Vlasov-Uehling-Uhlenbeck theory of medium energy heavy ion reactions: Role of mean field dynamics and two body collisions

H. Kruse,* B. V. Jacak,† J. J. Molitoris, G. D. Westfall, and H. Stöcker
National Superconducting Cyclotron Laboratory, Department of Physics and Astronomy and Department of Chemistry, Michigan State University, East Lansing, Michigan 48824

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The role of nonequilibrium and quantal effects in fast nucleus-nucleus collisions is studied via the Vlasov-Uehling-Uhlenbeck theory which includes the nuclear mean field dynamics, two-body collisions, and Pauli blocking. The intranuclear cascade model, where the dynamics is governed by independent NN collisions, and the Vlasov equation, where the nuclear mean field determines the collision dynamics, are also studied as reference cases. The Vlasov equation (no collision term) yields single particle distribution functions which—after the reaction—are only slightly modified in momentum space; even in central collisions, transparency is predicted. This is in agreement with the predictions of the quantal time-dependent Hartree-Fock method. In contrast, large momentum transfer is obtained when the Uehling-Uhlenbeck collision term is incorporated; then the final momentum distribution is nearly spherically symmetric in the center of mass and a well-equilibrated nuclear system is formed: the nuclei stop each other; the translational kinetic energy is transformed into randomized microscopic motion. The Vlasov-Uehling-Uhlenbeck theory is supplemented with a phase space coalescence model of fragment formation. Calculated proton spectra compare well with recent data for Ar(42, 92, and 137 MeV/nucleon) + Ca. Also the total yields of medium mass fragments are well reproduced in the present approach. The mean field dynamics without two-body collisions, on the other hand, exhibits forward peaked proton distributions, in contrast to the data. The cascade approach underpredicts the yields of low energy protons by more than an order of magnitude.

The recent interest in medium energy (20—200 MeV/nucleon) heavy ion collisions is motivated by the opportunity to study the transition from the Pauli principle dominated low energy region to a high energy region where two body collisions are important.[1] Time-dependent-Hartree-Fock and fluid-dynamical calculations have been applied in this energy region with drastically different results:[2] The mean field calculations exhibit transparency, while fluid dynamics predicts compound nucleus formation and rapid disintegration of the highly excited system. There is an obvious need to include the finite mean free path of nucleons, the interaction of nucleons with the nuclear mean field, and the two-particle viscosity due to NN collisions into a microscopic theory.[3] We present here a microscopic approach based on the Vlasov-Uehling-Uhlenbeck (VUU) equation[3] which incorporates both the nuclear mean field and NN collisions with an appropriate Pauli blocker. Recent data on inclusive light and heavy particle production from Ar(40—140 MeV/nucleon)+Ca reactions[4] provide a testing ground for the theory.

Recently we demonstrated that the mean field and the Pauli principle terms are important even at high bombarding energies, $E > 300$ MeV/nucleon.[5] The microscopic intranuclear cascade model,[6] which may loosely be viewed as a solution of the Boltzmann equation without the mean field term and Pauli blocking factors, has difficulties in reproducing high multiplicity selected data at these high energies.[7] This is surprising in view of the success of this model in describing inclusive data. At intermediate energies, $E_{\text{lab}} \sim 100$ MeV/nucleon, both effects are even more important: The potential field keeps the nuclei from expanding before collisions can occur, and also provides the one-body dissipation effects which dominate the dynamics at lower energies. Furthermore, respecting the Pauli principle is essential at these energies, where the incident nuclei are close or even overlapping in momentum space.

In the present work medium energy collisions are studied via the Vlasov-Uehling-Uhlenbeck equation, which includes the mean field dynamics and Pauli blocking of nucleon nucleon collisions. In this theory, the time evolution of the single particle distribution function $f(r,p,t)$ is given by[3]

$$\frac{\partial}{\partial t}f+\mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}f-\nabla U\cdot\frac{\partial}{\partial \mathbf{p}}f=-\int \frac{d^3p_2d^3p_1'd^3p_2'}{(2\pi)^6}\sigma v_{12}[ff_2(1-f_1')(1-f_2')-f_1'f_2'(1-f)(1-f_2)]\delta^3(p+p_2-p_1'-p_2') \,. \tag{1}$$

The single particle distribution function is obtained by ensemble averaging over the phase space distribution of test particles.[5,8,9] The motion of the test particles under the influence of the mean field is governed by the Vlasov equation, which is the classical analog to the time-dependent Hartree-Fock (TDHF) equations.[3]

The test particles are initially assigned random positions in a sharp sphere of nuclear radius. The center of mass of the individual ensembles are calculated and then shifted to their respective positions in the nucleus-nucleus ($\mathcal{N}$-$\mathcal{N}$) center of mass. Because of the finite number of particles in each ensemble the ensemble averaged configuration space distributions do exhibit—after shifting—a finite surface thickness of about 1.5 fm. Fermi momenta are also assigned randomly and then the individual ensembles are shifted, as is done in configuration space. The nuclei are then Lorentz boosted towards each other in the $\mathcal{N}$-$\mathcal{N}$ center of mass frame. Trajectories in configuration and momentum space are computed by assuming that each particle moves on a curved trajectory under the influence of an acceleration term generated by the gradient of the mean field. For the density dependent potential field, $U(\rho)$, a local Skyrme interaction is used:

$$U(\rho)=-124\rho/\rho_0+70.5(\rho/\rho_0)^2 \text{ MeV} , \qquad (2)$$

with a compressibility coefficient of $K=380$ MeV. The choice for this rather stiff equation of state is motivated by the results found at higher energy:[5,7] such a stiff equation of state seems necessary to describe the pion multiplicities and the transverse flow observed.

The long range Coulomb and Yukawa interaction are neglected here; they become increasingly important at lower bombarding energies and for fragments emitted in the projectile and target rapidity region. Furthermore, at low energies sequential particle emission is increasingly important, but not treated explicitly in the present approach: the collision calculations are stopped after 120 fm/c, i.e., when the number of NN collisions per fm/c becomes negligible, but before the unstable residual fragments have undergone substantial evaporation.

Fifteen collision simulations are followed in parallel with a total of 1200 test particles and the ensemble averaged phase space density in a sphere of radius 2 fm around each particle is computed.[5] The ensemble averaging results in statistical fluctuations at the 10% level (at normal density) and thus reasonably smooth single particle distribution functions, which are used to determine the mean field and the Pauli blocking probability.[5,9] About one hundred such parallel ensembles are followed to simulate an actual reaction.

A constant time-step integration routine is used to ensure synchronization of the ensembles.[5] The acceleration of the test particles due to the field gradient is calculated prior to each transport step, and is assumed to be constant within a synchronization time step. The local gradient of the field is computed via a finite difference method between two hemispheres centered around the test particle. This method[5] is analogous to Lagrange's method in fluid dynamics, in contrast to the space-fixed Eulerian mesh.

Protons, neutrons, deltas, and pions of different isospin are included separately with their experimental scattering cross sections.[5] The question of double counting of the mean field and the collision term is a basic restriction for the VUU approach. We take the following operational point of view: the phenomenological Skyrme potential incorporates the real part of the potential, i.e., the attractive one meson exchange (the linear term in U) and repulsive mean field interactions, while the two body scattering accounts for the residual interactions. It should be pointed out that energy conservation is fulfilled in the present ap-

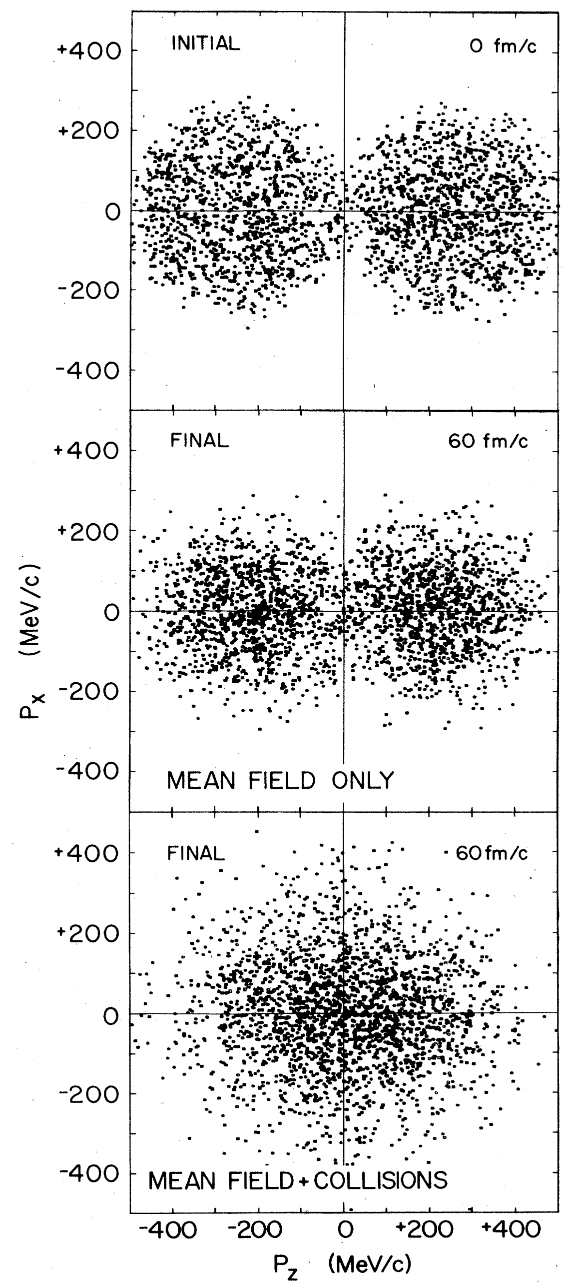

FIG. 1. The momentum space evolution of an Ar(137 MeV/nucleon)+Ca collision at impact parameter $b=0$ fm. The results from several parallel ensembles are superposed in order to represent the distribution function. (a) The initial state. (b) The final state without two body collisions. (c) The final state with two body collisions included.

proach for individual two body scatterings and for the ensemble average on the mean (but not within each separate ensemble, because of the coupling between different ensembles—energy conservation problems have been studied for a similar approach by Koehler *et al.*[3] using the relaxation time ansatz). The free particle cross sections have to be corrected for "in medium" effects, the most important one being the Pauli blocking of collisions. Two particles may undergo *s*-wave scattering if they approach each other with a minimum distance of less than $(\sigma/\pi)^{1/2}$ and if the final states are not Pauli blocked. The Pauli blocking factor for each nucleon is given by $(1-f)$, and the scattering probability is then reduced by the Uehling-Uhlenbeck factor $(1-f_1)(1-f_2)$. The Pauli blocker has been tested on ground state nuclei and has an efficiency of about 97%.

The Pauli blocking is very important at these bombarding energies: even at 137 MeV/nucleon, 80% of the attempted collisions are blocked due to lack of available final state configurations. Many of these attempted collisions are between nucleons of the same nucleus. The spectra of low energy ($E < 80$ MeV) nucleons are also influenced by Pauli blocking.

We have studied the same system Ar+Ca in the mean field approximation without two body collisions, thus mimicking TDHF by solving the Vlasov equation[3]—as far as we know this is the first time that a solution of the Vlasov equation in three dimensions has been done for nuclear collisions—the lack of two body collisions results in strongly forward peaked angular distributions, in qualitative agreement with three-dimensional TDHF calculations[2] in this energy regime. Figure 1(a) shows the initial state in momentum space for Ar(137 MeV/nucleon) +Ca; note that at this higher energy the Fermi spheres of target and projectile nuclei are well separated. The Ar projectile moves in the positive z direction, while the Ca target moves in the negative z direction in this center of mass frame. Figures 1(b) and (c) show the final state of this reaction as obtained in the present theory without and with the Uehling-Uhlenbeck collision term. Note that the momentum space distribution is practically unchanged in the mean field calculation—equilibration of the momenta is not observed—while the inclusion of the Uehling-Uhlenbeck collision term results in strong equilibration—the isotropy in Fig. 1(c) is indicative of substantial thermalization. A convenient way to compare the results is to use the ratio of transverse to longitudinal momenta,

$$R = 2/\pi \Sigma p_{\rm per} / \Sigma p_{\rm par} \ , \qquad (3)$$

where $p_{\rm per}$ and $p_{\rm par}$ are the momenta perpendicular to and parallel to the beam. Comparing the ratio of final to initial R values, we find 1.08 for the mean field only case and 2.05 for the mean field plus collisions approach. At lower energies, the comparison is not as dramatic; the initial R values are already high since the nuclei overlap more in momentum space—furthermore, most of the collisions are Pauli blocked. But the collision term always leads to increased isotropy. In Fig. 2 we show the reaction Ar(42 MeV/nucleon)+Ca as it develops in configuration space. Note that without the collision term [Fig. 2(b)], the nuclei tend to pass through one other, whereas a substantial degrading of the initial momentum occurs once the collision term [Fig. 2(a)] is included.

A generalized six-dimensional coalescence model is used to find the nucleons bound in clusters, and prevent them from contributing to the proton cross sections. This is important at medium energies, where a large fraction of the emitted protons are found to be bound in fragments.[4] In this scheme, a nucleon is part of a cluster if it is within a configuration space distance r_0 from any other member of the cluster, and within a momentum space distance p_0 from the center-of-momentum of the cluster. The sequential evaporation of protons from residual fragments is not included. The generalized coalescence prescription has been used to calculate inclusive proton spectra from the primordial nucleon distribution. We use $r_0 = 2.2$ fm and $p_0 = 200$ MeV/c. These parameters are adjusted to agree

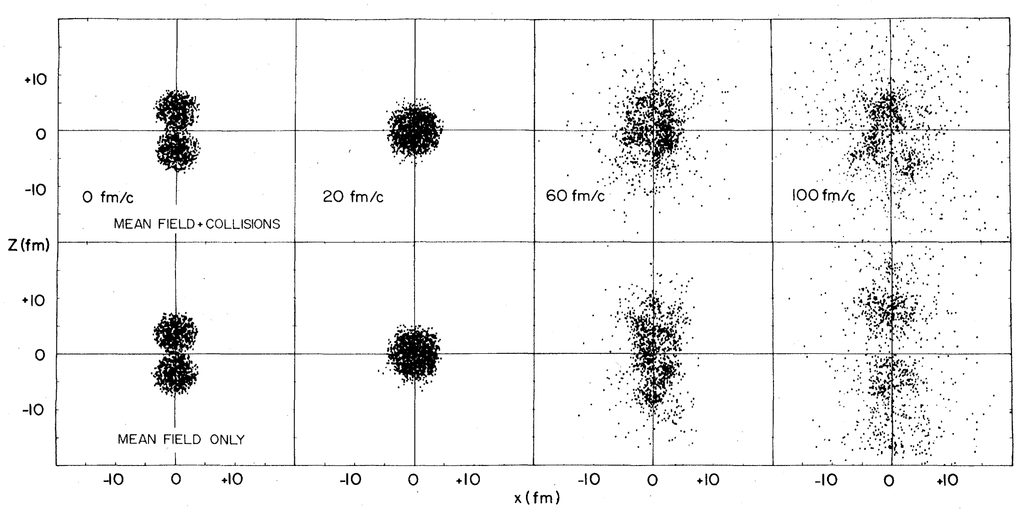

FIG. 2. Time development of an Ar(42 MeV/nucleon)+Ca collision in configuration space at $b = 0$ fm. Again the results from several ensembles are superposed. (a) The reaction develops with two body collisions included. (b) The reaction without two body collisions.

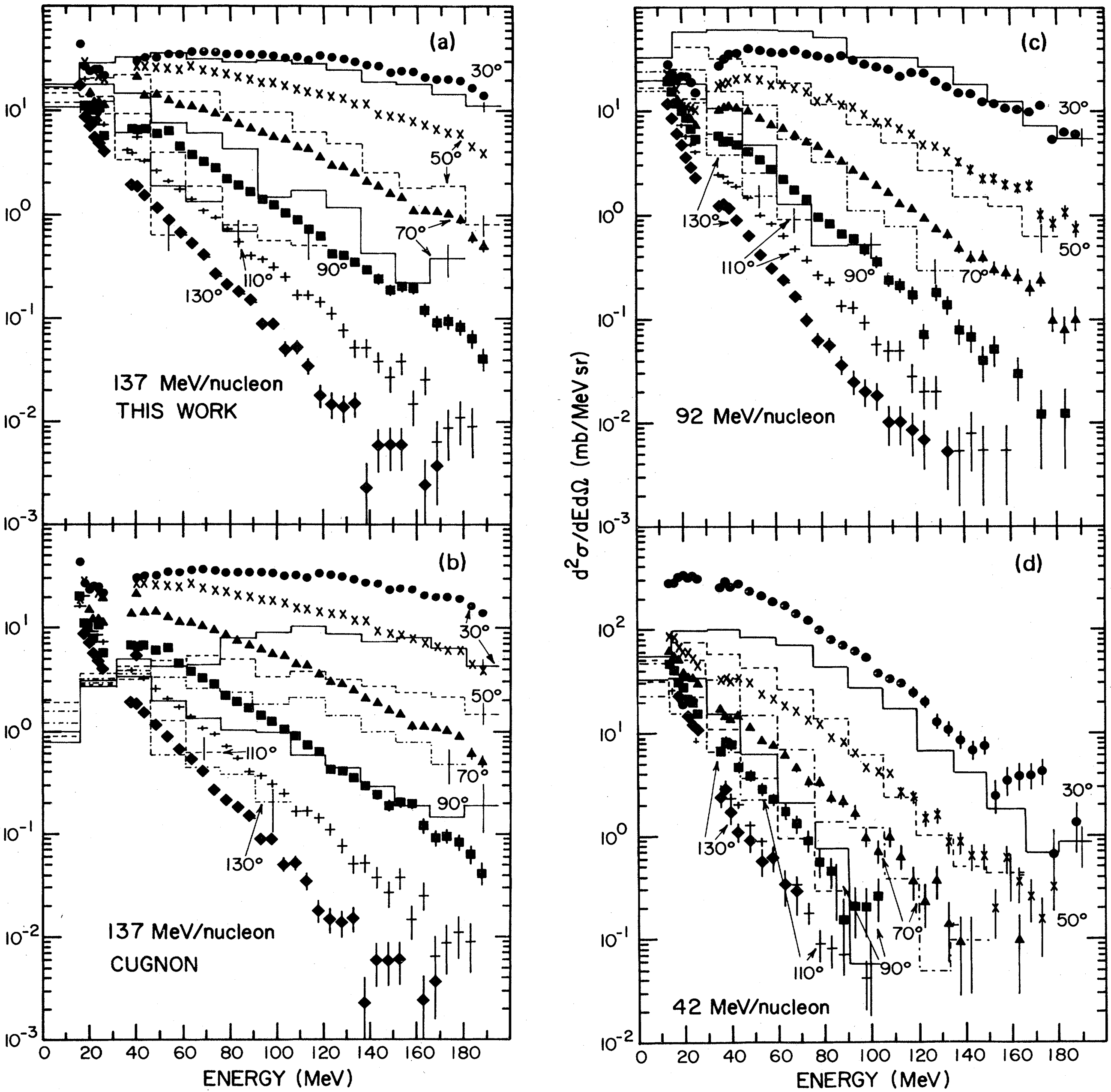

FIG. 3. Inclusive proton spectra from Ar(42, 92, and 137 MeV/nucleon)+Ca. The data (Ref. 4) are indicated by points, and the theory by histograms. The largest statistical errors in the calculation are shown on the histograms. The breaks in the data from 30 to 40 MeV are the result of dead layers in the detectors. (a) Comparison of the present work with the 137 MeV/nucleon data. (b) The same data compared to results obtained with the cascade mode. (c) The present theory compared to the 92 MeV/nucleon data. (d) The present theory compared to the 42 MeV/nucleon data.

with the experimentally observed total cross sections. These values also yield correct clustering at $t=0$ fm/c: two heavy clusters, namely the Ar and Ca nuclei are then obtained. Variation of the coalescence parameters changes the magnitude of the cross sections, but has a negligible effect on the shape of the spectra. The calculated neutron and proton distributions are practically identical, and have been combined to decrease the statistical uncertainty. It is interesting to point out that the phase space volume spanned by these values is very close to 4 h,[3] the volume occupied by a fourfold degenerate fermion. Our approach gets further support from the agreement of the predicted fragment yields as a function of fragment mass to the experimental data[4] for masses 1—14.

Figure 3(a) shows the comparison between calculated and measured proton spectra for 137 MeV/nucleon Ar+Ca. The calculated absolute cross sections and the slopes of the spectra agree reasonably well with the data.

Figure 3(b) shows the same data compared to the proton spectra calculated with our cascade mode, which serves as a reference to demonstrate the importance of the mean field and phase space Pauli blocking. For our cascade mode, we have turned off the Skyrme potential and used the simple Cugnon approximation to the Pauli blocking by excluding collisions with less than 24 MeV c.m. kinetic energy. The resulting nucleon distributions were analyzed via the same procedure as the Vlasov-Uehling-Uhlenbeck equation results, including the coalescence step. The simple cascade simulation, though appropriate for high energies, cannot reproduce the medium energy data.

The measured proton cross sections are known to within 20% for the 137 and 92 MeV/nucleon data, but are uncertain by a factor of 3 for the 42 MeV/nucleon data due to beam monitoring difficulties.[4] At 92 MeV/nucleon [Fig. 3(c)] the calculations agree with the data. The calculation at 42 MeV/nucleon [Fig. 3(d)] agrees well with the data except for the 30° spectra, which are underpredicted at the lower proton energies. This is probably due to our neglect of evaporation protons, which dominate the projectile and target rapidity regions.

In summary, the Boltzmann equation, including the nuclear mean field and Pauli blocking corrections, provides a new approach to intermediate energy heavy ion collisions. Inclusive proton spectra from 42, 92, and 137 MeV/nucleon Ar+Ca collisions as well as yields of medium mass fragments agree with the calculated cross sections. Nonequilibrium and quantum effects turn out to be important at these energies. Two body collisions yield a rapid approach towards equilibrium at these energies for medium mass systems. It will be interesting to study the equilibration at lower energies and/or for lighter systems. We are presently investigating the effect of the nuclear equation of state on intermediate energy collisions as well.

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*Present address: TELCO Research, Nashville, TN 37212.

†Present address: Los Alamos National Laboratory, Los Alamos, NM 87545.

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