Coulomb effects and charge symmetry breaking for the $A=4$ hypernuclei

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The effect ΔB_c of the Coulomb interaction on the Λ separation energy B_Λ of ${}^4_\Lambda$He was obtained by variational calculations made for ${}^4_\Lambda$He and ^{3}He. These calculations were made for several values of q^2 in the range $0 \le q^2 \le 9$ where qe is the proton charge, i.e., the Coulomb repulsion was artificially boosted. For $q^2 \lesssim 3$, the dependence on q^2 is linear, and interpolation to $q^2=1$ gives the physical values with improved accuracy: $-\Delta B_c = 0.05 \pm 0.02$ and 0.025 ± 0.015 MeV for the ground and excited state, respectively. This procedure also gives more accurate values for the differences between the proton and neutron radii of ^{3}He. The corresponding differences of B_Λ between ${}^4_\Lambda$He and ${}^4_\Lambda$H, to be attributed to charge symmetry breaking effects, are then 0.40 ± 0.06 and 0.27 ± 0.06 MeV. From these values we obtain a phenomenological charge symmetry breaking potential which is effectively spin independent. An examination of meson-exchange charge symmetry breaking models shows that these are consistent with the phenomenological charge symmetry breaking potential for the triplet but not for the singlet case.

I. INTRODUCTION

The mirror pair of hypernuclei ${}^4_\Lambda$H, ${}^4_\Lambda$He is the main source of information about the charge symmetry breaking (CSB) ΛN interaction (for a review see Ref. 1). Thus, the Λ separation energies[2,3] for the ground (0^+) state are

$$B_\Lambda({}^4_\Lambda\text{He}) = 2.39 \pm 0.03 \text{ MeV}, \quad B_\Lambda({}^4_\Lambda\text{H}) = 2.04 \pm 0.04 \text{ MeV}, \tag{1}$$

and for the excited (1^+) state are

$$B^*_\Lambda({}^4_\Lambda\text{He}) = 1.24 \pm 0.06 \text{ MeV}, \quad B^*_\Lambda({}^4_\Lambda\text{H}) = 1.00 \pm 0.06 \text{ MeV}. \tag{2}$$

The differences of the Λ separation energies

$$\Delta B^{\text{exp}}_\Lambda = B_\Lambda({}^4_\Lambda\text{He}) - B_\Lambda({}^4_\Lambda\text{H}) \tag{3}$$

are then

$$\Delta B^{\text{exp}}_\Lambda = 0.35 \pm 0.06 \text{ MeV}, \quad \Delta B^{*\text{exp}}_\Lambda = 0.24 \pm 0.06 \text{ MeV}. \tag{4}$$

The average values of B_Λ for ${}^4_\Lambda$H, ${}^4_\Lambda$He, to be identified with the charge symmetric (CS) values, are

$$B_\Lambda = 2.22 \pm 0.04 \text{ MeV}, \quad B^*_\Lambda = 1.12 \pm 0.06 \text{ MeV}. \tag{5}$$

However, the experimental values $\Delta B^{\text{exp}}_\Lambda$ must be corrected to include the difference ΔB_c due to the Coulomb interaction in order to obtain the values to be attributed to CSB effects. ΔB_c must be obtained by calculation,

$$\Delta B_c = B_\Lambda({}^4_\Lambda\text{He}) - B_\Lambda({}^4_\Lambda\text{H}), \tag{6}$$

$$B_\Lambda({}^4_\Lambda\text{He}) = -[E({}^4_\Lambda\text{He}) - E({}^3\text{He})], \tag{7}$$

$$B_\Lambda({}^4_\Lambda\text{H}) = -[E({}^4_\Lambda\text{H}) - E({}^3\text{H})], \tag{8}$$

with the energies E calculated using charge symmetric ΛN interactions.

To leading order in the Coulomb interaction

$$\Delta B_c = -\Delta E_c, \tag{9}$$

where

$$\Delta E_c = E_c({}^4_\Lambda\text{He}) - E_c({}^3\text{He}) \tag{10}$$

is the difference between the Coulomb energies of ${}^4_\Lambda$He and ^{3}He, i.e., ΔE_c is the change (increase) in Coulomb energy of the ^{3}He core due to the presence of the Λ which compresses the core. Thus ΔB_c is expected to be negative, and the value to be attributed to CSB effects is

$$\Delta B^{\text{CSB}}_\Lambda \equiv \Delta B_\Lambda = \Delta B^{\text{exp}}_\Lambda - \Delta B_c. \tag{11}$$

Existing estimates of ΔB_c for the ground state are very different: Dalitz *et al.*[4] obtain $|\Delta B_c| \approx 0.2$ MeV using variational Monte Carlo (MC) calculations with hard core ΛN and NN interactions, whereas Friar and Gibson[5] estimate only $\simeq 0.02$ MeV on the basis of the Coulomb energies of the $A=3$ and 4 nuclei. The smaller value is consistent with an earlier Hartree-Fock[6] calculation which used a ΛN interaction with a soft short-range repulsion. In the original paper on CSB effects, Dalitz and Von Hippel[7] obtained a value of ≈ 0.08 MeV from an estimate of the compression of the ^{3}He core by the Λ. This was based

on purely attractive ΛN and NN potentials, which would give a too easily compressible core and would thus overestimate $|\Delta B_c|$.

We have made variational MC calculations of the same general type as those of Dalitz *et al.*[4] for the $A=3$ nuclei, and for both the ground and excited states of the $A=4$ hypernuclei. For our ΛN potential we use an Urbana-type 2π-exchange potential[8] with a strongly repulsive core, and for our NN potential we use a central spin-isospin independent potential with a repulsive core (Mafliet-Tjon[9]). The latter gives an energy and radius of ^{3}He in reasonable agreement with experiment,[10] as will also be discussed in the following, so that the results for ΔB_c can be expected to be meaningful. We remark that the criticisms of Gibson and Lehman[11] about the adequacy of certain hypernuclear (and also nuclear) binding-energy calculations apply to the use of one-body wave functions such as shell-model wave functions or Hartree or Λ-nucleus wave functions using an effective Λ-nucleus potential, and which do not include two-body correlations, which are needed to, e.g., allow for range effects of the two-body forces. These criticisms do not apply to the correlated wave functions we use—as the excellent agreement between the variational and the exact Green function MC results for ^{3}He, discussed in the following, demonstrate.

A new feature of our calculations is that we boost the Coulomb effects by varying the square of the proton charge q^2e^2 from $q^2=0$ to 9. In particular, we thus obtain the energies $E_3=E(^3\text{He})$ and $E_4=E(^4_\Lambda\text{He})$, and hence B_Λ and ΔB_c, as functions of q^2. Thus

$$B_\Lambda(q^2)=-[E_4(q^2)-E_3(q^2)]\ , \tag{12}$$

$$\Delta B_c(q^2)=B_\Lambda(q^2)-B_\Lambda(q^2=0)\ , \tag{13}$$

and the desired value of ΔB_c is then $\Delta B_c(q^2=1)$. To lowest order in the Coulomb interaction all quantities and in particular the energies are proportional to q^2. Our approach thus provides a test of the accuracy and limits of lowest-order perturbation theory by testing the linearity of the results with q^2. In particular, our approach provides a more accurate value of ΔB_c, and also of other quantities such as the difference of radii between ^{3}He and ^{3}H, by amplifying the effect of the Coulomb interaction from the range $0\le q^2\le 1$ to the range of $q^2(\lesssim 3)$ for which the dependence on q^2 is linear. This is an important aspect of our calculations, because the statistical MC errors are appreciable, especially for $\Delta B_c(q^2=1)$, and by boosting $\Delta B_c(q^2)$ to values with $q^2>1$ and then interpolating for $q^2=1$ the effect of statistical errors is very much reduced. Even with this amplification it is necessary to obtain $E_3(q^2)$ and $E_4(q^2)$ to an accuracy of 0.02 MeV or better. In practice, because of limitations of computing time, this implies the use of central NN and ΛN potentials. In addition to ΔB_c we also obtain (for the Mafliet-Tjon potential) a more thoroughly optimized variational value of $E(^3\text{H})$, a value for Coulomb energy of ^{3}He, values of the differences in radii between ^{3}He and ^{3}H and between the nucleon, neutron, and proton radii of ^{3}He.

II. POTENTIALS

We use central NN and ΛN potentials for the reasons given in the preceding. Inclusion of noncentral and three-body forces is not expected to significantly affect ΔB_c since this is closely related to the Coulomb energy difference which, because of the long range of the Coulomb interaction, is expected to be insensitive to the short range and relatively weak noncentral and three-body correlations.

For the NN potential we use the local central spin-isospin independent Mafliet-Tjon (MT) potential (V) (Ref. 9)

$$V_{\text{NN}}=[7.39\exp(-3.11r)-2.93\exp(-1.55r)]\frac{\hbar c}{r}\ . \tag{14}$$

This gives a reasonable energy and radius for ^{3}He as discussed in the following. For the pp interaction we have in addition the Coulomb potential

$$V_{\text{pp}}=V_{\text{NN}}+V_c\ , \tag{15}$$

$$V_c=\frac{q^2e^2}{r}F(r)\quad\text{with } q^2=0,1,3,5,7,9\ , \tag{16}$$

where

$$F(r)=1(\text{NF}=0)\ ,\quad F(r)=\text{erf}(0.984r)(\text{NF}=1)\ . \tag{17}$$

NF=0 corresponds to point-charge protons and NF=1 to a Gaussian proton charge distribution of rms radius 0.8 fm.

For the ΛN (s-state) potential we use an Urbana-type 2π-exchange potential[8] of the same form as used in Ref. 12

$$V_{\Lambda\text{N}}=V_C-V_4T_\pi^2\ . \tag{18}$$

V_C is a Woods-Saxon repulsive core

$$V_C=W_C\left[1+\exp\left[\frac{r-R}{d}\right]\right]^{-1}\ , \tag{19}$$

with $W_C=2137$ MeV, $R=0.5$ fm, $d=0.2$ fm. These parameters are very close to those for the spin, isospin independent core of the NN potential of Ref. 8 and are the same as those used in Ref. 12. T_π is the one-pion exchange (OPE) tensor potential shape modified with a cutoff,

$$T_\pi(r)=\left[1+\frac{3}{x}+\frac{3}{x^2}\right]\frac{e^{-x}}{x}(1-e^{-cr^2})^2\ , \tag{20}$$

with $x=0.7r$, $c=2$ fm^{-2}. $V_{\Lambda\text{N}}$ corresponds to a two-pion exchange (TPE) mechanism due to OPE transition potentials $V_\pi(\Lambda\text{N}\leftrightarrow\Sigma\text{N},\Sigma\Delta)$ dominated by their tensor components. The intrinsic range of $V_{\Lambda\text{N}}$ is 2.0 fm, very close to that of the hard core potential of Ref. 4. (The intrinsic range is the effective range when V_4 just gives a bound state.) The strength V_4 is the spin average strength appropriate for $^4_\Lambda$H, $^4_\Lambda$He. In terms of the singlet and triplet strengths V_s and V_t,

$$V_4=\tfrac{1}{2}(V_s+V_t)\ ,\quad V_4^*=\tfrac{1}{6}(V_s+5V_t)\ . \tag{21}$$

We have made calculations for the values $V_4=6.2$ and

TABLE I. Variational results for ^{3}He. The variational parameters (also for Tables II and III) for NF=1 are the same as those for NF=0 for the same q^2. The values of c and a are the same for f_{np} and f_{pp} for all q^2. The radii are the same for NF=0 and NF=1. $\langle V \rangle$ is the expectation value of the potential energy and $\langle T \rangle$ that of the kinetic energy.

		Variational parameters						Energies (MeV)					Radii (fm)		
q^2	NF	κ_{np} (fm^{-1})	κ_{pp} (fm^{-1})	c (fm)	a (fm)	R_{np} (fm)	R_{pp} (fm)	$-E$		E_c	$-\langle V \rangle$	$\langle T \rangle$	$\langle r_N^2 \rangle^{1/2}$	$\langle r_n^2 \rangle^{1/2}$	$\langle r_p^2 \rangle^{1/2}$
0		0.28		2.5	0.6	1.3		8.25	±0.01	0	39.25	31.00	1.662	1.665	1.665
	0	0.27		2.5	0.6	1.3		7.52		0.723	38.20	29.96			
1									±0.01				1.702	1.695	2.623
	1							7.58		0.668	38.24	29.99			
	0	0.22	0.18	3.5	2.8	4.5		6.00		2.169	38.93	30.76			
3									±0.015				1.756	1.721	1.774
	1							6.17		1.999	38.89	30.72			
	0	0.21	0.13	4.5	2.5	5.5	3.5	4.61		3.431	36.05	28.00			
5									±0.017				1.897	1.814	1.937
	1							4.87		3.174	36.05	28.01			
	0	0.20	0.11	4.7	1.3	8.5	4.5	3.21		4.685	35.50	27.60			
7									±0.016				1.969	1.843	2.030
	1							3.56		4.337	35.48	27.58			
	0	0.19	0.01	5.0	1.0	5.0	3.0	1.98		5.484	31.15	23.69			
9									±0.015				2.224	2.033	2.313
	1							2.34		5.123	31.15	23.69			

6.05 MeV which encompass the ground and excited state energies. For $V_4=6.2$ MeV and $V_s=V_t$ the scattering length is $a=-1.87$ fm, and the effective range is $r_0=3.45$ fm. These values give a reasonable fit to the low energy Λp scattering data, in agreement with the well-known result that with no ΛN spin dependence analysis of the ground state of ${}^4_\Lambda$H, ${}^4_\Lambda$He gives agreement with ΛN scattering. For further discussion of analyses of Λp scattering and of the s-shell hypernuclei and of the associated scattering parameters, see Ref. 13 and Sec. VI.

III. TRIAL WAVE FUNCTIONS AND VARIATIONAL CALCULATIONS

We use trial functions of the standard product form for the spatial part of the three- and four-body wave functions, appropriate to symmetric $L=0$ states.

A. ^{3}He

We use

$$\Psi_3=f_{\rm np}(r_{12})f_{\rm np}(r_{13})f_{\rm pp}(r_{23})^2\chi_1 \ . \tag{22}$$

The spin function enters only through the choice of the appropriate NN potential, namely the MT potential of Eq. (14). For the calculation of the correlation functions we use the procedures of Refs. 10 and 14. For $f_{\rm np}=f_{\rm pp}$ our Ψ_3 is identical with that of Ref. 10. For $q^2<5$ this is an excellent approximation (Table I), but for $q^2\gtrsim 5$, i.e., for large Coulomb repulsions, a marginally lower energy is obtained with $f_{\rm pp}\neq f_{\rm np}$, in particular with $\kappa_{\rm pp}<\kappa_{\rm np}$, corresponding to $f_{\rm pp}$ being more extended than $f_{\rm np}$.

The correlation functions are determined by a Schrödinger-type equation. For $f_{\rm np}$ this is

$$\left[-\frac{\hbar^2}{2M({\rm n,p})}\nabla^2+V_{\rm np}+\Lambda_{\rm np}\right]f_{\rm np}=0 \ , \tag{23}$$

where $V_{\rm np}=V_{\rm NN}$, $M(i,j)$ is the reduced mass for the baryons i and j, and

$$\Lambda_{\rm np}=\frac{\hbar^2}{2M({\rm n,p})}A_{\rm np}(1-e^{-r^2/c_{\rm np}^2}) + \Lambda_{\rm np}\left[1+\exp\left[\frac{r-R_{\rm np}}{a_{\rm np}}\right]\right]^{-1} , \tag{24}$$

$$A_{\rm np}=\kappa_{\rm np}^2+\frac{2\kappa_{\rm np}}{r}(\nu_{\rm np}-1) + \frac{\nu_{\rm np}(\nu_{\rm np}-1)}{r^2} \ , \quad \text{with } \nu_{\rm np}=\tfrac{1}{2} \ . \tag{25}$$

The equation for $f_{\rm pp}$ is obtained by replacing all quantities indexed by np with the corresponding quantities indexed by pp and also with $\nu_{\rm pp}=\frac{1}{2}$. The form of the potential $\Lambda_{\rm np}$, $\Lambda_{\rm pp}$ through which the variational parameters enter is such that $f_{\rm np}$, $f_{\rm pp}$ have the form of asymptotic behavior required by the full three-body Schrödinger equation:

$$f_{\rm np}\sim r^{-\nu_{\rm np}}\exp(-\kappa_{\rm np}r) \ , \qquad f_{\rm pp}\sim r^{-\nu_{\rm pp}}\exp(-\kappa_{\rm pp}r) \ , \tag{26}$$

with $\kappa_{\rm n}=2\kappa_{\rm np}$, $\kappa_{\rm p}=\kappa_{\rm np}+\kappa_{\rm pp}$ related in the usual way to the n and p separation energies. (The latter are not constrained to the experimental values which may, however, provide a convenient starting point in the search for the optimum parameters $\kappa_{\rm np},\kappa_{\rm pp}$.) Ψ_3 thus depends on eight variational parameters: $\kappa_{\rm np}$, $c_{\rm np}$, $R_{\rm np}$, $a_{\rm np}$, $\kappa_{\rm pp}$, $c_{\rm pp}$, $R_{\rm pp}$, $a_{\rm pp}$.

B. $^4_\Lambda$He

The trial function is

$$\Psi_4=\left[f_{\rm np}(r_{12})f_{\rm np}(r_{13})f_{\rm pp}(r_{23})\prod_{i=1}^{3}f_{\Lambda\rm N}(r_{i4})\right]\chi \ . \tag{27}$$

χ is the spin function for the ground or excited state as appropriate; χ is taken into account through the appropriate numerical value of V_4 and need not be considered explicitly. The functions $f_{\rm np}$, $f_{\rm pp}$, $f_{\Lambda\rm N}$ are determined by a Schrödinger equation of the form of Eq. (23) but with

$$\nu_{\rm np}=\nu_{\rm pp}=\tfrac{1}{3} \text{ and } \nu_{\Lambda\rm N}=\tfrac{1}{4} \tag{28}$$

in Eqs. (25) and (26). The parameters are now those appropriate for ${}^4_\Lambda$He, and Ψ_4 depends on a total of 12 variational parameters. However, the energy is quite insensitive to many of these and there are different parameter sets which give almost equally low energies (e.g., sets for which $f_{\rm np}=f_{\rm pp}$ and $f_{\rm np}\neq f_{\rm pp}$).

The integrations needed to obtain $E=\langle\Psi\,|\,H\,|\,\Psi\rangle/\langle\Psi\,|\,\Psi\rangle$ and other expectation values were made with MC procedures. The optimizations were made for point charge protons (NF=0). However, we have made checks which indicate that for extended proton charges (NF=1) the point charge optimization is not changed within the statistical errors.

For a given set of variational parameters the statistical errors for $E(^3{\rm He})$ are $\simeq$0.01 MeV for $q^2=0,1$ and $\simeq$0.015 MeV for $q^2=3$–9 (Table I). For $E({}^4_\Lambda{\rm He})$ the errors are $\approx$0.02 MeV for all q^2 (Tables II and III). The accuracy of the optimization is comparable to the statistical errors. Thus the total error for $E(^3{\rm He})$ is $\approx$0.015 MeV for $q^2=0,1$ and $\approx$0.02 MeV for $q^2=3$–9, and for $E({}^4_\Lambda{\rm He})$ the total error is $\approx$0.03 MeV. Since B_Λ is the difference of the two variationally determined energies, its error is $\simeq$0.04 MeV.

IV. RESULTS FOR ^{3}H AND ^{3}He

Results are given in Table I and also displayed in Figs. 1 and 2. The Coulomb energy

$$E_c=\langle\Psi_3\,|\,V_c\,|\,\Psi_3\rangle/\langle\Psi_3\,|\,\Psi_3\rangle$$

is obtained for both NF=0 and 1 [Eq. (17)]. The point nucleon, neutron, and proton radii $\langle r_{\rm N}^2\rangle^{1/2}$, $\langle r_{\rm n}^2\rangle^{1/2}$, $\langle r_{\rm p}^2\rangle^{1/2}$, were also calculated. These radii satisfy

$$3\langle r_{\rm N}^2\rangle=\langle r_{\rm n}^2\rangle+2\langle r_{\rm p}^2\rangle \ . \tag{29}$$

Since $\langle r_{\rm N}^2\rangle$ is obtained as an average over all three nucleons, $\langle r_{\rm p}^2\rangle$ over the two protons and $\langle r_{\rm n}^2\rangle$ over the single neutron, the statistical errors of $\langle r_{\rm n}^2\rangle$ are reduced by obtaining it from $\langle r_{\rm N}^2\rangle,\langle r_{\rm p}^2\rangle$ using this relation. This is

TABLE II. Variational results for ${}^{4}_{\Lambda}$He for $V_4=6.2$ MeV. The values of c_N, a_N, and R_N are the same for f_{np} and f_{pp} for all q^2; $c_\Lambda=2$ fm, $a_\Lambda=R_\Lambda=1$ fm for all q^2. The radii are the same for NF=0 and NF=1. The values of B_Λ of Tables II and III correspond to the central values of $E({}^{4}_{\Lambda}\text{He})$ and the central values of $E({}^{3}\text{He})$ of Table I.

		Variational parameters						Energies (MeV)					Radii (fm)	
q^2	NF	κ_{np} (fm^{-1})	κ_{pp} (fm^{-1})	c_N (fm)	a_N (fm)	R_N (fm)	κ_Λ (fm)	$-E$	B_Λ	E_c	$-\langle V\rangle$	$\langle T\rangle$	$\langle r_N^2\rangle^{1/2}$	$\langle r_\Lambda^2\rangle^{1/2}$
0		0.31		2	0.6	1.3	0.12	10.65±0.018	2.40	0	55.08	44.42	1.54	3.56
1	0	0.30		2	0.6	1.5	0.125	9.89 ±0.02	2.37	0.777	55.71	45.05	1.54	3.43
	1							9.90	2.32	0.712	55.36	45.47		
3	0	0.2		4	2	4	0.115	8.25 ±0.022	2.25	2.243	52.99	42.50	1.66	3.71
	1							8.35	2.18	2.070	53.34	42.91		
5	0	0.185		4.5	2.5	5.5	0.125	6.84 ±0.022	2.23	3.650	53.85	43.36	1.68	3.54
	1							7.01	2.14	3.397	53.80	43.39		
7	0	0.19	0.15	4.7	2.1	6.5	0.125	5.39 ±0.022	2.18	4.913	51.43	41.13	1.75	3.53
	1							5.72	2.16	4.601	52.10	41.78		
9	0	0.18	0.13	5.0	1.0	7.0	0.125	4.00 ±0.02	2.02	5.993	49.32	39.32	1.82	3.50
	1							4.44	2.10	5.68	49.50	39.38		

TABLE III. Variational results for $^{4}_{\Lambda}$He for $V_4=6.05$ MeV. The values of c_N, a_N, and R_N are the same for f_{np} and f_{pp} for all q^2; $c_\Lambda=2$ fm, $a_\Lambda=R_\Lambda=1$ fm for all q^2. The radii are the same for NF=0 and NF=1.

		Variational parameters						Energies (MeV)			Radii (fm)			
q^2	NF	κ_{np} (fm^{-1})	κ_{pp} (fm^{-1})	c_N (fm)	a_N (fm)	R_N (fm)	κ_Λ (fm)	$-E$	B_Λ	E_c	$-\langle V\rangle$	$\langle T\rangle$	$\langle r_N^2\rangle^{1/2}$	$\langle r_\Lambda^2\rangle^{1/2}$
0	0	0.31		2	0.6	1	0.1	9.26±0.016	1.01	0	49.16	39.90	1.59	4.19
	0	0.3		2	0.6	1.5	0.09	8.53	1.01	0.756	48.63	39.35		
1								±0.017					1.60	4.37
	1							8.54	0.96	0.697	49.08	39.84		
	0	0.2		4	2	4	0.09	6.94	0.94	2.183	47.54	38.43		
3								±0.019					1.73	4.47
	1							7.13	0.96	2.022	47.07	37.91		
	0	0.195	0.175	4.5	2.5	5.5	0.08	5.54	0.93	3.460	44.21	35.21		
5								±0.018					1.81	5.07
	1							5.75	0.88	3.259	45.13	36.12		
	0	0.19	0.15	4.7	2.1	6.5	0.08	4.16	0.95	4.629	42.66	33.87		
7								±0.018					1.89	4.95
	1							4.49	0.93	4.398	43.66	34.77		
	0	0.18	0.13	5	1	7	0.085	2.83	0.85	5.657	40.40	31.92		
9								±0.018					1.98	4.92
	1							3.19	0.85	5.421	41.21	32.59		

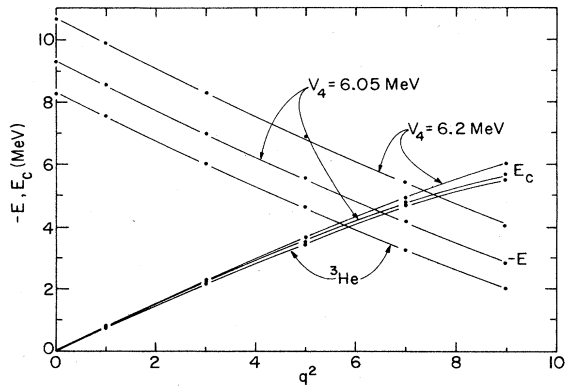

FIG. 1. The total energy E and the Coulomb energy E_c for ^{3}He and for $^4_\Lambda$He (for $V_4=6.2$ and 6.05 MeV) vs q^2 (q^2=the square of the proton charge/e^2).

the value in Table I which is consistent with that obtained by direct calculation. For E_c and the rms radii, the statistical errors (percentwise similar to those of the total energy) are considerably smaller than uncertainties resulting from the optimization since these quantities, in contrast to the total energy, are not stationary.

For $q^2<5$ the equality $f_{np}=f_{pp}$ is an excellent approximation, but for large Coulomb repulsions $q^2\geq 5$ a slightly lower energy is obtained for $f_{np}\neq f_{pp}$, although even for $q^2=9$ the gain obtained (≈ 0.1 MeV) by relaxing the equality $f_{np}=f_{pp}$ is slight.

A. Energy of ^{3}H

We obtain $E(^3\mathrm{H})=-8.25\pm0.015$ MeV for the appropriate ($q^2=0$) variational parameters given in Table I. This value is in good agreement with that of Ref. 10: -8.22 ± 0.02 MeV. Our slightly lower value could be because of improved optimization; however, the difference is hardly significant in view of the errors. The agreement with the Green function MC value of -8.26 ± 0.01 MeV (Ref. 15) is excellent. This is an "exact" value obtained by solution of the three-body Schrödinger equation. Similar excellent agreement is then also expected for ^{3}He for $q^2>0$ and for $^4_\Lambda$He for all q^2 since in both these cases central forces of similar short-range form as for ^{3}H are used.

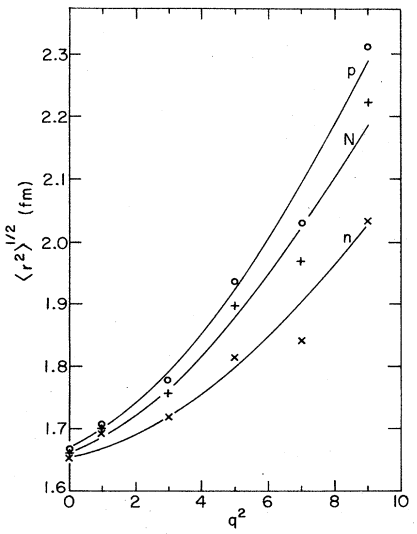

FIG. 2. The rms radii of the neutron (n), proton (p), and nucleon (N) distributions in ^{3}He vs q^2.

B. ^{3}He

The results, especially for E and E_c, but also for $\langle r^2\rangle^{1/2}$, show that there are no significant deviations from linearity with q^2 for $q^2\lesssim 3$. For such values of q^2 first-order perturbation theory in V_c, which predicts linearity with q^2, is then a good approximation and Coulomb distortion of the wave function is quite small.

Consistent with the accuracy of first-order perturbation theory, the relation

$$E(q^2=0)-E(q^2)=E_c(q^2)\ ,\quad \text{with } E_c(q^2)\equiv\langle V_c\rangle\ , \tag{30}$$

is satisfied for $q^2\lesssim 3$ (Table I) to within the statistical errors. In particular we have

$$\begin{aligned} E_c(^3\mathrm{He})&=E(^3\mathrm{H})-E(^3\mathrm{He})\\ &=0.73\pm0.01 \text{ MeV} \quad \text{for point charge protons}\\ &=0.67\pm0.01 \text{ MeV} \quad \text{for extended proton charges.}\end{aligned} \tag{31}$$

The variational calculations of Ref. 16 which use realistic two-body plus three-body forces give 0.74 and 0.69 MeV for point and extended charge protons, respectively, quite close to our values. The comparable (isoscalar) value (neglect of mixed symmetry effects and of the neutron charge distribution) for extended proton charges estimated from the experimental form factors is 0.67 MeV,[5,17] and the experimental binding energy difference between ^{3}H and ^{3}He is 0.76 MeV.

For $q^2\gtrsim 3$, and especially for $q^2\gtrsim 7$, deviations from linearity with q^2 become significant. The deviations are in the direction expected from higher-order contributions: E and E_c are less than the first-order result (dashed lines in Fig. 1), the radii (Fig. 2) increase with q^2, and $\langle r_p^2\rangle^{1/2}$ increases more rapidly than $\langle r_n^2\rangle^{1/2}$.

C. Radii of ^{3}H, ^{3}He

Our use of a central spin-isospin independent (MT) NN potential (small statistical MC errors) together with calculations for $q^2>1$ (amplification of Coulomb effects plus interpolation for $q^2=1$) allows an accurate determination of the radii of ^{3}He and ^{3}H and, in particular of the difference in radii between ^{3}He and ^{3}H, and also of the differences between the nucleon, neutron, and proton radii of ^{3}He. This is shown by Fig. 2. Thus we obtain

$$\langle r_N^2\rangle_{He}^{1/2}-\langle r_N^2\rangle_H^{1/2}=0.03\pm0.005 \text{ fm}\ , \tag{32}$$

and for ^{3}He

$$\langle r_p^2\rangle^{1/2}-\langle r_n^2\rangle^{1/2}\approx 0.02 \text{ fm} ,$$
$$\langle r_p^2\rangle^{1/2}-\langle r_N^2\rangle^{1/2}\approx 0.005 \text{ fm} , \quad (33)$$
$$\langle r_N^2\rangle^{1/2}-\langle r_n^2\rangle^{1/2}\simeq 0.015 \text{ fm} .$$

These results are mainly of interest in showing that a calculation of radii as a function of q^2 up to $q^2\approx 3$ could provide more accurate values, especially of small differences of radii, also for more realistic interactions.

For the point nucleon radii themselves, we then have $\langle r_N^2\rangle^{1/2}=1.66$ fm for ^{3}H and 1.69 fm for ^{3}He. The corresponding charge radii are 1.81 fm and 1.88 fm [using Eqs. (5.1) of Ref. 16], and the isoscalar charge radius is 1.86 fm. The experimental charge radii, which include mixed symmetry effects, are 1.70 ± 0.05 fm and 1.84 ± 0.05 fm, and the corresponding isoscalar charge radius is 1.79 ± 0.05 fm.

The rather good agreement obtained with the MT potential between the calculated and experimental values of the total and Coulomb energies and of the charge radii, implies that the local MT potential can also be expected to give a reasonable description of the $A=3$ nucleon core for the $A=4$ hypernuclei. In particular, Coulomb effects for the $A=4$ hypernuclei should be quite realistically obtainable by use of the MT potential. This satisfactory agreement for the MT potential could also imply that for the small differences of neutron and proton radii the deficiencies of this potential may be more than compensated for by the increased accuracy with which these differences have been obtained.

V. RESULTS for $^4_\Lambda$H and $^4_\Lambda$He

Results are shown in Tables II and III for $V_4=6.2$ and 6.05 MeV, respectively. There is a slight lowering of the energy for $q^2\gtrsim 5$ with $f_{np}\neq f_{pp}$, but this is barely outside the statistical MC errors. For $f_{\Lambda N}$ no significant improvement for any q^2 was found for values of c_Λ, a_Λ, and R_Λ different from those for $q^2=0$ ($c_\Lambda=2$ fm, $a_\Lambda=R_\Lambda=1$ fm). However, some slight differences were found in the optimum values of κ_Λ.

The radii $\langle r_N^2\rangle^{1/2}$, $\langle r_\Lambda^2\rangle^{1/2}$ are the point nucleon and Λ rms radii with respect to the c.m. of the nucleons. Since $\langle r_N^2\rangle^{1/2}$ is a measure of the radius of the ^{3}He core it is directly comparable with $\langle r_N^2\rangle^{1/2}$ for ^{3}He.

Our results for E and E_c are, as for ^{3}He, linear with q^2 for $q^2\lesssim 3$, implying the corresponding validity of first-order perturbation theory. For larger values of q^2 the deviations from linearity are again in the expected direction (more negative E and E_c, and larger radii) and increase with q^2. For $^4_\Lambda$H the values of $B_\Lambda(q^2=0)$ vs V_4 are shown in Table IV. We have also included a lower statistics result for $V_4=6.30$ MeV. These results are reproduced by

$$B_\Lambda=2.40+10.91(V_4-6.2)+10.93(V_4-6.2)^2 . \quad (34)$$

The values of $B_\Lambda(q^2)$, obtained using Eq. (12) with the appropriate values of $E_3(q^2)$ from Table I and $E_4(q^2)$ from Tables II and III, are shown vs q^2 in Figs. 3 and 4

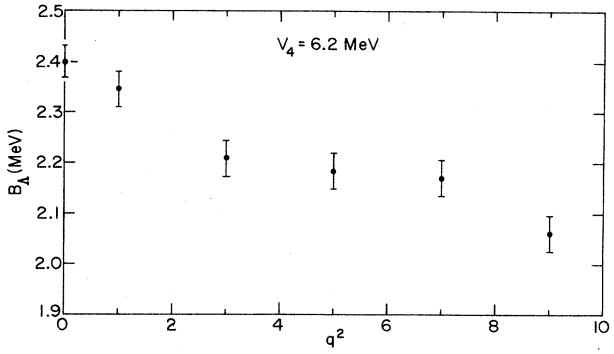

FIG. 3. B_Λ vs q^2 for $^4_\Lambda$He with $V_4=6.2$ MeV.

for $V_4=6.2$ and 6.05 MeV, respectively. (The value of B_Λ shown is the average for NF=0 and 1.) Because of the relatively large errors (± 0.04 MeV) one cannot draw unambiguous conclusions about the deviations from linearity of B_Λ with q^2 for larger q^2. Nevertheless, the dependence of B_Λ on q^2 for both $V_4=6.2$ and 6.05 MeV is quite consistent with the q^2 dependence of E_3 and E_4 separately, namely with a linear dependence for $q^2\lesssim 3$ and with increasing deviations from linearity for larger q^2.

We have, therefore, given most weight to the results for $q^2=0$, 1, and 3, and assuming linearity for $q^2\lesssim 3$, have obtained ΔB_c, using Eq. (13) and interpolating for $q^2=1$. This gives

$$-\Delta B_c=0.05\pm 0.02 \text{ MeV } (V_4=6.2 \text{ MeV})$$
$$=0.02\pm 0.015 \text{ MeV } (V_4=6.05 \text{ MeV}) . \quad (35)$$

Figures 3 and 4 show that the errors are considerably smaller than those obtained using only the results for $q^2=0$ and 1.

According to first-order perturbation theory in V_c the preceding values of ΔB_c are equal to the corresponding Coulomb-energy difference ΔE_c [Eq. (10)]. Indeed, the Coulomb energies of $^4_\Lambda$He are seen to be larger than those of ^{3}He and the core radii $\langle r_N^2\rangle^{1/2}$ are smaller than the radii $\langle r_N^2\rangle^{1/2}$ of ^{3}He, consistent with the expected compression of the core induced by the Λ. The differences of the directly calculated Coulomb energies [Eq. (10)], $\Delta E_c(q^2=1)\approx 0.05$ MeV and ≈ 0.03 MeV for $V_4=6.2$ and 6.05 MeV, respectively, are in agreement with the corresponding values of ΔB_c. However, the uncertainties in ΔE_c are in general considerably larger than those in ΔB_c

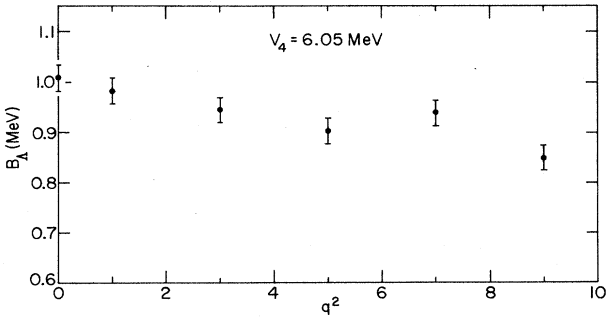

FIG 4. B_Λ vs q^2 for $^4_\Lambda$He with $V_4=6.05$ MeV.

TABLE IV. Results for ${}^4_\Lambda$H.

V_4 (MeV)	6.05	6.20	6.30
$-E$ (MeV)	9.26±0.016	10.65±0.018	11.85±0.07
B_Λ (MeV)	1.01±0.025	2.40±0.025	3.62±0.075
$\langle r_\Lambda^2\rangle^{1/2}$ (fm)	4.19	3.56	3.39
$\langle r_N^2\rangle^{1/2}$ (fm)	1.59	1.54	1.52

since the former are not obtained from the total energies which are optimized.

The decrease in the radius of the $A=3$ core induced by the Λ scales with the increase in Coulomb energy and is thus consistent with this. Thus, for He($q^2=1$) the decrease in radius due to the Λ is ≈ 0.16 fm for $V_4=6.2$ MeV and 0.10 fm for $V_4=6.05$ MeV, and the corresponding fractional decreases in radii, ≈ 0.09 and 0.06, are consistent with the corresponding fractional increases in E_c, namely ≈ 0.07 and 0.05.

Figure 5 shows ΔB_c vs the corresponding values of $B_\Lambda({}^4_\Lambda\text{H})$. The value $\Delta B_c=0$ is expected for $B_\Lambda=0$ and is seen to be very well consistent with the two calculated values. Interpolation to the experimental values of B_Λ gives

$$-\Delta B_c=0.050\pm0.02 \text{ MeV}, \quad -\Delta B_c^*=0.025\pm0.015 \text{ MeV}. \tag{36}$$

The corresponding values, Eq. (11), to be attributed to CSB effects are then quite close to the experimental values of ΔB_Λ and are

$$\Delta B_\Lambda=0.40\pm0.06 \text{ MeV}, \quad \Delta B_\Lambda^*=0.27\pm0.06 \text{ MeV}. \tag{37}$$

VI. PHENOMENOLOGICAL CHARGE SYMMETRY BREAKING INTERACTION

We obtain a phenomenological CSB potential and related quantities, based on the values of ΔB_Λ of Eq. (37).

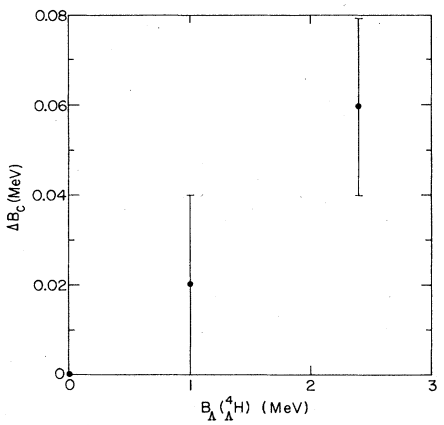

FIG. 5. ΔB_c vs $B_\Lambda({}^4_\Lambda\text{H})$.

The differences in the Λp and Λn scattering lengths are defined by

$$-\Delta a_s=a_s^p-a_s^n, \quad -\Delta a_t=a_t^p-a_t^n,$$

where a_s^p, a_t^p are the singlet and triplet Λp scattering lengths and a_s^n, a_t^n are the corresponding Λn quantities. (Δa is positive if the Λp potential is more attractive than the Λn one.) For the CSB potential we take

$$V^{\text{CSB}}=-\tfrac{1}{2}\tau_3 T_\pi^2(r)(V_s^{\text{CSB}}P_s+V_t^{\text{CSB}}P_t), \tag{38}$$

where P_s, P_t are the singlet and triplet ΛN projection operators and $V_s^{\text{CSB}}, V_t^{\text{CSB}}$ are the corresponding CSB strengths. For the potential shape we use that of the attractive part T_π^2 of the CS potential, with T_π given by Eq. (20). The total potential is then $V_{\Lambda N}+V^{\text{CSB}}$, with $V_{\Lambda N}$ given by Eq. (18). If the strength V is defined as by Eq. (18), i.e., by the potential $V_c-VT_\pi^2$, then V_s' and V_t' are defined by

$$V_s'=-\left.\frac{dV}{da}\right|_{a=a_s}, \quad V_t'=-\left.\frac{dV}{da}\right|_{a=a_t}, \tag{39}$$

where a is the scattering length corresponding to V. Because the potential shape is fixed, there is a corresponding unique relation between a and the effective range r_0 which is then not an independent parameter. The strengths $V_s^{\text{CSB}}, V_t^{\text{CSB}}$ may then be expressed in terms of $\Delta a_s, \Delta a_t$:

$$V_s^{\text{CSB}}=\Delta a_s V_s', \quad V_t^{\text{CSB}}=\Delta a_t V_t'. \tag{40}$$

A good fit to the calculated values of a (fm) as a function of V (MeV), for values of V not too different from 6.2 MeV, is

$$a=-1.88-4(V-6.2)+5(V-6.2)^2, \tag{41}$$

from which V_s', V_t' may be obtained. The corresponding relation for r_0 (fm) is

$$r_0=3.46-3.77(V-6.2)+3.78(V-6.2)^2. \tag{42}$$

The average singlet and triplet strengths for ${}^4_\Lambda$He, are, as follows from Eq. (21):

$$V_s=\tfrac{1}{3}V_s^p+\tfrac{2}{3}V_s^n, \quad V_t=V_t^p \text{ (ground state)}, \quad V_s^*=V_s^p, \quad V_t^*=\tfrac{3}{5}V_t^p+\tfrac{2}{5}V_t^n \text{ (excited state)}. \tag{43}$$

For ${}^4_\Lambda$H the indexes n and p are interchanged. Then

$$\Delta V_4=V_4({}^4_\Lambda\text{He})-V_4({}^4_\Lambda\text{H})=\tfrac{1}{6}[-(V_s^p-V_s^n)+3(V_t^p-V_t^n)], \tag{44}$$

and correspondingly

$$\Delta V_4^*=\tfrac{1}{6}[(V_s^p-V_s^n)+(V_t^p-V_t^n)]. \tag{45}$$

We then have $\Delta B_\Lambda=(dB_\Lambda/dV_4)\Delta V_4$ with $dB_\Lambda/dV_4=10.56$ and 7.94 for the ground and excited states, respectively [using Eq. (34) for the average values of B_Λ as given by Eq. (5)]. The potential V_{CSB} then gives

$$\Delta B_\Lambda=1.76(-\Delta a_s V_s'+3\Delta a_t V_t'), \tag{46}$$

$$\Delta B_\Lambda^* = 1.32(\Delta a_s V_s' + \Delta a_t V_t') \ . \tag{47}$$

Inverting these gives

$$\Delta a_s = (4V_s')^{-1}(-0.568\Delta B_\Lambda + 2.268\Delta B_\Lambda^*) \ , \tag{48}$$

$$\Delta a_t = (4V_t')^{-1}(0.568\Delta B_\Lambda + 0.756\Delta B_\Lambda^*) \ . \tag{49}$$

$V^{\rm CSB}$, given by Eqs. (38) and (40), may then be written as

$$V^{\rm CSB} = -\tau_3 T_\pi^2 \tfrac{1}{8}[(0.568\Delta B_\Lambda + 0.756\Delta B_\Lambda^*) + (0.568\Delta B_\Lambda - 0.756\Delta B_\Lambda^*)\boldsymbol{\sigma}_\Lambda\cdot\boldsymbol{\sigma}_{\rm N}] \ , \tag{50}$$

or with the values of Eq. (37),

$$V^{\rm CSB} = -0.054\tau_3 T_\pi^2[(1\pm 0.11) + (0.054\pm 0.14)\boldsymbol{\sigma}_\Lambda\cdot\boldsymbol{\sigma}_{\rm N}] \ , \tag{51}$$

and from Eqs. (48) and (49),

$$\Delta a_s = (4V_s')^{-1}(0.39\pm 0.14) \ , \qquad \Delta a_t = (4V_t')^{-1}(0.43\pm 0.08) \ . \tag{52}$$

For $a_s = a_t \approx -1.9$ fm: $V_s' = V_t' \approx 0.25$ MeV fm^{-1} and $\Delta a_s \approx 0.39$ fm, $\Delta a_t \approx 0.43$ fm. *These results clearly show that V^{CSB} is effectively spin independent.* In fact, the spin dependence is even less than would be suggested simply by the values of $\Delta B_\Lambda, \Delta B_\Lambda^*$, because dB_Λ/dV_4 is less for the excited state than for the ground state. [In the analysis of Ref. 12 the spin dependence was neglected and $V^{\rm CSB} = -0.05\tau_3 T_{2\pi}^2$ was used which is a good approximation to Eq. (51).] As we shall discuss in detail in Sec. VII, this spin independence of the CSB interaction is in strong disagreement with the predictions of existing meson-exchange models.

In order to check whether the expressions (46) and (47), and hence also Eqs. (48) and (49), are reasonable approximations, with the specific potential shape T_π^2 used, we consider the four one-boson exchange (OBE) potentials (A,B,D,F) of Nagels *et al.*[18] which include CSB effects, together with the calculations of Gibson and Lehman[19] for these potentials. Table V shows the average Λn, Λp singlet and triplet scattering lengths a_s and a_t and the values of $\Delta a_s, \Delta a_t$ of Ref. 18, and also the corresponding values of V_s', V_t' obtained using Eq. (41), and the values of $\Delta B_\Lambda, \Delta B_\Lambda^*$ obtained with Eqs. (46) and (47). We also show the average Λn, Λp effective ranges of Ref. 18 together with our values of r_0 [Eq. (42)] which correspond to the average values of the scattering lengths. Also shown are the results of Gibson and Lehman who made Faddeev-type calculations of ΔB_Λ using separable (two-parameter) potentials fitted to the values of a, r_0 of the corresponding potential of Ref. 18. Our values of ΔB_Λ are seen to be in quite satisfactory agreement with those of Gibson and Lehman. Thus Eq. (46), and therefore presumably also Eq. (47) and Eqs. (48) and (49), and also Eq. (50), is a reasonable approximation to a calculation (such as that of Gibson and Lehman) using the correct potential shapes. For such a calculation, the results depend appreciably also on the effective range (for a given scattering length), and the reason for the good agreement is that our potential shape also gives effective ranges (for potentials B, D, F) in reasonable agreement with those of Nagels *et al.* However, even for potential A the agreement for ΔB_Λ is not bad despite the poor agreement between the effective ranges.

VII. COMPARISON WITH MESON-EXCHANGE MODELS OF THE CSB INTERACTION

To obtain some insight into the preceding results, and also as a further check on our procedure for obtaining ΔB_Λ, we consider the following simple potential models. For the CSB potential we use the OPE potential,[1,7]

$$V_\pi^{\rm CSB} = -0.19\tau_3 V_\pi \ , \qquad V_\pi = V_\pi^\sigma + V_\pi^T = f_{{\rm NN}\pi}^2 \frac{m_\pi}{3}[\boldsymbol{\sigma}_\Lambda\cdot\boldsymbol{\sigma}_{\rm N} + T(r)S_{\Lambda{\rm N}}]Y(r) \ , \tag{53}$$

where $Y(r), T(r)$ are the usual Yukawa and tensor shape functions (no cutoffs) and where $f_{{\rm NN}\pi}^2 = 0.082$. For the (strong) CS potential $V_{\Lambda{\rm N}}$ we consider the following two potentials: (1) The purely central potential $V_{\Lambda{\rm N}} = V_{2\pi}$ of Eq. (18) with $V_4 = V = 6.2$ MeV. (2) An OBE potential $V_{\Lambda{\rm N}} = V_{\sigma{\rm K}}$ (Ref. 20) which has a hard core of radius 0.43 fm and which includes kaon and σ-meson exchange with a $\Lambda{\rm N}\bar{\rm K}$ coupling constant $g_{{\rm N}\Lambda\bar{\rm K}}^2 = 16$; the σ coupling is then adjusted so that $V_{\sigma{\rm K}}$ gives $a \approx -2$ fm. This gives a (^3S) ΛN potential with a tensor component of reasonable range and strength due to K exchange. We use this poten-

TABLE V. ΛN scattering lengths and effective ranges (in fm) and $A = 4$ CSB energy differences ΔB_Λ, ΔB_Λ^* (MeV) calculated from Eqs. (46) and (47) for the potentials A, B, D, F of Nagels *et al.* (Ref. 18), and for the potential models of Sec. VII with the OPE CSB potential of Eq. (53). The errors of ΔB_Λ, ΔB_Λ^* for potentials A–F are discussed in Sec. VII. $\Delta B_\Lambda^{\rm GL}$ is the value calculated by Gibson and Lehman (Ref. 19). $\bar a_s, \bar a_t$ are the averages of the Λp and Λn scattering lengths and $\bar r_{0s}$, $\bar r_{0t}$ the average effective ranges. Our values of $\bar r_0$, obtained with Eqs. (41) and (42) for the values of $\bar a$ shown, are given in parentheses.

Model	$-\bar a_s$	$\bar r_{0s}$	Δa_s	$-\bar a_t$	$\bar r_{0t}$	Δa_t	$\Delta B_\Lambda^{\rm GL}$	ΔB_Λ	ΔB_Λ^*
A	2.42	2.04(3.10)	−0.51	1.17	2.43(4.50)	0.3	1.32	1.16	0.15
B	2.29	3.14(3.16)	−0.36	1.77	3.25(3.58)	0.22	0.47	0.44	−0.02
D	1.90	3.72(3.43)	−0.26	1.95	3.25(3.40)	0.22	0.43	0.38	−0.02
F	1.96	3.17(3.39)	−0.22	1.89	3.36(3.45)	0.09	0.19	0.20	−0.03
$V_{2\pi}+V_\pi^\sigma$	1.87	(3.45)	−0.09	1.89	(3.45)	0.03		0.070	−0.020
$V_{\sigma{\rm K}}+V_\pi^\sigma$	1.96	(3.39)	−0.09	1.96	(3.39)	0.03		0.076	−0.019
$V_{\sigma{\rm K}}+V_\pi$	1.96	(3.39)	−0.09	1.96	(3.39)	0.15		0.228	0.019

tial for both the ^1S and ^3S channels; for the former the tensor component acts only to give an effective central potential ($\sim V_K^{T^2}$). The results for these potentials $V_{\Lambda N}+V^{CSB}$ are also shown in Table V. The calculations of the low-energy s-wave scattering parameters for potentials with a tensor component V^T are made in the standard way for coupled s-d channels. The relation of such calculations with perturbation theory is extensively discussed in Ref. 20. In particular to leading order in V^T such s-d scattering calculations are equivalent to second-order perturbation theory in V^T which can be represented by use of an effective central s-state potential $(V^T)^2/\bar{E}$ where $\bar{E}$ is an appropriate mean excitation energy. The coupled s-d calculations, of course, also include higher-order contributions in V^T as discussed in detail in Ref. 20. For $g^2_{N\Lambda\bar{K}}=16$ these higher-order effects (e.g., the higher-order effects which occur through the potential in the d state) are quite small.

It is important to note that the CSB tensor component $V_T^{CSB}\propto V_\pi^T$ will effectively contribute to the triplet interaction in first order by acting in conjunction with the strong CS tensor force V_T^{CS}. Thus with $V_{\sigma K}$ one has a (lowest order) contribution $\propto V_T^{CS}V_T^{CSB}\propto V_K^T V_\pi^T$ (which depends on the relative sign of V_K^T and V_π^T). V_T^{CSB} acting twice gives a negligible contribution because of the small CSB strength. In particular for $V_{\Lambda N}=V_{2\pi}$ there will be no contribution from $V_T^{CSB}\propto V_\pi^T$ since in this case there is no CS tensor force; for $V_{\sigma K}$ the contribution from V_T^{CSB} will be proportional to V_K^T, i.e., to $g^2_{N\Lambda\bar{K}}$. (No cutoff is needed for V_π^T since this contributes only for $V_{\sigma K}$ which provides a cutoff through the hard core.) Because V_π^T dominates V_π its contribution to the effective CSB triplet interaction may be comparable to that due to V_π^σ. In fact, as is seen from Table V, the CSB tensor part V_π^T is seen to give the major contribution (of 0.12 fm) to Δa_t and also to ΔB_Λ.

Table V shows that for the central part V_π^σ of V_π^{CSB}, the CSB results are the same for $V_{2\pi}$ and $V_{\sigma K}$. Furthermore, as a check on our procedure, calculations for $^4_\Lambda$H with $V_{2\pi}+V_\pi^\sigma$ (for the ground state parameters of Table II) give $\Delta B_\Lambda\approx 0.06$ MeV in fair agreement with the value 0.08 MeV obtained with the CSB potential of Eq. (50) with a T_π^2 shape.

Qualitatively, the results for $V_{\sigma K}+V_\pi$ are similar to those for the potentials B,D,F of Nagels *et al.* The differences are due to their inclusion of CSB effects due to ρ and δ exchange and also to the effects of the Σ^+, Σ^- mass difference which enter through their coupled channel calculations (and which will make a small positive contribution to Δa_t). Some further differences in the triplet CSB result could arise from CS tensor force contributions due to K* and TPE exchange which are included in Ref. 18.

We now discuss the differences between the phenomenological and meson-exchange CSB results, the latter being represented by the results of Nagels *et al.*,[18] as exhibited in Table V. It should be noted that for the potentials of Nagels *et al.*, the errors in $\Delta B_\Lambda,\Delta B_\Lambda^*$ which result from the errors in their scattering parameters are quite small. Thus, for potential D the errors are ≈ 0.02 MeV for ΔB_Λ and ≈ 0.005 MeV for ΔB_Λ^*. This is because for any one of their potentials the CSB potentials are fixed by the choice of CSB coupling constants. Thus, the errors occur only through the CS potentials whose parameters are in part determined by fits to the scattering data. Thus, any errors in $\Delta B_\Lambda,\Delta B_\Lambda^*$, as obtained using Eqs. (46) and (47), occur directly through the errors in the CS values $\bar{a}_s,\bar{a}_t$ (and hence through V_s',V_t'), and also through the induced errors in $\Delta a_s,\Delta a_t$ ($\Delta a_s=-0.26\pm0.04$ fm, $\Delta a_t=0.22\pm0.02$ fm for C) which result from the errors in $\bar{a}_s,\bar{a}_t$ for a fixed CSB potential.

Triplet CSB interaction. The triplet meson-exchange value Δa_t will have uncertainties due to uncertainties in the CS tensor force, e.g., from uncertainties in $g^2_{\Lambda N\bar{K}}$, K* exchange, TPE, etc., contributions. Thus, e.g., from Table V, the CSB tensor force contribution of our $V_{\sigma K}+V_\pi$ model to Δa_t is $\Delta a_t^T\simeq 0.12$ fm. This is proportional to $g^2_{\Lambda N\bar{K}}$ and if this is increased from 16 to the not unreasonable value of 24, then $\Delta a_t^T\approx 0.18$ fm and $\Delta a_t\approx 0.21$ fm. Thus, any uncertainty in V_T^{CS} gives corresponding uncertainties in the meson-theoretical estimates of Δa_t. Because of this dependence on V_T^{CS} the meson-theoretical estimates of Δa_t are then independent of those of Δa_s.

Furthermore (probably moderate) differences between the (calculated) ΛN values of Δa_t and the phenomenological values obtained from $\Delta B_\Lambda,\Delta B_\Lambda^*$ can arise from many-body and nuclear structure effects. Dispersive effects for the intermediate Λ or N in the tensor force contributions $\propto V_T^{CSB}V_T^{CS}$ are expected to reduce Δa_t by perhaps 15—20%, intermediate to the reduction for (CS) ΛN-ΣN OPE tensor couplings (Ref. 21) and the considerably smaller reduction for a K-exchange tensor force (Ref. 20). There will also be ΛNN contributions with the CS transition ΛN-ΣN potential acting twice for different nucleons. Because of the long range and strong OPE tensor coupling such contributions could possibly be significant but have not been estimated. Finally NN tensor forces could contribute. Although it seems unlikely that a relatively small admixture of D states could significantly change the CSB contribution for $A=4$, this remains to be shown.

In view of this discussion, and since in any case there is no major discrepancy between the meson-exchange values (≈ 0.1—0.3 fm) and the phenomenological values (≈ 0.25—0.55 fm) of Δa_t, we may conclude that the triplet CSB interaction obtained from the $A=4$ hypernuclei is consistent with meson-exchange models.

Singlet CSB interaction. For the singlet value Δa_s there is now no uncertainty corresponding to that arising from V_T^{CS} for Δa_t. In the triplet case it was possible to compensate any uncertainty in V_T^{CS} (e.g., in V_K) by adjustments in the central components (V_σ) to give a total effective CS potential in agreement with the data (i.e., $a_t\approx -2$ fm). This freedom does not exist for the singlet case where only the total effective singlet potential is relevant for the CSB contribution. Furthermore, dispersive and ΛNN effects, and probably also those of NN tensor forces, are expected to be less than for Δa_t.

The large difference for Δa_s between the meson-exchange values (≈ -0.2 to -0.45 fm) and the phenomenological values (≈ 0.35 to 0.5 fm), which are even of opposite sign, then strongly suggests that meson-

exchange models of the singlet CSB interaction are inconsistent with the $A=4$ hypernuclei, indicating that there may be important quark structure contributions. Of course complete calculations with ΛN and NN tensor forces are required for the $A=4$ hypernuclei in order to definitely establish that nuclear structure effects do not change the preceding conclusions.

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