Coherent $K_L^0 \to K_S^0$ regeneration on Cu

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The amplitudes $F_{21}(0)=\frac{1}{2}[F(0)-\overline{F(0)}]$ for the coherent $K_L^0 \to K_S^0$ regeneration on Cu in the momentum region of $600 \le P_K \le 1400$ MeV/c are calculated in the optical model. The many-body corrections for Y^* resonances in the nuclear medium are taken into account in the calculations of the strangeness -1 component $\overline{F(0)}$ in $F_{21}(0)$. Within the framework of the phenomenological isobar-doorway model, the many-body corrections for the resonances are expressed in terms of the resonance energy shifts and width-modification factors. The optical potentials for the scattering of $\overline{K}^0$ and K^0 on the nucleus are constructed with the $\overline{K}^0$- and K^0-nucleon partial wave amplitudes parametrized by Gopal *et al.* and by Nakajima *et al.*, respectively. The calculated results are compared with the experimental data measured by Birnbaum *et al.* The inclusion of the many-body effects on the Y^*'s is successful in explaining the striking energy dependence of the data around $P_K = 1250$ MeV/c.

I. INTRODUCTION

One of the interesting phenomena in neutral kaon physics is the regeneration of short-lived kaons (K_S^0's) through the interactions of long-lived kaons (K_L^0's) with matter, as noted by Dover and Walker.[1] In the present paper, we focus our attention on the $K_L^0 \to K_S^0$ elastic regeneration in nuclei at medium energies.

Birnbaum *et al.*[2] have measured the magnitude of the forward amplitude

$$F_{21}(0)=\frac{1}{2}[F(0)-\overline{F(0)}]$$

for the coherent regeneration of K_S^0 on Cu, in the region of momenta $600 \le P_K \le 1400$ MeV/c. Here, $\overline{F(0)}$ and $F(0)$ are the forward amplitudes for the $\overline{K}^0$ (strangeness $S=-1$) and K^0 ($S=+1$) scattering on Cu. The energy dependence of the data shows a rapid falloff around $P_K = 1250$ MeV/c. The optical model calculations in the eikonal limit have been performed by Birnbaum *et al.*[2] To obtain the parameters of the optical potentials for the interactions of $\overline{K}^0$ and K^0 with Cu, they used, as inputs, the kaon-nucleon (KN) amplitudes obtained from the total cross sections measured for the charged kaons, by the use of the charge independence of the KN amplitudes, the optical theorem, and the forward dispersion relations. Although the parameter-free prediction reproduces the data in the momentum region below 1000 MeV/c, it does not explain the striking energy dependence of the data around 1250 MeV/c. The discrepancy could not be removed by including the effects of the nuclear Fermi motion.[2]

The $\overline{K}^0$-nucleon ($\overline{K}^0$N) interactions are dominated by the formation of many Y^* resonances (Y^*'s) in the medium energy region considered here. Therefore, it is worthwhile to investigate the effects of the many-body corrections for the Y^*'s, in the calculation of the $S=-1$ component $\overline{F(0)}$ in $F_{21}(0)$. At present, the existence of Z^* resonances in the K^0-nucleon (K^0N) system seems to be inconclusive.

The main purpose of the present work is to examine if an attempt to include the many-body corrections for the Y^*'s is successful in removing the discrepancy between the predictions and the data around 1250 MeV/c. In this work, the many-body corrections are represented with the resonance energy shifts ΔE and width-modification factors β, within the framework of the phenomenological isobar-doorway (ID) model which was originally formulated by Kisslinger for π-nucleus interactions[3] and extended to $\overline{K}$-nucleus scattering and reactions.[4] Recently, the ID model was applied to the analyses of the $^{12}C(K^-,\pi^-)^{12}_{\Lambda}C^*$ reaction[5] and K^--nucleus elastic scattering[6] at 800 MeV/c. The calculations taking account of the effects of ΔE and β reproduce the observed data better than the first order calculations. In the present work, the amplitudes $\overline{F(0)}$ and $F(0)$, of which $F_{21}(0)$ consists, are evaluated by solving the Klein-Gordon equations with the optical potentials of a Kisslinger form.[4] The parameters of the optical potentials are constructed with the parametrized $\overline{K}^0$N and K^0N partial wave amplitudes.

II. CALCULATIONS

The optical potential of Kisslinger form[4] is

$$2(E/\hbar c)U_N(r)= b_0 k^2\rho(r)+b_1 \vec{\nabla}\cdot\rho(r)\vec{\nabla} + b_2 \vec{\nabla}^2\rho(r)+b_3 \vec{\nabla}^4\rho(r) , \qquad (1)$$

where E and k are the kaon total energy and wave number in the K-nucleus center of mass system (c.m.s.), and $\rho(r)$ is the nuclear density normalized to unity. The contributions of the third and fourth terms in Eq. (1) to the amplitudes $\overline{F(\theta)}$ and $F(\theta)$ are expected to be small at forward scattering angles, since the terms stem from the Ansätze[4]

TABLE I. Resonance parameters for the Y^*'s taken from the $\bar{K}^0N$ (K^-N) amplitudes parametrized by Gopal *et al.*[7]

l_{I2j}	M^* (MeV)	Γ (MeV)	t	l_{I2j}	M^* (MeV)	Γ (MeV)	t
S_{01}	1670±5	45±10	0.20±0.03	D_{13}	1670±5	50±5	0.08±0.03
S_{01}	1825±20	230±20	0.37±0.05	D_{05}	1825±10	94±10	0.04±0.03
S_{11}	1770±15	60±10	0.15±0.03	D_{15}	1774±5	130±10	0.41±0.03
S_{11}	1955±15	170±40	0.44±0.05	F_{05}	1822±2	81±5	0.57±0.02
P_{01}	1573±25	147±50	0.24±0.04	F_{05}	2100±50	200±50	0.07±0.03
P_{01}	1853±20	166±20	0.21±0.04	F_{15}	1920±10	130±10	0.05±0.03
P_{11}	1738±10	72±10	0.14±0.04	F_{17}	2040±5	190±10	0.24±0.02
P_{03}	1900±5	72±10	0.18±0.02	G_{07}	2110±10	250±30	0.30±0.03
D_{03}	1690±5	60±5	0.24±0.03	G_{09}	1808±5	27±5	0.04±0.01

$$|\vec{k}-\vec{k}'|^2 \text{ and } |\vec{k}-\vec{k}'|^4$$

for the off-shell behavior of the partial wave KN t matrices in the momentum space. Then, the third and forth terms are neglected in the evaluation of the forward amplitude

$$F_{21}(0)=\tfrac{1}{2}[F(0)-\overline{F(0)}]$$

for the coherent regeneration. However, the effects of the terms are examined numerically at several points of energies. The parameters b_0 and b_1 are

$$b_0=-B\sum_{l\neq 1}\bar{a}_l \tag{2a}$$

and

$$b_1=B\bar{a}_1 , \tag{2b}$$

with

$$\bar{a}_l=N_1[(l+1)f^1(l,j^+)+lf^1(l,j^-)] + \frac{N_2}{2}\{(l+1)[f^0(l,j^+)+f^1(l,j^+)] + l[f^0(l,j^-)+f^1(l,j^-)]\} . \tag{3}$$

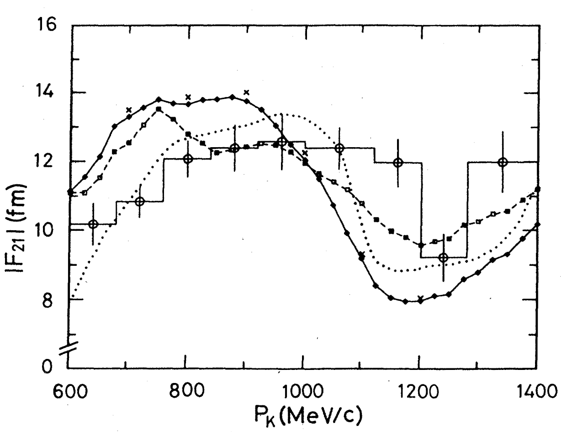

FIG. 1. Magnitude of the elastic regeneration amplitude $F_{21}(0)$ at the forward direction. The curves are explained in the text. The symbol $\times$ represens the results of calculations with the optical potential of Eq. (1) including the terms $b_2\vec{\nabla}^2\rho(r)$ and $b_3\vec{\nabla}^4\rho(r)$ where $(\Delta E,\beta)=(0$ MeV, 1) for all Y^*'s. The difference between $|F_{21}(0)|$ calculated by using the optical potential with and without the terms is less than 2%. Data are taken from Ref. 2.

Here $N_1=Z$ and $N_2=N$ for the $S=-1$ $\bar{K}^0$-nucleus scattering and $N_1=N$ and $N_2=Z$ for the $S=+1$ K^0-nucleus scattering (N and Z are the neutron and proton numbers, respectively). The factor B in Eq. (2) represents the transformation of the KN amplitudes $f^I(l,j^{\pm})$ in the KN c.m.s. into those in the K-nucleus c.m.s. The KN amplitudes parametrized by Gopal *et al.*[7] and Nakajima *et al.* (solution-N)[8] are used for the $S=-1$ $\bar{K}^0N$ and $S=+1$ K^0N amplitudes, respectively.

In Ref. 7, the $S=-1$ partial wave amplitude $f^I(l,j^{\pm})$ is, in each (I,l,j) state, separated into a background and a resonant part. The resonant part is expressed as a sum of Breit-Wigner resonances of the form

$$f_{BW}=-\frac{t\Gamma(w)}{2\kappa[w-M^*+i\frac{1}{2}\Gamma(w)]} , \tag{4}$$

where κ is the wave number of the KN c.m.s., w is the invariant mass in the KN system, t is the elasticity, and M^* and Γ are the mass and width of a resonance (Y^*). In the ID model, the resonance amplitude f_{BW} is replaced by

$$f(\Delta E,\beta)=\frac{w-M^*+i\frac{1}{2}\Gamma(w)}{w-M^*-\Delta E+i\frac{1}{2}\beta\Gamma(w)}f_{BW} . \tag{5}$$

The parameters ΔE and β represent the average many-body effects on the Y^* in a nuclear medium.

The Klein-Gordon equations with the optical potentials described here are solved with the algorithm adopted in the program PIRK (Ref. 9) for a coordinate-space optical potential, to evaluate the amplitudes $\overline{F(0)}$ and $F(0)$.

III. RESULTS AND DISCUSSION

The Y^*'s taken into account in the present calculation are listed in Table I together with their properties given in Ref. 7. The nuclear density for Cu is chosen to be the Woods-Saxon form with $R=4.215$ fm and $a=0.546$ fm.[10] The calculated amplitudes $F_{21}(0)$ are compared, in Fig. 1, with the experimental data[2] in the region of $600\leq P_K\leq 1400$ MeV/c. The solid curve represents the results of calculation using $\Delta E=0$ MeV and $\beta=1.0$ for all the Y^*'s listed in Table I. This parameter-free calculation gives rise to a large discrepancy between the predictions and the data, especially around 1250 MeV/c. The effects of the third and fourth terms in Eq. (1) are small as expected. This is shown, in Fig. 1, by the symbol $\times$

TABLE II. Values of the parameters ΔE and β used in the present calculation.

Y^*'s	ΔE (MeV)	β	Refs.
$S_{01}(1670)$, $D_{03}(1690)$	10	1.3	4–6
$D_{13}(1670)$, $P_{11}(1738)$, $S_{11}(1770)$	25	1.2	4–6
$D_{15}(1774)$	40	2.0	4–6
The other Y^*'s	30	1.3	

which represents the results of calculation with the optical potential of Eq. (1). The forms of b_2 and b_3 used in the calculation are the same as those given in Ref. 4. The dotted curve shows the results of the parameter-free calculation in the eikonal limit done by Birnbaum *et al.*[2] For the real parts of the optical potential parameters for the scattering of $\overline{\mathrm{K}}^0$ and K^0 on Cu, they used the real parts of the KN amplitudes evaluated by using kaon forward dispersion relations. They took the nuclear density for Cu to be the Woods-Saxon form with $R = 4.576$ fm and $a = 0.57$ fm. The general behavior of the model prediction is in agreement with the data in the momentum region below 1000 MeV/c. However, there is a large discrepancy between the predictions and the data around 1250 MeV/c. The discrepancy is not removed by the modification of the density parameters. These parameter-free calculations fail to reproduce the striking energy dependence of the data around 1250 MeV/c.

We now turn to a discussion about the effects of ΔE and β for the Y^*'s on the calculated $F_{21}(0)$. The dashed curve represents the results calculated by taking account of the effects of ΔE and β. The values of ΔE and β used in the calculation are listed in Table II. For six Y^*'s [$S_{01}(1670)$, $S_{11}(1770)$, $P_{11}(1738)$, $D_{13}(1670)$, $D_{03}(1690)$, and $D_{15}(1774)$] in the vicinity of the energy 1700 MeV corresponding to $P_{\mathrm{K}} = 800$ MeV/c, we use the same values of ΔE and β as those used in the analyses[6] of the K^- elastic scattering on ^{12}C and ^{40}Ca at 800 MeV/c. For the other twelve Y^*'s we take $(\Delta E,\beta)$ to be (30 MeV, 1.3) which are similar to the values used for the six Y^*'s in the vicinity of 1700 MeV. As seen in Fig. 1, the agreement between the predictions and the data around 1250 MeV/c is improved when the effects of ΔE and β are taken into account.

Figure 2 shows the energy dependence of b_i/A, the optical potential parameters divided by the mass number A, for the $\overline{\mathrm{K}}^0$-Cu scattering. The lines without symbols represent the parameters b_i/A where $(\Delta E,\beta)$ are set to be (0 MeV, 1) for all the Y^*'s. The parameter b_0 exhibits a striking energy dependence. On the other hand, the energy dependence of b_1 is weak in spite of the existence of the p-state resonances [$P_{01}(1573)$, $P_{11}(1738)$, $P_{01}(1853)$, and $P_{03}(1900)$]. The lines with the symbol ♦ represent the parameters calculated by using the values of ΔE and β listed in Table II. The parameter b_0 is affected by ΔE and β for all the Y^*'s except the p-state resonances. On the other hand, b_1 reflects the effects of ΔE and β for only the p-state resonances. Because of the combined effects of ΔE and β for several Y^*'s in the vicinity of a certain energy, the dependence of b_i on ΔE and β at the energy is complicated. Due to five Y^*'s [$S_{01}(1825)$, $S_{11}(1955)$, $D_{15}(1774)$, $F_{05}(1822)$, and $G_{07}(2110)$] with relatively large resonance amplitudes, the variations of b_0 with ΔE and β are remarkable. However, b_1 is not very affected by ΔE and β. This is because of the fact that the

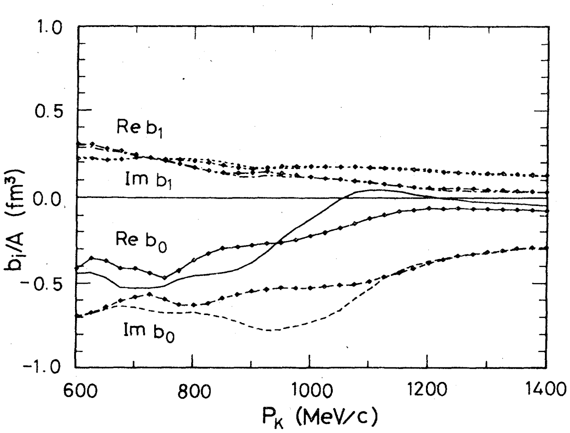

FIG. 2. Parameters b_i/A of the optical potential for the $\overline{\mathrm{K}}^0$-Cu scattering. The lines without symbols show b_i/A calculated with $\Delta E = 0$ MeV and $\beta = 1$ for all the Y^*'s listed in Table I. The lines with the symbol ♦ represent b_i/A calculated with the values of ΔE and β listed in Table II. The $\overline{\mathrm{K}}^0$N amplitudes of Gopal *et al.* (Ref. 7) are used for $f^I(l,j^{\pm})$.

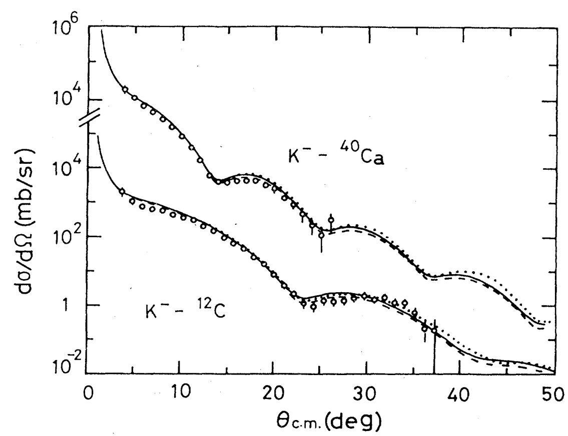

FIG. 3. Differential cross sections for the K^- elastic scattering on ^{12}C and ^{40}Ca at 800 MeV/c (Ref. 6). The solid curve shows the results calculated using $(\Delta E,\beta) = (0$ MeV, 1) for six Y^*'s in the vicinity of 1700 MeV (see Ref. 6). The dashed curve represents the results using the values of ΔE and β listed in Table II for the six Y^*'s. The optical potential of Eq. (1) is used to calculate these curves. The dotted curves represent the results calculated with the optical potential of the form $b_0k^2\rho(r) + b_1\vec{\nabla}\cdot\rho(r)\vec{\nabla}$ where $(\Delta E,\beta) = (0$ MeV, 1) for the six Y^*'s. The nuclear density for ^{12}C is taken to be the modified harmonic-oscillator form with the length parameter $c = 1.649$ fm and free parameter $\alpha = 1.247$ (Ref. 11), and that for ^{40}Ca is taken to be the three-parameter Fermi type with $R = 3.766$ fm, $a = 0.586$ fm, and $w = -0.164$ (Ref. 11). Data are taken from Ref. 11.

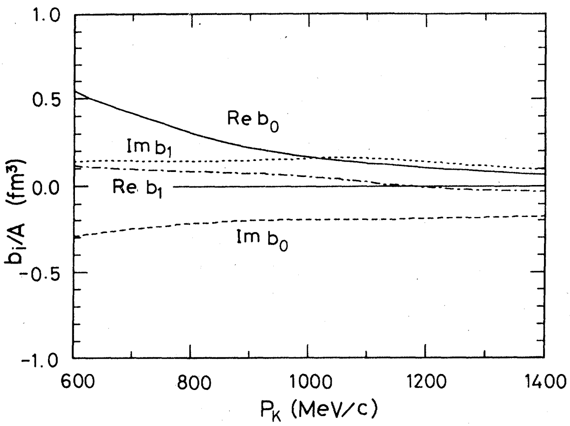

FIG. 4. Parameters b_i/A of the optical potential for the K^0-Cu scattering. The K^0N amplitudes of Nakajima *et al.* (Ref. 8) are used for $f^I(l,j^{\pm})$.

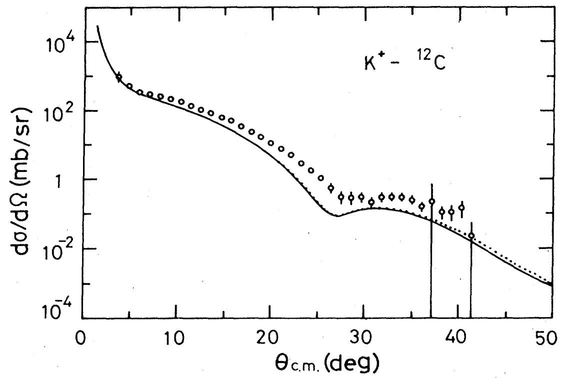

FIG. 5. Differential cross sections for the K^+ elastic scattering on ^{12}C at 800 MeV/c. The solid curve is the result calculated with the optical potential of Eq. (1). The dotted curve represents the results of calculation with the optical potential of the form $b_0 k^2 \rho(r) + b_1 \vec{\nabla} \cdot \rho(r) \vec{\nabla}$. The nuclear density for ^{12}C used is the same as that in Fig. 3. Data are taken from Ref. 11.

resonance amplitudes parametrized for the p-state resonances are relatively small, compared to those for the above-mentioned five Y^*'s in the S, D, F, and G states.

The optical potentials constructed with the $S=-1$ KN amplitudes parametrized by Gopal *et al.*[7] used here reproduce the data[11] on the differential cross sections for the K^--nucleus elastic scattering at 800 MeV/c well. For reference, we show, in Fig. 3, some of the results[6] calculated for the K^--nucleus elastic scattering at 800 MeV/c in the ID model. Good fits to the data are obtained when the same values of ΔE and β as those used in the present calculation are used for the six Y^*'s in the vicinity of 1700 MeV. Thus, the inclusion of the effects of the many-body corrections for the Y^*'s is successful in consistently explaining the data for both the $K_L^0 \to K_S^0$ elastic regeneration and K^--nucleus elastic scattering, although the effects are not very large at 800 MeV/c.

Figure 4 shows b_i/A of the optical potential for the K^0-Cu interaction. The parameters b_i exhibit the smooth energy dependence, reflecting the nonresonant structure of the K^0N interaction. Figure 5 shows the differential cross sections for the K^+ elastic scattering on ^{12}C at 800 MeV/c. The curve is calculated with the optical potential constructed with the $S=+1$ KN amplitudes parametrized by Nakajima *et al.*[8] As seen in the figure, the K^+-nucleus optical potential does not reproduce the data on the angular distribution of the cross sections for the K^+-nucleus elastic scattering at 800 MeV/c well. The optical potentials obtained with all the available $S=+1$ KN amplitudes cannot reproduce the data for the K^+-nucleus elastic scattering,[11–13] not only those obtained with the $S=+1$ KN amplitudes used in the present calculations. In order to explain the data, one must use K^+N amplitudes modified dramatically from those obtained by analyses of the two-body K^+N scattering data.[14] At present, this problem is not settled.

IV. SUMMARY AND CONCLUSIONS

The forward amplitudes for the $K_L^0 \to K_S^0$ elastic regeneration on Cu are calculated in the optical model, in the momentum range of $600 \le P_K \le 1400$ MeV/c. The many-body corrections for the Y^*'s in the nuclear medium are taken into account to explain the striking energy dependence of the data around 1250 MeV/c. The corrections are expressed in terms of the resonance energy shifts ΔE and width-modification factors β, within the framework of the phenomenological ID model.

The inclusion of the many-body corrections for the Y^*'s improves the agreement with the data around 1250 MeV/c. The phenomenological values of ΔE and β used in the present calculation are consistent with those used in the analyses of the $^{12}C(K^-,\pi^-)^{12}_{\Lambda}C^*$ reaction and K^--nucleus elastic scattering at 800 MeV/c. Although, in general, the values of ΔE and β may be chosen differently for each Y^*, it is difficult to extract definite values of ΔE and β for the individual Y^*'s, because the effects of ΔE and β for several Y^*'s in the vicinity of a chosen energy are always mixed up. The consideration of the many-body corrections for the Y^*'s is indispensable for explaining the striking energy dependence of the data around 1250 MeV/c.

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