Inelastic scattering of pions on ^{12}C at 800 MeV/c

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The cross sections for the $^{12}C(\pi,\pi')^{12}C^*$ (2^+, 4.44 MeV; 3^-, 9.64 MeV) reactions at 800 MeV/c are calculated in the distorted-wave impulse approximation. Distortion effects are taken through the eikonal form. Data are reproduced with realistic transition densities and the πN scattering amplitudes fitted to πN data, except for the region of low momentum transfers.

Recently, a precise measurement of the cross sections has been performed[1] for the inelastic scattering of π's from ^{12}C at an incident momentum of 800 MeV/c. At an energy such as 675 MeV, corresponding to 800 MeV/c, the $\Delta(1232)$ resonance is no longer dominant, although around 800 MeV/c there are several other N* and Δ resonances which are weak compared to the $\Delta(1232)$ resonance. The scattering at the energy gives rise to large momentum transfers even at small scattering angles; therefore, it is worthwhile to investigate π-nucleus scattering from the point of view of nuclear structure studies. An eikonal approximation for the pion waves and the impulse approximation are expected to be valid for π-nucleus scattering. In the present note, the differential cross sections are calculated for the $^{12}C(\pi,\pi')^{12}C^*$(2^+, 4.44 MeV; 3^-, 9.64 MeV) reactions at 800 MeV/c.

The amplitude for the inelastic scattering is[2]

$$F_{|\Delta J|,M}(\mathbf{q})=\frac{ik}{2\pi}\int d^3r\exp[i\mathbf{q}\cdot\mathbf{r}]D\langle J_fM_f|V(\mathbf{r})|J_iM_i\rangle \ , \quad (1)$$

where $\mathbf{q}$ is the momentum transfer $\mathbf{k}_f-\mathbf{k}_i$, $\mathbf{k}_i$ and $\mathbf{k}_f$ being the momenta of projectiles and scattered particles. The matrix element for the transition from the ground state $J^\pi=0^+$ to the excited state $J=L$, M is

$$\langle JM|V|00\rangle=\xi\rho_{\rm tr}(r)\,Y_{LM}(\mathbf{r}/r) \ , \quad (2)$$

where $\rho_{\rm tr}(r)$ is the transition density and the overall strength ξ of the transition is

$$\xi=-\frac{2\pi i}{\kappa}[Zf_{\rm p}+(A-Z)f_{\rm n}] \ . \quad (3)$$

Here, κ is the projectile-nucleon c.m. momentum; $f_{\rm p,n}$ are the scattering amplitudes averaged over nuclear Fermi motion for the projectile proton and neutron and Z and A the atomic and mass numbers of the target nucleus. The factor D which takes account of distortion effects is expressed, in the eikonal form, as

$$D=\exp[-\chi(b)] \ , \quad (4)$$

where

$$\chi(b)=\xi\int_{-\infty}^{+\infty}\rho(r)\,dz \ . \quad (5)$$

Here, b is the impact parameter and $\rho(r)$ the nuclear ground-state density normalized to unity.

The transition densities obtained by analyzing electron inelastic scattering data are of the form in configuration space[3]

$$\rho_{\rm tr}(r)=r^L(\alpha+\beta r^2)\exp(-\gamma r^2) \ . \quad (6)$$

With the densities, the amplitudes $F_{|\Delta J|,M}(q)$ are explicitly given by, in the impact parameter representation,

$$F_{2,0}(q)=\frac{ik}{4}\xi\left[\frac{5}{\gamma}\right]^{1/2}\int_0^\infty db\,bJ_0(qb)\left[\frac{\alpha}{\gamma}+\frac{3\beta}{2\gamma^2}+\left[\frac{\beta}{2\gamma}-\alpha\right]b^2-\beta b^4\right]\exp[-\chi(b)-\gamma b^2] \ , \quad (7a)$$

$$F_{2,\pm2}(q)=\frac{ik}{4}\xi\left[\frac{15}{2\gamma}\right]^{1/2}\int_0^\infty db\,b^3J_2(qb)\left[\alpha+\frac{\beta}{2\gamma}+\beta b^2\right]\exp[-\chi(b)-\gamma b^2] \ , \quad (7b)$$

$$F_{2,\pm1}(q)=0 \quad (7c)$$

for the transition from the $J^\pi=0^+$ to the 2^+ state, and

$$F_{3,\pm1}(q)=\mp\frac{ik}{8}\xi\left[\frac{21}{\gamma}\right]^{1/2}\int_0^\infty dbb^2J_1(qb)\left[\frac{2\alpha}{\gamma}+\frac{3\beta}{\gamma^2}+\left[\frac{3\beta}{2\gamma}-\alpha\right]b^2-\beta b^4\right]\exp[-\chi(b)-\gamma b^2] \ , \quad (8a)$$

$$F_{3,\pm3}(q)=\mp\frac{ik}{8}\xi\left[\frac{35}{\gamma}\right]^{1/2}\int_0^\infty db\,b^4J_3(qb)\left[\alpha+\frac{\beta}{2\gamma}+\beta b^2\right]\exp[-\chi(b)-\gamma b^2] \ , \quad (8b)$$

$$F_{3,0}(q)=F_{3,\pm2}(q)=0 \quad (8c)$$

for the transition from the 0^+ to the 3^- state.

The parameters of $\rho_{tr}(r)$ are $\alpha=0.003\,18$ fm^{-5}, $\beta=0.001\,55$ fm^{-7}, $\gamma=0.437$ fm^{-2} for the ground state $J^\pi=0^+$ to the $J=2^+(4.44$ MeV) excitation, and $\alpha=0.001\,57$ fm^{-6}, $\beta=0.0$ fm^{-8}, $\gamma=0.325$ fm^{-2} for the $J^\pi=0^+$ to the $3^-(9.64$ MeV) excitation of ^{12}C. These parameters α, β, and γ are consistent with those obtained by taking the Fourier transform of the transition form factor,[4]

$$\rho_{tr}(q)=q^L(A+Bq^2)\exp(-Cq^2) \ . \tag{9}$$

It is noted that the parameters α and β used here differ from those given in Ref. 3 by a factor of $1/Z(Z=6)$. The ground-state density of ^{12}C is taken to be

$$\rho(r)=\rho_0\left[1+w\frac{r^2}{a^2}\right]\exp\left[-\frac{r^2}{a^2}\right] \ , \tag{10}$$

where ρ_0 is the normalization constant. The parameters are taken to be $a=1.51$ fm and $w=2.33$.[5] The amplitudes $f_{p,n}$ are evaluated by using the CERN theoretical phase shifts[6] for πN scattering data.

The differential cross sections predicted are compared with the data[1] for the inelastic scattering of π from ^{12}C in Figs. 1 and 2. The cross sections for the π^+ and π^- scattering on self-conjugate nuclei are identical because of the isospin invariance, except for effects due to Coulomb interactions. The contributions of Coulomb interactions to the cross sections are small.

The isospin averaged cross sections σ_{tot} for πN scattering are very close[7] to those for K^-N scattering around 800 MeV/c. Hence, the mean-free path of π is nearly the same[8] as that of K^-. The cross section σ_{tot} fixes the real part of ξ. It is also related to the real part of $\chi(b)$. The cross sections for π scattering are similar to those for K^- scattering[5,9,10] in their momentum transfer dependences and even in their absolute magnitudes at 800 MeV/c.

To see how the cross sections predicted depend on the transition density, the cross sections are also calculated with the Tassie model.[11] The model gives the transition density

$$\rho_{tr}^T(r)=\zeta_L r^L \frac{1}{r}\frac{d}{dr}\rho(r)$$

for the $J^\pi=0^+$ to the $J=L$ state, where ζ_L is the nuclear structure strength of the transition which can be expressed[2] in terms of the reduced transition matrix element $B(EL)$. The explicit form of the amplitudes in the impact parameter representation is given in Ref. 9 for the transition density of the derivative form. The density $\rho_{tr}^T(r)$ has an oscillatory structure in the region of small r, for the modified Gaussian density of the ground state given by Eq. (10) when $w>1$. The oscillatory structure acts to cancel the contribution of the transition from the nuclear interior region to the inelastic scattering amplitudes as if the cutoff approximation were

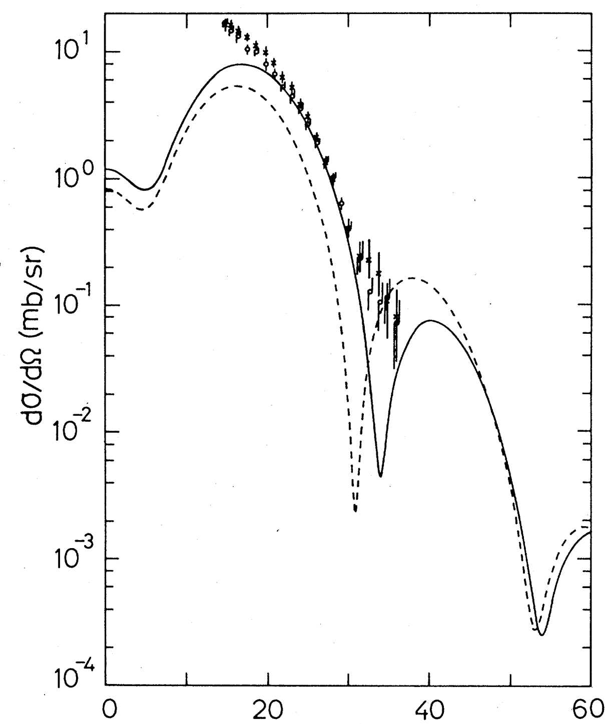

FIG. 1. The cross sections for the ^{12}C$(\pi,\pi')^{12}$C$^*(2^+$, 4.44 MeV) reaction at 800 MeV/c. The solid and dashed curves are, respectively, calculated with the transition densities fitted to electron inelastic scattering and proportional to derivative form of the ground-state density. Data are taken from Ref. 1. ϕ and ∗ represent the data for the π^+ and π^- scattering, respectively.

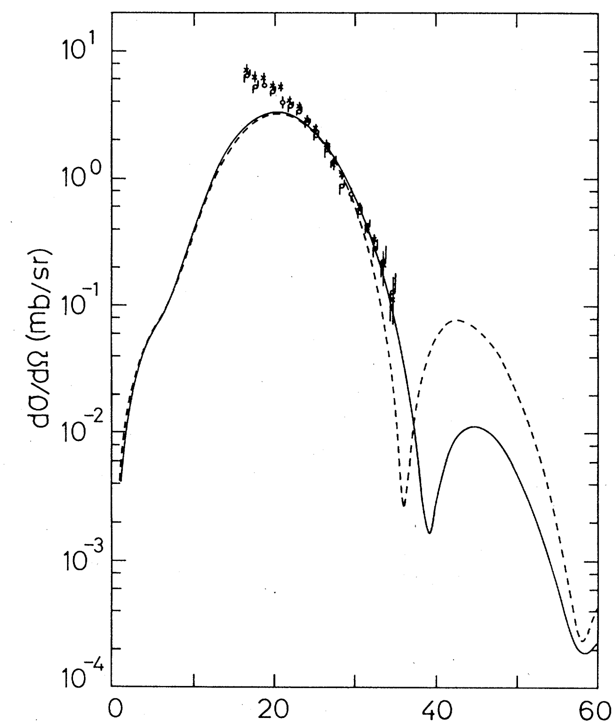

FIG. 2. The cross sections for the ^{12}C$(\pi,\pi')^{12}$C$^*(3^-$, 9.64 MeV) reaction at 800 MeV/c. The curves correspond to those in Fig. 1. Data are taken from Ref. 1. ϕ and ∗ represent the data for the π^+ and π^- scattering, respectively.

made irrespective to the mean-free path of projectiles. The maxima and minima in the differential cross sections predicted with the Tassie model shift to small angles, compared with the prediction of the phenomenological transition density.

In the energy region below and at the $\Delta(1232)$ resonance the basic πN interaction is dominated by a few partial waves, the dominant one being the P_{33} which has the $\Delta(1232)$ resonance. The role of $\Delta(1232)$ in π-nucleus scattering is extensively investigated. However, at the energy considered here the $\Delta(1232)$ resonance is no longer dominant. The conventional static distorted-wave impulse approximation calculation works to explain substantially the π-nucleus interactions, when the transition densities obtained by analyzing electron inelastic scattering data and πN scattering amplitudes fitted to πN data are used. The empirical nuclear density and πN scattering amplitudes fitted to πN data also describes[12] the elastic scattering of π on light nuclei at energies above $\Delta(1232)$ resonance. However, as seen in Figs. 1 and 2, there is a discrepancy between the cross sections measured and predicted in the region of low momentum transfers. The discrepancy might be an interesting problem. Such a discrepancy does not occur in the case for the K^--nucleus scattering at 800 MeV/c.[5,9,10] Before tackling the problem, one should note that, as stated in Ref. 1, the large elastic scattering peak in the excitation spectra makes the measurement of the cross sections for the inelastic scattering at forward angles difficult.

[1]D. Marlow *et al.*, Phys. Rev. C **30**, 1662 (1984).
[2]R. D. Amado, F. Lenz, J. A. McNeil, and D. A. Sparrow, Phys. Rev. C **22**, 2094 (1980).
[3]C. Gustaffson and E. Lambert, Ann. Phys. **111**, 304 (1978).
[4]J. Saudinos and C. Wilkin, Annu. Rev. Nucl. Sci. **24**, 341 (1974).
[5]D. Marlow *et al.*, Phys. Rev. C **25**, 2619 (1982).
[6]D. J. Herndon, A. Barbaro-Galtieri, and A. H. Rosenfeld, Lawrence Radiation Laboratory Report No. UCRL-20030 πN, 1970 (unpublished).
[7]Particle Data Group, Rev. Mod. Phys. **56**, No. 2, Part II, (1984).
[8]C. B. Dover and G. E. Walker, Phys. Rev. C **19**, 1393 (1979).
[9]Y. Sakamoto and F. M. Toyama, Lett. Nuovo Cimento **32**, 465 (1981).
[10]Y. Sakamoto, Y. Hatsuda, and F. M. Toyama, Phys. Rev. C **25**, 2695 (1982).
[11]L. J. Tassie, Aust. J. Phys. **9**, 407 (1956).
[12]A. K. Dozier and J. S. Chalmers, Phys. Rev. C **23**, 399 (1981).