Method for observation of neutrinos and antineutrinos

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Theory is given for momentum transfer to an ensemble of particles by incident neutrinos or antineutrinos in such a way that subsequent measurements cannot reveal the detailed characteristics of this transfer. It is shown that large scattering cross sections may be obtained, proportional to the square of the number of scatterers.

INTRODUCTION

Large numbers of interacting particles and long observation times have been required for weak interaction experiments at low energies. Total cross sections are proportional to the number of scatterers.

For the scattering of electromagnetic waves by macroscopic quantities of matter, the total cross sections in the x ray region are also proportional to the number of scatterers.[1,2] However, for wavelengths large in comparison with dimensions of a macroscopic volume of scatterers, the total cross section may be proportional to the square[3] of the number of scatterers.

Research reported here explores a new method for obtaining weak interaction cross sections proportional to the square of the number of scatterers. In order to understand how this might be accomplished, we consider first the nonrelativistic theory of scattering by a two-dimensional array of scattering potentials.

SCATTERING BY A PLANAR ARRAY

Let us imagine that there are N scatterers equally spaced along the x and y directions (Fig. 1). The x and y scatterer spacing is length b. A beam of particles has incident momentum $\bar{p}_{IO}$ and momentum $\bar{p}_{IF}$ after elastic scattering. The interactions occur in a volume V. Incident and scattered particles are represented by the wave functions

$$\psi_O = \frac{1}{\sqrt{V}} e^{i\bar{p}_{IO}\cdot\bar{r}/\hbar - iEt/\hbar} ,$$
$$\psi_F = \frac{1}{\sqrt{V}} e^{i\bar{p}_{IF}\cdot\bar{r}/\hbar - iEt/\hbar} , \tag{1}$$

respectively.

Let the scattering potential be $U(\bar{r})$. The interaction matrix element is then

$$H' = \frac{1}{V}\int e^{-i\bar{p}_{IF}\cdot\bar{r}/\hbar} U(\bar{r}) e^{i\bar{p}_{IO}\cdot\bar{r}/\hbar} d^3x . \tag{2}$$

Suppose that each scatterer interacts via a delta function potential with integrated value B. Then $U(\bar{r})$ is given by

$$U(\bar{r}) = B \sum_{n_x=1}^{n_x=N^{1/2}} \sum_{m_y=1}^{m_y=N^{1/2}} \delta(x-n_x b)\delta(y-m_y b)\delta(z) . \tag{3}$$

For (3), H' is evaluated as

$$H' = \frac{B}{V} \sum_{m_y=1}^{m_y=N^{1/2}} \sum_{n_x=1}^{n_x=N^{1/2}} e^{i(\bar{p}_{IO}-\bar{p}_{IF})_x n_x b/\hbar} \times e^{i(\bar{p}_{IO}-\bar{p}_{IF})_y m_y b/\hbar} . \tag{4}$$

In (4), $(\bar{p}_{IO}-\bar{p}_{IF})_x$ and $(\bar{p}_{IO}-\bar{p}_{IF})_y$ are the x and y components of $\bar{p}_{IO}-\bar{p}_{IF}$, respectively.

Fermi's golden rule gives a transition probability W with

$$W = \frac{2\pi}{\hbar} |H'|^2 \rho(E) . \tag{5}$$

The density of states $\rho(E)$ is computed by noting that in a range dE the total number of states for the outgoing particles is, for solid angle $d\Omega$,

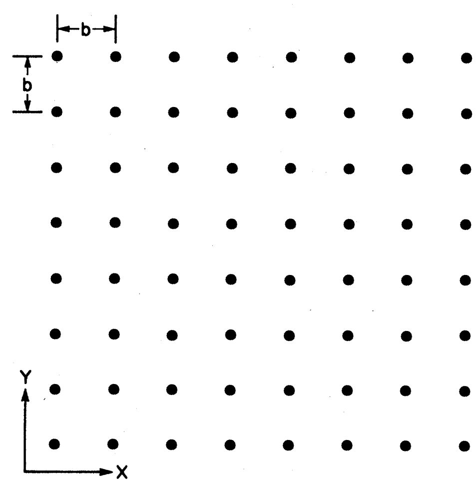

FIG. 1. A two-dimensional array of delta function potential scatterers.

$$\rho(E)dE=\frac{V}{(2\pi\hbar)^3}|p_{IF}|^2 dp_{IF}d\Omega \ . \tag{6}$$

For zero rest mass particles, $dE=cdp$. Expression (6) then gives

$$\rho(E)=\frac{V}{c(2\pi\hbar)^3}|p_{IF}|^2 d\Omega \ . \tag{7}$$

The incident particle velocity c and normalization imply an incident particle flux

$$\frac{c}{V} \ . \tag{8}$$

The interaction matrix element (4) is the product of two geometric progressions which are readily summed. The scattering cross section σ is the quotient of (5) and (8), with

$$\sigma=\frac{|p_{IO}|^2B^2}{4\pi^2\hbar^4c^2}\int\frac{\sin^2[\frac{1}{2}N^{1/2}b(\bar{p}_{IO}-\bar{p}_{IF})_x/\hbar]\sin^2[\frac{1}{2}N^{1/2}b(\bar{p}_{IO}-\bar{p}_{IF})_y/\hbar]}{\sin^2[\frac{1}{2}b(\bar{p}_{IO}-\bar{p}_{IF})_x/\hbar]\sin^2[\frac{1}{2}b(\bar{p}_{IO}-\bar{p}_{IF})_y/\hbar]}d\Omega \ . \tag{9}$$

The differential cross section in (9) has a maximum value proportional to N^2, given by

$$\left[\frac{d\sigma}{d\Omega}\right]_{\max}=\frac{|p_{IO}|^2B^2N^2}{4\pi^2\hbar^4c^2} \ . \tag{10}$$

For $\bar{p}_{IO}$ in a direction normal to the array, in the z direction, (10) corresponds to forward scattering with $\bar{p}_{IF}$ also in the z direction. For the forward scattering peak, the solid angle $d\Omega$ is determined by the first zeros of the integrand of (9). These occur for

$$(\delta p_x)_{\substack{\text{numerator}\\ \text{zero}}}=|(\bar{p}_{IO}-\bar{p}_{IF})_x|=\frac{2\pi\hbar}{N^{1/2}b} \ ,$$
$$(\delta_{p_y})_{\substack{\text{numerator}\\ \text{zero}}}=|(\bar{p}_{IO}-\bar{p}_{IF})_y|=\frac{2\pi\hbar}{N^{1/2}b} \ . \tag{11}$$

Equation (11) gives a solid angle

$$d\Omega\approx\pi\left|\frac{2\pi\hbar}{N^{1/2}bp_{IO}}\right|^2 \ . \tag{12}$$

The total cross section associated with this forward scattering peak is the product of (10) and (12), $\Delta\sigma_F$, given by

$$\Delta\sigma_F=\frac{\pi B^2N}{\hbar^2b^2c^2} \ . \tag{13}$$

A study of (9) indicates that there are other peaks in addition to the forward peak.

There will be a peak for each value of $\bar{p}_{IO}-\bar{p}_{IF}$ which gives a zero in the denominator of (9). These occur at intervals defined by

$$(\delta p_x)_{\substack{\text{denominator}\\ \text{zero}}}=\frac{2\pi\hbar}{b} \ ,$$
$$(\delta p_y)_{\substack{\text{denominator}\\ \text{zero}}}=\frac{2\pi\hbar}{b} \ . \tag{14}$$

The total number of peaks n_p is the number of cells of area $(\delta p_x\delta p_y)_{\text{denominator zero}}$ contained within the circle in the xy momentum space plane with elastic scattering momentum radius p_{IF},

$$n_p=\frac{b^2p_{IF}^2}{4\pi\hbar^2} \ . \tag{15}$$

The total cross section, σ_{total}, is then given approximately by the product of (15) and (13) as

$$\sigma_{\text{total}}\approx\frac{|p_{IO}|^2B^2N}{4\hbar^4c^2} \ . \tag{16}$$

Equation (16) is proportional to N in consequence of the fact that the peak values (10) in the differential cross section are multiplied by a solid angle for each peak, inversely proportional to N. A similar result is obtained for one- and three-dimensional scatterer arrays.

Expressions (9) and (13) are given in the literature and describe the scattering[4] of x rays very well.

A METHOD OF OBTAINING CROSS SECTIONS PROPORTIONAL TO N^2

In order to obtain a total cross section proportional to N^2, a method is required which does not lead to the very small solid angles of (12).

Each scatterer should be represented by a wave function and exchange of momentum with the scatterer must be taken into account. If a scatterer exchanges momentum $\Delta\bar{p}$, the expectation value of its momentum after scattering is altered by $\Delta\bar{p}$. This requires the scatterer wave function ψ_{SF} after scattering to be related to the wave function ψ_{SO} before scattering by

$$\psi_{SF}=\psi_{SO}e^{i\Delta\bar{p}\cdot\bar{r}/\hbar} \ . \tag{17}$$

For such exchange by the nth scatterer, the integral of (2) would therefore contain a term

$$e^{i(\bar{p}_{IO}-\bar{p}_{IF}-\Delta\bar{p})\cdot\bar{r}_n/\hbar} \ . \tag{18}$$

Equation (18) suggests that the solid angle Eq. (12) will be modified. To explore this possibility and for later applications, we employ the relativistic quantum mechanics.

INTERACTION OF FOUR-CURRENT DENSITIES

Let us consider the S matrix for interaction of two four-current densities[5,6] given by

$$S=\frac{1}{\hbar c}\int \langle F\,|\,\bar{\psi}_S\Gamma\psi_S\bar{\psi}_I\Xi\psi_I\,|\,O\rangle d^4x\ , \tag{19}$$

$|F\rangle$ is the final state, $|0\rangle$ is the original state. $\bar{\psi}_S$ is a creation operator for scatterer S, $\bar{\psi}_I$ is a creation operator for incident particle I. ψ_S and ψ_I are the corresponding annihilation operators. Γ and Ξ are position independent operators.

The operators $\bar{\psi}_S$ and $\bar{\psi}_I$ are represented[5] by the following expansions:

$$\bar{\psi}_S=\sum_n\sum_j \psi^*_{Sjn}(\bar{\mathbf{r}}-\bar{\mathbf{r}}_n)a^\dagger_{jn}\ , \tag{20}$$

$$\bar{\psi}_I=\frac{1}{\sqrt{V}}\sum_k \bar{U}_{Ik}e^{-(i/\hbar)\bar{\mathbf{p}}_{Ik}\cdot\bar{\mathbf{r}}}d^\dagger_k\ . \tag{21}$$

Here $\bar{\mathbf{r}}$ is again the position three vector and $a^\dagger_{jn}$ is a creation operator for the state with wave function ψ^*_{Sjn}; as before, n refers to the nth scattering site. $d^\dagger_k$ is a creation operator for an incident particle with known momentum $\bar{\mathbf{p}}_{Ik}$. U_{Ik} is an incident particle spinor.

We consider N scatterers in a solid. For the states ψ_{Sjn}, harmonic oscillator states are selected. For a harmonic oscillator wave function centered at radius vector $\bar{\mathbf{r}}_n$,

$$\psi_{SOn}=K^{3/2}\pi^{-3/4}e^{-(K^2/2)|\bar{\mathbf{r}}-\bar{\mathbf{r}}_n|^2}\ . \tag{22}$$

In (22), K specifies the volume of each scatterer. For the N scatterers, the original state is taken to be

$$a^\dagger_{O1}a^\dagger_{O2}a^\dagger_{O3}\cdots a^\dagger_{ON}\,|\,{}^{\text{vaccum}}_{\text{state}}\rangle\ . \tag{23}$$

For nuclei in a solid, the wave functions of different scatterers will not overlap to a significant degree, and the symmetry of the many particle wave function need not be considered.

Let us assume now that the scatterer position probability distribution $\psi^*_{Sjn}\psi_{Sjn}$ is not changed by the scattering,

$$(\psi^*_{Sjn}\psi_{Sjn})_{\substack{\text{before}\\ \text{scattering}}}=(\psi^*_{Sjn}\psi_{Sjn})_{\substack{\text{after}\\ \text{scattering}}}\ . \tag{24}$$

Equation (24) implies that each final scatterer state $(\psi_{Sjn})_F$ may be related to the original state by

$$(\psi_{Sjn})_F=(\psi_{Sjn})_O e^{i(\Delta p_\mu)_n x^\mu/\hbar}\ . \tag{25}$$

Equation (25) implies that each component in the momentum decomposition of the nth scatterer is shifted by the momentum $(\Delta p_\mu)_n$, corresponding to momentum exchange Δp_μ.

Suppose there is exchange of momentum $(\Delta p_\mu)_n$ at the nth site, from (25), $\bar{\psi}_S$ in (19) must then be replaced by

$$\bar{\psi}'_S=\sum_n \psi^*_{SOn}a^\dagger_{On}e^{-i(\Delta p_\mu)_n x^\mu/\hbar}\ . \tag{26}$$

Expressions (20)—(26) are employed to evaluate the S matrix (19) for initial and final scatterer states which are harmonic oscillator ground states. Let us now consider the case of spin zero scatterers, $\Gamma=1$.

$$S=\frac{\bar{U}_{IF}\Xi U_{IO}}{\hbar c V}\int\sum_{n=1}^{n=N}K^3\pi^{-3/2}e^{-K^2|\bar{\mathbf{r}}-\bar{\mathbf{r}}_n|^2+(i/\hbar)(p_{IO}-p_{IF}-\Delta p_n)_\mu x^\mu}d^4x\ . \tag{27}$$

SCATTERING CROSS SECTIONS

Suppose now that we have scatterers in a cubic crystal with N identical cells, each with length b. For these assumptions the S matrix (27) is integrated over the crystal volume, and over the time interval $-\tau/2$ to $+\tau/2$. τ is a time long compared with any relevant energy level periods. The result is

$$S=\bar{U}_{IF}\Xi U_{IO}XYZT\left[\frac{1}{\hbar V}\right]\ , \tag{28}$$

with

$$X=\sum_{n=1}^{n=N^{1/3}}e^{(i/\hbar)(\bar{\mathbf{p}}_{IO}-\bar{\mathbf{p}}_{IF}-\Delta\bar{\mathbf{p}}_n)_x X_n-(1/K^2)[(\bar{\mathbf{p}}_{IO}-\bar{\mathbf{p}}_{IF}-\Delta\bar{\mathbf{p}})/2\hbar]^2_x}\ . \tag{29}$$

In (29), $X_n=nb$, with corresponding definitions for Y and Z.

$$T=\frac{\sin\left[\dfrac{(E_{IF}-E_{IO}+E_{SF}-E_{SO})\tau}{2\hbar}\right]}{\left[\dfrac{E_{IF}-E_{IO}+E_{SF}-E_{SO}}{2\hbar}\right]}\ . \tag{30}$$

E_{IF} and E_{SF} are the final state energies of the incident particle and ensemble of scatterers, respectively, E_{IO} and E_{SO} are the corresponding original energies.

The scattering cross section is given by[7] σ, with

$$\sigma=\sum\frac{V(S-1)^2}{c\tau}=\frac{V}{(2\pi)^6 c\tau\hbar^8}\int|\bar{U}_{IF}\Xi U_{IO}XYZT|^2 d\bar{\mathbf{p}}_S d\bar{\mathbf{p}}_I\ . \tag{31}$$

In (31), $d\bar{\mathbf{p}}_S$ is the element of momentum space for the final state of the ensemble of scatterers, $d\bar{\mathbf{p}}_I$ is the element of momentum space for the final state of the incident particle. T in (28)—(31) is a function of the momentum variables in X, Y, and Z. The integration is carried out in the following way.

The length L of the crystal is given by $L=N^{1/3}b$; to evaluate (31) we must make an assumption regarding $\Delta\bar{\mathbf{p}}_n$. Let us assume that each scatterer exchanges an equal amount of momentum $\Delta\bar{\mathbf{p}}_\alpha$. $\bar{\mathbf{p}}_s$ is therefore a function of $\Delta\bar{\mathbf{p}}_\alpha$. This gives for certain integrals the approximate value

$$\frac{L}{2\pi\hbar}\int X^2 d\bar{\mathbf{p}}_{Sx}=\frac{L}{2\pi\hbar}\int\left\{\frac{\sin\left[\dfrac{N^{1/3}b(\bar{\mathbf{p}}_{IO}-\bar{\mathbf{p}}_{IF}-\Delta\bar{\mathbf{p}}_\alpha)_x}{2\hbar}\right]}{\sin\left[\dfrac{b(\bar{\mathbf{p}}_{IO}-\bar{\mathbf{p}}_{IF}-\Delta\bar{\mathbf{p}}_\alpha)_x}{2\hbar}\right]}\right\}^2 e^{-(2/K^2)[(\bar{\mathbf{p}}_{IO}-\bar{\mathbf{p}}_{IF}-\Delta\bar{\mathbf{p}}_\alpha)/2\hbar]^2}dp_{Sx}$$

$$=N^{2/3}\ . \tag{32}$$

The integration (32) is exact in the limit $K\to\infty$ and an excellent approximation for expected values of $K\to 10^8$. Integrations over $\bar{\mathbf{p}}_{Sy}$ and $\bar{\mathbf{p}}_{Sz}$ give similar results.

Combining (31) and (32) then gives

$$\sigma=\frac{N^2}{(2\pi)^3 c\hbar^5\tau}\int(\bar{U}_{IF}\Xi U_{IO}T)^2 d\bar{\mathbf{p}}_I$$
$$=\frac{N^2}{(2\pi)^3 c\hbar^5\tau}\int(\bar{U}_{IF}\Xi U_{IO}Tp_I)^2\frac{d|p_I|}{dE}dE\,d\Omega_I\ , \tag{33}$$

with $E=E_I+E_s$, $d\Omega_I$ is the element of solid angle into which the incident particle is scattered.

In the center of mass system[7]

$$\frac{d|p_I|}{dE}=\frac{E_{IF}E_{SF}}{c^2 p_I(E_{IF}+E_{SF})}\ . \tag{34}$$

Equation (34) is integrated over E first

$$\sigma=\frac{N^2}{4\pi^2c^3\hbar^4}\int\frac{(\bar{U}_{IF}\Xi U_{IO})^2 P_I E_{IF}E_{SF}}{(E_{IF}+E_{SF})}d\Omega_I\ . \tag{35}$$

Suppose that the incident beam of particles is again in the Z direction. The solid angle associated with the forward peak is given by the first zeros of $\sin^2[\frac{1}{2}N^{1/3}b(\bar{\mathbf{p}}_{IO}-\bar{\mathbf{p}}_{IF}-\Delta\bar{\mathbf{p}}_\alpha)_x/\hbar]$ and $\sin^2[\frac{1}{2}N^{1/3}b(\bar{\mathbf{p}}_{IO}-\bar{\mathbf{p}}_{IF}-\Delta\bar{\mathbf{p}}_\alpha)_y/\hbar]$. These give

$$\tfrac{1}{2}N^{1/3}b(\bar{\mathbf{p}}_{IO}-\bar{\mathbf{p}}_{IF}-\Delta\bar{\mathbf{p}}_\alpha)_x/\hbar=\pi$$

and

$$\tfrac{1}{2}N^{1/3}b(\bar{\mathbf{p}}_{IO}-\bar{\mathbf{p}}_{IF}-\Delta\bar{\mathbf{p}}_\alpha)_y/\hbar=\pi$$

and

$$|\bar{\mathbf{p}}_{IO}-\bar{\mathbf{p}}_{IF}|^2=\left[\frac{2\pi\hbar}{N^{1/3}b}+\Delta\bar{\mathbf{p}}_{\alpha x}\right]^2+\left[\frac{2\pi\hbar}{N^{1/3}b}+\Delta\bar{\mathbf{p}}_{\alpha y}\right]^2,$$

therefore

$$d\Omega\approx\pi|\bar{\mathbf{p}}_{IO}-\bar{\mathbf{p}}_{IF}|^2/|\bar{\mathbf{p}}_{IO}|^2$$
$$=\left[\left[\frac{2\pi\hbar}{N^{1/3}b}+\Delta\bar{\mathbf{p}}_{\alpha x}\right]^2+\left[\frac{2\pi\hbar}{N^{1/3}b}+\Delta\bar{\mathbf{p}}_{\alpha y}\right]^2\right]\Big/|\bar{\mathbf{p}}_{IO}|^2\ . \tag{36}$$

Since momentum is conserved, and each scatterer was assumed to transfer equal momentum in a single scattering, it follows that

$$N\Delta\bar{\mathbf{p}}_\alpha=\bar{\mathbf{p}}_{IO}-\bar{\mathbf{p}}_{IF}\ . \tag{37}$$

For large N, $\Delta\bar{\mathbf{p}}_\alpha$ is very small and the term $2\hbar\pi/N^{1/3}b$ in (36) will be much larger. The solid angle implied by (36) will therefore be very small and the total cross section Eq. (35) will be very small.

MOMENTUM EXCHANGE POSSIBILITIES

Any number of scatterers may exchange momentum in a scattering process. The total cross section must consider all possibilities. If the scatterers are electrons, as in the case of x rays, each scatterer is usually bound to a particular site and the coupling of electrons on different sites with each other is small. Under these conditions each electron may be expected to exchange any amount of momentum. If such exchange is a random process, each electron would exchange approximately $\Delta\bar{\mathbf{p}}_B$ with

$$\Delta\bar{\mathbf{p}}_B\approx\frac{\bar{\mathbf{p}}_{IO}-\bar{\mathbf{p}}_{IF}}{\sqrt{N}}\ . \tag{38}$$

For large N, (38) is so small that the momentum transfer does not play a significant role. The total cross section has the very small value implied by the small solid angle into which an incident particle is scattered.

Suppose that the nuclei of a solid are the scatterers. These may be very tightly coupled to each other. If the incident particles have very low energy, the following process may occur. All of the momentum may be exchanged at a single nucleus. The tight binding of that nucleus to other nuclei would result in the momentum being quickly transferred to the entire lattice. (Tight binding implies scatterer quantum states with well-defined positions. In Appendix A it is shown that scatterer states of well-defined momenta give small total cross sections.)

For exchange of momentum at a single scatterer at site $\overline{\mathbf{r}}_n$, Eq. (26) will be replaced by

$$\bar{\psi}'_{Sn}=\psi^*_{SOn}a^\dagger_{On}e^{-i\Delta p_\mu x^\mu/\hbar}+\sum_{i\neq n}\psi^*_{SOi}a^\dagger_{Oi}\ . \tag{26a}$$

Equation (26a) may be written in a more illuminating form by adding $\psi^*_{SOn}a^\dagger_{On}$ to the last term and subtracting it from the first term to give

$$\bar{\psi}'_{Sn}=\psi^*_{SOn}a^\dagger_{On}(e^{-i\Delta p_\mu x^\mu/\hbar}-1)+\sum_{\text{all } n}\psi^*_{SOi}a^\dagger_{Oi}\ . \tag{26b}$$

In (26b) the last term gives a probability amplitude for the possible process where no momentum is exchanged at any site. The first term then represents the contribution to the amplitude for exchange of momentum at the nth site. We assume strong coupling of nuclei to each other with no possible way of identifying the scattering site. Therefore, we must sum only the first term in (26b) over all possible sites. Carrying out this sum then gives

$$\bar{\psi}''_s=\sum_j\psi^*_{SOj}a^\dagger_{Oj}e^{-i\Delta p_\mu x^\mu/\hbar}\ . \tag{26c}$$

Equation (26c) gives a solid angle

$$d\Omega\approx\pi\left[\left[\Delta p_x+\frac{2\hbar\pi}{N^{1/3}b}\right]^2+\left[\Delta p_y+\frac{2\hbar\pi}{N^{1/3}b}\right]^2\right]\Big/\ |p_{IO}|^2\ . \tag{39}$$

If $\overline{\mathbf{p}}_{IO}$ is sufficiently small, a total momentum transfer with $\Delta\overline{\mathbf{p}}\rightarrow 2\overline{\mathbf{p}}_{IO}$ is possible without the momentum transfer changing the coupled scatterer wave function enough to permit identifying that scatterer after scattering.

Under these conditions, Eq. (39) may approach 4π and Eq. (35) may approach the value

$$\sigma\approx\frac{|U_{IF}\Xi U_{IO}|^2E_{IF}^2N^2}{\pi\hbar^4c^4}\ . \tag{40}$$

The large cross section Eq. (40) implies that the kinematics of the exchange does not restrict the value of the solid angle into which an incident particle is scattered. In Appendix B it is shown that this is indeed the case.

We may also imagine processes in which two, three, or any number of unidentified scatterers exchange all the momentum. In Appendix C these possibilities are considered, and it is shown that the single unidentifiable scatterer case gives the largest cross section.

LIMITS OF VALIDITY OF THE FORMULA FOR THE TOTAL CROSS SECTION

A crystal would have to be infinitely stiff for every incident particle to be scattered with the large cross section Eq. (40).

Available crystals might be expected to have cross sections approaching Eq. (40) if: (a) the energy of interaction of an incident particle with a scatterer is small compared with the binding energy of each scatterer to other scatterers; (b) the recoil energy of each scatterer is small compared with the "Debye" temperature energy kT_{Debye}. This follows from the theory[8–10] of the Mössbauer effect. This theory gives the fraction of gamma ray emissions which results in recoil of the crystal as a whole, and the fraction which results in recoil of the emitting nucleus exciting lattice vibrations. Clearly the recoil of the crystal as a whole corresponds to the infinite stiffness case discussed here. The same theory must apply for momentum transfer by an incident scatterer.

At temperature T small compared with the Debye temperature T_{Debye}, the fraction of Mössbauer gamma ray emissions which results in recoil of the entire crystal is calculated to be f with[10]

$$f=e^{-(E_R/kT_{\text{Debye}})[(3/2)+(\pi^2T^2/T^2_{\text{Debye}})]}\ . \tag{41}$$

In (41), E_R is the recoil energy given in terms of the individual scatterer mass μ_S by $(\Delta p)^2/2\mu_S$.

If Eq. (41) approaches unity this is clearly sufficient to guarantee a very large total cross section. It is not certain that this is necessary.

In the Mössbauer effect, the narrow line widths are associated with the recoil of the entire crystal with no phonon excitation. If phonons are excited, each gamma ray would have energy shared with a given type of phonon excitation. Since there are many ways of exciting the lattice, this will give a larger line breadth than excitation of no phonons.

For the single scatterer momentum exchange discussed here, it is only necessary that after scattering, the single scatterer wave function should not be changed so much that its identity may be established by subsequent measurements. It remains to be proved that this can or cannot be done if phonons are excited.

Another issue is the possibility of processes in which n_S unidentified nuclei exchange all of the momentum as discussed in Appendix C. The n_S particle exchange leads to a cross section smaller than for the one particle exchange. However the recoil momentum is reduced by the factor $1/n_S$ and the recoil energy is reduced by a factor $1/n_S^2$. Therefore the reduction in cross section is approximately compensated by an increased factor f in (41). For these reasons it is expected that the cross section will not decrease with exchange of momentum as rapidly as implied by Eq. (41) for a one particle exchange process.

COHERENT INELASTIC SCATTERING

The Copenhagen interpretation of quantum mechanics permits a coherent scattering process in which all of the momentum is exchanged by certain unidentified scatterers while other unidentified scatterers may exchange energy.

COHERENT SCATTERING OF NEUTRINOS AND ANTINEUTRINOS

Let us apply Eq. (40) to the scattering of neutrinos and antineutrinos. The neutral current interaction then gives

$$\sigma=\frac{G_W^2N^2}{8\pi^2\hbar^4c^4}\int E_\nu^2\langle\,|\,\overline{U}_{SF}\gamma^\alpha(1+\gamma_5)U_{\nu O}\overline{U}_{\nu F}\gamma_\alpha(1+\gamma_5)U_{SO}\,|^2\rangle d\Omega_\nu\,. \tag{42}$$

It is possible to show that[11]

$$\overline{U}_{SF}\gamma^\alpha(1+\gamma_5)U_{\nu O}\overline{U}_{\nu F}\gamma_\alpha(1+\gamma_5)U_{SO} =-\overline{U}_{SF}\gamma^\alpha(1+\gamma_5)U_{SO}\overline{U}_{\nu F}\gamma_\alpha(1+\gamma_5)U_{\nu O}\,. \tag{43}$$

In "spinor" representation

$$\gamma_5=\begin{vmatrix}-1 & 0\\ 0 & 1\end{vmatrix},\ \ \overline{\gamma}=\begin{vmatrix}0 & -\overline{\sigma}\\ \overline{\sigma} & 0\end{vmatrix}\,. \tag{44}$$

All elements here are 2×2 matrices

$$\overline{\gamma}(1+\gamma^5)=\begin{vmatrix}0 & -2\overline{\sigma}\\ 0 & 0\end{vmatrix}\,. \tag{45}$$

Let

$$U_S=\begin{vmatrix}n_S\\ X_S\end{vmatrix},\ \ U_\nu=\begin{vmatrix}n_\nu\\ X_\nu\end{vmatrix}\,; \tag{46}$$

n and X are two-component spinors

$$\overline{U}_S\overline{\gamma}(1+\gamma_5)U_S=-2X_S^\dagger\overline{\sigma}X_S\,, \tag{47}$$

$$\overline{U}_S\gamma^0(1+\gamma_5)U_S=2X_S^\dagger X_S\,, \tag{48}$$

therefore,

$$\frac{G_W}{\sqrt{2}}\overline{U}_S\gamma^\alpha(1+\gamma_5)U_\nu\overline{U}_\nu\gamma_\alpha(1+\gamma_5)U_S =\frac{4G_W}{\sqrt{2}}(X_{SF}^\dagger X_{SO}X_{\nu F}^\dagger X_{\nu O}-X_{SF}^\dagger\overline{\sigma}X_{SO}X_{\nu F}^\dagger\overline{\sigma}X_{\nu O})\,. \tag{49}$$

For unpolarized scatterers, the last (spin terms) in (49) average to zero.

Suppose the incident direction is again the z direction. For scattering through an angle θ, the spinor transformation law leads to

$$X_{\nu F}'^{\dagger}X_{\nu O}=\cos\frac{\theta}{2}\,. \tag{50}$$

Integration of (42) then gives for the total cross section

$$\sigma=\frac{4G_W^2E_\nu^2N^2}{\pi\hbar^4c^4}\,. \tag{51}$$

Equation (51) is the total cross section for N identical scatterers. Required modifications for quark models will be considered in another paper. Equation (51) is the same for both neutrinos and antineutrinos. In general, if all terms in (49) contribute significantly, the neutrino and antineutrino cases would not be identical.

EXPERIMENTS

A number of experiments have confirmed theoretical predictions of relatively large cross sections. One series observed heating of a nuclear spin system in a target crystal, associated with inelastic coherent scattering of antineutrinos from the ten megawatt reactor at the National Bureau of Standards in Gaithersburg, Maryland.

A second experiment observed a repulsive force of 4×10^{-7} dyn on a 12 g crystal elastic scattering antineutrinos from a 600 Ci tritium source. This corresponds to a total cross section approximately 1.5 cm^2.

A third experiment also employed antineutrinos from the ten megawatt reactor at the National Bureau of Standards. Elastic scattering was observed, with a cross section approximately 2 cm^2, for a 100 g crystal. A larger crystal was employed as a shield. Repulsive force changes, approximately 3×10^{-5} dyn, were observed as the shield was placed between the reactor and the target crystal.

These experiments will be described in detail in forthcoming papers.

CONCLUSION

Theory predicts large cross sections for tightly coupled nuclei interacting with low energy neutrinos and antineutrinos.

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APPENDIX A: COHERENT SCATTERING WITH WELL-DEFINED SCATTERER MOMENTA

The large cross sections may be observed only under some very restricted conditions. One such condition is that the scatterers have well-defined positions. In this appendix it is demonstrated that the method will not give large cross sections if the scatterers have well-defined momenta. Consider again the S matrix.

$$S=\frac{1}{\hbar c}\int\langle F\,|\,\overline{\psi}_S\Gamma\psi_S\overline{\psi}_I\Xi\psi_I\,|\,O\rangle d^4x d^3x_c\,. \tag{A1}$$

For well-defined scatterer momenta, it is convenient to discuss the elastic scattering case in terms of the center of mass motion.

The following kinds of quantum states are chosen for the operators $\bar{\psi}_S$ and $\bar{\psi}_I$.

$$\bar{\psi}_S=\sum\sum \psi^*_{Sj}(\bar{\mathbf{r}})a_j^\dagger \psi^*_{SCn}(\bar{\mathbf{r}}_C)b_n^\dagger \; ,$$
$$\bar{\psi}_I=\sum_k \bar{U}_{Ik}e^{-i\bar{\mathbf{p}}_{IK}\cdot\bar{\mathbf{r}}/\hbar}d_k^\dagger/\sqrt{V} \; . \tag{A2}$$

In (A2), $\bar{\mathbf{r}}$ is the position three vector, $a_j^\dagger$ is a creation operator for the state with wave function ψ^*_{Sj}. $b_n^\dagger$ is a creation operator for the center of mass state with wave function $\psi^\dagger_{SCn}(\bar{\mathbf{r}}_C)$. $\bar{\mathbf{r}}_C$ is the center of mass coordinate vector. $d_k^\dagger$ is a creation operator for an incident particle with known momentum $\bar{\mathbf{p}}_{Ik}$. $\bar{U}_{Ik}$ may be a scalar, tensor, or spinor required to describe the incident particles.

The wave functions ψ^*_{SCn} for the center of mass are then $e^{-i\bar{\mathbf{p}}_C\cdot\bar{\mathbf{r}}_C/\hbar}$ and the three space part of the integral (A1) may be written as

$$S_3\rightarrow\bar{U}_{IF}\Xi U_{IO}\int \psi^*_{SjF}(\bar{\mathbf{r}})\Gamma\psi_{SIO}(\bar{\mathbf{r}})e^{(i/\hbar)[(\bar{\mathbf{p}}_{CO}-\bar{\mathbf{p}}_{CF})\cdot\bar{\mathbf{r}}_C+(\bar{\mathbf{p}}_{IO}-\bar{\mathbf{p}}_{IF})\cdot\bar{\mathbf{r}}]}d\bar{\mathbf{r}}\,d\bar{\mathbf{r}}_C \; . \tag{A3}$$

$\bar{\mathbf{p}}_{CO}$ and $\bar{\mathbf{p}}_{IO}$ are the original momenta of the center of mass and incident particle, respectively, and $\bar{\mathbf{p}}_{CF}$ and $\bar{\mathbf{p}}_{IF}$ are the final state values. Let $\bar{\mathbf{r}}'$ be the three vector from the center of mass to the three volume element $d\bar{r}$.

$$\bar{\mathbf{r}}_C=\bar{\mathbf{r}}-\bar{\mathbf{r}}' \; . \tag{A4}$$

Substituting (A4) into (A3) and carrying out the integration gives

$$S_3\rightarrow\bar{U}_{IF}\Xi U_{IO}\delta_3(\bar{\mathbf{p}}_{CO}-\bar{\mathbf{p}}_{CF}+\bar{\mathbf{p}}_{IO}-\bar{\mathbf{p}}_{IF})\int \psi^*_{SjF}(\bar{\mathbf{r}}')\Gamma\psi_{SIO}(\bar{\mathbf{r}}')e^{(i/\hbar)(\bar{\mathbf{p}}_{IO}-\bar{\mathbf{p}}_{IF})\cdot\bar{\mathbf{r}}'}d\bar{\mathbf{r}}' \; . \tag{A5}$$

The quantity $\Delta p_\mu=\bar{\mathbf{p}}_{CO}-\bar{\mathbf{p}}_{CF}$ will then disappear in the subsequent integrations.

For elastic scattering the "internal" state $\psi_{Sj}(\bar{\mathbf{r}}')$ is not changed by the scattering and $\psi^*_{SjF}(\bar{\mathbf{r}}')=\psi^*_{SjO}(\bar{\mathbf{r}}')$. In practice (A5) will give an extremely small total cross section, because the solid angle into which scattering may occur is limited as in (12).

APPENDIX B: SOME KINEMATICAL CONSIDERATIONS FOR ZERO REST MASS PARTICLES

Suppose a beam of zero rest mass particles is scattered by a large crystal with mass M, initially at rest. If momentum and energy are strictly conserved and the internal degrees of freedom of M are not excited, it may be shown that

$$\frac{1}{|p_{IF}|}-\frac{1}{|p_{IO}|}-\frac{1}{Mc}\{1-[1-(p_S^2/p_{IF}^2)(1-\cos^2\phi)]^{1/2}\}=0 \; , \tag{B1}$$

$$p_{IF}^2-p_{IO}^2-p_S^2+2\,|p_{IO}|\,|p_S|\cos\phi=0 \; . \tag{B2}$$

In (B1) and (B2), following earlier definitions, p_{IF} and p_{IO} refer to the final and original incident particle momenta, and p_S refers to the final momentum of the center of mass of M. ϕ is the angle which p_S makes with the incident particle momentum. For a given value of p_{IO} it is clear that p_{IF} and ϕ may have a wide range of values. An even wider range is possible in practice, since the interaction time is smaller than the length of M divided by c and the internal degrees of freedom of M may share the energy. (B1) implies, for elastic scattering, that $|p_{IF}|\approx|p_{IO}|$, and (B2) requires that either $p_S\approx 0$ or $p_S\approx 2\,|p_{IO}|\cos\phi$. ϕ can therefore vary over a wide range. It follows that there are no serious restrictions on the integration (14).

APPENDIX C: OTHER MOMENTUM EXCHANGE POSSIBILITIES

Most of the present paper treats the case where a single unidentified scatterer exchanges all of the momentum. Clearly other processes might occur in which any number of unidentifiable scatterers exchange all of the momentum. All possible kinds of exchange must contribute to the total cross section. Suppose that an unidentified number of scatterers, n_S, exchange total momentum Δp_μ, not necessarily in equal fractions, so that

$$\Delta p_x=\sum_{j=1}^{j=n_S}\frac{\Delta p_x}{n_{jx}} \; ,$$
$$\Delta p_y=\sum_{j=1}^{j=n_S}\frac{\Delta p_y}{n_{jy}} \; ,$$
$$\Delta p_z=\sum_{j=1}^{j=n_S}\frac{\Delta p_z}{n_{jz}} \; ,$$
$$\Delta E=\sum_{j=1}^{j=n_S}\frac{\Delta E}{n_{jO}} \; . \tag{C1}$$

Corresponding to (26A), we have

$$\bar{\psi}''_{Sn}=\sum \psi^*_{SOn}a^\dagger_{On}(e^{-i[(\Delta p_x x/n_{jx})+(\Delta p_y y/n_{jy})+(\Delta p_z z/n_{jz})+(\Delta Et/n_{jt})]/\hbar}-1)+\sum_N \psi^*_{SOi}a^\dagger_{Oi} \; . \tag{C2}$$

For construction of the S matrix each of the N particles must be summed over because the scatterers cannot be identified. Care is required to sum over each particle no more than once for a particular value of n_{jx}, n_{jy}, n_{jz}. For the quantity X of Eq. (29) this will give

$$X=\sum_{j=1}^{j=n_S}\left[\frac{(1-e^{-iN^{1/3}b[\bar{p}_{IO}-\bar{p}_{IF}-(\Delta\bar{p}/n_{jx})]_x/\hbar})}{(1-e^{-ib[\bar{p}_{IO}-\bar{p}_{IF}-(\Delta\bar{p}/n_{jx})]_x/\hbar})}\right]e^{-i[\bar{p}_{IO}-\bar{p}_{Ib}-(\Delta\bar{p}/n_{jx})]_x^2/2\hbar^2K^2}\ . \tag{C3}$$

$X^2Y^2Z^2T^2$ is required for the cross section. When squared, the cross product terms in (C3) are expected to sum to a small value. The momentum space integrals will then consist of sums $X_{njx}^2\,Y_{njy}^2\,Z_{njz}^2\,T_{njO}^2$, approximately.

The boundary conditions restrict the n_{jx},n_{jy},n_{jz} to rational numbers exceeding 1. Δp_x, Δp_y, Δp_z are summed over and have values determined by boundary conditions. Consideration of these requirements indicates that all possible momentum transfer combinations will be included if the n_{jx}, n_{jy}, n_{jz} have integral values from 1 to $(pN^{1/3}b/2\pi\hbar)$.

A given set of n_{jx}, n_{jy}, n_{jz}, will lead to a solid angle given by:

$$d\Omega=\left[\frac{(\Delta p_x^2)}{n_{jx}^2}+\frac{(\Delta p_y^2)}{n_{jy}^2}+\frac{(\Delta p_z^2)}{n_{jz}^2}\right]\Big/p_{IO}^2\ . \tag{C4}$$

For infinitely stiff material, assuming no restrictions on $\Delta\bar{p}$ and all directions equally likely,

$$\langle(\Delta p_x)^2\rangle=\langle(\Delta p_y)^2\rangle=\langle(\Delta p_z)^2\rangle=p_{IO}^2/3\ . \tag{C5}$$

A total cross section Eq. (35), obtained from (C5) and a summation of (C4) over all positive and negative n_j is then

$$\sigma\to\sigma_1[1+2(\tfrac{1}{4}+\tfrac{1}{9}+\tfrac{1}{16}+\cdots)]\ . \tag{C6}$$

In (C6), σ_1 is the cross section for the process in which all momentum is exchanged by a single unidentified scatterer. The series

$$1+\tfrac{1}{4}+\tfrac{1}{9}+\tfrac{1}{16}+\cdots=\frac{\pi^2}{6}\ , \tag{C7}$$

Eqs. (C6) and (C7) then give

$$\sigma\to\sigma_1\left[\frac{\pi^2}{3}-1\right]\ . \tag{C8}$$

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