Polarization observables in $\pi d \to \pi d$ and $pp \to \pi d$ and the NN-πNN equations

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Here we compare the results from the NN-πNN equation with some of the available data on $pp \to \pi d$ and $\pi d \to \pi d$ polarization observables. In particular, we investigate some of the uncertainties in the theoretical results due to variations in the input to the calculation. We find no major discrepancy between theory and experiment that warrants the introduction of new degrees of freedom such as dibaryons.

I. INTRODUCTION

The recent explosion in polarization measurements in both π-d scattering[1–3] and $pp \to \pi d$ reactions[4–11] has been motivated by the possible observation of dibaryon resonances and their interpretation in terms of six-quark states. Although the measurements to date predominantly favor the absence of such resonances, as we will show, the large body of data accumulated can give us considerable insight into the mechanism for pion scattering and absorption in the lightest nucleus, i.e., the deuteron. This in turn may shed more light on the mechanisms for these reactions in heavier nuclei. Furthermore, a major discrepancy between the experimental results and theories based on the conventional degrees of freedom (i.e., $N,\Delta,\pi,\rho,\ldots$), could be taken as evidence for the need to introduce explicit quark degrees of freedom. This comparison between theory and experiment is possible because we have a theory (the NN-πNN equations[12,13]) that describes these reactions in a unified framework, satisfies two- and three-body unitarity, and can incorporate nucleon, Δ, π, and ρ degrees of freedom.

The aim of the present investigation is twofold:

(i) To examine the convergence of the NN-πNN calculation to the input π-N and N-N partial wave expansion. Here, we find that for π-d elastic scattering, we need to include all S- and P-wave, π-N and N-N interactions. However, for pion production, there is a non-negligible contribution from the D waves.

(ii) To study the sensitivity of the observables to uncertainties in the π-N and N-N input amplitudes. We observed that for π-d elastic scattering, the tensor polarization T_{20} is the most sensitive observable to the choice of the π-N interaction, particularly the P_{11} partial wave. For $pp \leftrightarrow \pi d$ the differential cross section seems to be more sensitive to the P_{11} interaction, and its off-shell behavior, than the polarization observables. In general, within the uncertainty introduced by the input amplitudes, we find no major discrepancy between theory and experiment. This is particularly the case as there is scope for improvement in the theory.

In Sec. II, we present a brief summary of the NN-πNN equations. Since we will be examining the sensitivity of our results to the P_{11} interaction, we present, in this section, the parametrization of this interaction. We also specify how the inelasticity is included in the π-N amplitudes. We then proceed, in Sec. III, to a discussion of our results. Finally, in Sec. IV, we present some concluding remarks.

II. THE BASIC FORMULATION

The reactions $\pi d \to \pi d$ and $pp \to \pi d$ are naturally coupled, in that the reaction cross section for πd elastic scattering is related to the total cross section for $\pi d \to pp$. This suggests the need for a unified description of the three reactions $NN \to NN$, $pp \to \pi d$, and π-d elastic scattering. Over the past few years, several groups[12–15] have developed a set of equations for the NN-πNN system that satisfy two- and three-body unitarity and describe all three of the above reactions. Although, in the final analysis, the equations developed by the different groups are identical,[16] the implementations of these equations for practical calculations[17–24] are different enough to give different results for the observables in π-d elastic scattering and $pp \leftrightarrow \pi d$ reactions. To define the input to our calculations, we need to summarize the NN-πNN equations used, and the two-body input to these equations, particularly the P_{11} π-N amplitude. It is this amplitude which gives the coupling between the π-d and N-N channels, and the final results can be sensitive to the way this amplitude is divided into a pole and nonpole part.

Assuming separable potentials for the input π-N and N-N amplitude, we can write the NN-πNN equation as a set of coupled integral equations similar to the Faddeev[25] equations and of the form (see, e.g., Ref. 17, hereafter referred to as I),

$$\begin{aligned}
\chi_{dd} &= Z_{d\Delta}\tau_\Delta\chi_{\Delta d} + Z_{dN}\tfrac{1}{2}d_N\chi_{Nd}\,,\\
\chi_{\Delta d} &= Z_{\Delta d}(1+\tau_d\chi_{dd}) + Z_{\Delta\Delta}\tau_\Delta\chi_{\Delta d} + Z_{\Delta N}\tfrac{1}{2}d_N\chi_{Nd}\,, \qquad (1)\\
\chi_{Nd} &= Z_{Nd}(1+\tau_d\chi_{dd}) + Z_{N\Delta}\tau_\Delta\chi_{\Delta d} + Z_{NN}\tfrac{1}{2}d_N\chi_{Nd}\,,
\end{aligned}$$

for $\pi d \to \pi d$ and $\pi d \to pp$. Here, d refers to the π(NN) channels, which include not only the π-d, but all channels with the N-N interacting and a spectator pion. In a similar manner Δ refers to channels with the π-N interacting and a spectator nucleon. Since we have assumed separable potentials of the form

$$V_\alpha(k,k')=g_\alpha(k)\lambda_\alpha g_\alpha(k') \quad (\alpha=\mathrm{d},\Delta) \tag{2}$$

for the input two-body interaction, the corresponding amplitudes are

$$t_\alpha(k,k',E)=g_\alpha(k)\tau_\alpha(E)g_\alpha(k') \quad (\alpha=\mathrm{d},\Delta)\ . \tag{3}$$

The nucleon propagator $d_{\rm N}$ will be defined when we discuss the P_{11} amplitude later in this section. The one particle exchange amplitudes $Z_{\alpha\beta}$ in Eq. (1) are fully antisymmetrized and are given explicitly in I.[17] Throughout the present investigation, we use semirelativistic kinematics, i.e., we treat the pion relativistically, while the nucleons are treated nonrelativistically. In this way, we maintain the correct clustering properties for the three-body amplitudes.[26]

For the N-N input amplitude, we use the separable potentials of Mongan[27] in all channels other than the deuteron ($^3S_1$-$^3D_1$). To test the sensitivity of the polarization observables to the deuteron D-state probability, we have used the unitary pole approximation[28] to the Reid soft core,[29] Bryan-Scott,[30] and Tourreil-Sprung[31] potentials. These have D-state probabilities of 6.56%, 5.36%, and 4.09%, respectively. For the π-N potentials other than the P_{11}, we have used the parametrization of Thomas.[32]

To include the coupling between the NN and πNN channels, the P_{11} amplitude needs to have the structure of a pole term plus a nonpole term.[12,13] This structure can be achieved by taking an energy dependent rank-two potential of the form[33–35]

$$v(k,k',E)=f_0(k)\frac{1}{E-m_0}f_0(k')+g(k)\lambda g(k')\ . \tag{4}$$

For the form factors, we have chosen

$$f_0(k)=\frac{C_1 k}{[\omega(k)]^{1/2}}\frac{1}{(k^2+\alpha^2)^{n_1}} \tag{5a}$$

and

$$g(k)=\frac{C_2 k}{[\omega(k)]^{1/2}}\left[\frac{1}{k^2+\beta_1^2}+C_3\frac{k^{2n_2}}{(k^2+\beta_2^2)^{n_3}}\right] \tag{5b}$$

with $\omega(k)=(k^2+m_\pi^2)^{1/2}$. The corresponding amplitude can be written as[35]

$$t(k,k',E)=f(k,E)d(E)f(k',E)+g(k)\tau(E)g(k')\ , \tag{6}$$

where the second term on the right-hand side (rhs) of Eq. (6) is the amplitude corresponding to the potential $g(k)\lambda g(k')$. The first term on the rhs of Eq. (6) has the nucleon pole in $d(E)$ which is given by

$$d(E)=[(E-m_{\rm N})[1-(E-m_{\rm N})\Gamma_2(E)]^{-1}\ , \tag{7}$$

$$\begin{aligned}\Gamma_2(E)=&\langle f(m_{\rm N})\,|\,[G(m_{\rm N})]^2G(E)\,|\,f(E)\rangle\\ &+\langle f(m_{\rm N})\,|\,[G(m_{\rm N})]^2g\rangle\tau(m_{\rm N})\\ &\times\langle g\,|\,G(m_{\rm N})G(E)\,|\,f(E)\rangle\ .\end{aligned} \tag{8}$$

Here, the πNN form factor $f(k,E)$ is given by

$$\begin{aligned}f(k,E)&=\langle k\,|\,f(E)\rangle\\ &=Z_2^{1/2}\{f_0(k)+g(k)\tau(E)\langle g\,|\,G(E)\,|\,f_0\rangle\}\end{aligned} \tag{9}$$

with the wave function renormalization constant given by

$$Z_2=1-\Gamma_1(m_{\rm N}) \tag{10}$$

with

$$\Gamma_1(m_{\rm N})=\langle f(m_{\rm N})\,|\,[G(m_{\rm N})]^2\,|\,f(m_{\rm N})\rangle\ . \tag{11}$$

The parameter of the potential m_0 has been adjusted,[36] so that $d(E)$ has a pole at the physical nucleon mass $m_{\rm N}$, i.e.,

$$m_0=m_{\rm N}-Z_2^{-1/2}\langle f_0\,|\,G(m_{\rm N})\,|\,f(m_{\rm N})\rangle\ , \tag{12}$$

and C_1 is chosen so that the residue at the pole is given in terms of the πNN coupling constant by the relation[36]

$$f_{\pi{\rm NN}}^2(k)=\frac{\pi}{3}\frac{m_\pi^2}{2m_{\rm N}^2}\omega(k)E(k)[E(k)+m_{\rm N}]\times\left[\frac{m_{\rm N}+W(k)}{2W(k)}\right]\left[\frac{f(k,m)}{k}\right]^2, \tag{13}$$

where

$$W(k)=E(k)+\omega(k)$$

with

$$E(k)=(k^2+m_{\rm N}^2)^{1/2}\ .$$

The πNN coupling constant is then given by $f_{\pi{\rm NN}}^2(k_0)=0.079$ with

$$k_0^2=-m_\pi^2\left[1-\frac{m_\pi^2}{4m_{\rm N}^2}\right]. \tag{14}$$

To study the sensitivity of the results for π-d elastic scattering, and pion production, to the choice of the P_{11} interaction, we present in Table I the parameters[36] of such potentials and their prediction for Z_2, $f_{\pi{\rm NN}}^2(0)$, and the π-N scattering length a_{11}. In Fig. 1(a), we present the corresponding phase shifts as compared with the latest π-N phase shift analysis,[37–39] while in Fig. 1(b) we present the corresponding πNN form factor $f(k,m_{\rm N})$ normalized to one at $k=0$. In general, the agreement with the phase shifts is very good. However, we will see in Sec. III that the results of the NN-πNN calculation are sensitive to the fit at low energies ($T_\pi<150$ MeV). At this stage, we would like to point out that the πNN form factor $f(k,m_{\rm N})$ changes rapidly with k between $k=k_0$ and 0. This rapid fall is a result of taking Yamaguchi form factors rather than the Gaussian[40] or Bessel function, which arise in the bag model of hadrons.[41] These latter form factors are relatively flat, near $k=0$. Any attempt at reducing this fall in the form factor has destroyed the fit

TABLE I. Parameters of the P_{11} π-N interaction used in the present investigation. All these potentials give a πNN coupling constant $f^{(0)}_{\pi NN}(k_0)=0.079$. Potential $E2$ has the inelasticity included (i.e., $\hat{\eta}\neq 1$).

Potential	n_1	n_2	n_3	α	β_1	β_2	C_1	C_2	C_3	m_0	Z_2	$f^2_{\pi NN}(0)$	a_{11}
$E2$	2	1	4	3.4983	1.2278	4.5545	30.5768	0.3233	21 326.9	5.2062	0.7939	0.0510	−0.0710
PJ	2	1	3	3.8206	1.2689	5.181	43.5646	0.2907	1420.59	5.1574	0.8059	0.0520	−0.0706
$M1$	1	2	3	2.7703	1.4422	2.1982	1.0726	0.3433	7.4026	5.4314	0.7273	0.0488	−0.0721
$P6$	2			6.6448	3.3617		256.749	2.4871	0.0	5.8118	0.6073	0.0697	−0.1079

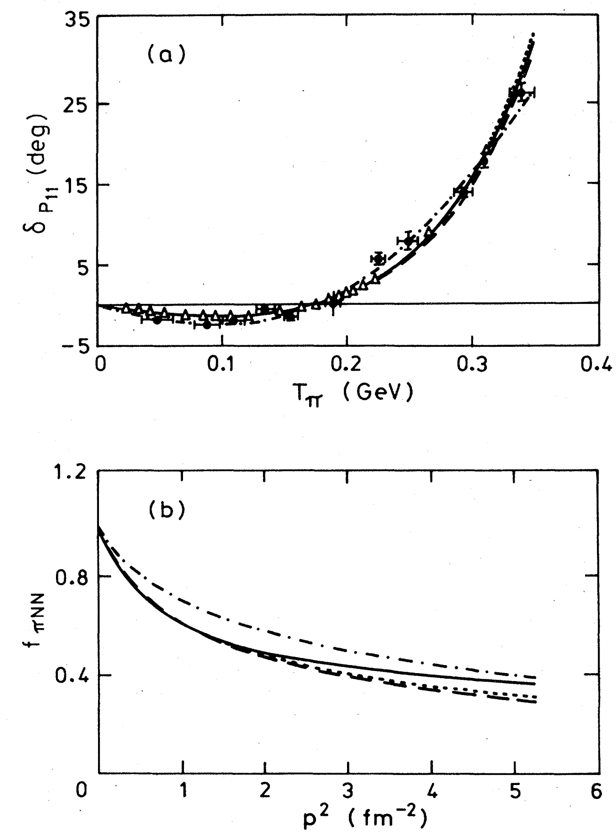

FIG. 1. The phase shifts (a), and πNN form factor $f(k_0,m_N)/f(0,m_N)$ (b) for the P_{11} potentials $M1$ (——), $E2$ (— — —), PJ (· · ·), and $P6$ (— · — · — ·). The experimental data are those of Ref. 37 (△) and Ref. 38 (●).

to the P_{11} phase shifts. We will see in Sec. III that the pion production cross section is sensitive to $f^2_{\pi NN}(0)$.

Since some of the P- and D-wave π-N phase shifts become suddenly large just above the threshold for pion production, it is important to know how rapid changes in the phase shifts change the parametrization of the π-N amplitudes. To fit these changes in the phase shifts we need to include the effect of inelasticity. This is most simply achieved by a modification of the two-body Green's function[36,40] so that

$$G(E)=\int_0^\infty dk\,k^2 \frac{|k\rangle\langle k|}{\hat{\eta}(k)[E-W(k)]}\,, \tag{15}$$

where

$$\hat{\eta}(k)=\frac{\sigma_{el}(k)}{\sigma_T(k)} \tag{16}$$

and the on-shell T matrix is given by

$$t(k)=-\frac{\hat{\eta}(k)}{\rho(k)}e^{i\hat{\delta}(k)}\sin\hat{\delta}(k)\,. \tag{17}$$

This parametrization of $t(k)$ in terms of $\hat{\eta}(k)$ and $\hat{\delta}(k)$

guarantees the proper inclusion of the pion production threshold, and is consistent with unitarity. In particular, below the pion production threshold our amplitude $t(k)$ satisfies two-body unitarity. Here $\hat{\eta}(k)$ and $\hat{\delta}(k)$ are extracted from the experimental results and $\hat{\eta}(k)$ is parametrized to fit this data. Given $\hat{\eta}(k)$ we can construct separable potentials that fit the experimental phase shifts and have the production threshold included.

III. NUMERICAL RESULTS

Before we can proceed to a discussion of our final results for π-d elastic scattering and $pp \rightarrow \pi d$, it is imperative that we specify the numerical procedure used in solving the NN-πNN equation. Since the kernels of the integral equations have moving singularities on the real axis, we have used the method of contour rotation to avoid these singularities. This allows us to get three significant figure accuracy in the amplitudes with 26-point Gauss Legendre quadratures, appropriately mapped onto 0 to ∞. For the partial wave projection of the one-particle exchange amplitudes (i.e., $Z_{\alpha\beta}$), we find 16-point quadrature is more than sufficient, while for the integrals to determine $\tau(E)$ and $d(E)$, we have used 48-point Gauss Legendre quadratures. In the case when $\hat{\eta} \neq 1$, for the π-N amplitude, we could not rotate the integration contour into the calculation of $\tau(E)$, and the integral along the real axis required a careful choice of quadratures.

The inclusion of all input S- and P-wave π-N and N-N interactions leads to as many as 27 coupled integral equations. This number can increase to as many as 36 coupled integral equations, if one includes all D-wave N-N interactions. To solve such a large number of equations, we have found it convenient to iterate the equations and at the same time use a diagonal Pade approximation of order $N/2$ after the Nth iteration to evaluate the amplitude and test for convergence. We find that this scheme converges for all partial waves with $J \leq 7$. In fact, for $J > 4$, the first few iterations are all we need.

Having established the stability of our numerical procedure, we turn to the convergence of the results with respect to the N-N and π-N partial wave expansions. In I, we carried out extensive tests to see which three-body channels were important, given that we want to include all S- and P-wave, π-N and S-, P-, and D-wave N-N channels. Although this procedure reduces the number of coupled integral equations, it has to be tested every time the two-body interaction is changed. This can be a time consuming procedure. We therefore have taken the philosophy of truncating the partial wave expansion in the two-body system and then including all the three-body channels allowed. Here, we would like to point out that different groups have taken different approaches to this problem. The Lyon group[18,19,22] and the Weizmann group[20,21,24] have taken the procedure we have used in the present paper, but have included S- and P-wave pions and only the $^3S_1$-$^3D_1$ (deuteron) N-N interaction. As we will

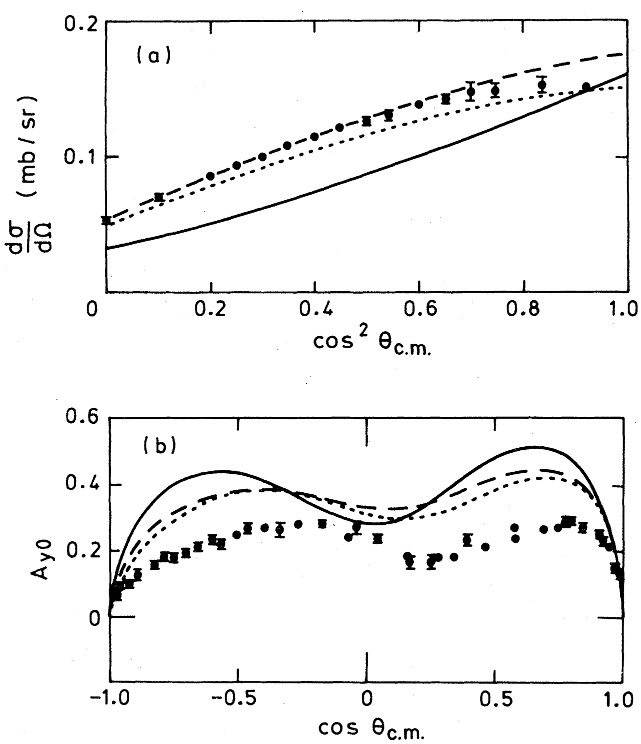

FIG. 2. The differential cross section and A_{y0} for $pp \rightarrow \pi d$ at $T_p = 800$ MeV. The theoretical results correspond to including all S- and P-wave π-N amplitudes, and N-N amplitudes in $^3S_1$-$^3D_1$ (——), $^3S_1$-$^3D_1$, S- and P-wave (— — —), and all S-, P-, and D-wave (· · ·) channels. The experimental results are those of Nann *et al.* (Ref. 8), and Saha *et al.* (Ref. 5).

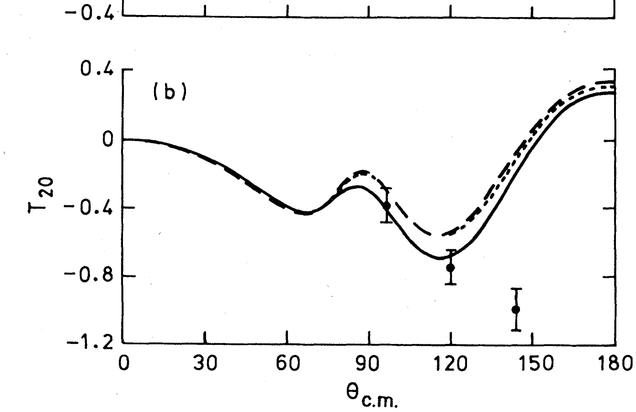

FIG. 3. Comparison of iT_{11} (a) and T_{20} (b) for π-d elastic scattering at $T_\pi = 256$ MeV, with different choices of N-N input partial waves included. The curves are labeled as in Fig. 2. The experimental results are those of Ref. 1 for iT_{11}, and Ref. 3 for T_{20}.

show, this is not sufficient at $T_\pi = 256$ MeV. On the other hand, the Osaka group[42] has selected only a few three-body channels. This procedure can be highly dangerous unless one carries out extensive tests to make sure that the neglected three-body channels are negligible.

In Fig. 2(a), we present the differential cross section for $pp \rightarrow \pi d$ at $T_p = 800$ MeV, when all S- and P-wave π-N interactions are included, and when ${}^3S_1$-${}^3D_1$ (solid); ${}^3S_1$-${}^3D_1$ and all S and P wave N-N (dashed); and all S-, P-, and D-wave N-N (dotted) channels are included. It is clear from these results that the inclusion of the deuteron only is not sufficient, and the cross section is underestimated by a considerable factor. Furthermore, the shape of the angular distribution is not correct. This need to include both S- and P-wave N-N interaction might go part of the way in explaining the observed discrepancy between theory and experiment in the work of the Lyon group[19,22] and the Weizmann group.[20,21,24] This sensitivity to the inclusion of P-wave N-N is also present in A_{y0}, as demonstrated in Fig. 2(b). Although the contribution of D-wave N-N is not negligible, we have decided to neglect it in the present investigation as it has a small effect on the π-d elastic scattering observables. This is illustrated in Fig. 3, where we present π-d polarization results for the vector polarization iT_{11}, and the tensor polarization T_{20}. The neglect of D-wave N-N amplitudes will reduce the number of coupled integral equations to be solved from 36 to 27 for most partial waves. In this way, we can reduce our computing time by a factor of 2 at least.

We now turn to the D-wave π-N amplitude and its contribution to π-d elastic scattering. The lowest energy D-wave π-N resonance is the D_{13} with a pole at $(1440 - 60i)$ MeV. We might expect the threshold for the production of this resonance to have a similar effect to the $\Delta(1232)$ threshold. Furthermore, the position of this resonance threshold is close to where a ${}^1G_4$ "dibaryon" resonance in N-N scattering has been found. This suggests that the D_{13} might have some influence on π-d elastic scattering, and in particular the vector polarization results (iT_{11}). To test this possibility we have included two possible D_{13} potentials. The first includes the effect of inelasticity (i.e., $\hat{\eta} \neq 1$), the second has no inelasticity (i.e, $\hat{\eta} = 1$). Both of these potentials have resonance poles at about the same position in the complex E plane. In Fig. 4 we present the results of including the D_{13} π-N amplitude on π-d elastic scattering, the vector polarization iT_{11}, and the tensor polarization T_{20} at $T_\pi = 256$ MeV. The effect here is negligible as we are more than 100 MeV below the D_{13} resonance threshold. Thus for π-d scattering at the energies presently considered, the contribution of higher π-N resonances is negligible.

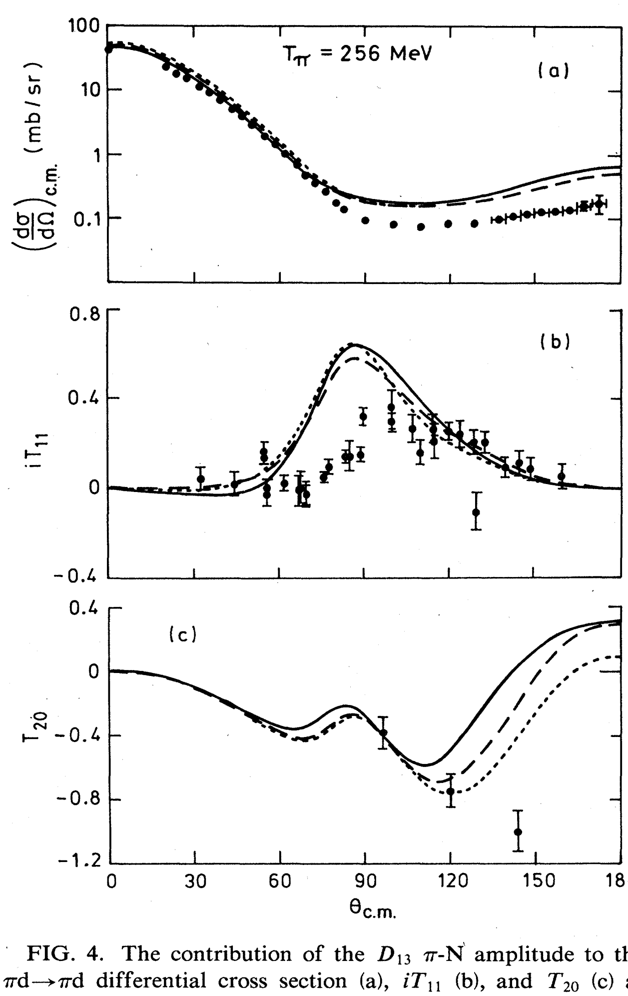

FIG. 4. The contribution of the D_{13} π-N amplitude to the $\pi d \rightarrow \pi d$ differential cross section (a), iT_{11} (b), and T_{20} (c) at $T_\pi = 256$ MeV. The curves are labeled such that those with D_{13} and $\hat{\eta} = 1$ (——), no D_{13} (— — —), and with D_{13} and $\hat{\eta} \neq 1$ (· · ·). The experimental results for the differential cross section are those of Ref. 38, the iT_{11} data are those of Ref. 1, while the T_{20} data are from Refs. 2 and 3.

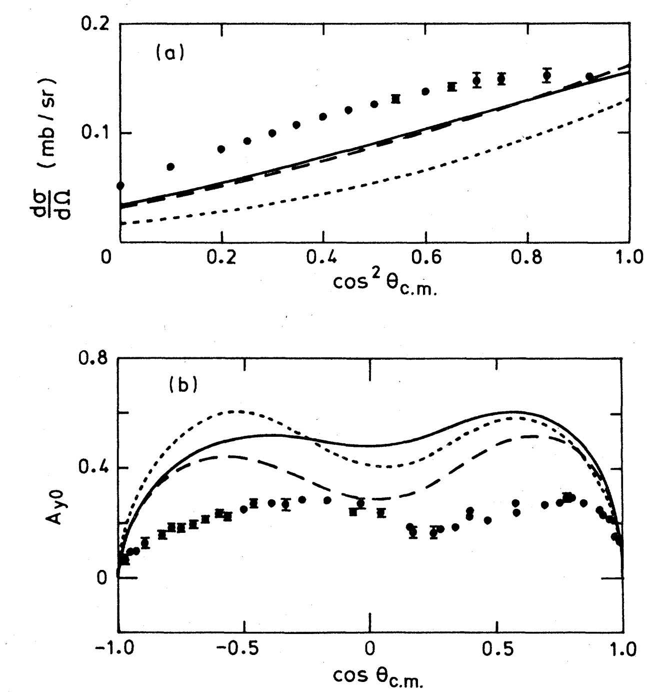

FIG. 5. The contribution of the D_{13} π-N amplitude to the $pp \rightarrow \pi d$ differential cross section (a), and A_{y0} (b) at $T_p = 800$ MeV. The experimental results are those in Fig. 2. Curves labeled as in Fig. 4.

If we examine the above effects in the pp→πd (Fig. 5), we find that for the differential cross section, the inclusion of inelasticity in the π-N amplitudes gives a large reduction in the cross section, while the D_{13} amplitude with no inelasticity leads to a very small contribution to the differential cross section. This is mainly due to the fact that with $\hat{\eta}\neq 1$, the off-shell behavior of the interaction is quite different, which is the result of the fact that the form factor $g(k)$ has to reproduce the rapid variation in the phase shifts. We note here that the inelasticity is included in all π-N channels. On the other hand, for the asymmetry A_{y0} [Fig. 5(b)], the inclusion of the D_{13} with and without inelasticity has a major effect. Here again, we see that a *D*-wave π-N amplitude can have a significant contribution to the polarization cross section at this energy. This suggests that one needs to include *D*-wave N-N and π-N amplitudes if theory is to reproduce experiment. For this to be achieved, we need to develop perturbative methods of including these channels.

So far, we have examined the role of higher partial waves in the two-body input amplitudes. We now turn to the role of the lower partial waves, and in particular, the

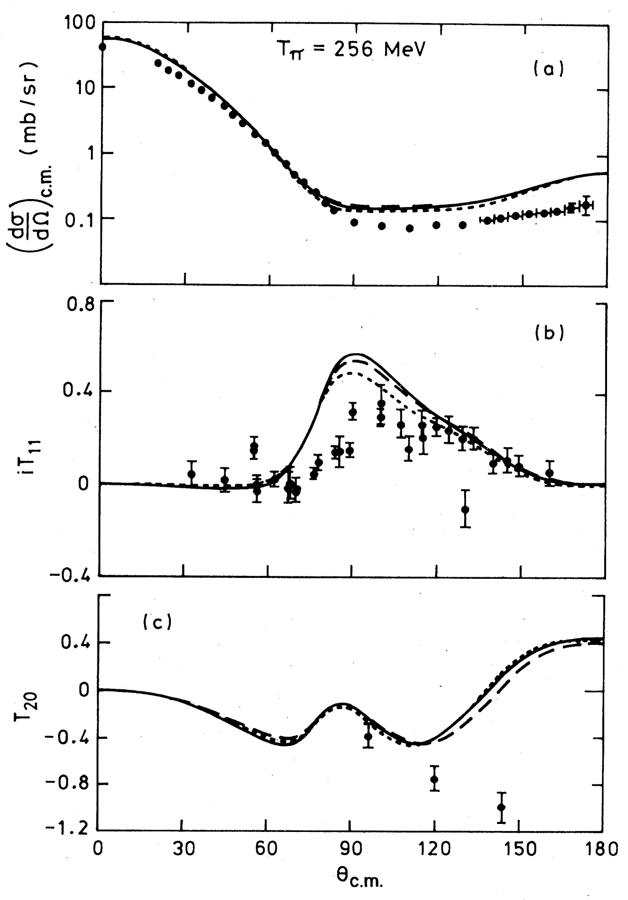

FIG. 6. The sensitivity to the *D*-state probability of the deuteron, of the π-d elastic scattering differential cross section (a), iT_{11} (b), and T_{20} (c). The curves correspond to the *D*-state probability of 6.56% (——), 5.36% (— — —), and 4.09% (· · ·). The experimental data are those in Figs. 3 and 4.

$^3S_1$-$^3D_1$ N-N and P_{11} π-N channels. The idea of measuring the polarization T_{20} in π-d elastic scattering[43] was first suggested as a method of determining the *D*-state probability (P_D) of the deuteron. To test this idea at higher energies, we have calculated both the π-d elastic scattering cross section and the pion production cross section for three different $^3S_1$-$^3D_1$ potentials with *D*-state probabilities of 6.56%,[29] 5.36%,[30] and 4.09%.[31] We find that for $T_\pi = 256$ MeV, the π-d results are not sensitive to the *D*-state probability of the deuteron (Fig. 6). On the other hand, for pp→πd ($T_p = 800$ MeV), the differential cross section has no great variation with P_D, but A_{y0} favors a lower value of P_D (see Fig. 7).

The most controversial partial wave in all the NN-πNN calculations is the P_{11} channel. It is this amplitude, and particularly its division into a pole and nonpole, that determines the coupling between the NN and πNN channels. The different formulations of the NN-πNN equations[12,13,15] have all given the same procedure for dividing this amplitude, and this division is consistent with two- and three-body unitarity.[16] More recently, three-body formulation of the π-N system,[44] based on a Chew-Low Hamiltonian, give a P_{11} amplitude that is a sum of a pole and nonpole part. The problem is that the experimental π-N phase shifts constrain the sum of the on-shell pole and nonpole amplitudes. Since the pole part alone is repulsive, and the nonpole is attractive, we have a delicate cancellation between the two parts to give the small phase shifts one observes at low energies. When this amplitude

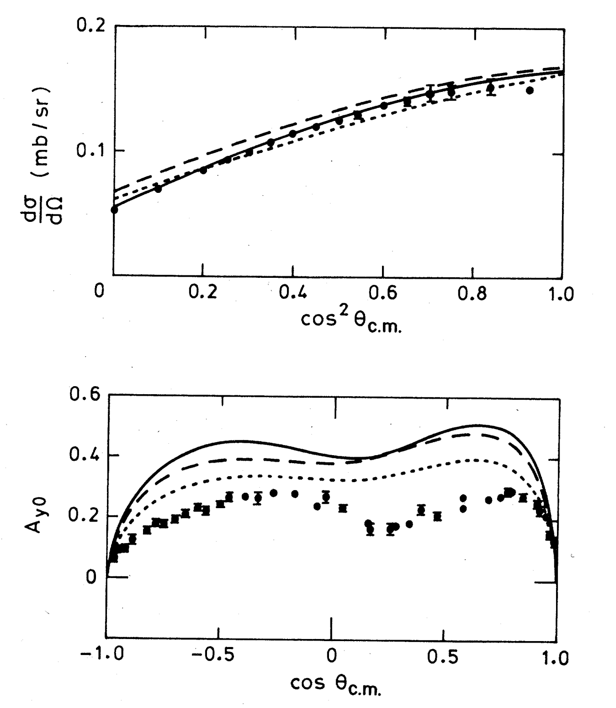

FIG. 7. The sensitivity of the differential cross section (a), and A_{y0} (b) in pp→πd at $T_p = 800$ MeV to the *D*-state probability of the deuteron. Curves are labeled in Fig. 6. The data are those in Fig. 2.

is used in the NN-πNN equations, the delicate cancellation is not always present. In particular, for all channels other than 0^+, 2^-, 4^-, and 6^- we have a three-body channel with a contribution from the nonpole part of the amplitude only. This arises as a result of the fact that N-N intermediate states have to satisfy the Pauli exclusion principle and this excludes the pole part of the amplitude which gives the N-N intermediate states. Thus, in these partial waves, we get no cancellation between the two parts of the P_{11} amplitude, and the final results depend very intricately on how the amplitude was divided. To give a measure of this cancellation, we have excluded those three-body channels in which both the pole and nonpole are not present. In Figs. 8–10, we present the π-d differential cross section, vector polarization iT_{11}, and tensor polarization T_{20}, for the case when all three-body channels have both pole and nonpole amplitudes (dashed curve), and the case where we allow the nonpole amplitude in three-body channels forbidden by the Pauli principle (solid curve). From this comparison, it is clear that at $T_\pi = 140$ MeV, the vector polarization does not discriminate between the two cases, while the tensor polarization data of Holt *et al.*[3] prefer the case when the nonpole amplitude is not included in the Pauli forbidden channels. At the higher energy of $T_\pi = 256$ MeV, the situation is more confusing, in that the vector polarization prefers one solution, while the tensor polarization suggests the second solution. This has led some people[45] to suggest that the coupling between the NN and πNN channels, as given by the NN-πNN equation, is not the complete story. We feel that this cancellation problem will impose a constraint on the division of the amplitude into a pole and nonpole part, and should be further investigated within a more fundamental theory of π-N scattering.

Having established the fact that the results of NN-πNN equations can be sensitive to the P_{11} amplitude, we turn to a comparison of the observables for different P_{11} interactions. We have chosen, for a start, a set of potentials that give almost the same phase shifts for $T_\pi < 400$ MeV. The phase shifts for these potentials are in good agreement with the available phase shifts.[37–39] Here we note that for $T_\pi < 150$ MeV, the two sets[37,38] of recent phase

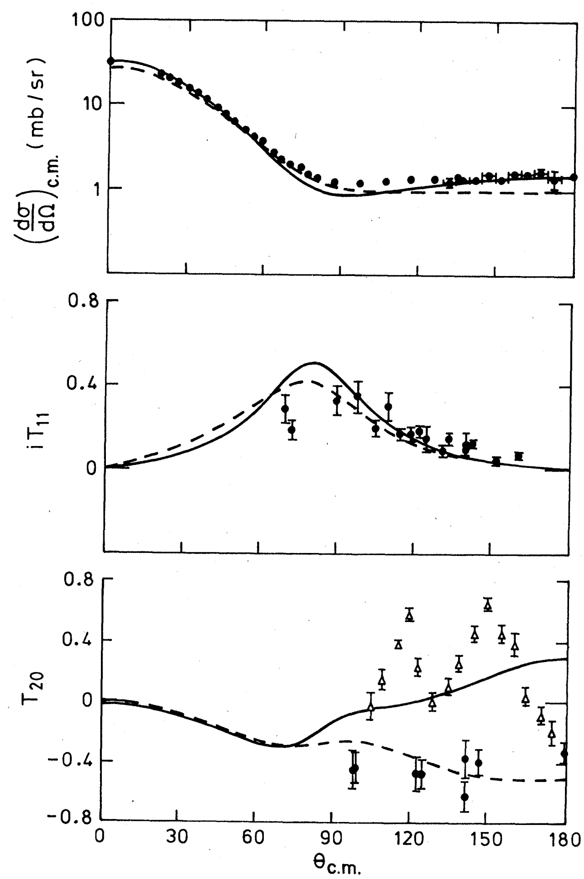

FIG. 8. The effect of excluding (— — —), or including (——) the nonpole part of the P_{11} interaction in three-body channels that violate the Pauli principle in π-d elastic scattering at $T_\pi = 140$ MeV. The experimental results are Refs. 48 and 49 for the differential cross section, Ref. 1 for iT_{11}, and Ref. 2 (△) and Ref. 3 (●) for T_{20}.

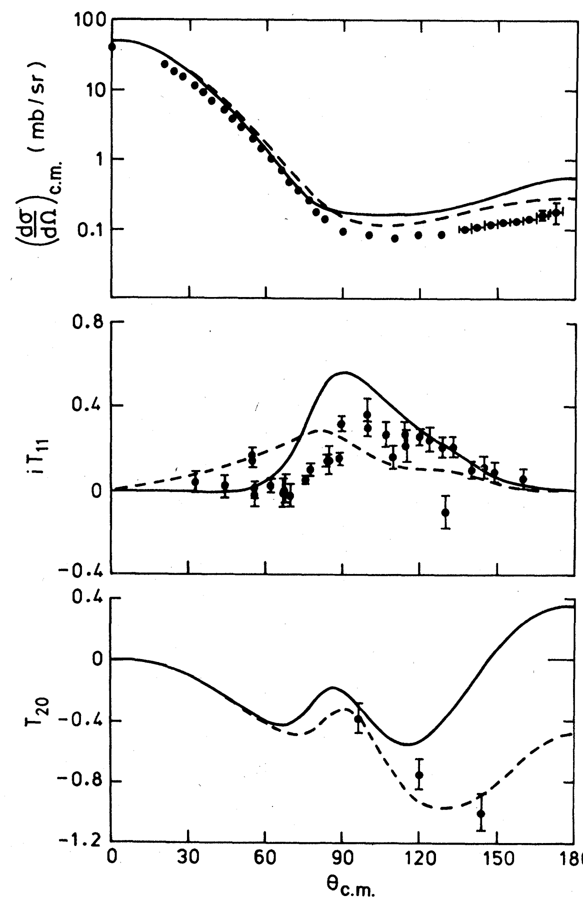

FIG. 9. The effect of excluding the nonpole part of the P_{11} interaction in three-body channels that violate the Pauli principle in π-d elastic scattering at $T_\pi = 256$ MeV. The curves are labeled as in Fig. 8. The experimental results are Ref. 48 for the cross section, Ref. 1 for iT_{11}, and Ref. 3 for T_{20}.

shifts analysis are not in complete agreement. Thus, potential $P6$ fits the phase shifts of Zidell *et al.*[38] and has a larger scattering length ($a_{11}=0.1m_\pi^{-3}$) than the potentials $M1$, $E2$, and PJ, which fit the analysis of Koch and Pietariner[37] and give a scattering length of $0.07m_\pi^{-3}$. In Figs. 11—15, we present the results for $\pi d \to \pi d$ at $T_\pi=140$, 256, and 325 MeV and $pp \to \pi d$ at $T_p=567$ and 800 MeV. Here, we observe that for the differential cross section and vector polarization iT_{11}, the agreement is reasonably good. In particular, we seem to get the energy dependence of the angular distribution for iT_{11}, and especially, the dip around 80°. This is contrary to the claim that the NN-πNN equations cannot reproduce the energy dependence of iT_{11}.[45] Comparing the results for the different P_{11} potentials, we observe that at $T_\pi=140$ MeV, potential $P6$ gives a different result from the other three. This could be due to the larger scattering length, and more negative low energy phase shifts. Also, this potential has a t (nonpole) that is much more attractive than the other three potentials. At higher energies, this difference is not as pronounced, but then the phase shifts for $P6$ are similar to those of the other potential. For T_{20} the agreement between theory and experiment is not as good. This is mainly the result of the sensitivity of T_{20} at back-

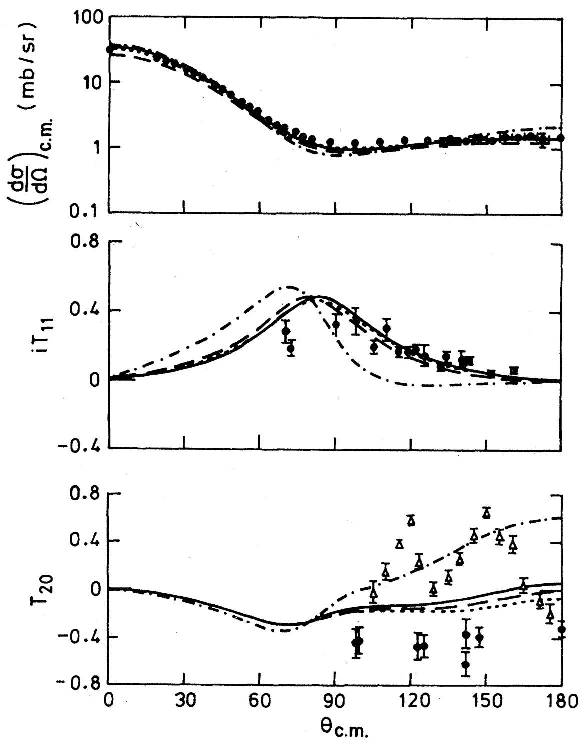

FIG. 11. A comparison of π-d elastic observables at $T_\pi=140$ MeV for different P_{11} interactions. The curves are labeled as in Fig. 1. The experimental results are those in Fig. 8.

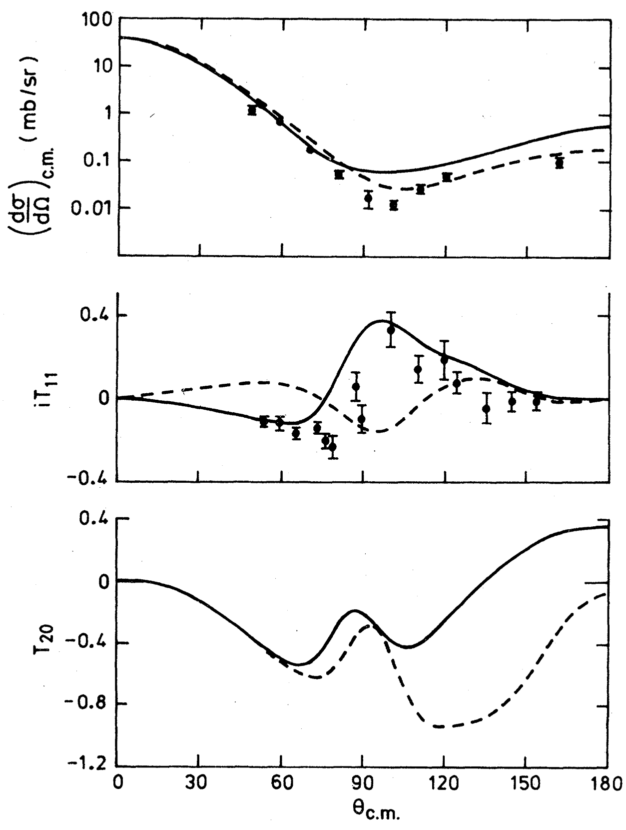

FIG. 10. The effect of excluding the nonpole part of the P_{11} interaction in three-body channels that violate the Pauli principle in π-d elastic scattering at $T_\pi=325$ MeV. The experimental results are Ref. 50 for the differential cross section, Ref. 1 for iT_{11}. The curves are labeled as in Fig. 8.

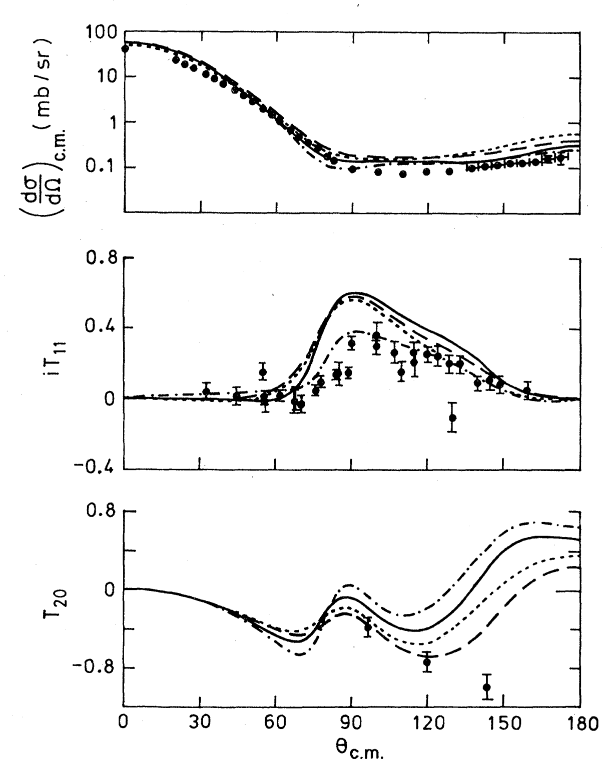

FIG. 12. The same comparison as in Fig. 11 at $T_\pi=256$ MeV. The experimental results are the same as those in Fig. 9.

ward angles to the P_{11} interaction, and in particular, the way the amplitude is divided into a pole and nonpole part. Finally, we observe that the inclusion of inelasticity in all π-N channels (potential $E2$) does not effect the π-d results.

Turning to $pp \to \pi d$, we find, with the exclusion of the potential $E2$, which includes the effect of inelasticity ($\hat{\eta} \neq 1$), the different potentials which give approximately the same polarization results (Figs. 14 and 15). Here again, the potential $P6$ gives a larger cross section at the lower energy than the other two potentials, while that distinction is considerably reduced at the higher energy. If we compare the results for $pp \to \pi d$ at $T_p = 567$ MeV with $\pi d \to \pi d$ at $T_\pi = 140$ MeV, we find that π-d elastic data favor the potentials $M1$ and PJ, while the pion production data favor $P6$. This suggests that there is some missing physics in the NN-πNN equations, which would change the results for one of the reactions. Finally, we note that including inelasticities in the π-N amplitudes gives different results from the other potentials in $pp \to \pi d$ observables, while that was not the case for π-d elastic scattering. This difference is mainly due to the fact that the $pp \to \pi d$ amplitudes are more sensitive to the off-shell behavior of the π-N amplitude. The inclusion of inelasticity in the construction of these amplitudes changes the off shell behavior because of the rapid variation in the phase shifts near the production threshold.

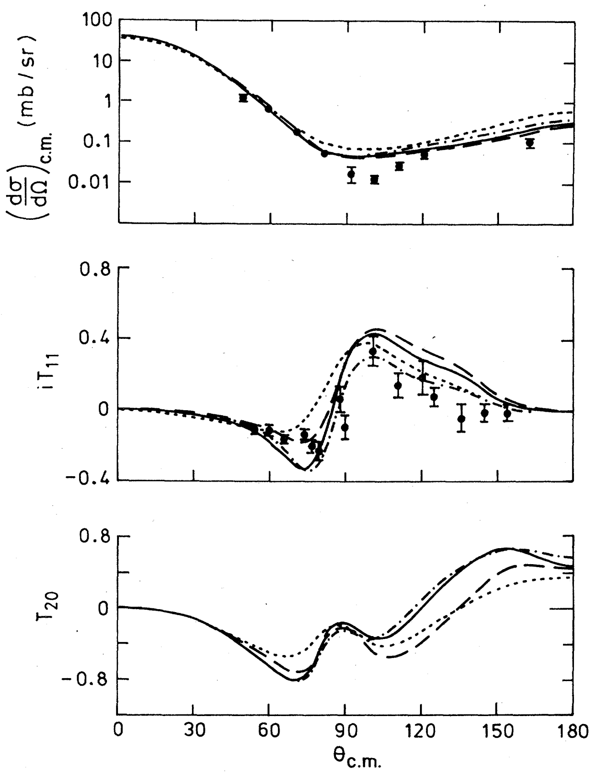

FIG. 13. The same comparison as in Fig. 12 at $T_\pi = 325$ MeV. The experimental results are the same as those in Fig. 10.

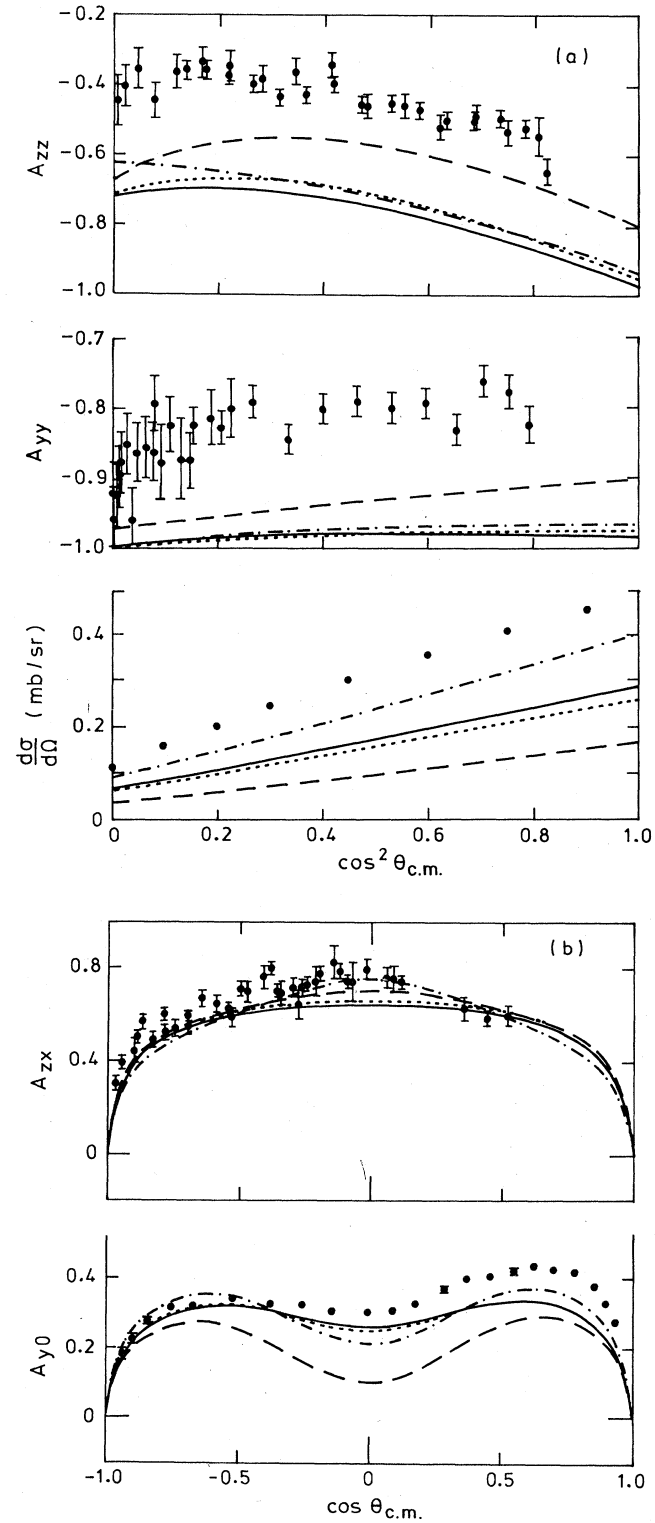

FIG. 14. A comparison of $pp \to \pi d$ observables at $T_p = 567$ MeV for different P_{11} interactions. The curves are labeled as in Fig. 1. The experimental results are those of Ref. 6 for the differential cross section, Ref. 6 for A_{y0}, Ref. 4 for A_{yy}, Ref. 7 for A_{zz}, and Refs. 6 and 7 for A_{zx}. Note that the energy at which the experiments were carried out is within 10 MeV of the theory.

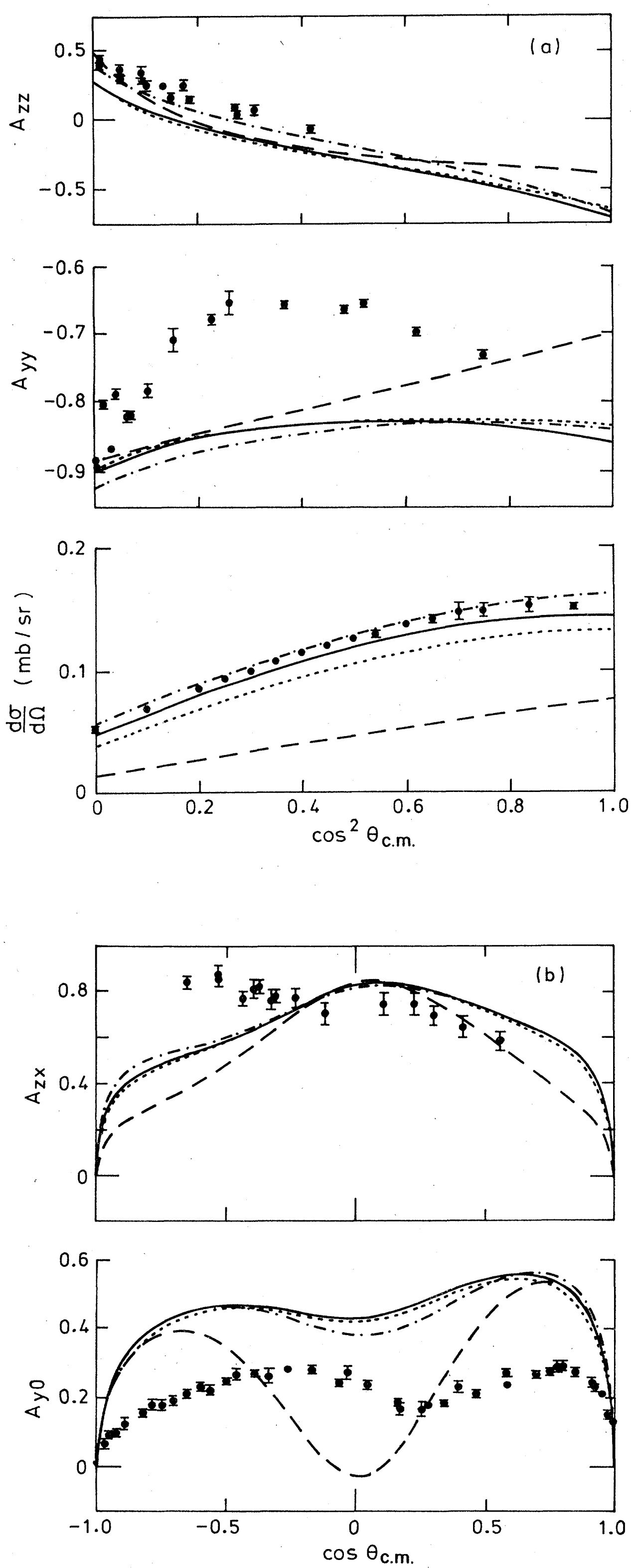

FIG. 15. The same comparison as in Fig. 14 for $T_p = 800$ MeV. The experimental results are those of Ref. 8 for the differential cross section, Refs. 5 and 8 for A_{y0}, and Ref. 9 for A_{yy}, A_{zz}, A_{zx}.

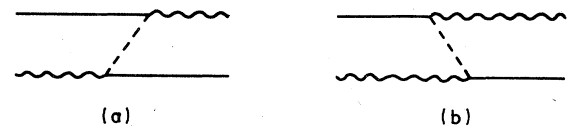

FIG. 16. Mechanism included (a) and excluded (b) in the present formulation for the NN-πNN equations. Both of these diagrams are included in a theory in which the N and Δ are treated on equal footing, Refs. 46 and 47.

IV. CONCLUSION

From the above detailed comparison of the results from the NN-πNN equations with experiment, we can draw two general conclusions.

(i) Overall, the NN-πNN calculations give a reasonable description of the experimental results, and the agreement between theory and experiment would improve if we could constrain the input π-N amplitudes particularly in the P_{11} channel. It is also clear that for pp$\to\pi$d we need to include the D-wave N-N and π-N amplitude before we can reduce the theoretical uncertainty to the level of the error in the experimental data.

(ii) At this stage it is clear from the comparison between theory and experiment that one need not introduce new degrees of freedom associated with dibaryon resonances. This is particularly the case as agreement between theory and experiment for the energy dependence of iT_{11} in π-d elastic scattering is very good. The only serious discrepancy between theory and experiment is in the T_{20} for π-d scattering. Here we might be able to adjust the P_{11} interaction to get better agreement with the results of Holt *et al.*[3] for T_{20}; however, it would be very difficult to get a similar agreement with the SIN results[2] without introducing major new degrees of freedom into the theory.

Although the overall agreement between theory and experiment is good there are some features of the theory that need to be improved.

(i) We need a fully relativistic theory at these energies. Although there are a number of "covariant" calculations of both π-d and pp$\to\pi$d, the equations used do not satisfy proper clustering for the S matrix, and it is not clear how good an approximation these equations are, since we have no relativistic three-body theory that is computationally viable.

(ii) Having established the sensitivity of the results to the P_{11} interaction, we should point out that the change in the πNN coupling constant $f^2_{\pi\mathrm{NN}}(k)$ between $k = k_0$ and 0 [i.e., $f^2_{\pi\mathrm{NN}}(0)/f^2_{\pi\mathrm{NN}}(k_0)$] is significantly larger than the limit set by the partial conservation of axial-vector current (PCAC) of $<7\%$ (see Table I). Any attempt to reduce this difference with the present Yamaguchi form factors has led to a poor fit to the P_{11} phase shifts. This suggests that we need a different choice of form factor.

(iii) The present formulation of the equations treats the $\Delta(1232)$ as a pure π-N resonance, while quantum chromodynamics (QCD) requires that the nucleon and $\Delta(1232)$ be a spin singlet and triplet three-quark state with the mass difference arising from one-gluon exchange and renormalization. The implication of this idea to the NN-πNN system[46,47] shows that the present equation leads to undercounting. In particular, the $\Delta(1232)$ can only emit a forward pion but not a backward pion as the latter corresponds to two-pion intermediate states. This is illustrated in Fig. 16, where (a) is included, while (b) is not. Such undercounting leads to an incomplete description of the distortion in the NN channel, which in turn can effect our results for both N-N scattering and pion production.

Considering the above problems with the theory as it stands, the agreement between theory and experiment is very good.

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