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Brief Reports

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Systematics of fission fragment total kinetic energy release

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Recent data for the most probable total kinetic energy release in fission $\langle E_K \rangle$, have been combined with earlier results in order to reevaluate the systematic dependence of these values on the Coulomb parameter $Z^2/A^{1/3}$. A least-squares analysis of the current data base yields $\langle E_K \rangle = 0.1189Z^2/A^{1/3} + 7.3$ MeV, where $Z^2/A^{1/3}$ refers to the fissioning nucleus. This expression exhibits a somewhat steeper slope than previous fits.

Studies of the systematic behavior of the most probable total kinetic energy release in fission $\langle E_K \rangle$ have shown that the data can be rather accurately described by a simple model based on Coulomb repulsion between prolate spheriods.[1-3] Such a model predicts that $\langle E_K \rangle$ depends linearly on the Coulomb parameter, $Z^2/A^{1/3}$, of the fissioning nucleus. The coefficients derived from a least-squares fit to the data can be directly associated with the separation distance between the charge centers of the two fragments at the onset of acceleration and the deformation of the fragments.[3] It is expected that this relationship should describe liquid drop fission energetics, i.e., fission well above the barrier. Since the dependence of $\langle E_K \rangle$ on excitation energy is rather weak, such a fitting procedure succeeds relatively well for spontaneous fission as well. Nonetheless, any rigorous description of the total kinetic energy release in fission must also account for the effects of nuclear structure and pairing associated with both the fissioning nucleus and the nascent fragments.[4-7]

In general there is quite satisfactory agreement between the simple semiempirical model based on Coulomb spheriods and liquid drop model calculations.[8-10] Hence, for the purposes of comparison with data, the $Z^2/A^{1/3}$ parametrization serves as a useful approximation to detailed calculations. For example, strong deviations from systematic behavior for ^{258}Fm and ^{259}Fm have been interpreted in terms of shell effects.[6,7] Also, comparisons with the Coulomb spheriod predictions have become increasingly important in evaluating the degree of equilibration achieved in fissionlike phenomena associated with complex nucleus-nucleus collisions, e.g., in strongly damped collisions,[11] quasifission[12] and intermediate-mass ($6 \lesssim A \lesssim 30$) fragment emission from highly excited nuclei.[13]

In Ref. 3, which was based on $\langle E_K \rangle$ data available prior to 1966, the following expression for the most probable total kinetic energy release in fission was derived:

$$\langle E_K \rangle = 0.1071Z^2/A^{1/3} + 22.2 \text{ MeV}. \tag{1}$$

This relationship has been widely used for comparison with data, generally quite successfully. However, subsequent data for light fissioning systems, in particular that of Namboodiri *et al.*,[14] and also high precision data on actinide elements[5-7,15-17] suggested strongly some years ago that the slope for Eq. (1) was not sufficiently steep. Based on data available up to 1981, a new least-squares evaluation[18] led to the expression

$$\langle E_K \rangle = 0.1166Z^2/A^{1/3} + 9.0 \text{ MeV}. \tag{2}$$

In addition to providing a more general fit to the data over a significantly extended range of $Z^2/A^{1/3}$ values, this result also yielded a reduced value of the constant term. Since $\langle E_K \rangle$ must vanish as Z approaches zero, the reduction of this term appears to be a correction in the right direction. On the other hand, it must also be appreciated that the constant term is related to the distance between deformed fragment charge centers, and that this simple model must eventually break down at low $Z^2/A^{1/3}$ values due to the diffuse nature of light nuclei and the associated perturbations to the necking degree of freedom. Consideration of these effects in liquid drop model calculations[10] indeed predicts a change in slope at low $Z^2/A^{1/3}$ in the direction of vanishing $\langle E_K \rangle$ values at $Z=0$.

The present reexamination of the data was stimulated by two recent publications which reported new data at the lower[19] and upper[20] extremes of $Z^2/A^{1/3}$. Both of these experiments employed the kinematic coincidence technique along with mass or charge identification to minimize contributions to the spectra from competing processes, such as damped collisions. These data are in generally good agreement with Eq. (2), but differ significantly from Eq. (1), as shown in Table I. Conducting a weighted least-squares fit which includes the data of Refs. 19 and 20, as well as previous data quoted in Refs. 3, 5–7, and 14–18, yields the following result:

$$\langle E_K \rangle = (0.1189 \pm 0.0011)Z^2/A^{1/3} + 7.3(\pm 1.5) \text{ MeV}. \tag{3}$$

In performing these fits, a single value for $\langle E_K \rangle$ was entered for each fissioning nucleus; where several reported

TABLE I. Comparison of data of Refs. 19 and 20 with Eqs. (1)–(3).

System	$Z^2/A^{1/3}$	$\langle E_K \rangle_{\mathrm{expt}}$ (MeV)	$\langle E_K \rangle$ predicted (MeV) Eq. (1)	 Eq. (2)	 Eq. (3)
$^{52}_{26}$Fe	181	30.2 ± 2	41.6	30.1	28.9
		34.0 ± 3	41.6	30.1	28.9
$^{49}_{24}$Cr	157	29.6 ± 5.0	39.1	27.3	26.0
$^{46}_{23}$V	148	28.6 ± 7.0	38.0	26.2	25.9
$^{265}_{105}X$	1716	214 ± 4	206.0	209.1	211.3
$^{286}_{112}X$	1904	237 ± 4	226.1	231.0	233.6
$^{283}_{113}X$	1945	244 ± 4	230.5	235.8	238.5
$^{286}_{114}X$	1973	246 ± 4	233.5	239.0	241.8
$^{296}_{118}X$	2089	254 ± 5	246.0	252.6	255.7
$^{302}_{120}X$	2146	263 ± 4	252.1	259.3	262.4

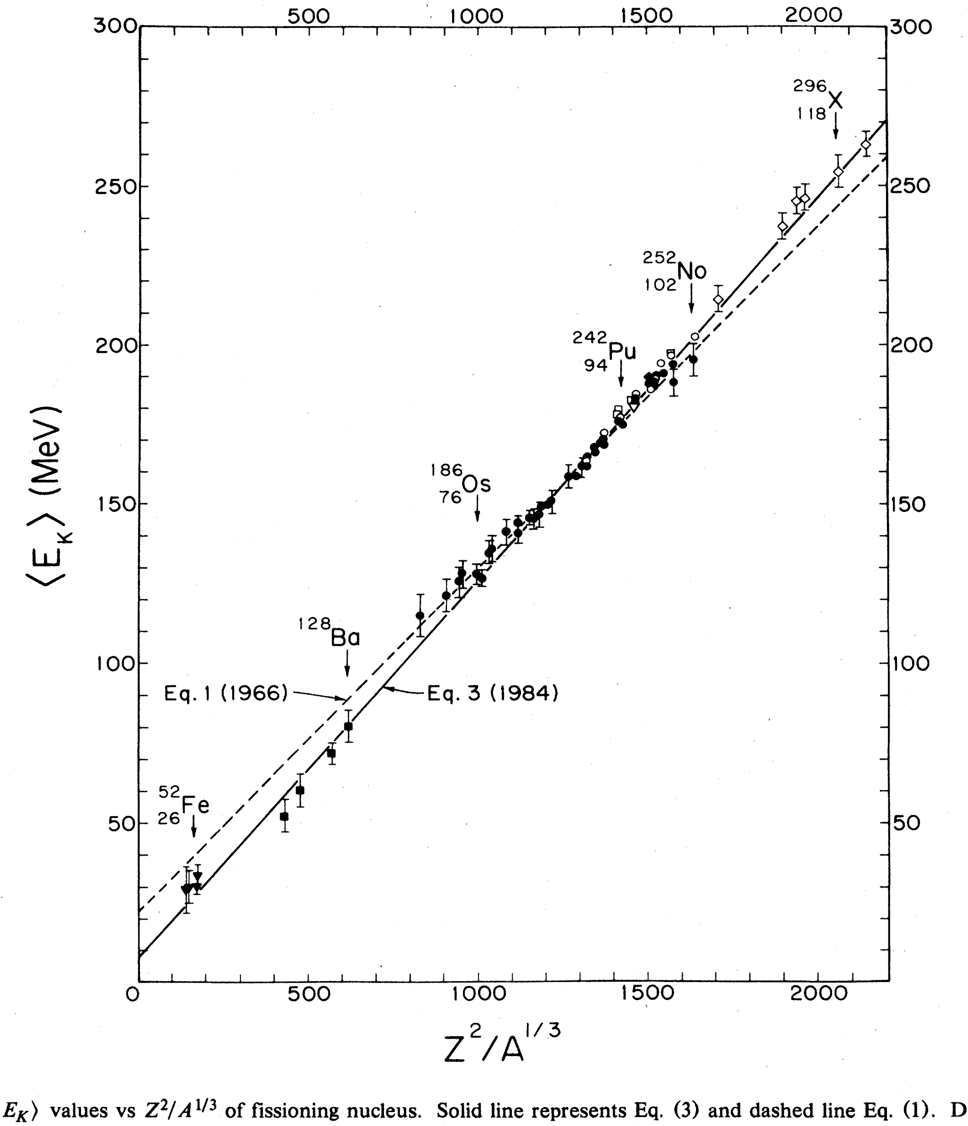

FIG. 1. Plot of $\langle E_K \rangle$ values vs $Z^2/A^{1/3}$ of fissioning nucleus. Solid line represents Eq. (3) and dashed line Eq. (1). Data are indicated as follows: ●—Ref. 3; ■—Ref. 14; ◆—Ref. 7; ▽—Ref. 15; ○—Ref. 5; □—Ref. 16; ▼—Ref. 19; ◇—Ref. 20; ⊗ —Ref. 21. Error bars are indicated only for errors greater than ±2 MeV.

values of $\langle E_K \rangle$ existed, a weighted average of these was used, with corresponding standard deviations. A minimum error of ±1 MeV was assumed in the fitting procedure. Also included in the data base were results for spontaneous fission, although $\langle E_K \rangle$ values for excited nuclei usually lie 1–2 MeV higher than for the fission of nuclei in their ground state.[5] The exception to this procedure was the omission of spontaneous fission values for ^{258}Fm and ^{259}Fm (238 and 242 MeV, respectively), which fall almost 40 MeV above the predictions, presumably due to the influence of shell effects as ^{264}Fm is approached.[4] Removal of the remaining spontaneous fission data from the data set had virtually no effect on the resulting parameters of Eq. (3), yielding predictions to within ±0.1 MeV of the fit with all data included over the entire range of experimental values. Equation (3) is in very good agreement with liquid drop model calculations based on a one-body dissipation theory.[9,10] The χ-squared value per degree of freedom for the fit is 1.54. In Fig. 1 the experimental data are compared with Eqs. (1) and (3). It is clear that Eq. (3) provides a significant improvement in the description of the data.

In summary, a reevaluation of the dependence of $\langle E_K \rangle$ values on $Z^2/A^{1/3}$ yields a somewhat stronger dependence on this parameter than previously used. This relationship [Eq. (3)] thus serves as a more general estimate of the most probable kinetic energy release in fission and fissionlike phenomena than that of Ref. 3, particularly for very light or very heavy systems. In addition, excellent agreement with fission kinetic energy release values predicted by the liquid drop model is observed.

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