\gamma}\,|\,\exp(i\delta)\ , \tag{2.23}$$

$$t^B_{\pi\gamma}=|\,t^B_{\pi\gamma}\,|\,\exp(i\delta_e)\ . \tag{2.24}$$

Notice that the norms are understood to include signs hereafter. Equation (2.23) states that the photoproduction amplitude carries the same phase as the corresponding elastic amplitude irrespective of the model for electromagnetic interactions. This implies that $t^\Delta_{\pi\gamma}$ has the phase δ. The rescattering term brings about the phase change in $t^R_{\pi\gamma}$. We now define the background photoproduction phase shift δ_p by the equation

$$t^R_{\pi\gamma}=|\,t^R_{\pi\gamma}\,|\,\exp[i(\delta+\delta_p)]\ . \tag{2.25}$$

Putting Eqs. (2.23)—(2.25) into Eq. (2.17), we find

$$|\,t_{\pi\gamma}\,|=|\,t^R_{\pi\gamma}\,|\,\frac{\sin(\delta+\delta_p-\delta_e)}{\sin(\delta-\delta_e)}\ , \tag{2.26}$$

$$|\,t^B_{\pi\gamma}\,|=|\,t^R_{\pi\gamma}\,|\,\frac{\sin\delta_p}{\sin(\delta-\delta_e)}\ . \tag{2.27}$$

The above two equations are equivalent to the results derived by Wittmann, Davidson, and Mukhopadhyay.[10] Since our approach is based on the dynamical equations, we can further study the content of $|\,t^R_{\pi\gamma}\,|$. Substituting Eqs. (2.21) and (2.22) into Eq. (2.9), we find

$$\tilde{v}^\dagger_\pi G_\Delta \tilde{v}_\pi=-\frac{1}{\rho}\exp[i(\delta+\delta_e)]\sin(\delta-\delta_e)\ . \tag{2.28}$$

Comparing this equation with Eq. (2.20), $|\,t^\Delta_{\pi\gamma}\,|$ is written as

$$|\,t^\Delta_{\pi\gamma}\,|=-\frac{|\,v_\gamma\,|}{\rho\,|\,\tilde{v}_\pi\,|}\sin(\delta-\delta_e)\ . \tag{2.29}$$

Here v_γ and $\tilde{v}_\pi$ are read as the on-shell form factors of their matrix elements. The effect of the rescattering term on the resonant amplitude is expressed by introducing a renormalization factor

$$\gamma=\frac{|\,t^R_{\pi\gamma}\,|}{|\,t^\Delta_{\pi\gamma}\,|}\ . \tag{2.30}$$

With the aid of the above equations the final expressions for the photoproduction amplitudes are obtained

$$t_{\pi\gamma}=-\frac{1}{\rho}\tilde{N}\exp(i\delta)\sin(\delta+\delta_p-\delta_e)\ , \tag{2.31}$$

$$t^B_{\pi\gamma}=-\frac{1}{\rho}\tilde{N}\exp(i\delta_e)\sin\delta_p\ . \tag{2.32}$$

The common factor $\tilde{N}$ is given by

$$\tilde{N}=\frac{\gamma\,|\,v_\gamma\,|}{|\,\tilde{v}_\pi\,|}\ . \tag{2.33}$$

Equations (2.31) and (2.32) correspond, respectively, to Eqs. (46) and (43) in Ref. 8 by Olsson. One notices that Olsson's final result is given in terms of the unrenormalized form factors as $N=|\,v_\gamma\,|\,/\,|\,v_\pi\,|$ instead of our formula (2.33). However, in his prescription the form factors v_π and v_γ must be recognized as dressed ones. The precise meaning of his result is made transparent by our expression (2.33). Moreover, it must be recapitulated that in contrast to approaches of other authors, our approach explicitly shows how the photoproduction amplitude is affected by the background interactions.

III. DETERMINATION OF THE ELEMENTARY INTERACTIONS

In Sec. II πN scattering and photoproduction amplitudes are expressed in terms of the four elementary interactions v_π, $v^B_{\pi\pi}$, v_γ, and $v^B_{\pi\gamma}$. In this section these interactions are phenomenologically determined by fitting the amplitudes to data.

A. Momentum space representations

In the momentum space, πN, γN, and Δ states are expressed as $|\,\mathbf{q}\rangle$, $|\,\mathbf{k},\boldsymbol{\epsilon}\rangle$, and $|\,\Delta\rangle$. The matrix elements of the free propagator in πN and Δ states are written as

$$\langle\,\mathbf{q}'\,|\,G_0\,|\,\mathbf{q}\,\rangle=\delta^3(\mathbf{q}-\mathbf{q}')\times[W-E_{\rm N}(q)-\omega(q)+i\epsilon]^{-1}\ , \tag{3.1}$$

$$\langle\,\Delta\,|\,G_0\,|\,\Delta\,\rangle=(W-m_\Delta+i\epsilon)^{-1} \tag{3.2}$$

with the usual definitions

$$E_{\rm N}(q)=(m^2_{\rm N}+q^2)^{1/2}\ , \tag{3.3}$$

$$\omega(q)=(m^2_\pi+q^2)^{1/2}\ . \tag{3.4}$$

W, m_Δ, $m_{\rm N}$, and m_π are the total energy, bare Δ mass, and physical nucleon and pion masses, respectively. The matrix element of the πNΔ interaction v_π has the form

$$\langle\,\Delta\,|\,v_\pi\,|\,\mathbf{q}\,\rangle=v_\pi(q){\rm O}_{\pi{\rm N}\Delta}(\mathbf{q})\ , \tag{3.5}$$

where $v_\pi(q)$ is the form factor and

$$O_{\pi N\Delta}(\mathbf{q})=i\mathbf{S}\cdot\mathbf{q}T_{\alpha} \tag{3.6}$$

is the $\pi N\to\Delta$ transition operator, respectively. **S** (**T**) is the $N\to\Delta$ transition spin (isospin) operator. The matrix element of $v^B_{\pi\pi}$ is proportional to $O^\dagger_{\pi N\Delta}O_{\pi N\Delta}$, since it is a projection operator into the πN P_{33} channel:

$$\langle \mathbf{q}' \mid V^B_{\pi\pi} \mid \mathbf{q}\rangle = v^B_{\pi\pi}(q',q)O^\dagger_{\pi N\Delta}(\mathbf{q}')O_{\pi N\Delta}(\mathbf{q}) \ . \tag{3.7}$$

Similarly the matrix element of $t_{\pi\pi}$ is of the form

$$\langle \mathbf{q}' \mid t_{\pi\pi} \mid \mathbf{q}\rangle = t_{\pi\pi}(q',q)O^\dagger_{\pi N\Delta}(\mathbf{q}')O_{\pi N\Delta}(\mathbf{q}) \ . \tag{3.8}$$

For convenience, we assume $v^B_{\pi\pi}(q',q)$ to be separable

$$v^B_{\pi\pi}(q',q)=h(q')\lambda^{-1}h(q) \ . \tag{3.9}$$

This assumption makes it easy to calculate $t_{\pi\pi}(q',q)$ and leads us to

$$t_{\pi\pi}(q',q)=\frac{1}{D}\sum_{i,j}F_i(q')N_{ij}F_j(q) \ . \tag{3.10}$$

Here F_i ($i=1,2$) is defined by

$$F_1(q)=v_\pi(q), \quad F_2(q)=h(q) \ . \tag{3.11}$$

N_{ij} is given by a loop integral σ_{ij} as follows,

$$\begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix} = \begin{bmatrix} \lambda-\sigma_{22} & \sigma_{21} \\ \sigma_{12} & W-m_\Delta-\sigma_{11} \end{bmatrix} \ . \tag{3.12}$$

σ_{ij} is written explicitly as

$$\sigma_{ij}=\frac{4\pi}{3}\int_0^\infty \frac{1}{(2\pi)^3}q^4dq\frac{1}{2\omega(q)}\frac{m_N}{E_N(q)}F_i(q)F_j(q) \times\frac{1}{W-E_N(q)-\omega(q)+i\epsilon} \ . \tag{3.13}$$

D in Eq. (3.10) is the determinant of N_{ij},

$$D=N_{11}N_{22}-N_{12}N_{21} \ . \tag{3.14}$$

The phases δ_e and δ in Sec. II are expressed by N_{11} and D, respectively,

$$N_{11}=\mid N_{11}\mid \exp(-i\delta_e) \ , \tag{3.15}$$

$$D=\mid D\mid \exp(-i\delta) \ . \tag{3.16}$$

The last equation follows from the fact that the on-shell value of the sum over i and j in Eq. (3.10) is real.

From the usual decomposition of the photoproduction amplitude,[15] the matrix element of $t_{\pi\gamma}$ is found to be proportional to the projection operators

$$\begin{aligned} &[\mathbf{q}\cdot(\mathbf{k}\times\boldsymbol{\epsilon})-\tfrac{1}{3}\boldsymbol{\sigma}\cdot\mathbf{q}\,\boldsymbol{\sigma}\cdot(\mathbf{k}\times\boldsymbol{\epsilon})]T^\dagger_\alpha T_3 \quad [\text{for } M_{1+}(\tfrac{3}{2})] \ , \\ &-\frac{i}{2}(\mathbf{q}\cdot\mathbf{k}\,\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}+\mathbf{q}\cdot\boldsymbol{\epsilon}\,\boldsymbol{\sigma}\cdot\mathbf{k})T^\dagger_\alpha T_3 \quad [\text{for } E_{1+}(\tfrac{3}{2})] \ . \end{aligned} \tag{3.17}$$

We can write these operators in a compact form $O^\dagger_{\pi N\Delta}O_{\gamma N\Delta}$ using the $\gamma N\to\Delta$ transition operator

$$O_{\gamma N\Delta}(\mathbf{k},\boldsymbol{\epsilon})=\begin{cases} i\mathbf{S}\cdot(\mathbf{k}\times\boldsymbol{\epsilon})T_3 \quad [\text{for } M_{1+}(\tfrac{3}{2})] \ , \\ \mathbf{k}\cdot\overleftrightarrow{\Sigma}\cdot\boldsymbol{\epsilon}T_3 \quad [\text{for } E_{1+}(\tfrac{3}{2})] \ . \end{cases} \tag{3.18}$$

The tensor transition spin operator $\overleftrightarrow{\Sigma}$ is defined by

$$\Sigma_{ij}=\tfrac{1}{2}(S_i\sigma_j+S_j\sigma_i) \ . \tag{3.19}$$

Thus the matrix element of $t_{\pi\gamma}$ is expressed in the form

$$\langle \mathbf{q} \mid t_{\pi\gamma} \mid \mathbf{k},\boldsymbol{\epsilon}\rangle = t_{\pi\gamma}(q,k)O^\dagger_{\pi N\Delta}(\mathbf{q})O_{\gamma N\Delta}(\mathbf{k},\boldsymbol{\epsilon}) \ . \tag{3.20}$$

The matrix elements of v_γ and $v^B_{\pi\gamma}$ are written in terms of $O_{\gamma N\Delta}$ and $O_{\pi N\Delta}$ as

$$\langle \Delta \mid v_\gamma \mid \mathbf{k},\boldsymbol{\epsilon}\rangle = v_\gamma(k)O_{\gamma N\Delta}(\mathbf{k},\boldsymbol{\epsilon}) \ , \tag{3.21}$$

$$\langle \mathbf{q} \mid v^B_{\pi\gamma} \mid \mathbf{k},\boldsymbol{\epsilon}\rangle = v^B_{\pi\gamma}(q,k)O^\dagger_{\pi N\Delta}(\mathbf{q})O_{\gamma N\Delta}(\mathbf{k},\boldsymbol{\epsilon}) \ . \tag{3.22}$$

The amplitude $t_{\pi\gamma}(q,k)$ is evaluated from Eq. (2.31), once the dressed $\pi N\Delta$ vertex function is calculated from

$$\tilde{v}_\pi(q)=v_\pi(q)+\sigma_{12}h(q)/N_{11} \tag{3.23}$$

and the renormalization factor γ and the photoproduction phase shift δ_p are fixed by

$$\gamma\exp(i\delta_p)=1+v_\gamma(k)^{-1}[\sigma_{1\gamma}(k)+\sigma_{12}\sigma_{2\gamma}(k)/N_{11}] \ . \tag{3.24}$$

$\sigma_{i\gamma}(k)$ is calculated replacing $F_j(q)$ in Eq. (3.13) by

$$v_{\pi\gamma}(q,k)=v^B_{\pi\gamma}(q,k)+v_\pi(q)(W-m_\Delta)^{-1}v_\gamma(k) \ . \tag{3.25}$$

In Sec. II, we were led to Eq. (2.31) exploiting unitarity, i.e., Watson's theorem. For completeness we verify Watson's theorem without referring explicitly to the unitarity equation. The amplitude $t_{\pi\gamma}(q,k)$ can be cast in a simpler form by rewriting Eq. (2.15) as

$$t_{\pi\gamma}(q,k)=\frac{1}{D}\left[D\cdot v_{\pi\gamma}(q,k)+\sum_{i,j}F_i(q)N_{ij}\sigma_{j\gamma}(k)\right] \ . \tag{3.26}$$

We can prove that the on-shell value of the quantity in the brackets is real. Therefore the phase of $t_{\pi\gamma}(q,k)$ is equal to $\arg(D^{-1})=\delta$.

B. πN scattering in the P_{33} channel

The partitioning of the πN scattering amplitude into a resonant and background part is evidently model dependent. We can fit the πN scattering phase shift without the background $v^B_{\pi\pi}$. Alternatively, Théberge, Thomas, and Miller[16] presented the cloudy bag model (CBM) that contains both resonant and background πN interactions. In the present paper we do not calculate coupling constants and form factors in a particular quark model but simply parametrize them and fit them to data. Since our model has essentially the same dynamical structure as CBM, we expect that the extracted interactions can be compared to the predictions of CBM.

To examine the effect of the elastic background, we give two different models. First we fit the phase shift without the background interaction. The form factor $v_\pi(q)$ is parametrized as

$$v_\pi(q)=\frac{f_{\pi N\Delta}}{m_\pi}\left[\frac{b^2_\Delta}{b^2_\Delta+q^2}\right]^2 , \tag{3.27}$$

TABLE I. Parameters for v_π and $v^B_{\pi\pi}$. A refers to the model without the elastic background and B to the model incorporating the background.

	m_Δ (MeV)	$f_{\pi N\Delta}$	b_Δ (MeV/c)	f_B	b_B (MeV/c)
A	1284	2.915	497.2		
B	1365	1.913	880.8	2.063	1212

where $f_{\pi N\Delta}$ and b_Δ are a coupling constant and a cutoff mass, respectively. We have three parameters m_Δ, $f_{\pi N\Delta}$, and b_Δ and the resulting parameters are listed in row A of Table I. The phase shift is shown in Fig. 5.

Next, the elastic background interaction $v^B_{\pi\pi}$ is introduced in addition to the resonant interaction v_π. The main contribution to the background interaction is supposed to be from the driving term in the Chew-Low model[15] (Fig. 4). This term becomes a separable form by using an approximation for the propagator[16]

$$[W-E_N(|\mathbf{q}-\mathbf{q}'|)-\omega(q)-\omega(q')]^{-1} \cong -\frac{\omega(q_0)}{\omega(q)\omega(q')}\,, \quad (3.28)$$

where q_0 is the on-shell pion momentum. Then $v^B_{\pi\pi}$, Eq. (3.9), is parametrized by

$$\lambda=-\omega(q_0)^{-1}\,, \quad (3.29)$$

$$h(q)=\frac{f_B}{m_\pi}\left[\frac{b_B^2-m_\pi^2}{b_B^2+q^2}\right]^2\frac{1}{\omega(q)}\,. \quad (3.30)$$

We do not use the Chew-Low value $f_B=2f=2.0$ ($f^2/4\pi=0.08$) but leave it as a free parameter. The parameter set to be fixed is m_Δ, $f_{\pi N\Delta}$, b_Δ, f_B, and b_B. We list them in row B of Table I. The resulting phase shift is shown in Fig. 5 but we cannot distinguish the difference between model A and B. The elastic background phase shift δ_e for model B is also exhibited in the figure.

Though both of the models are well fitted, their parameter values are quite different: The cutoff mass b_Δ for the $\pi N\Delta$ vertex of model B is large compared with that of model A. The larger b_Δ of model B approximately corresponds to the CBM form factor with the bag radius 0.72 fm. The $\pi N\Delta$ coupling constant $f_{\pi N\Delta}$ of model B resembles the static quark model prediction $f_{\pi N\Delta}=(\frac{72}{25})^{1/2}f=1.7$. One also sees thats f_B virtually coincides with the Chew-Low value which is used in CBM.

It is reasonable that the parameter values for model B and CBM are close since the treatment of the πN scattering in model B is equivalent to CBM.

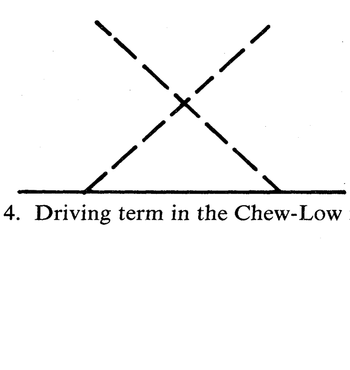

FIG. 4. Driving term in the Chew-Low model.

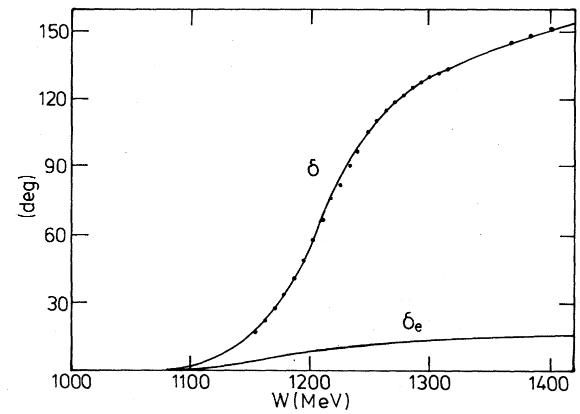

FIG. 5. Fits to the $\pi N P_{33}$ phase shift of Ref. 17. The elastic background phase shift δ_e for model B is also shown.

C. Pion photoproduction amplitude

We proceed to determine v_γ and $v^B_{\pi\gamma}$. The k dependence of $v_\gamma(k)$ is only through the total energy W, because the incoming photon is real. The total energy varies from 1080 to 1400 MeV in the region of our interest. We neglect this k dependence and write

$$v_\gamma(k)\simeq f_{\gamma N\Delta}\frac{e}{2m_N} \quad (3.31)$$

in units of the nuclear magneton. The dominant contribution to $v^B_{\pi\gamma}$ supposedly comes from the sum of the Born terms $v^{\rm Born}_{\pi\gamma}$ depicted in Fig. 6. The amplitude $v^B_{\pi\gamma}(q,k)$ is assumed to be proportional to $v^{\rm Born}_{\pi\gamma}(q,k)$,

$$v^B_{\pi\gamma}(q,k)=f_p\left[\frac{b_p^2}{b_p^2+q^2}\right]^2 v^{\rm Born}_{\pi\gamma}(q,k)\,. \quad (3.32)$$

The form factor is introduced to cut off $v^{\rm Born}_{\pi\gamma}(q,k)$ at high momenta. The factor f_p is inserted to take into account effectively the ω meson exchange and other complex mechanisms. We justify such a procedure seeing that the ω effect is simply to renormalize the Born terms slightly (see Ref. 9). $v^{\rm Born}_{\pi\gamma}(q,k)$ is the $M_{1+}(\frac{3}{2})$ or $E_{1+}(\frac{3}{2})$ multipole calculated from

$$\langle \mathbf{q}\,|\,v^{\rm Born}_{\pi\gamma}\,|\,\mathbf{k},\boldsymbol{\epsilon}\rangle=\langle \mathbf{q}\,|\,v_a+v_b+v_c\,|\,\mathbf{k},\boldsymbol{\epsilon}\rangle\,. \quad (3.33)$$

Here v_a, v_b, and v_c correspond to the diagrams in Figs. 6(a)—(c):

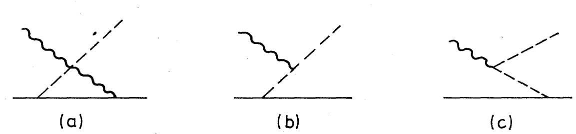

FIG. 6. Born terms which contribute to $M_{1+}(\frac{3}{2})$ and $E_{1+}(\frac{3}{2})$ multipoles.

TABLE II. Parameters for v_γ and $v^B_{\pi\gamma}$.

		$f_{\gamma N\Delta}$	f_p	b_p (MeV/c)
$M_{1+}(\frac{3}{2})$	A	-4.07 ± 0.03	0.94 ± 0.03	900 ± 100
	B	$-3.1\ \pm0.2$	1.19 ± 0.02	200 ± 1400
$E_{1+}(\frac{3}{2})$	A	0.11 ± 0.02	1.27 ± 0.07	750 ± 90
	B	0.23 ± 0.03	1.27 ± 0.06	750 ± 80

$$\langle \mathbf{q} \,|\, v_a \,|\, \mathbf{k},\boldsymbol{\epsilon} \rangle = -\frac{\kappa e f}{m_N m_\pi} \boldsymbol{\sigma}\cdot(\mathbf{k}\times\boldsymbol{\epsilon})\boldsymbol{\sigma}\cdot\mathbf{q}\tau_3\tau_\alpha \times \frac{1}{E_N(k)-\omega(q)-E_N(|\mathbf{k}+\mathbf{q}|)} , \tag{3.34}$$

$$\langle \mathbf{q} \,|\, v_b \,|\, \mathbf{k},\boldsymbol{\epsilon} \rangle = \frac{ef}{2m_\pi} \boldsymbol{\sigma}\cdot(\mathbf{k}-\mathbf{q})\boldsymbol{\epsilon}\cdot(2\mathbf{q}-\mathbf{k})\epsilon_{3\beta\alpha}\tau_\beta \times \frac{1}{E_N(k)-E_N(q)-\omega(|\mathbf{q}-\mathbf{k}|)} \times \frac{1}{\omega(|\mathbf{q}-\mathbf{k}|)} , \tag{3.35}$$

$$\langle \mathbf{q} \,|\, v_c \,|\, \mathbf{k},\boldsymbol{\epsilon} \rangle = \frac{ef}{2m_\pi} \boldsymbol{\sigma}\cdot(\mathbf{k}-\mathbf{q})\boldsymbol{\epsilon}\cdot(2\mathbf{q}-\mathbf{k})\epsilon_{3\beta\alpha}\tau_\beta \times \frac{1}{\omega(q)+\omega(|\mathbf{q}-\mathbf{k}|)-k} \times \frac{1}{\omega(|\mathbf{q}-\mathbf{k}|)} . \tag{3.36}$$

We use $\kappa=\frac{1}{2}(\mu_p-\mu_n)=1.85$ and $e^2/4\pi=\frac{1}{137}$.

We adjust $f_{\gamma N\Delta}$, f_p, and b_p to fit $t_{\pi\gamma}$ to each of observed $M_{1+}(\frac{3}{2})$ and $E_{1+}(\frac{3}{2})$ multipoles.[17] Parameter fits are performed in two cases (A and B) depending on the model for πN scattering in the previous section. The best fit parameters are listed in Table II and corresponding amplitudes are shown in Fig. 7. Both models A and B reproduce the data very well, being indistinguishable in the figures. The disagreement of the $E_{1+}(\frac{3}{2})$ multipole between the fits and the data around $E_\gamma=400\sim450$ MeV is due to the fact that the data in the lower energy region do not smoothly continue to the data at higher energies.

Koch and Moniz[11] simply parametrized $\delta_p-\delta_e$ and fitted it to the $M_{1+}(\frac{3}{2})$ data. In Fig. 8 our results for $\delta_p-\delta_e$ are given together with δ_p. The phase $\delta_p-\delta_e$ for the $M_{1+}(\frac{3}{2})$ amplitude is about 8 deg on the resonance. This makes the maximum of Im$M_{1+}(\frac{3}{2})$ shift about 7 MeV lower from the position of the Δ resonance, because the photoproduction amplitude is proportional to $\sin(\delta+\delta_p-\delta_e)$.

IV. HELICITY AMPLITUDES FOR THE BARE Δ

From the γNΔ vertex interaction v_γ, photoexcitation amplitudes for the Δ resonance can be estimated. Clearly, model B should be compared to the quark models. The renormalization factor γ for the resonant amplitude causes a significant change in the amplitudes. In the case of the $M1$ multipole, since $\gamma\cong1.8$ on the Δ resonance (see Fig. 9), it is expected to be instrumental in reducing considerably the result of the data analyses which do not take account of the renormalization effect. Concerning the $E2$ multipole, since γ is negative, our $E2$ amplitude will become completely different from other estimations.

The definition of helicity amplitudes is

$$A_{1/2}=\frac{1}{\sqrt{2k_\Delta}}\left[\frac{m_N}{E_N(q_\Delta)}\right]^{1/2}\langle S_z=\tfrac{1}{2};\Delta \,|\, v_\gamma \,|\, \mathbf{k}_\Delta,\boldsymbol{\epsilon},s_z=-\tfrac{1}{2};\mathrm{N}\rangle , \tag{4.1}$$

$$A_{3/2}=\frac{1}{\sqrt{2k_\Delta}}\left[\frac{m_N}{E_N(q_\Delta)}\right]^{1/2}\langle s_z=\tfrac{3}{2};\Delta \,|\, v_\gamma \,|\, \mathbf{k}_\Delta,\boldsymbol{\epsilon},s_z=\tfrac{1}{2};\mathrm{N}\rangle \tag{4.2}$$

where k_Δ (q_Δ) is the photon (pion) momentum on the Δ resonance. $A_{1/2}(M1)$, $A_{3/2}(M1)$, $A_{1/2}(E2)$, and $A_{3/2}(E2)$ are not independent of each other but related by

$$A_{3/2}(M1)=\sqrt{3}A_{1/2}(M1) , \tag{4.3}$$

$$A_{3/2}(E2)=-\frac{1}{\sqrt{3}}A_{1/2}(E2) . \tag{4.4}$$

Two independent quantities $A_{1/2}(M1)$ and the $E2/M1$ ratio R are listed in Table III, where

$$R=\frac{A_{1/2}(E2)}{3A_{1/2}(M1)} . \tag{4.5}$$

It can be seen that compared to the compilation by the Particle Data Group[13] our amplitude $A_{1/2}(M1)$ gets much closer in magnitude to the prediction of the MIT bag model.[18] Similar reduction of the $M1$ amplitude is deduced by Olsson[8] using the unitarity relation and a pole model for v_π and v_γ. However, as we mentioned in Sec.

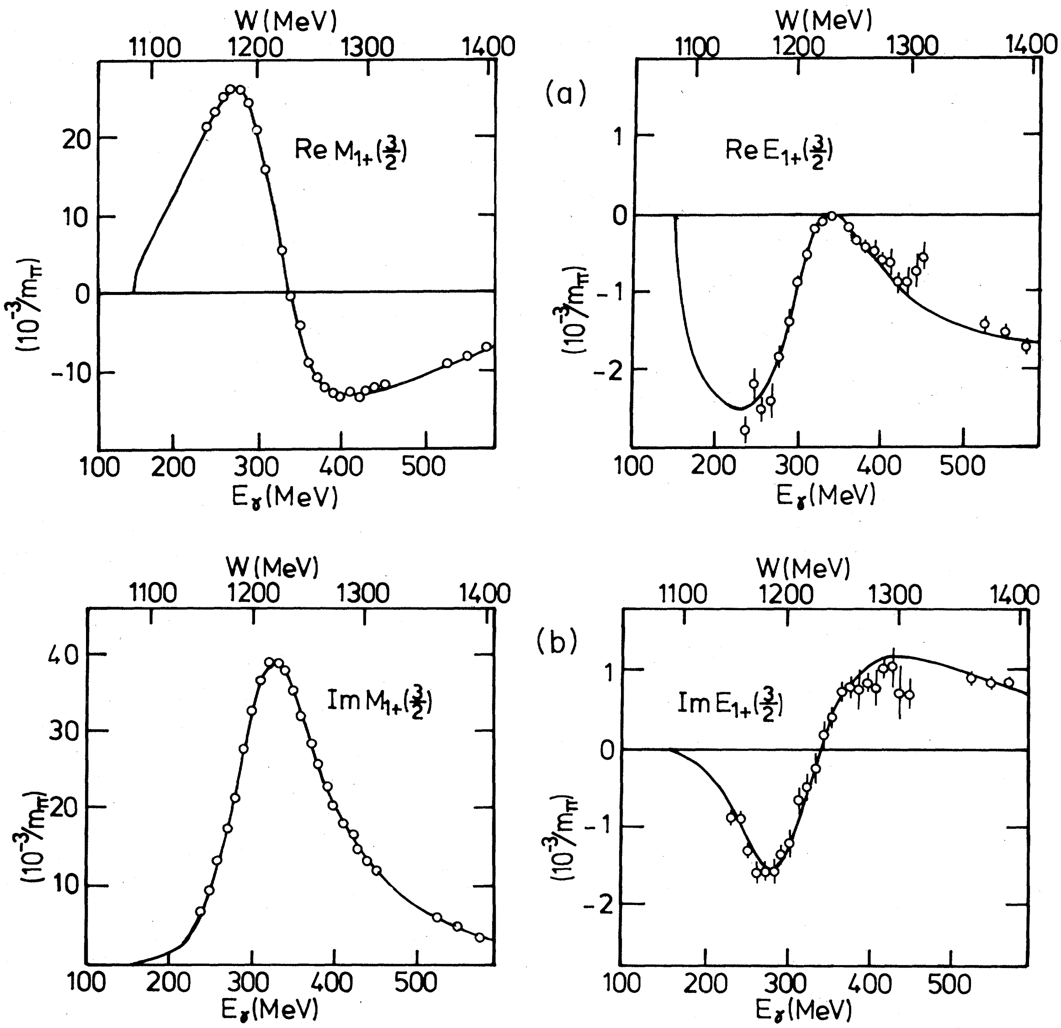

FIG. 7. Fits to the real and imaginary parts of the $M_{1+}(\frac{3}{2})$ and $E_{1+}(\frac{3}{2})$ multipoles of Ref. 17.

II, he adapted the pole model to $\tilde{N}$ rather than N [see discussion below Eq. (2.33)]. Such a procedure is not satisfactory, when we try to compare the resulting $\gamma N\Delta$ coupling constant with the quark model prediction. From the standpoint of the chiral bag model, Kälbermann and Eisenberg[3] discussed the pionic correction to the $\gamma N\Delta$ vertex interaction, a part of which corresponds to the rescattering term in our model. Their result also shows the enhancement of the helicity amplitude $A_{1/2}(M1)$ compared with the MIT bag model prediction.

On the other hand, there is a drastic change in the $E2$ amplitude. The sign and magnitude of our $E2$ amplitude are different from the data analyses[6,7] which ignored the interference between the resonant and background amplitudes. The quark model prediction of the $E2$ amplitude is uncertain and currently in dispute. Isgur, Karl, and Koniuk[2] used the nonrelativistic constituent quark model with the standard strength of the hyperfine interaction and estimated $R=-0.4\%$. However, Dey and Dey[4] introduced the finite range spin-spin interaction and got a completely different result $R=-0.05\sim0.8\ \%$ depending on the quantum chromodynamics (QCD) running coupling constant $\alpha_s=0.5\sim1.12$. According to Kälbermann and Eisenberg,[3] the contribution of the pion cloud yields $R=-0.9\%$. It is difficult to compare our result with their prediction, because their model involves the rescat-

TABLE III. A comparison of $A_{1/2}(M1)$ and $R=E2/M1$ of the previous analysis with those obtained here and quark model predictions.

	Particle Data Group	This work (Ref. 13)	Quark model
$A_{1/2}(M1)(\times10^{-3}\ \mathrm{GeV}^{-1/2})$	-147 ± 5	-84 ± 5	-102 (Ref. 18)
R (%)	-1.4 ± 1.4	3.7 ± 0.4	-0.4 (Ref. 2)

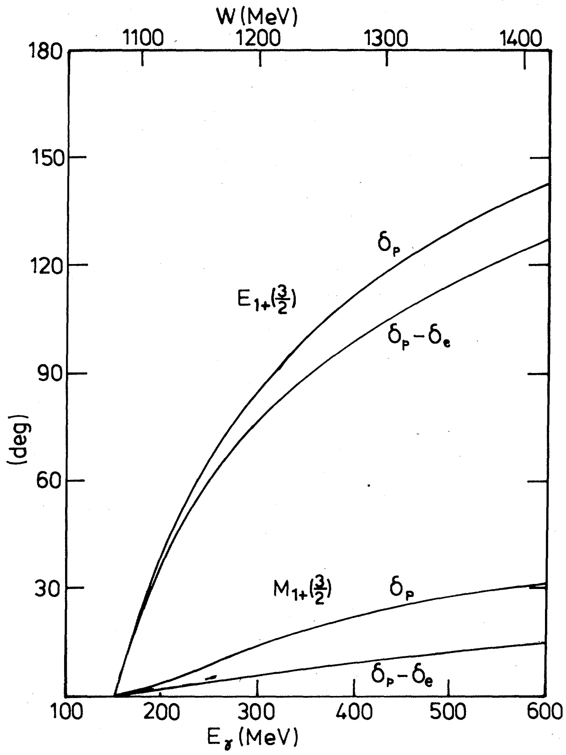

FIG. 8. Phase angles appearing in the photoproduction multipole formula (2.31).

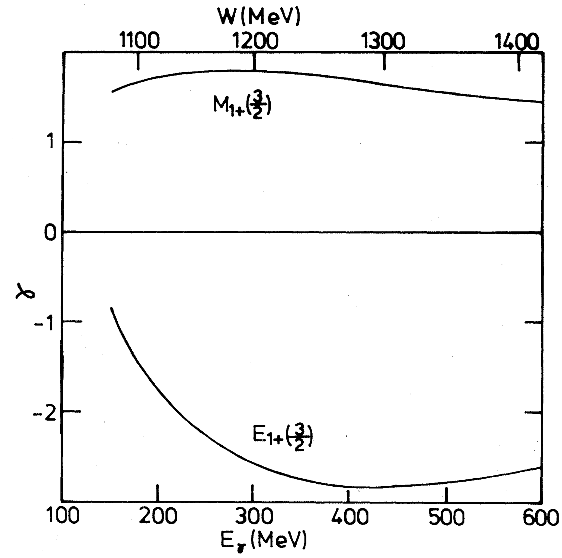

FIG. 9. Renormalization factor $\gamma = |t^R_{\pi\gamma}| / |t^\Delta_{\pi\gamma}|$.

tering term as an effect of the pion cloud. The remaining effect of the pion cloud which does not correspond to the rescattering term should be absorbed into the "bare" $\gamma N\Delta$ coupling constant in our model. These corrections might be important for the small $E2$ amplitude. Recently Drechsel and Giannini[5] reported that the ratio R varies an order of magnitude depending on whether one uses the charge or current operator for the $E2$ $\gamma N\Delta$ vertex interaction. More effort will be needed to make a sound prediction of the $E2$ amplitude.

V. SUMMARY AND CONCLUSIONS

We obtained a dynamical model for pion photoproduction. The important ingredient is the background interaction in addition to the Δ resonant interaction. The fit to the πN P_{33} phase shift is done by introducing the background interaction. Resulting parameters are close to those of CBM as is naturally expected. Our model automatically satisfies Watson's theorem and is closely related to Olsson's model. Olsson, however, neglected the renormalization factor and we find it plays an important role in the estimation of the bare $\gamma N\Delta$ coupling constant. The longstanding discrepancy between the quark model prediction and the empirical value for the $M1$ transition strength is resolved as the effect of the πN rescattering term. However, the $E2$ amplitude is predicted differently from other authors. The quark model prediction of the $E2$ amplitude is problematic and has not settled yet. We hope the progress of the theory will make a definitive prediction of the $E2$ amplitude.

It is noted that our model is equipped with the off-shell transition matrix elements of the elementary photoproduction process. We confined ourselves to the P_{33} channel in this paper, but extension to other partial waves is straightforward. We intend to apply this model to the calculation of the many body photonuclear reaction.

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