Relationship between partial wave amplitudes and polarization observables in $pp \to d\pi^+$ and $\pi d \to \pi d$

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The polarization observables of the reactions $\vec{p}\,\vec{p} \to \pi^+ d$, $\vec{p}p \to \vec{d}\pi^+$, and $\pi\vec{d} \to \pi\vec{d}$ are investigated. Expressions relating these observables directly to (LSJ) partial wave amplitudes are derived and tabulations of the partial wave contributions are given for some of the observables. Examples are given of how such tabulations can be useful for optimizing the connection between theory and experiment and in suggesting possible new experiments. All observables are also calculated numerically using a unitary few-body model of the NN-πNN system to generate the amplitudes. Sensitivity to the choice of P_{11} interaction is investigated.

I. INTRODUCTION

The study of the πNN system (including true pion absorption) is of fundamental importance in intermediate energy physics. The few-body nature of this system makes it amenable to precise microscopic descriptions while at the same time providing a useful test case for models of more complicated systems. As a prime example of the sophistication that such descriptions have attained in recent years, we mention the "unitary model."[1–7] This model describes all the processes $\pi d \to \pi d$, $NN \to \pi d$, and $NN \to NN$ with the one set of coupled linear equations, it preserves two- and three-body unitarity, and it effectively sums the whole multiple scattering series. In addition, there has been much interest in the πNN system regarding the existence of dibaryons[8] (or more generally, evidence for quark degrees of freedom[9]) and most recently, regarding relativistic effects.[10] This interest is also reflected in the large amount of accurate data that has become available in the last few years. For NN→NN this data is essentially complete and allows a reconstruction of amplitudes.[11] For $pp \to d\pi^+$ and $\pi d \to \pi d$ the data is almost exclusively[12] restricted to observables having particles polarized in either initial or final states but not both. There is no doubt therefore that in the forthcoming years there will be increasing interest in measuring polarization transfer observables for these two reactions; indeed there are currently proposals for measuring $\vec{p}p \leftrightarrow \vec{d}\pi$ at all the pion factories. One purpose of this paper is to provide predictions of transfer observables for both $\vec{p}p \to \vec{d}\pi$, and $\pi\vec{d} \to \pi\vec{d}$.

Much of the current experimental data is well understood in terms of Δ-isobar formation in intermediate states; nevertheless, there are large and intriguing discrepancies with data. A particularly disturbing example[13] is the asymmetry A_{y0} in $\vec{p}p \to d\pi^+$ at proton (laboratory) energies above 700 MeV. For example, when compared with the new LAMPF data[14] at 800 MeV, most of the current theories[1,15] tend to overestimate the asymmetry by about 100%. As the theoretical models are diverse in their various amounts of pion rescattering, heavy meson exchanges, and relativity, it is very difficult to determine the origin of the discrepancy. Similar cases of diverse theories agreeing more with each other than with experiment may be found for some spin correlation coefficients[16] in $pp \to d\pi^+$, as well as in some observables in π-d elastic scattering.[17] In view of this uncertain connection between experiment and the underlying physics, it is most important that we have a good understanding of the relationship between a measured observable (e.g., $d\sigma/d\Omega$, it_{11}, t_{20}, etc.) and the central quantity of a theoretical prediction, namely a partial wave amplitude. It is the other and perhaps more important purpose of this work to provide the basis for such an understanding by examining explicit expressions relating partial wave amplitudes *directly* to observables. [As most calculations employ a nonrelativistic (LSJ) partial wave expansion, it is these amplitudes that we will address in this paper.] Thus our approach is significantly different from the recent works that emphasize the relationship between observables and spin amplitudes.[18] Although working with spin amplitudes has the advantage that they are finite in number, the connection with theories that utilize a partial wave decomposition is not optimal. We may also contrast our approach with the alternative one of performing an amplitude search to fit experimental data. The latter approach is indispensible in extracting numerical values for amplitudes; however, much valuable information that gave rise to a particular solution is buried in the numerical analysis. By examining the explicit mathematical relationship between experimental quantities and partial wave amplitudes, we hope to obtain a deeper insight than is generally afforded by other methods. Although the ultimate (as yet unrealized) goal is to isolate those amplitudes (hopefully few) which are incorrectly predicted by present theories, a study such as ours also provides us with a better understanding of what one learns in general (if anything) from a particular measurement, and can therefore be useful in suggesting significant experiments yet to be done.

To express any observable O_ν (ν just labels a particular observable) in terms of the partial wave amplitudes a_I ($I=\{L_f,S_f,L_i,S_i,J\}$) we follow the proposal of Niskanen[19] and perform an expansion in terms of orthogonal functions $P_L^\nu(\theta)$,

$$4\pi\sigma_{00}O_\nu=\sum_L A_L^\nu P_L^\nu(\theta) , \tag{1}$$

where σ_{00} is the cross section, θ is the scattering angle, and $P_L^\nu(\theta)$ is typically an associated Legendre function (more generally it is a Wigner rotation function). The coefficients A_L^ν are then given by bilinear combinations of the amplitudes,

$$A_L^\nu=\sum_{I,I'} C_L^\nu(I,I')a_I a_{I'}^* , \tag{2}$$

where $C_L^\nu(I,I')$ are factors made up of appropriate angular momentum recoupling coefficients. It is clear that an advantage of investigating this expansion lies in the fact that one can see directly and independently of experimental results which combinations of amplitudes contribute to an observable, their relative strength in the contribution, and those which do not contribute at all. Moreover, the expansion allows one to directly ascertain the effect of dominant amplitudes on some particular observable. For example, we find that the existence of cross terms between the large 2^+ amplitude (N-Δ in relative s state) and the smaller 0^+, 2^+, and 4^+ amplitudes points to it_{11} as being the only observable in $pp\rightarrow d\pi^+$ highly sensitive to these amplitudes. Another interesting consequence of our expansion is that, once the coefficients $C_L^\nu(I,I')$ are tabulated, it becomes possible to look for linear combinations of observables that eliminate the contribution of certain amplitudes. That this can be useful is well illustrated by the combination $\sigma_{00}(1-A_{zz}+A_{xx}+A_{yy})$ which we have found to depend solely on $J^\pi=2^-,4^-,\dots,$ amplitudes. At intermediate energies $J^\pi\geq 4^-$ amplitudes are insignificant and the above linear combination provides an empirical way to constrain the two $J^\pi=2^-$ amplitudes. It is recognized that for interpreting data, the expansion given by Eqs. (1) and (2) is less useful if there are too many non-negligible terms in the series. Unfortunately this is the case for π-d elastic scattering where many partial waves contribute. In this case a blind amplitude search may prove unavoidable. Nevertheless, the information provided by the expansion should prove invaluable to our basic understanding of many results.

In Sec. II we derive explicit expressions for the factors $C_L^\nu(I,I')$. For $\vec{p}\vec{p}\rightarrow\pi^+d$ we give separate expressions for each observable. However, by considering the spherical tensor form of polarization transfer observables we are able to derive one expression for all the observables of a general reaction $\vec{a}+b\rightarrow\vec{c}+d$.

In Sec. III A we discuss our results for the reaction $pp\rightarrow\pi^+d$. A tabulation of the factors $C_L^\nu(I,I')$ is presented and after consideration of what one can learn from such tabulations we present numerical results for all the correlation and polarization transfer observables of $pp\rightarrow\pi^+d$. For this, the partial wave amplitudes are taken from our calculations using the unitary few-body model. Both energy and model dependence is investigated. For the model dependence we choose three different parametrizations of the P_{11} interaction. We find remarkable insensitivity of the tensor observables t_{20}, t_{21}, and t_{22} to both the energy and our choice of model.

Section III B is devoted to our results for $\pi d\rightarrow\pi d$. In view of the large number of partial waves we do not discuss the factors $C_L^\nu(I,I')$ for this reaction; instead, we concentrate on numerical results for the observables. We find that many observables are large and show substantial sensitivity to our choice of model. If we limit ourselves to those polarization transfer observables that may be measurable with present technology, then we find that t_{1-1}^{11} at backward angles might be useful. We present our conclusions in Sec. IV.

II. THE ORTHOGONAL FUNCTION EXPANSION OF OBSERVABLES

In this section we derive expressions for observables of $pp\rightarrow d\pi^+$ and $\pi d\rightarrow\pi d$ that are of the form given by Eqs. (1) and (2). Our aim is to provide "look up tables" for the factors $C_L^\nu(I,I')$. Thus our approach is very similar to that originally used by Mandl and Regge[20] for $\vec{p}p\rightarrow\pi^+d$. However, in their analysis they expressed the observables ($d\sigma/d\Omega$ and A_{y0}) in terms of an expansion in powers of $\cos\theta$. It has since become clear that an expansion in $\cos\theta$ suffers from slow convergence[7] and from large correlation errors when used to parametrize data.[21] The use of orthogonal functions in the expansion improves the situation greatly (although not completely) and seems now to be a standard feature of data analysis in $pp\rightarrow d\pi^+$—particularly in extracting the angle independent coefficients A_L^ν of Eq. (1).

Before proceeding with the derivation, we first need to state some of our conventions. We shall often denote a general $2\rightarrow 2$ reaction by $a+b\rightarrow c+d$, where particles a,b,c,d have masses m_a,m_b,m_c,m_d and spins s_a,s_b,s_c,s_d. In order to make a close connection with the results of Mandl and Regge, we choose their definition of the amplitude when we are dealing with the reaction $pp\rightarrow d\pi^+$. The relation between their amplitude, denoted by a_{fi} $(=\langle f\,|\,a\,|\,i\,\rangle)$, and the usual reduced t matrix T_{fi} that satisfies[22]

$$S_{fi}=\delta_{fi}-2\pi i\delta(E_f-E_i)\delta(\mathbf{P}_f-\mathbf{P}_i)T_{fi} , \tag{3}$$

is given, after partical wave decomposition, by

$$a_I=[4\pi^3(p_f/p_i)\mu_i(p_i)\mu_f(p_f)(2L_i+1)]^{1/2}T_I , \tag{4}$$

where $I=\{L_f,S_f,L_i,S_i,J\}$ labels the LSJ quantum numbers, p_i and p_f are the initial and final center of mass momenta, and

$$\mu_i(p_i)=\frac{(p_i^2+m_a^2)^{1/2}(p_i^2+m_b^2)^{1/2}}{(p_i^2+m_a^2)^{1/2}+(p_i^2+m_b^2)^{1/2}} , \tag{5a}$$

$$\mu_f(p_f)=\frac{(p_f^2+m_c^2)^{1/2}(p_f^2+m_d^2)^{1/2}}{(p_f^2+m_c^2)^{1/2}+(p_f^2+m_d^2)^{1/2}} . \tag{5b}$$

For the case of π-d elastic scattering it is more convenient if we use the amplitude b_I defined by

$$b_I=[4\pi^3(p_f/p_i)\mu_i(p_i)\mu_f(p_f)]^{1/2}T_I \ . \tag{6}$$

The symmetry of this definition under the interchange of i and f and the assumption of time reversal invariance implies (for elastic scattering)

$$b_{\{L_f,S_f,L_i,S_i,J\}}=b_{\{L_i,S_i,L_f,S_f,J\}} \tag{7}$$

which in turn shall result in fewer partial wave channels that need to be considered explicitly.

We shall use the Madison convention[23] throughout.

A. Polarization transfer

Here we shall consider the general reaction

$$\vec{a}+b\rightarrow\vec{c}+d \ , \tag{8}$$

in the center of mass system with the "incident" particle a and the "scattered" particle c assumed to be polarized. In describing the polarization transfer observables we follow the conventions of Simonius.[24] In particular, we use spherical tensors (rather than cartesian) to describe the polarization of every particle (even if spin $\frac{1}{2}$). This choice results in simple transformation properties of observables under rotations and moreover, leads to a single expression for all the possible transfer coefficients.

In accordance with the Madison convention, the polarization of particle a is specified in a coordinate frame (frame I) that has the z axis along the momentum $\mathbf{p}_a$ of particle a. Similarly for particle c the frame to be used (frame II) has the z axis pointing along $\mathbf{p}_c$. Frames I and II have a common y axis pointing in the direction $\mathbf{p}_i\times\mathbf{p}_f$ where $\mathbf{p}_i$ and $\mathbf{p}_f$ specify those incoming and outgoing particles between which the scattering angle is measured. Unless otherwise specified, in the following we shall take

$$\mathbf{p}_i=\mathbf{p}_a \text{ and } \mathbf{p}_f=\mathbf{p}_c \ . \tag{9}$$

For the reaction in Eq. (8), the transfer coefficients $t_{k_cq_c}^{k_aq_a}$ are defined in terms of the polarizations $t^a_{k_aq_a}$ and $t^c_{k_cq_c}$ of particles a and c, respectively, by

$$t^{c\mathrm{II}}_{k_cq_c}=\sum_{k_aq_a}t^{a\mathrm{I}}_{k_aq_a}t^{k_aq_a}_{k_cq_c}\{\theta;b(\vec{a},\vec{c})d\} \ ; \tag{10}$$

the labels II and I specify the coordinate frames in which the polarizations must be given, and the argument $\{\theta;b(\vec{a},\vec{c})d\}$ further specifies the scattering angle and the identity of the incoming and outgoing particles.

In order to relate the transfer coefficients to the amplitudes a_I for the reaction, we make use of the density matrices $\rho^a_{\mu_a\mu_a'}$ and $\rho^c_{\mu_c\mu_c'}$ for the ensemble of particles a and c, respectively. It can be shown that these are related (up to a constant) via

$$\rho^c_{\mu_c\mu_c'}=\sum_{\substack{\mu_a\mu_a'\\ \mu_b\mu_d}}\langle\mu_c\mu_d\,|\,a\,|\,\mu_a\mu_b\rangle\rho^a_{\mu_a\mu_a'}\langle\mu_a'\mu_b\,|\,a^\dagger\,|\,\mu_c'\mu_d\rangle \ . \tag{11}$$

We emphasize that all quantities in this equation refer to the one coordinate system (our amplitudes are not helicity amplitudes). If we now make use of the expressions relating the density matrix $\rho_{\mu\mu'}$ to the spherical tensors t_{kq},

$$t_{kq}=\hat{s}\sum_{\mu\mu'}(-1)^{s-\mu}(s\mu's-\mu\,|\,kq)\rho_{\mu\mu'} \ , \tag{12a}$$

$$\rho_{\mu\mu'}=(1/\hat{s})\sum_{kq}(-1)^{s-\mu}(s\mu's-\mu\,|\,kq)t_{kq} \ , \tag{12b}$$

where s is the spin of the particles in the ensemble and $\hat{s}=(2s+1)^{1/2}$, then we obtain that

$$t^c_{k_cq_c}=\sum_{k_aq_a}t^a_{k_aq_a}\tilde{t}^{k_aq_a}_{k_cq_c}\{\theta;b(\vec{a},\vec{c})d\} \ , \tag{13}$$

where

$$\tilde{t}^{k_aq_a}_{k_cq_c}=\hat{s}_a\hat{s}_c\sum_{\mu_a\mu_a'}(-1)^{s_a-\mu_a}(s_a\mu_a's_a-\mu_a\,|\,k_aq_a)\sum_{\mu_c\mu_c'}(-1)^{s_c-\mu_c}(s_c\mu_c's_c-\mu_c\,|\,k_cq_c)$$
$$\times\sum_{\mu_b\mu_d}\langle\mu_c\mu_d\,|\,a\,|\,\mu_a\mu_b\rangle\langle\mu_c'\mu_d\,|\,a\,|\,\mu_a'\mu_b\rangle^*/\mathrm{Tr}(aa^\dagger) \ , \tag{14}$$

which has specially been normalized so that $t^{00}_{00}=\tilde{t}^{00}_{00}=1$. Again we emphasize that all quantities relate to the same coordinate system. If we choose this system to be frame II say, then Eq. (13) can be related to Eq. (10) by rotating $t^a_{k_aq_a}$ through an angle θ around the y axis. As usual, this is achieved through the use of Wigner rotation matrices (in our case d functions), and we obtain that

$$t^{k_aq_a}_{k_cq_c}=\sum_{q_a'}d^{k_a}_{q_aq_a'}(\theta)\tilde{t}^{k_aq_a'}_{k_cc_c} \ , \tag{15}$$

where the d function is given in the convention of Brink and Satchler.[25] Equations (14) and (15) give the relation between spin amplitudes and transfer coefficients. In Eq. (14) we can now use the partial wave decomposition

$$\langle\mu_c\mu_d\,|\,a\,|\,\mu_a\mu_b\rangle=\sum_{\substack{S_fS_i\\ M_{S_f}M_{S_i}}}\sum_{\substack{L_fL_i\\ M_{L_f}M_{L_i}}}\sum_{JM}(s_c\mu_cs_d\mu_d\,|\,S_fM_{S_f})(s_a\mu_as_b\mu_b\,|\,S_iM_{S_i})$$
$$\times(L_fM_{L_f}S_fM_{S_f}\,|\,JM)(L_iM_{L_i}S_iM_{S_i}\,|\,JM)Y_{L_fM_{L_f}}(\hat{p}_f)Y^*_{L_iM_{L_i}}(\hat{p}_i)a_{\{L_fS_fL_iS_iJ\}} \tag{16}$$

and since the coordinate system was chosen to be frame II, $\hat{p}_f=(0,0)$ and $\hat{p}_i=(\theta,\pi)$, and it is clear we obtain a relation for $\tilde{t}\,^{k_aq_a}_{k_cq_c}$ of the form given by Eq. (1) with the orthogonal functions being associated Legendre polynomials. The sums over magnetic quantum numbers can be done, and after the further sum of Eq. (15) we obtain the expression we seek,

$$4\pi\sigma_{00}t^{k_aq_a}_{k_cq_c}=\sum_{L,I,I'}T_L(I,I')a_Ia^*_{I'}d^L_{q_cq_a}(\theta)\,, \tag{17}$$

where

$$T_L(I,I')=\hat{k}_a\hat{s}_a\hat{k}_c\hat{s}_c\hat{S}_i\hat{S}\,'_i\hat{L}_f\hat{S}_f\hat{L}\,'_f\hat{S}\,'_f(2L+1)(2J+1)(2J'+1)(2s_a+1)^{-1}(2s_b+1)^{-1}$$
$$\times(-1)^{s_a+s_b+s_c+s_d+L'_f+S'_f+L'_i+S'_i}\begin{Bmatrix}S_i & S'_i & k_a\\ s_a & s_a & s_b\end{Bmatrix}\begin{Bmatrix}S_f & S'_f & k_c\\ s_c & s_c & s_d\end{Bmatrix}$$
$$\times\sum_{X_i,X_f}(2X_i+1)(2X_f+1)\begin{pmatrix}L_i & L'_i & X_i\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}L_f & L'_f & X_f\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}X_i & k_a & L\\ 0 & -q_a & q_a\end{pmatrix}\begin{pmatrix}X_f & k_c & L\\ 0 & -q_c & q_c\end{pmatrix}$$
$$\times\begin{Bmatrix}L_i & L'_i & X_i\\ S_i & S'_i & k_a\\ J & J' & L\end{Bmatrix}\begin{Bmatrix}L_f & L'_f & X_f\\ S_f & S'_f & k_c\\ J & J' & L\end{Bmatrix}. \tag{18}$$

We note that the orthogonal functions in which we are expanding are now the Wigner d functions $d^L_{q_cq_a}(\theta)$ which are directly related to associated Legendre polynomials only when either of q_c or q_a are zero.

B. Polarization correlations

Unlike in the previous section, we make no attempt at a general formulation; instead, we give expressions only for the reaction $\vec{p}\vec{p}\to\pi^+d$. Furthermore, we use Cartesian tensors exclusively and give separate expressions for the cross section and all the independent correlation coefficients.

Denoting the polarization of the beam by $\mathbf{P}^{(a)}$ and the polarization of the target by $\mathbf{P}^{(b)}$, the cross section is given by

$$d\sigma/d\Omega=C\sum_{\text{all}\,\mu}\langle\mu'_a\,|\,1+\mathbf{P}^{(a)}\cdot\boldsymbol{\sigma}^{(a)}\,|\,\mu_a\rangle\langle\mu'_b\,|\,1+\mathbf{P}^{(b)}\cdot\boldsymbol{\sigma}^{(b)}\,|\,\mu_b\rangle\langle\mu_c\mu_d\,|\,T\,|\,\mu'_a\mu'_b\rangle\langle\mu_c\mu_d\,|\,T\,|\,\mu_a\mu_b\rangle^*\,, \tag{19}$$

where $\boldsymbol{\sigma}^{(a)}$ and $\boldsymbol{\sigma}^{(b)}$ are the Pauli spin matrices for particles a and b, respectively, and

$$C=\frac{1}{(2s_a+1)(2s_b+1)}(2\pi)^4(p_f/p_i)\mu_i(p_i)\mu_f(p_f)\,. \tag{20}$$

Rewriting Eq. (19) in terms of partial cross sections σ_{ij} ($i,j=0,1,2,3$; with σ_{00} being the spin-averaged cross section):

$$d\sigma/d\Omega=\sigma_{00}+\sum_{i=1}^{3}P_i^{(a)}\sigma_{i0}+\sum_{j=1}^{3}P_j^{(b)}\sigma_{0j}+\sum_{i=1}^{3}\sum_{j=1}^{3}P_i^{(a)}P_j^{(b)}\sigma_{ij}\,, \tag{21}$$

leads to the usual definition of the correlation tensor A_{ij}, namely

$$A_{ij}=\sigma_{ij}/\sigma_{00}\,. \tag{22}$$

By parity invariance, of the 16 possible A_{ij} (including the trivial $A_{00}\equiv1$), only eight are nonzero. In addition, the identity of the two initial particles reduces the number of independent correlations to six. With the choice of coordinate frame having the z axis along the beam direction (frame I), the use of Eq. (16) in Eq. (21) leads to the following expressions for six independent correlations:

$$4\pi\sigma_{00}=\tfrac{1}{4}\sum_{I,I',L}(2J+1)(2J'+1)(2L+1)\hat{L}_f\hat{L}\,'_f\begin{pmatrix}L_i & L'_i & L\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}L_f & L'_f & L\\ 0 & 0 & 0\end{pmatrix}$$
$$\times\delta_{S_i,S'_i}(-1)^{S_f-S_i}\begin{Bmatrix}L_i & L'_i & L\\ J' & J & S_i\end{Bmatrix}\begin{Bmatrix}L_f & L'_f & L\\ J' & J & S_f\end{Bmatrix}P_L(\cos\theta)a_Ia^*_{I'}\,, \tag{23}$$

$$4\pi\sigma_{00}A_{y0}=-i\left(\tfrac{3}{8}\right)^{1/2}\sum_{I,I',L}(2J+1)(2J'+1)(2L+1)\hat{S}_i\hat{S}'_i\hat{L}_f\hat{L}'_f\hat{L}(-1)^J$$
$$\times\begin{bmatrix}L_i & L_i' & L\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}L_f & L_f' & L\\ 0 & 0 & 0\end{bmatrix}\begin{Bmatrix}1 & S_i & S_i'\\ \tfrac{1}{2} & \tfrac{1}{2} & \tfrac{1}{2}\end{Bmatrix}\begin{Bmatrix}J' & L & J\\ L_f & S_f & L_f'\end{Bmatrix}\begin{Bmatrix}1 & L & L\\ S_i' & J' & L_i'\\ S_i & J & L_i\end{Bmatrix}$$
$$\times[L(L+1)]^{-1/2}P_{L1}(\cos\theta)a_I a_{I'}^{*}\ , \tag{24}$$

$$4\pi\sigma_{00}A_{zx}=-\left(\tfrac{3}{4}\right)^{1/2}\sum_{I,I',L}(2J+1)(2J'+1)(2L+1)\hat{S}_i\hat{S}'_i\hat{L}_f\hat{L}'_f\begin{bmatrix}L_f & L_f' & L\\ 0 & 0 & 0\end{bmatrix}$$
$$\times\left[(-1)^{S_i'}\begin{bmatrix}1 & S_i & S_i'\\ 1 & 0 & -1\end{bmatrix}\begin{bmatrix}J' & L & J\\ 1 & -1 & 0\end{bmatrix}\begin{bmatrix}L_i & S_i & J\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}L_i' & S_i' & J'\\ 0 & 1 & -1\end{bmatrix}\right.$$
$$\left.+(-1)^{S_i}\begin{bmatrix}1 & S_i' & S_i\\ 1 & 0 & -1\end{bmatrix}\begin{bmatrix}J & L & J'\\ 1 & -1 & 0\end{bmatrix}\begin{bmatrix}L_i' & S_i' & J'\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}L_i & S_i & J\\ 0 & 1 & -1\end{bmatrix}\right]$$
$$\times[L(L+1)]^{-1/2}\begin{Bmatrix}1 & S_i & S_i'\\ \tfrac{1}{2} & \tfrac{1}{2} & \tfrac{1}{2}\end{Bmatrix}\begin{Bmatrix}J' & L & J\\ L_f & S_f & L_f'\end{Bmatrix}P_{L1}(\cos\theta)a_I a_{I'}^{*}\ , \tag{25}$$

$$4\pi\sigma_{00}A_{xx}=\tfrac{1}{4}\sum_{I,I',L}(2J+1)(2J'+1)(2L+1)\hat{L}_f\hat{L}'_f(-1)^{S_i}\delta_{S_i,S_i'}a_I a_{I'}^{*}\begin{bmatrix}L_f & L_f' & L\\ 0 & 0 & 0\end{bmatrix}\begin{Bmatrix}J' & L & J\\ L_f & S_f & L_f'\end{Bmatrix}$$
$$\times\left\{\begin{bmatrix}J' & L & J\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}L_i & S_i & J\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}L_i' & S_i & J'\\ 0 & 0 & 0\end{bmatrix}P_L(\cos\theta)\right.$$
$$\left.-2\begin{bmatrix}J' & L & J\\ 1 & -2 & 1\end{bmatrix}\begin{bmatrix}L_i & S_i & J\\ 0 & -1 & 1\end{bmatrix}\begin{bmatrix}L_i' & S_i & J'\\ 0 & 1 & -1\end{bmatrix}[(L+2)!/(L-2)!]^{-1/2}P_{L2}(\cos\theta)\right\}, \tag{26}$$

$$4\pi\sigma_{00}A_{yy}=\tfrac{1}{4}\sum_{I,I',L}(2J+1)(2J'+1)(2L+1)\hat{L}_f\hat{L}'_f(-1)^{S_i}\delta_{S_i,S_i'}a_I a_{I'}^{*}\begin{bmatrix}L_f & L_f' & L\\ 0 & 0 & 0\end{bmatrix}\begin{Bmatrix}J' & L & J\\ L_f & S_f & L_f'\end{Bmatrix}$$
$$\times\left\{\begin{bmatrix}J' & L & J\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}L_i & S_i & J\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}L_i' & S_i & J'\\ 0 & 0 & 0\end{bmatrix}P_L(\cos\theta)\right.$$
$$\left.+2\begin{bmatrix}J' & L & J\\ 1 & -2 & 1\end{bmatrix}\begin{bmatrix}L_i & S_i & J\\ 0 & -1 & 1\end{bmatrix}\begin{bmatrix}L_i' & S_i & J'\\ 0 & 1 & -1\end{bmatrix}[(L+2)!/(L-2)!]^{-1/2}P_{L2}(\cos\theta)\right\}, \tag{27}$$

$$4\pi\sigma_{00}A_{zz}=\tfrac{1}{4}\sum_{I,I',L}(2J+1)(2J'+1)(2L+1)\hat{L}_f\hat{L}'_f\delta_{S_i,S_i'}\begin{bmatrix}L_f & L_f' & L\\ 0 & 0 & 0\end{bmatrix}\begin{Bmatrix}J' & L & J\\ L_f & S_f & L_f'\end{Bmatrix}$$
$$\times\left[\begin{bmatrix}J' & L & J\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}L_i & S_i & J\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}L_i' & S_i & J'\\ 0 & 0 & 0\end{bmatrix}\right.$$
$$\left.+2\begin{bmatrix}J' & L & J\\ 1 & 0 & -1\end{bmatrix}\begin{bmatrix}L_i & S_i & J\\ 0 & 1 & -1\end{bmatrix}\begin{bmatrix}L_i' & S_i & J'\\ 0 & 1 & -1\end{bmatrix}\right]P_L(\cos\theta)a_I a_{I'}^{*}\ . \tag{28}$$

III. RESULTS AND DISCUSSION

In each of the orthogonal function expansions Eqs. (17) and (23)—(28), the angular momentum factors are either symmetric or antisymmetric under the interchange of I and I'. For example, the factor $T_L(I,I')$, given by Eq. (18), satisfies

$$T_L(I,I')=(-1)^{k_a+k_c}T_L(I',I) . \quad (29)$$

Thus we may explicitly sum over the two orderings of channels I and I' giving expansions in terms of either $\mathrm{Re}(a_I a_{I'}^*)$ or $\mathrm{Im}(a_I a_{I'}^*)$. In this case the coefficients, which we write generically as $c_L^{\gamma}(I,I')$, are related to the $C_L^{\gamma}(I,I')$ by at most a factor of 2. Specifically, we shall write the orthogonal function expansions as follows:

For polarization correlations in $\vec{p}\vec{p}\to\pi^+ d$,

$$4\pi\sigma_{00}=\sum_{\substack{L\ \mathrm{even}\\ I'\geq I}} a_L^{00}(I,I')P_L(\cos\theta)\,\mathrm{Re}(a_I a_{I'}^*) , \quad (30)$$

$$4\pi\sigma_{00}A_{y0}=\sum_{L,I'>I} b_L^{y0}(I,I')P_{L1}(\cos\theta)\,\mathrm{Im}(a_I a_{I'}^*) , \quad (31)$$

$$4\pi\sigma_{00}A_{zx}=\sum_{L,I'>I} b_L^{zx}(I,I')P_{L1}(\cos\theta)\,\mathrm{Re}(a_I a_{I'}^*) , \quad (32)$$

$$4\pi\sigma_{00}A_{xx}=\sum_{\substack{L\ \mathrm{even}\\ I'\geq I}} [a_L^{xx}(I,I')P_L(\cos\theta) + c_L^{xx}(I,I')P_{L2}(\cos\theta)]\,\mathrm{Re}(a_I a_{I'}^*) , \quad (33)$$

$$4\pi\sigma_{00}A_{yy}=\sum_{\substack{L\ \mathrm{even}\\ I'\geq I}} [a_L^{xx}(I,I')P_L(\cos\theta) - c_L^{xx}(I,I')P_{L2}(\cos\theta)]\,\mathrm{Re}(a_I a_{I'}^*) , \quad (34)$$

$$4\pi\sigma_{00}A_{zz}=\sum_{\substack{L\ \mathrm{even}\\ I'\geq I}} a_L^{zz}(I,I')P_L(\cos\theta)\,\mathrm{Re}(a_I a_{I'}^*) . \quad (35)$$

For tabulation purposes it is convenient to write

$$A_{xx}=\tilde{A}_{xx}+\tilde{C}_{xx} , \quad (36a)$$

$$A_{yy}=\tilde{A}_{xx}-\tilde{C}_{xx} , \quad (36b)$$

where the definition of $\tilde{A}_{xx}$ and $\tilde{C}_{xx}$ is clear from Eqs. (33) and (34).

For polarization transfers with $q_c\neq 0$ and $q_a\neq 0$,

$$4\pi\sigma_{00}t_{k_c q_c}^{k_a q_a}=\sum_{L,I'\geq I} T_L(I,I')d_{q_c q_a}^{L}(\theta)\,\mathrm{Re}(a_I a_{I'}^*) , \quad (37)$$

if k_a+k_c is even,

$$4\pi\sigma_{00}it_{k_c q_c}^{k_a q_a}=\sum_{L,I'>I} T_L(I,I')d_{q_c q_a}^{L}(\theta)\,\mathrm{Im}(a_I a_{I'}^*) , \quad (38)$$

if k_a+k_c is odd. If either $q_c=0$ or $q_a=0$ then we use Legendre functions $P_L\,|q_c-q_a|$ instead of the Wigner functions in Eqs. (37) and (38). For example, for unpolarized initial states

$$4\pi\sigma_{00}it_{11}=\sum_{L,I'>I} b_L^{11}(I,I')P_{L1}(\cos\theta)\,\mathrm{Im}(a_I a_{I'}^*) , \quad (39)$$

$$4\pi\sigma_{00}t_{20}=\sum_{L,I'\geq I} a_L^{20}(I,I')P_L(\cos\theta)\,\mathrm{Re}(a_I a_{I'}^*) , \quad (40)$$

$$4\pi\sigma_{00}t_{21}=\sum_{L,I'\geq I} b_L^{21}(I,I')P_{L1}(\cos\theta)\,\mathrm{Re}(a_I a_{I'}^*) , \quad (41)$$

$$4\pi\sigma_{00}t_{22}=\sum_{L,I'\geq I} c_L^{22}(I,I')P_{L2}(\cos\theta)\,\mathrm{Re}(a_I a_{I'}^*) . \quad (42)$$

Expressions for the coefficients $a_L^{00}(I,I')$, $b_L^{zx}(I,I')$, etc., are trivially found from Eqs. (18) and (23)—(28). We note that the tabulation of these coefficients not only gives a better insight into the nature of an observable, but it also provides an efficient method for calculating the observables—the angular momentum algebra need only be done once. This may be of practical importance as for example in fitting data with an amplitude search.

All the above results implicitly refer to the center of mass (c.m.) system. In the case of correlation observables the results are also valid in the laboratory frame as the c.m. and laboratory frames share a common quantization axis (along the beam direction). However, for a polarized particle in the final state the quantization axis in the c.m. and laboratory frames will be different. A simple, and strictly nonrelativistic, way to obtain the laboratory transfers $t_{k_c q_c}^{k_a q_a\,\mathrm{lab}}\{\theta_L;b(\vec{a},\vec{c})d\}$ in terms of the center of mass transfers $t_{k_c q_c}^{k_a q_a}\{\theta;b(\vec{a},\vec{c})d\}$ is as follows. Equation (10) is the working definition of the polarization transfers both in the c.m. and laboratory systems. Both the c.m. and laboratory observers will use a common frame, frame I, to describe the polarization state of particles a. However, in describing particles c, each observer will use a frame II that has its z axis along the momentum of particles c. This is illustrated in Fig. 1. All frames use a common y axis. Thus, in describing the polarization state of particles c, going from the c.m. to laboratory system is effectively the same as rotating the state of particles c by the angle

$$\omega=\theta-\theta_L \quad (43)$$

around the y axis. Thus we have that[24]

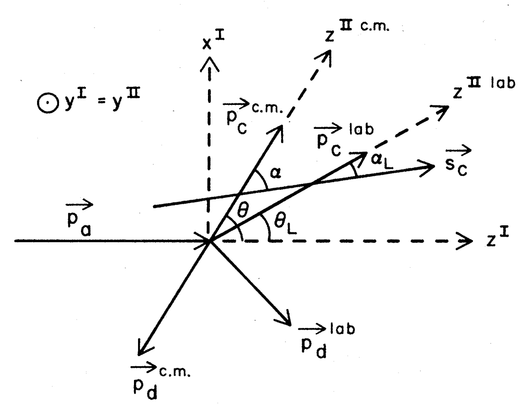

FIG. 1. A top view of the scattering plane for the reaction $b(\vec{a},\vec{c})d$ comparing c.m. and laboratory system quantities. The polarization of particle c is represented by $\mathbf{s}_c$. Frame II is defined as having its z axis along $\mathbf{p}_c$ in either system; consequently a c.m. to laboratory transformation is equivalent to a rotation of the states $|s_c\rangle$ around the y axis by the angle $|\alpha-\alpha_L|=\theta-\theta_L$.

$$t_{k_c q_c}^{c\,\mathrm{lab\,II}} = \sum_{q_c'} d^{k_c}_{q_c q_c'}(\omega) t^{c\,\mathrm{c.m.\,II}}_{k_c q_c'} , \quad (44)$$

and the further use of Eq. (10) gives

$$t_{k_c q_c}^{k_a q_a\,\mathrm{lab}}\{\theta_L; b(\vec{a},\vec{c})d\} = \sum_{q_c'} d^{k_c}_{q_c q_c'}(\omega) t^{k_a q_a\,\mathrm{c.m.}}_{k_c q_c'}\{\theta; b(\vec{a},\vec{c})d\} . \quad (45)$$

A more rigorous and relativistically correct derivation involves the examination of how helicity states rotate in going from the c.m. to laboratory system.[26] The final result is still given by Eq. (45), however, ω is now defined by the equations

$$\cos\omega = \cos\theta\cos\theta_L + \gamma_{\mathrm{c.m.}}\sin\theta\sin\theta_L , \quad (46a)$$

$$\sin\omega = (m/E)(\sin\theta\cos\theta_L - \gamma_{\mathrm{c.m.}}\cos\theta\sin\theta_L) , \quad (46b)$$

where $\gamma_{\mathrm{c.m.}} = (1 - v^2_{\mathrm{c.m.}}/c^2)^{-1/2}$, and m and E refer to the mass and total energy of particle c. In the nonrelativistic limit we of course recover the expression for ω given in Eq. (43).

A. $pp \to \pi^+ d$

In this section we concentrate specifically on the $pp \to \pi^+ d$ reaction. In Eqs. (30)–(35) we have already specified the six (counting the cross section σ_{00} for convenience) independent correlation coefficients that we consider. Assuming parity invariance there are 16 independent transfer observables (excluding the cross section and it^{11}_{00} which is related to A_{y0} by a factor of $\sqrt{2}$). These we choose to be it_{11}, t_{20}, t_{21}, t_{22}, t^{10}_{10}, t^{10}_{11}, it^{10}_{21}, it^{10}_{22}, t^{11}_{10}, t^{11}_{11}, t^{11}_{1-1}, it^{11}_{20}, it^{11}_{21}, it^{11}_{2-1}, it^{11}_{22}, and it^{11}_{2-2}. Before presenting our results for these particular observables, we should note that although the spherical form for the transfer observables is convenient for us, experimentally it is frequently easier to measure Cartesian tensor observables $P^{i_a j_a}_{i_c j_c}$. The relationship between the two is easily determined from the defining relations

$$t^{k_a q_a}_{k_c q_c} = \mathrm{Tr}(\tau_{k_c q_c} F \tau^{\dagger}_{k_a q_a} F^{\dagger}) / \mathrm{Tr}(FF^{\dagger}) , \quad (47a)$$

$$P^{i_a j_a}_{i_c j_c} = \mathrm{Tr}(\pi_{i_c j_c} F \pi_{i_a q_a} F^{\dagger}) / \mathrm{Tr}(FF^{\dagger}) , \quad (47b)$$

where F are the helicity amplitudes and τ_{kq}, π_{ij}, and the relationship between them is given in the Madison convention.[23] We note that

$$\langle \mu' | \tau_{kq} | \mu \rangle = \hat{s}(-)^{s-\mu}(s\mu' s - \mu | kq) \quad (48)$$

and Eq. (14) is just a version of Eq. (47a) but with single frame amplitudes replacing the helicity amplitudes. For completeness we give the relations for the reaction $p(\vec{p},\vec{d})\pi^+$:

$$P^x_x = (\tfrac{2}{3})^{1/2}(t^{11}_{11} - t^{11}_{1-1}) \quad (\equiv K_{SS}) , \quad (49a)$$

$$P^y_y = (\tfrac{2}{3})^{1/2}(t^{11}_{11} + t^{11}_{1-1}) \quad (\equiv K_{NN}) , \quad (49b)$$

$$P^z_z = (\tfrac{2}{3})^{1/2} t^{10}_{10} \quad (\equiv K_{LL}) , \quad (49c)$$

$$P^x_z = -2/\sqrt{3}\, t^{11}_{10} , \quad (49d)$$

$$P^z_x = -2/\sqrt{3}\, t^{10}_{11} , \quad (49e)$$

$$P^y_{zz} = -2it^{11}_{20} , \quad (49f)$$

$$P^z_{yz} = \sqrt{3}\, it^{10}_{21} , \quad (49g)$$

$$P^z_{xy} = -\sqrt{3}\, it^{10}_{22} , \quad (49h)$$

$$P^x_{yz} = -(\tfrac{3}{2})^{1/2} i(t^{11}_{21} + t^{11}_{2-1}) , \quad (49i)$$

$$P^y_{xz} = (\tfrac{3}{2})^{1/2} i(t^{11}_{21} - t^{11}_{2-1}) , \quad (49j)$$

$$P^x_{xy} = (\tfrac{3}{2})^{1/2} i(t^{11}_{22} - t^{11}_{2-2}) , \quad (49k)$$

$$P^y_{xx} - P^y_{yy} = -\sqrt{6}\, i(t^{11}_{22} + t^{11}_{2-2}) . \quad (49l)$$

We follow our previous notation[7] and sequentially order the partial wave amplitudes a_I as in Table I. Although

TABLE I. The partial wave channels, up to h-wave pions, of the reaction $pp \to \pi^+ d$.

a_I	J^π	Pion wave	L_f (πd)	S_f (πd)	L_i (pp)	S_i (pp)	p-p state
a_0	0^+	p	1	1	0	0	$^1S_0$
a_1	1^-	s	0	1	1	1	$^3P_1$
a_2	2^+	p	1	1	2	0	$^1D_2$
a_3	1^-	d	2	1	1	1	$^3P_1$
a_4	2^-	d	2	1	1	1	$^3P_2$
a_5	2^-	d	2	1	3	1	$^3F_2$
a_6	3^-	d	2	1	3	1	$^3F_3$
a_7	2^+	f	3	1	2	0	$^1D_2$
a_8	4^+	f	3	1	4	0	$^1G_4$
a_9	3^-	g	4	1	3	1	$^3F_3$
a_{10}	4^-	g	4	1	3	1	$^3F_4$
a_{11}	4^-	g	4	1	5	1	$^3H_4$
a_{12}	5^-	g	4	1	5	1	$^3H_5$
a_{13}	4^+	h	5	1	4	0	$^1G_4$
a_{14}	6^+	h	5	1	6	0	$^1I_6$

we retain up to h-wave pions (a_0—a_{14}) for numerical calculation of observables, it is not practical to present a full tabulation of all the corresponding coefficients $c_L^\gamma(I,I')$ here. However, one can ascertain most features from a smaller tabulation based on a more limited number of partial waves. Indeed, we find that for proton energies below 800 MeV, most of the numerical contribution comes from amplitudes a_0—a_8 (at most f-wave pions), so we retain only these nine amplitudes in presenting our tables.

For correlation observables in $\vec{p}(\vec{p},\pi^+)d$ the coefficients $c_L^\gamma(I,I')$, as defined in Eqs. (30)—(36), are enumerated in Table II. Lack of space precludes us from presenting corresponding tables of coefficients for all the polarization transfers in $p(\vec{p},\vec{d})\pi^+$; however, the coefficients for the simplest of these, namely the vector and tensor polarizations as defined in Eqs. (39)—(42), are also tabulated in Table II. We now discuss the way in which Table II may help illuminate the relationship between theoretical amplitudes and a measured observable. Firstly, we know both on physical grounds and from calculations that the dominant amplitude at intermediate energies is a_2. This is attributed to the fact that the $J=2^+$ channel is the only one in which a nucleon and a delta (formed in the scattering process) can be in relative s state. From Table II we can directly ascertain the consequences of a dominant a_2. We would expect that those observables with $|a_2|^2$ contributions in the numerator (there will always be a $|a_2|^2$ contribution coming from the division by the cross section) will mostly be determined by the magnitude of a_2. Those not involving $|a_2|^2$ in the numerator but with cross terms involving a_2 (A_{y0}, A_{zx}, and it_{11} in Table II) should be especially sensitive to those amplitudes that multiply a_2. In Table III we summarize all the independent correlation and transfer observables in $pp\rightarrow\pi^+d$ that do not have $|a_2|^2$ terms in the numerator—amplitudes having cross terms with a_2 are also listed. We see that it_{11} is unique in that it is the only such observable having the large a_2 multiplying the amplitudes a_0, a_7, and a_8. We therefore expect that accurate measurements of it_{11} will be crucial in the determination of a_0, a_7, and a_8. Indeed the recent iT_{11} measurements of Smith *et al.*[27] have already had a significant constraining effect on amplitude searches[28]—especially regarding the previously highly undetermined phase of a_0. Another feature apparent from Table III is that the polarization transfers involving a longitudinally polarized proton beam, namely t_{10}^{10}, t_{11}^{10}, t_{21}^{10}, and t_{22}^{10}, will depend rather sensitively on the coupled $J=2^-$ amplitudes a_4 and a_5; in contrast, the other observables have large contributions coming also from a_1, a_6, and perhaps to a lesser extent from a_3, thereby making a determination of a_4 and a_5 more difficult (although not impossible, as we shall see below).

We note from Eq. (45) that the above observations hold equally well for the laboratory as for the c.m. system. On the other hand we should temper any conclusions by the realization that there are amplitudes besides a_2, particularly a_1, a_4, a_6, and a_8, that can give non-negligible contributions all on their own. In fact if we take the extreme stance that all amplitudes are negligible compared to a_2 then, using Table II, we would get $A_{xx}=A_{yy}=A_{zz}=-1$ for the correlations and

$$t_{20}=-\frac{\sqrt{2}}{70}\,\frac{7+25P_2(\cos\theta)+108P_4(\cos\theta)}{1+P_2(\cos\theta)}\,, \qquad (50a)$$

$$t_{21}=-\frac{\sqrt{3}}{35}\,\frac{5P_{21}(\cos\theta)+9P_{41}(\cos\theta)}{1+P_2(\cos\theta)}\,, \qquad (50b)$$

$$t_{22}=-\frac{3\sqrt{3}}{140}\,\frac{5P_{22}(\cos\theta)+2P_{42}(\cos\theta)}{1+P_2(\cos\theta)}\,, \qquad (50c)$$

for the polarization tensors in the center of mass.

To compare these "predictions" with an actual calculation, and indeed to provide the first calculation of many of the transfer observables, we use a few-body unitary model to calculate the amplitudes a_I. The description of this model, including full details of the actual calculations used in the present discussion, may be found in Refs. 7 and 29. Here we mention only those aspects of the few-body calculation that are most germane to the present discussion. As we have already noted, the unitary model provides a simultaneous description of the reactions $pp\rightarrow\pi^+d$, $\pi d\rightarrow\pi d$, and $NN\rightarrow NN$, and includes, at least in principle, much of the physics in an exact way. Perhaps the single most important difference between all the unitary few-body calculations is in the precise treatment of the P_{11} channel. For this reason we choose three different parametrizations of the P_{11} interaction in order to test the model sensitivity of polarization observables. Two of the interactions,[29] denoted by $M1$ and $P6$, describe both the pole and nonpole pieces of the P_{11} t matrix by separable potentials in such a way that the nonpole piece is used in explicit dressings of the πNN vertex as well as the nucleon propagator. The third interaction,[7] denoted by $BO8$, describes the full t matrix by a two-term separable potential and no explicit dressings are performed (they are effectively included through the use of physical masses). Although explicit dressings are required by unitarity,[30] interaction $BO8$ has the virtue that it is able to describe the pion production cross section while the more precise descriptions ($M1$ and $P6$ for example) underestimate the cross section by 20—50 %.

We first examine the energy dependence of the polarization observables. For this we choose interaction $M1$ for the P_{11} channel, and the three-proton laboratory energies $T_p=383$, 567, and 800 MeV which span the 33-resonance region. In Fig. 2 we present results for the correlation observables at the three energies showing experimental data where available. We see that the cross section below 800 MeV is badly underestimated by the assumed model. This is typical of all the latest calculations using the unitary model and is believed to be caused by neglect of states having backward going pions in the $NN\rightarrow N\Delta$ amplitude.[15] Nevertheless, the shapes of the distributions are reasonably well reproduced so it is hoped that the predictions for spin dependent observables may be more realistic. It is evident from both the calculation and experiment that A_{xx} as well as A_{zz} deviate substantially from -1 so that a_2 is definitely not the sole contributing amplitude. Considering this conclusion the behavior of A_{yy} is enigmatic. A_{yy} is largely energy and angle independent and moreover, for energies below 800 MeV, is in the re-

TABLE II. Orthogonal polynomial coefficients, as defined by Eqs. (30)–(36) and (39)–(42), for the correlation observables in $\vec{p}(\vec{p},\pi^+)d$, and for the center of mass tensor observables in $p(p,\vec{d})\pi^+$. A square root is implied for numbers to the right of the colon, thus $-\frac{1}{2}:\frac{3}{4}$ means $(-\frac{1}{2})(\frac{3}{4})^{1/2}$.

$4\pi\sigma_{00}$	Amplitude product	a_0^{00}	a_2^{00}	a_4^{00}	a_6^{00}	$4\pi\sigma_{00}A_{zz}$	a_0^{zz}	a_2^{zz}	a_4^{zz}	a_6^{zz}
	$\lvert a_0\rvert^2$	$\frac{1}{4}$	0	0	0		$-\frac{1}{4}$	0	0	0
	$\lvert a_1\rvert^2$	$\frac{1}{4}$	0	0	0		$\frac{1}{4}$	0	0	0
	$\lvert a_2\rvert^2$	$\frac{1}{4}$	$\frac{1}{4}$	0	0		$-\frac{1}{4}$	$-\frac{1}{4}$	0	0
	$\lvert a_3\rvert^2$	$\frac{1}{4}$	$-\frac{1}{8}$	0	0		$\frac{1}{4}$	$-\frac{1}{8}$	0	0
	$\lvert a_4\rvert^2$	$\frac{5}{12}$	$\frac{5}{24}$	0	0		$\frac{1}{12}$	$-\frac{5}{168}$	$\frac{4}{7}$	0
	$\lvert a_5\rvert^2$	$\frac{5}{28}$	$\frac{5}{49}$	$-\frac{5}{49}$	0		$-\frac{1}{28}$	$-\frac{5}{98}$	$\frac{13}{49}$	0
	$\lvert a_6\rvert^2$	$\frac{1}{4}$	$\frac{3}{14}$	$\frac{1}{28}$	0		$\frac{1}{4}$	$\frac{3}{14}$	$\frac{1}{28}$	0
	$\lvert a_7\rvert^2$	$\frac{1}{4}$	$\frac{2}{7}$	$\frac{3}{14}$	0		$-\frac{1}{4}$	$-\frac{2}{7}$	$-\frac{3}{14}$	0
	$\lvert a_8\rvert^2$	$\frac{1}{4}$	$\frac{25}{84}$	$\frac{81}{308}$	$\frac{25}{132}$		$-\frac{1}{4}$	$-\frac{25}{84}$	$-\frac{81}{308}$	$-\frac{25}{132}$
	$\mathrm{Re}(a_0a_2^*)$	0	$-1:\frac{1}{2}$	0	0		0	$:\frac{1}{2}$	0	0
	$\mathrm{Re}(a_0a_7^*)$	0	$\frac{1}{2}:3$	0	0		0	$-\frac{1}{2}:3$	0	0
	$\mathrm{Re}(a_0a_8^*)$	0	0	-1	0		0	0	1	0
	$\mathrm{Re}(a_1a_3^*)$	0	$\frac{1}{2}:\frac{1}{2}$	0	0		0	$\frac{1}{2}:\frac{1}{2}$	0	0
	$\mathrm{Re}(a_1a_4^*)$	0	$\frac{1}{2}:\frac{5}{2}$	0	0		0	$\frac{1}{2}:\frac{5}{2}$	0	0
	$\mathrm{Re}(a_1a_5^*)$	0	$\frac{1}{2}:\frac{5}{7}$	0	0		0	$\frac{1}{2}:\frac{5}{7}$	0	0
	$\mathrm{Re}(a_1a_6^*)$	0	$:\frac{1}{2}$	0	0		0	$:\frac{1}{2}$	0	0
	$\mathrm{Re}(a_2a_7^*)$	0	$-\frac{1}{7}:\frac{3}{2}$	$-\frac{3}{7}:6$	0		0	$\frac{1}{7}:\frac{3}{2}$	$\frac{3}{7}:6$	0
	$\mathrm{Re}(a_2a_8^*)$	0	$\frac{9}{7}:\frac{1}{2}$	$\frac{5}{7}:\frac{1}{2}$	0		0	$-\frac{9}{7}:\frac{1}{2}$	$-\frac{5}{7}:\frac{1}{2}$	0
	$\mathrm{Re}(a_3a_4^*)$	0	$\frac{1}{4}:5$	0	0		0	$\frac{1}{4}:5$	0	0
	$\mathrm{Re}(a_3a_5^*)$	0	$\frac{1}{2}:\frac{5}{14}$	0	0		0	$\frac{1}{2}:\frac{5}{14}$	0	0
	$\mathrm{Re}(a_3a_6^*)$	0	$-\frac{1}{7}$	$\frac{9}{14}$	0		0	$-\frac{1}{7}$	$\frac{9}{14}$	0
	$\mathrm{Re}(a_4a_5^*)$	0	$-\frac{5}{14}:\frac{1}{14}$	$\frac{10}{7}:\frac{2}{7}$	0		$:\frac{2}{7}$	$\frac{15}{14}:\frac{1}{14}$	$-\frac{2}{7}:\frac{2}{7}$	0
	$\mathrm{Re}(a_4a_6^*)$	0	$\frac{1}{7}:5$	$\frac{5}{14}:5$	0		0	$\frac{1}{7}:5$	$\frac{5}{14}:5$	0
	$\mathrm{Re}(a_5a_6^*)$	0	$\frac{1}{7}:\frac{10}{7}$	$\frac{5}{7}:\frac{5}{14}$	0		0	$\frac{1}{7}:\frac{10}{7}$	$\frac{5}{7}:\frac{5}{14}$	0
	$\mathrm{Re}(a_7a_8^*)$	0	$-\frac{1}{7}:\frac{1}{3}$	$-\frac{15}{77}:3$	$-\frac{25}{11}:\frac{1}{3}$		0	$\frac{1}{7}:\frac{1}{3}$	$\frac{15}{77}:3$	$\frac{25}{11}:\frac{1}{3}$

$4\pi\sigma_{00}\widetilde{A}_{xx}$	Amplitude product	a_0^{xx}	a_2^{xx}	a_4^{xx}	a_6^{xx}	$4\pi\sigma_{00}\widetilde{C}_{xx}$	Amplitude product	c_2^{xx}	c_4^{xx}
	$\lvert a_0\rvert^2$	$-\frac{1}{4}$	0	0	0		$\lvert a_3\rvert^2$	$\frac{1}{16}$	0
	$\lvert a_2\rvert^2$	$-\frac{1}{4}$	$-\frac{1}{4}$	0	0		$\lvert a_4\rvert^2$	$-\frac{5}{112}$	$\frac{1}{42}$
	$\lvert a_4\rvert^2$	$\frac{1}{6}$	$\frac{5}{42}$	$-\frac{2}{7}$	0		$\lvert a_5\rvert^2$	$-\frac{5}{392}$	$\frac{1}{147}$
	$\lvert a_5\rvert^2$	$\frac{3}{28}$	$\frac{15}{196}$	$-\frac{9}{49}$	0		$\lvert a_6\rvert^2$	$\frac{1}{14}$	$\frac{1}{84}$
	$\lvert a_7\rvert^2$	$-\frac{1}{4}$	$-\frac{2}{7}$	$-\frac{3}{14}$	0				
	$\lvert a_8\rvert^2$	$-\frac{1}{4}$	$-\frac{25}{84}$	$-\frac{81}{308}$	$-\frac{25}{132}$		$\mathrm{Re}(a_1a_3^*)$	$-\frac{1}{4}:\frac{1}{2}$	0
							$\mathrm{Re}(a_1a_4^*)$	$\frac{1}{12}:\frac{5}{2}$	0
	$\mathrm{Re}(a_0a_2^*)$	0	$:\frac{1}{2}$	0	0		$\mathrm{Re}(a_1a_5^*)$	$\frac{1}{12}:\frac{5}{7}$	0
	$\mathrm{Re}(a_0a_7^*)$	0	$-\frac{1}{2}:3$	0	0		$\mathrm{Re}(a_1a_6^*)$	$-\frac{1}{12}:\frac{1}{2}$	0
	$\mathrm{Re}(a_0a_8^*)$	0	0	1	0		$\mathrm{Re}(a_3a_4^*)$	$\frac{1}{24}:5$	0

TABLE II. (*Continued*).

$4\pi\sigma_{00}\tilde{A}_{xx}$	Amplitude product	a_0^{xx}	a_2^{xx}	a_4^{xx}	a_6^{xx}
	$\mathrm{Re}(a_2a_7^*)$	0	$\frac{1}{7}:\frac{3}{2}$	$\frac{3}{7}:6$	0
	$\mathrm{Re}(a_2a_8^*)$	0	$-\frac{9}{7}:\frac{1}{2}$	$-\frac{5}{7}:\frac{1}{2}$	0
	$\mathrm{Re}(a_4a_5^*)$	$-1:\frac{1}{14}$	$-\frac{5}{7}:\frac{1}{14}$	$\frac{6}{7}:\frac{2}{7}$	0
	$\mathrm{Re}(a_7a_8^*)$	0	$\frac{1}{7}:\frac{1}{3}$	$\frac{15}{77}:3$	$\frac{25}{11}:\frac{1}{3}$

$4\pi\sigma_{00}\tilde{C}_{xx}$	Amplitude product	c_2^{xx}	c_4^{xx}
	$\mathrm{Re}(a_3a_5^*)$	$\frac{1}{12}:\frac{5}{14}$	0
	$\mathrm{Re}(a_3a_6^*)$	$\frac{1}{84}$	$-\frac{3}{56}$
	$\mathrm{Re}(a_4a_5^*)$	$-\frac{5}{28}:\frac{1}{14}$	$\frac{1}{21}:\frac{2}{7}$
	$\mathrm{Re}(a_4a_6^*)$	$-\frac{1}{28}:5$	$-\frac{1}{168}:5$
	$\mathrm{Re}(a_5a_6^*)$	$-\frac{1}{14}:\frac{5}{14}$	$-\frac{1}{84}:\frac{5}{14}$

$4\pi\sigma_{00}A_{y0}$	Amplitude product	b_1^{y0}	b_2^{y0}	b_3^{y0}	b_4^{y0}	b_5^{y0}
	$\mathrm{Im}(a_0a_1^*)$	$-\frac{1}{2}:\frac{1}{2}$	0	0	0	0
	$\mathrm{Im}(a_0a_3^*)$	$\frac{1}{2}$	0	0	0	0
	$\mathrm{Im}(a_0a_6^*)$	0	0	$-\frac{1}{4}$	0	0
	$\mathrm{Im}(a_1a_2^*)$	$\frac{1}{4}$	0	0	0	0
	$\mathrm{Im}(a_1a_4^*)$	0	$\frac{1}{6}:\frac{5}{2}$	0	0	0
	$\mathrm{Im}(a_1a_5^*)$	0	$-\frac{1}{4}:\frac{5}{7}$	0	0	0
	$\mathrm{Im}(a_1a_7^*)$	0	0	$\frac{1}{2}:\frac{1}{6}$	0	0
	$\mathrm{Im}(a_1a_8^*)$	0	0	$\frac{1}{4}:\frac{1}{2}$	0	0
	$\mathrm{Im}(a_2a_3^*)$	$\frac{1}{20}:\frac{1}{2}$	0	$-\frac{3}{10}:\frac{1}{2}$	0	0
	$\mathrm{Im}(a_2a_4^*)$	$-\frac{3}{4}:\frac{1}{10}$	0	$-\frac{1}{2}:\frac{1}{10}$	0	0
	$\mathrm{Im}(a_2a_5^*)$	$-\frac{3}{4}:\frac{1}{35}$	0	$-\frac{1}{2}:\frac{1}{35}$	0	0
	$\mathrm{Im}(a_2a_6^*)$	$\frac{3}{5}:\frac{1}{2}$	0	$\frac{3}{20}:\frac{1}{2}$	0	0
	$\mathrm{Im}(a_3a_4^*)$	0	$\frac{1}{12}:5$	0	0	0
	$\mathrm{Im}(a_3a_5^*)$	0	$-\frac{1}{4}:\frac{5}{14}$	0	0	0
	$\mathrm{Im}(a_3a_7^*)$	$\frac{3}{20}:3$	0	$-\frac{1}{5}:\frac{1}{3}$	0	0
	$\mathrm{Im}(a_3a_8^*)$	0	0	$-\frac{1}{24}$	0	$\frac{1}{6}$
	$\mathrm{Im}(a_4a_5^*)$	0	$\frac{25}{84}:\frac{1}{14}$	0	$-\frac{5}{7}:\frac{1}{14}$	0
	$\mathrm{Im}(a_4a_6^*)$	0	$-\frac{1}{21}:5$	0	$-\frac{1}{28}:5$	0
	$\mathrm{Im}(a_4a_7^*)$	$\frac{1}{4}:\frac{3}{5}$	0	$\frac{1}{2}:\frac{1}{15}$	0	0
	$\mathrm{Im}(a_4a_8^*)$	0	0	$\frac{5}{72}:5$	0	$\frac{1}{18}:5$
	$\mathrm{Im}(a_5a_6^*)$	0	$\frac{1}{7}:\frac{5}{14}$	0	$\frac{3}{28}:\frac{5}{14}$	0
	$\mathrm{Im}(a_5a_7^*)$	$\frac{1}{2}:\frac{3}{70}$	0	$:\frac{1}{210}$	0	0
	$\mathrm{Im}(a_5a_8^*)$	0	0	$\frac{5}{36}:\frac{5}{14}$	0	$\frac{1}{9}:\frac{5}{14}$
	$\mathrm{Im}(a_6a_7^*)$	$\frac{1}{70}:3$	0	$\frac{1}{10}:\frac{1}{3}$	0	$\frac{5}{14}:\frac{1}{3}$
	$\mathrm{Im}(a_6a_8^*)$	$\frac{9}{28}$	0	$-\frac{1}{36}$	0	$-\frac{11}{252}$

$4\pi\sigma_{00}A_{zx}$	Amplitude product	b_1^{zx}	b_2^{zx}	b_3^{zx}	b_4^{zx}	b_5^{zx}
	$\lvert a_4 \rvert^2$	0	$-\frac{5}{84}$	0	$\frac{1}{7}$	0
	$\lvert a_5 \rvert^2$	0	$\frac{5}{196}$	0	$-\frac{3}{49}$	0
	$\mathrm{Re}(a_0a_1^*)$	$-\frac{1}{2}:\frac{1}{2}$	0	0	0	0
	$\mathrm{Re}(a_0a_3^*)$	$\frac{1}{2}$	0	0	0	0

TABLE II. (*Continued*).

$4\pi\sigma_{00}A_{zx}$	Amplitude product	b_1^{zx}	b_2^{zx}	b_3^{zx}	b_4^{zx}	b_5^{zx}
	$\mathrm{Re}(a_0a_6^*)$	0	0	$-\frac{1}{4}$	0	0
	$\mathrm{Re}(a_1a_2^*)$	$-\frac{1}{4}$	0	0	0	0
	$\mathrm{Re}(a_1a_4^*)$	0	$\frac{1}{6}:\frac{5}{2}$	0	0	0
	$\mathrm{Re}(a_1a_5^*)$	0	$-\frac{1}{4}:\frac{5}{7}$	0	0	0
	$\mathrm{Re}(a_1a_7^*)$	0	0	$-\frac{1}{2}:\frac{1}{6}$	0	0
	$\mathrm{Re}(a_1a_8^*)$	0	0	$-\frac{1}{4}:\frac{1}{2}$	0	0
	$\mathrm{Re}(a_2a_3^*)$	$\frac{1}{20}:\frac{1}{2}$	0	$-\frac{3}{10}:\frac{1}{2}$	0	0
	$\mathrm{Re}(a_2a_4^*)$	$-\frac{3}{4}:\frac{1}{10}$	0	$-\frac{1}{2}:\frac{1}{10}$	0	0
	$\mathrm{Re}(a_2a_5^*)$	$-\frac{3}{4}:\frac{1}{35}$	0	$-\frac{1}{2}:\frac{1}{35}$	0	0
	$\mathrm{Re}(a_2a_6^*)$	$\frac{3}{5}:\frac{1}{2}$	0	$\frac{3}{20}:\frac{1}{2}$	0	0
	$\mathrm{Re}(a_3a_4^*)$	0	$\frac{1}{12}:5$	0	0	0
	$\mathrm{Re}(a_3a_5^*)$	0	$-\frac{1}{4}:\frac{5}{14}$	0	0	0
	$\mathrm{Re}(a_3a_7^*)$	$-\frac{3}{20}:3$	0	$\frac{1}{5}:\frac{1}{3}$	0	0
	$\mathrm{Re}(a_3a_8^*)$	0	0	$\frac{1}{24}$	0	$-\frac{1}{6}$
	$\mathrm{Re}(a_4a_5^*)$	0	$\frac{5}{84}:\frac{1}{14}$	0	$-\frac{1}{7}:\frac{1}{14}$	0
	$\mathrm{Re}(a_4a_6^*)$	0	$\frac{1}{21}:5$	0	$\frac{1}{28}:5$	0
	$\mathrm{Re}(a_4a_7^*)$	$-\frac{1}{4}:\frac{3}{5}$	0	$-\frac{1}{2}:\frac{1}{15}$	0	0
	$\mathrm{Re}(a_4a_8^*)$	0	0	$-\frac{5}{72}:5$	0	$-\frac{1}{18}:5$
	$\mathrm{Re}(a_5a_6^*)$	0	$-\frac{1}{7}:\frac{5}{14}$	0	$-\frac{3}{28}:\frac{5}{14}$	0
	$\mathrm{Re}(a_5a_7^*)$	$-\frac{1}{2}:\frac{3}{70}$	0	$-1:\frac{1}{210}$	0	0
	$\mathrm{Re}(a_5a_8^*)$	0	0	$-\frac{5}{36}:\frac{5}{14}$	0	$-\frac{1}{9}:\frac{5}{14}$
	$\mathrm{Re}(a_6a_7^*)$	$-\frac{1}{70}:3$	0	$-\frac{1}{10}:\frac{1}{3}$	0	$-\frac{5}{14}:\frac{1}{3}$
	$\mathrm{Re}(a_6a_8^*)$	$-\frac{9}{28}$	0	$\frac{1}{36}$	0	$\frac{11}{252}$

$4\pi\sigma_{00}it_{11}$	Amplitude product	b_2^{11}	b_4^{11}	b_6^{11}
	$\mathrm{Im}(a_0a_2^*)$	$-\frac{1}{4}:\frac{3}{2}$	0	0
	$\mathrm{Im}(a_0a_7^*)$	$-\frac{1}{4}$	0	0
	$\mathrm{Im}(a_0a_8^*)$	0	$-\frac{1}{8}:3$	0
	$\mathrm{Im}(a_1a_3^*)$	$-\frac{1}{8}:\frac{3}{2}$	0	0
	$\mathrm{Im}(a_1a_4^*)$	$-\frac{1}{8}:\frac{5}{6}$	0	0
	$\mathrm{Im}(a_1a_5^*)$	$-\frac{1}{8}:\frac{5}{21}$	0	0
	$\mathrm{Im}(a_1a_6^*)$	$\frac{1}{2}:\frac{1}{6}$	0	0
	$\mathrm{Im}(a_2a_7^*)$	$\frac{5}{28}:\frac{1}{2}$	$\frac{9}{28}:\frac{1}{2}$	0
	$\mathrm{Im}(a_2a_8^*)$	$\frac{3}{14}:\frac{3}{2}$	$\frac{1}{28}:\frac{3}{2}$	0
	$\mathrm{Im}(a_3a_4^*)$	$\frac{1}{8}:\frac{5}{3}$	0	0
	$\mathrm{Im}(a_3a_5^*)$	$\frac{1}{4}:\frac{5}{42}$	0	0
	$\mathrm{Im}(a_3a_6^*)$	$-\frac{5}{28}:\frac{1}{3}$	$\frac{9}{112}:3$	0
	$\mathrm{Im}(a_4a_6^*)$	$\frac{1}{28}:15$	$\frac{3}{112}:15$	0

$4\pi\sigma_{00}t_{20}$	Amplitude product	a_0^{20}	a_2^{20}	a_4^{20}	a_6^{20}	a_8^{20}
	$\|a_0\|^2$	$-\frac{1}{2}:\frac{1}{2}$	0	0	0	0
	$\|a_1\|^2$	0	$\frac{1}{4}:\frac{1}{2}$	0	0	0
	$\|a_2\|^2$	$-\frac{1}{20}:\frac{1}{2}$	$-\frac{5}{28}:\frac{1}{2}$	$-\frac{27}{35}:\frac{1}{2}$	0	0
	$\|a_3\|^2$	$-\frac{1}{4}:\frac{1}{2}$	$\frac{3}{8}:\frac{1}{2}$	0	0	0
	$\|a_4\|^2$	$\frac{5}{12}:\frac{1}{2}$	$\frac{5}{24}:\frac{1}{2}$	0	0	0
	$\|a_5\|^2$	$\frac{5}{28}:\frac{1}{2}$	$\frac{5}{49}:\frac{1}{2}$	$-\frac{5}{49}:\frac{1}{2}$	0	0
	$\|a_6\|^2$	$-\frac{1}{14}:\frac{1}{2}$	$-\frac{3}{28}:\frac{1}{2}$	$-\frac{2}{77}:2$	$\frac{225}{308}:\frac{1}{2}$	0
	$\|a_7\|^2$	$-\frac{1}{5}:\frac{1}{2}$	$-\frac{5}{14}:\frac{1}{2}$	$-\frac{33}{35}:\frac{1}{2}$	0	0
	$\|a_8\|^2$	$-\frac{1}{12}:\frac{1}{2}$	$-\frac{125}{924}:\frac{1}{2}$	$-\frac{81}{364}:\frac{1}{2}$	$-\frac{5}{12}:\frac{1}{2}$	$-\frac{245}{429}:2$
	$\mathrm{Re}(a_0a_2^*)$	0	1	0	0	0
	$\mathrm{Re}(a_0a_7^*)$	0	$-1:\frac{3}{2}$	0	0	0
	$\mathrm{Re}(a_0a_8^*)$	0	0	:2	0	0
	$\mathrm{Re}(a_1a_3^*)$	$\frac{1}{2}$	$-\frac{1}{4}$	0	0	0
	$\mathrm{Re}(a_1a_4^*)$	0	$\frac{1}{4}:5$	0	0	0

TABLE II. (*Continued*).

$4\pi\sigma_{00}it_{11}$	Amplitude product	b_2^{11}	b_4^{11}	b_6^{11}
	$\mathrm{Im}(a_5a_6^*)$	$\frac{1}{14}:\frac{15}{14}$	$\frac{3}{56}:\frac{15}{14}$	0
	$\mathrm{Im}(a_7a_8^*)$	$-\frac{1}{12}$	$-\frac{9}{88}$	$-\frac{25}{132}$

$4\pi\sigma_{00}t_{20}$	Amplitude product	a_0^{20}	a_2^{20}	a_4^{20}	a_6^{20}	a_8^{20}
	$\mathrm{Re}(a_1a_5^*)$	0	$\frac{1}{2}:\frac{5}{14}$	0	0	0
	$\mathrm{Re}(a_1a_6^*)$	0	$-\frac{1}{7}$	$\frac{9}{14}$	0	0
	$\mathrm{Re}(a_2a_7^*)$	$\frac{3}{10}:3$	$\frac{5}{14}:3$	$\frac{12}{35}:3$	0	0
	$\mathrm{Re}(a_2a_8^*)$	0	$-\frac{3}{14}$	$-\frac{65}{154}$	$-\frac{15}{11}$	0
	$\mathrm{Re}(a_3a_4^*)$	0	$\frac{1}{4}:\frac{5}{2}$	0	0	0
	$\mathrm{Re}(a_3a_5^*)$	0	$\frac{1}{4}:\frac{5}{7}$	0	0	0
	$\mathrm{Re}(a_3a_6^*)$	0	$\frac{4}{7}:2$	$-\frac{9}{14}:\frac{1}{2}$	0	0
	$\mathrm{Re}(a_4a_5^*)$	0	$-\frac{5}{28}:\frac{1}{7}$	$\frac{10}{7}:\frac{1}{7}$	0	0
	$\mathrm{Re}(a_4a_6^*)$	0	$\frac{1}{7}:\frac{5}{2}$	$\frac{5}{14}:\frac{5}{2}$	0	0
	$\mathrm{Re}(a_5a_6^*)$	0	$\frac{1}{7}:\frac{5}{7}$	$\frac{5}{14}:\frac{5}{7}$	0	0
	$\mathrm{Re}(a_7a_8^*)$	0	$\frac{17}{7}:\frac{1}{6}$	$\frac{45}{77}:\frac{3}{2}$	$\frac{10}{11}:\frac{2}{3}$	0

$4\pi\sigma_{00}t_{21}$	Amplitude product	b_2^{21}	b_4^{21}	b_6^{21}	b_8^{21}
	$\lvert a_1\rvert^2$	$\frac{1}{8}:\frac{1}{3}$	0	0	0
	$\lvert a_2\rvert^2$	$-\frac{1}{28}:3$	$-\frac{9}{140}:3$	0	0
	$\lvert a_3\rvert^2$	$-\frac{1}{8}:\frac{1}{3}$	0	0	0
	$\lvert a_6\rvert^2$	$-\frac{1}{56}:3$	$-\frac{3}{616}:3$	$\frac{25}{616}:3$	0
	$\lvert a_7\rvert^2$	$\frac{1}{28}:3$	$\frac{9}{140}:3$	0	0
	$\lvert a_8\rvert^2$	$-\frac{25}{462}:\frac{1}{3}$	$-\frac{81}{4004}:3$	$-\frac{5}{66}:\frac{1}{3}$	$-\frac{245}{1716}:\frac{1}{3}$
	$\mathrm{Re}(a_0a_2^*)$	$\frac{1}{4}:\frac{3}{2}$	0	0	0
	$\mathrm{Re}(a_0a_7^*)$	$\frac{1}{4}$	0	0	0
	$\mathrm{Re}(a_0a_8^*)$	0	$\frac{1}{8}:3$	0	0
	$\mathrm{Re}(a_1a_3^*)$	$-\frac{1}{8}:\frac{1}{6}$	0	0	0
	$\mathrm{Re}(a_1a_4^*)$	$\frac{1}{8}:\frac{5}{6}$	0	0	0
	$\mathrm{Re}(a_1a_5^*)$	$\frac{1}{8}:\frac{5}{21}$	0	0	0
	$\mathrm{Re}(a_1a_6^*)$	$-\frac{1}{14}:\frac{1}{6}$	$\frac{3}{28}:\frac{3}{2}$	0	0
	$\mathrm{Re}(a_2a_7^*)$	$\frac{1}{28}:\frac{1}{2}$	$\frac{9}{140}:\frac{1}{2}$	0	0
	$\mathrm{Re}(a_2a_8^*)$	$-\frac{1}{14}:\frac{1}{6}$	$-\frac{23}{308}:\frac{3}{2}$	$-\frac{5}{11}:\frac{1}{6}$	0
	$\mathrm{Re}(a_3a_4^*)$	$-\frac{1}{8}:\frac{5}{3}$	0	0	0
	$\mathrm{Re}(a_3a_5^*)$	$-\frac{1}{4}:\frac{5}{42}$	0	0	0
	$\mathrm{Re}(a_3a_6^*)$	$\frac{11}{28}:\frac{1}{3}$	$-\frac{3}{112}:3$	0	0
	$\mathrm{Re}(a_4a_6^*)$	$\frac{1}{28}:15$	$\frac{3}{112}:15$	0	0
	$\mathrm{Re}(a_5a_6^*)$	$\frac{1}{14}:\frac{15}{14}$	$\frac{3}{56}:\frac{15}{14}$	0	0
	$\mathrm{Re}(a_7a_8^*)$	$\frac{23}{84}$	$\frac{39}{616}$	$\frac{5}{132}$	0

$4\pi\sigma_{00}t_{22}$	Amplitude product	c_2^{22}	c_4^{22}	c_6^{22}	c_8^{22}
	$\lvert a_1\rvert^2$	$\frac{1}{16}:\frac{1}{3}$	0	0	0
	$\lvert a_2\rvert^2$	$-\frac{3}{112}:3$	$-\frac{3}{280}:3$	0	0
	$\lvert a_3\rvert^2$	$\frac{1}{32}:\frac{1}{3}$	0	0	0
	$\lvert a_4\rvert^2$	$\frac{5}{32}:\frac{1}{3}$	0	0	0
	$\lvert a_5\rvert^2$	$\frac{5}{196}:3$	$\frac{5}{392}:\frac{1}{3}$	0	0
	$\lvert a_6\rvert^2$	$-\frac{1}{56}:3$	$-\frac{1}{308}:\frac{1}{3}$	$\frac{5}{1232}:3$	0
	$\lvert a_7\rvert^2$	$-\frac{1}{56}:3$	$-\frac{1}{140}:3$	0	0
	$\lvert a_8\rvert^2$	$-\frac{125}{1848}:\frac{1}{3}$	$-\frac{45}{8008}:3$	$-\frac{5}{528}:\frac{1}{3}$	$-\frac{35}{3432}:\frac{1}{3}$
	$\mathrm{Re}(a_1a_3^*)$	$\frac{1}{8}:\frac{1}{6}$	0	0	0
	$\mathrm{Re}(a_1a_4^*)$	$-\frac{1}{8}:\frac{5}{6}$	0	0	0
	$\mathrm{Re}(a_1a_5^*)$	$-\frac{1}{8}:\frac{5}{21}$	0	0	0
	$\mathrm{Re}(a_1a_6^*)$	$\frac{1}{14}:\frac{1}{6}$	$\frac{1}{56}:\frac{3}{2}$	0	0
	$\mathrm{Re}(a_2a_7^*)$	$-\frac{3}{28}:\frac{1}{2}$	$-\frac{3}{70}:\frac{1}{2}$	0	0
	$\mathrm{Re}(a_2a_8^*)$	$\frac{5}{56}:\frac{1}{6}$	$-\frac{9}{616}:\frac{3}{2}$	$-\frac{1}{22}:\frac{1}{6}$	0
	$\mathrm{Re}(a_3a_4^*)$	$-\frac{1}{16}:\frac{5}{3}$	0	0	0
	$\mathrm{Re}(a_3a_5^*)$	$-\frac{1}{8}:\frac{5}{42}$	0	0	0
	$\mathrm{Re}(a_3a_6^*)$	$\frac{1}{28}:\frac{1}{3}$	$\frac{1}{112}:3$	0	0
	$\mathrm{Re}(a_4a_5^*)$	$-\frac{5}{56}:\frac{3}{14}$	$-\frac{5}{14}:\frac{1}{42}$	0	0
	$\mathrm{Re}(a_4a_6^*)$	$\frac{1}{56}:15$	$\frac{1}{112}:\frac{5}{3}$	0	0
	$\mathrm{Re}(a_5a_6^*)$	$\frac{1}{28}:\frac{15}{14}$	$\frac{1}{56}:\frac{5}{42}$	0	0
	$\mathrm{Re}(a_7a_8^*)$	$\frac{5}{168}$	$-\frac{9}{616}$	$-\frac{1}{66}$	0

gion of -0.9 qualitatively agreeing with our naive a_2 dominance prediction. A very similar and, as it turns out, related phenomenon occurs for the tensor polarizations. In Fig. 3 we present results for the polarization transfer observables specifically expressed in the laboratory frame in order to be most useful for considerations of possible experiments. Again the three curves on each graph correspond to the three energies 383, 567, and 800 MeV. For each of the tensor observables t_{20}, t_{21}, and t_{22}, we note the similarity between the different energy curves. Moreover, the a_2 dominance predictions corresponding to the c.m. relations of Eq. (50) also follow the same shapes (in

TABLE III. Observables in $pp \to d\pi^+$ having no contribution from $|a_2|^2$, together with a list of those amplitudes that, in the contribution to an observable, have products with the large a_2.

Observable	Amplitudes having cross terms with a_2	Observable	Amplitudes having cross terms with a_2
A_{y0}	a_1,a_3,a_4,a_5,a_6	t_{10}^{11}	a_1,a_3,a_4,a_5,a_6
A_{zx}	a_1,a_3,a_4,a_5,a_6	t_{11}^{11}	a_1,a_3,a_4,a_5,a_6
$\widetilde{C}_{xx}$	none	t_{1-1}^{11}	a_1,a_3,a_4,a_5,a_6
it_{11}	a_0,a_7,a_8	it_{20}^{11}	a_1,a_3,a_4,a_5,a_6
t_{10}^{10}	a_4,a_5	it_{21}^{11}	a_1,a_3,a_4,a_5,a_6
t_{11}^{10}	a_4,a_5	it_{2-1}^{11}	a_1,a_3,a_4,a_5,a_6
t_{21}^{10}	a_4,a_5	it_{22}^{11}	a_1,a_3,a_4,a_5,a_6
t_{22}^{10}	a_4,a_5	it_{2-2}^{11}	a_1,a_3,a_4,a_5,a_6

fact they are virtually indistinguishable from the 567 MeV results). The relative constancy of these curves is in marked contrast to the large energy dependence of it_{11}, t_{11}^{11}, and indeed most other transfer observables. Although there are as yet no measurements of any of the polarization tensors, there is nevertheless indirect experimental confirmation of the constancy of these quantities. This is provided by the relation[31]

$$1+3A_{yy}=2\sqrt{3}t_{22}+\sqrt{2}t_{20} \tag{51}$$

which holds rigorously for the reaction $pp \to \pi^+ d$. We note that Eq. (51) applies equally well for the c.m. and laboratory frames since each side of the equation is invariant under rotations about the y axis. Of course Eqs. (50) and (51), taken together, imply that $A_{yy}=-1$ independent of energy and angle; as we have noted, this is at least qualitatively what is measured. To reconcile this success of the a_2 dominance assumption for the tensor polarizations with its failure for A_{xx} and A_{zz}, we are led to assume that there must be some type of cancellation taking place within the expressions for the tensor observables. Unfortunately, the exact nature of this cancellation is not easily seen from Table II.

The model dependence of the above observables is addressed in Figs. 4 and 5 where we compare calculations resulting from the three choices $M1$, $P6$, and $BO8$ of the input P_{11} interaction. The energy chosen for this comparison is 567 MeV although we note that comparisons at other energies do not result in substantially different conclusions. Again the most striking observation from Figs. 4 and 5 is the constancy of the tensor polarizations even

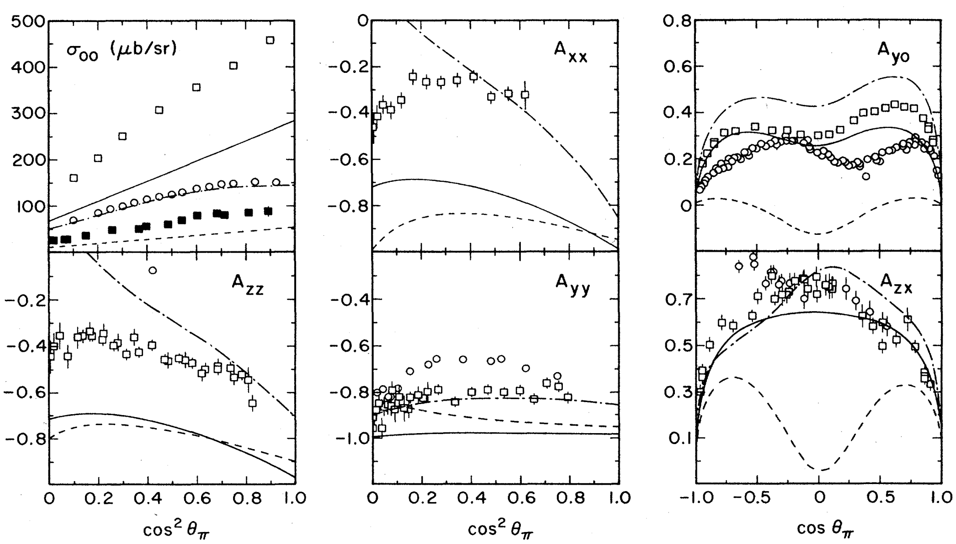

FIG. 2. Energy dependence of the center of mass differential cross section (σ_{00}) and the five correlation observables for the reaction $\vec{p}(\vec{p},\pi^+)d$. The three curves are results from a few-body calculation with the "$M1$" P_{11} interaction (Ref. 29) and correspond to the proton laboratory kinetic energies of 567 MeV (solid curve), 383 MeV (dash curve), and 800 MeV (dash-dot curve). The experimental points are distinguished only by energy as follows: 567 MeV (open squares; Refs. 34—38), 383 MeV (solid squares; Ref. 39), and 800 MeV (open circles; Refs. 14, and 40—42).

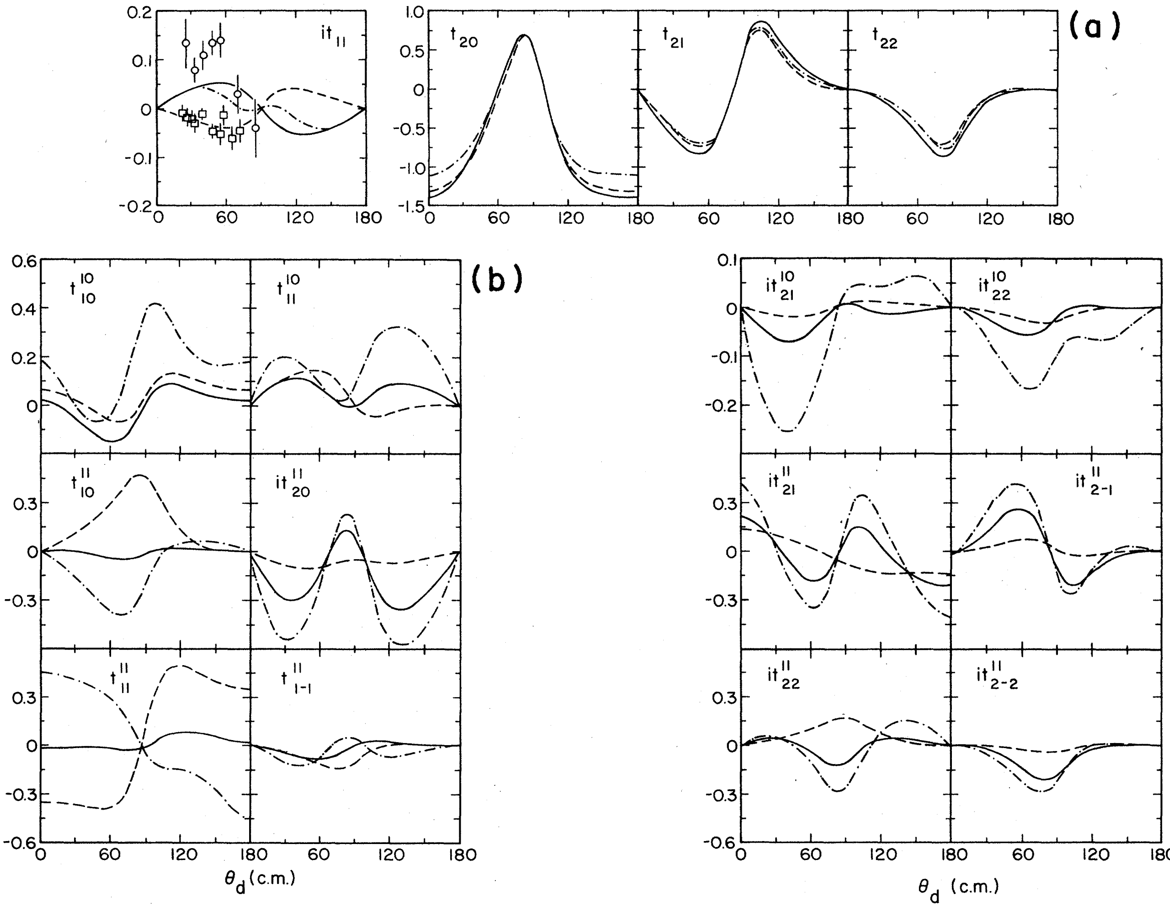

FIG. 3. Energy dependence of the laboratory-system polarization transfer observables for the reaction $p(\vec{p},\vec{d})\pi^+$. The three curves are results from a few-body calculation with the $M1$ P_{11} interaction (Ref. 29) and correspond to the proton laboratory kinetic energies of 567 MeV (solid curve), 383 MeV (dash curve), and 800 MeV (dash-dot curve). The experimental points for it_{11} are from Ref. 27 and are labeled as follows: 567 MeV (open squares) and 800 MeV (open circles).

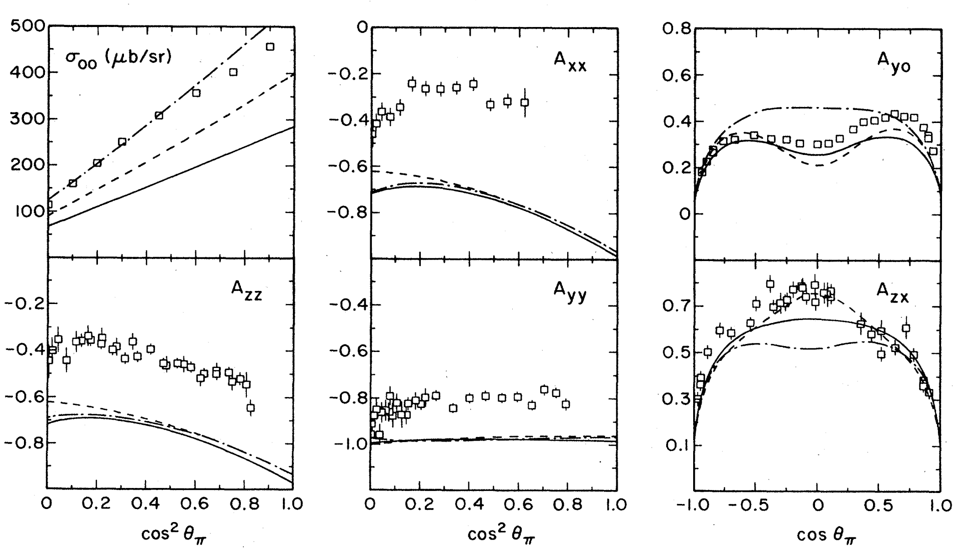

FIG. 4. Model dependence of the center of mass differential cross section (σ_{00}) and the five correlation observables for the reaction $\vec{p}(\vec{p},\pi^+)d$ at the proton laboratory kinetic energy of 567 MeV. The three curves are results from few-body calculations using the P_{11} interactions $M1$ (solid curve) (Ref. 29), $P6$ (dash curve) (Ref. 29), and $BO8$ (dash-dot curve) (Ref. 7). The experimental points are from Refs. 34–38.

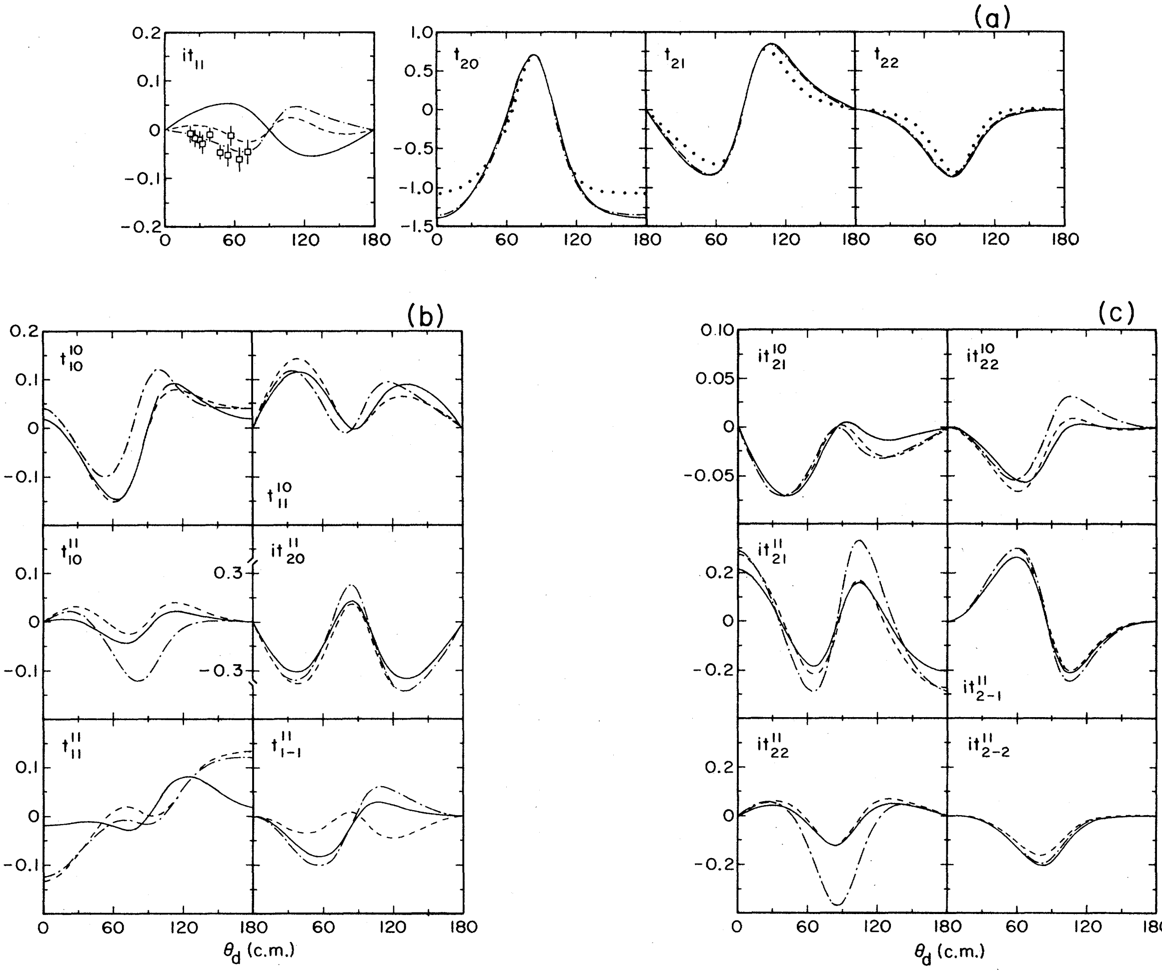

FIG. 5. Model dependence of the laboratory-system polarization transfer observables for the reaction $p(\vec{p},\vec{d})\pi^+$ at the proton laboratory kinetic energy of 567 MeV. Three of the curves are results from few-body calculations using the P_{11} interactions $M1$ (solid curve) (Ref. 29), $P6$ (dash curve) (Ref. 29), and $BO8$ (dash-dot curve) (Ref. 7). The dotted curve is the 575 MeV result of the Osaka group's amplitude search (Ref. 28). The experimental points are from Ref. 27.

though the corresponding cross sections differ by as much as 100%. In fact many of the spin-dependent observables display weak model dependence, and those that have $|a_2|^2$ contributions in the numerator (A_{xx}, A_{yy}, A_{zz}, t_{20}, t_{21}, and t_{22}) are particularly insensitive to the differences in our choice of models. Thus the main effect of choosing different P_{11} interactions appears to be a simple scaling of the partial wave amplitudes and any hope of agreement with A_{xx}, A_{yy}, and A_{zz} data will need to come from a mechanism that is probably missing from the unitary models (perhaps the backward going pions?). One may consequently wonder whether an experimental determination of the tensor polarizations could yield results different from our predictions despite the fact that they are totally insensitive to our models. One possible way to try and answer this question is to compare our predictions with that of an amplitude search solution that fits available data. Such a comparison is included in Fig. 5 where we plot the tensor polarizations resulting from the Osaka group's amplitude search[28] at 575 MeV (dotted curve). For t_{20}, differences between our predictions and those of the amplitude search are small for angles between 40 and 120 degrees. The largest difference occurs at extreme forward and backward angles; there, t_{22} is approximately zero and by Eq. (51) t_{20} is determined by the existing experiments for A_{yy}. It therefore seems likely that unless one can do a very accurate experiment, no new information will be obtained from measuring t_{20}. Similar arguments follow for t_{22}. For t_{21}, there are small but significant differences and again accurate measurements would be needed for their determination. Some of the observables not involving $|a_2|^2$ in the numerator are much more able to discriminate between our three models. In this respect A_{y0}, A_{zx}, it_{11}, t^{11}_{11}, t^{10}_{10}, t^{11}_{10}, it^{11}_{21}, and it^{11}_{22} appear to be the most useful.

Another way to exploit Table II is to look for useful combinations of observables. For example, it is apparent that the combinations $\sigma_{00}(1+A_{zz})$, $\sigma_{00}(1+A_{xx})$, and

$\sigma_{00}(1+A_{yy})$ depend only on triplet amplitudes (i.e., initial protons are in a triplet state). The complementary quantities $\sigma_{00}(1-A_{zz})$, $\sigma_{00}(1-A_{xx})$, and $\sigma_{00}(1-A_{yy})$ depend mainly on singlet amplitudes—the triplet contribution coming only from the $J^\pi=2^-,4^-,\ldots$ channels. Thus the use of these quantities for comparing theory and experiment can focus more directly on the physical content of a calculation; in this case, for example, we would be able to say something about the spin-orbit force in the calculation. Indeed such a comparison, using $\sigma_{00}(1\pm A_{zz})$ has already been used by Glass *et al.*[41] in presenting their data. From Table II we find that the combination $\frac{1}{2}\sigma_{00}(1-A_{zz}+A_{xx}+A_{yy})$ depends only on $J^\pi=2^-,4^-,\ldots$ amplitudes; moreover it forms precisely the triplet component of $\sigma_{00}(1-A_{zz})$. Thus it is $\sigma_{00}(1-A_{zz}-A_{xx}-A_{yy})$ which depends purely on singlet amplitudes. At medium energies where we can neglect the channels $4^-,6^-,\ldots$, we get the remarkable result that $\sigma_{00}(1-A_{zz}+A_{xx}+A_{yy})$ depends only on the two 2^- amplitudes a_4 and a_5. As $A_{yy}\sim-1$ and $A_{xx}\sim A_{zz}$, rather accurate experimental results are needed to determine a_4 and a_5 from this linear combination. Although other combinations like $\sigma_{00}(1-\sqrt{2}t_{20})$ (does not depend on $J^\pi=2^-,4^-,\ldots$) seem less interesting, these might still have use in future amplitude searches as they can reduce the number of amplitudes that need to be varied.

B. $\pi d\to\pi d$

Present polarization measurements of π-d elastic scattering have generated much speculation about the existence of dibaryons. This was initially sparked by a measurement of iT_{11} at 256 MeV that appeared to confirm the oscillatory behavior predicted in a prior calculation having dibaryon admixtures. Since then, iT_{11} has been remeasured with the most significant change being that there no longer appears to be an oscillation at backward angles. At forward angles the oscillation is now smaller than previously observed and overall the necessity for a dibaryon has all but disappeared. For the other currently measured polarization observable t_{20} there is remarkable disagreement between the results of Gruebler *et al.*[32] and Ungricht *et al.*[33] The results of the former group still hold out promise of an exotic phenomenon in π-d elastic scattering. At present we can only speculate as to the eventual outcome of these dilemmas. If there is something exotic then one would also expect to see a self-consistent behavior of other π-d elastic observables; indeed there may be observables where a signature might be better seen. Even in the absence of exotic effects the polarization measurements of $\pi\vec{d}\to\pi\vec{d}$ are still very interesting in that they may provide a way of resolving the ambiguity inherent in the separation of the P_{11} interaction into pole and nonpole pieces. In $\pi d\to\pi d$ the pole piece is responsible for true pion absorption while the nonpole piece contributes to multiple scattering in the P_{11} channel. These two pieces are expected to cancel to various extents for the different observables. Comparison with experimental cross sections seems to indicate a need for a rather complete cancellation between these two parts. If one believes the measurement of Ungricht *et al.* then this appears to be also true for t_{20}; however, there is no reason why there should be complete cancellation for all observables and one can envisage learning about the separate pieces of the P_{11} by a sum of polarization measurements. For these reasons we deem it timely to examine all the observables in $\pi\vec{d}\to\pi\vec{d}$ despite the obvious experimental difficulties of doing polarization transfer experiments for this reaction.

There are 25 independent observables describing $\pi\vec{d}\to\pi\vec{d}$ (as usual we assume parity conservation). Unfortunately, an analysis of observables in terms of the underlying bilinear combinations of partial wave amplitudes is not as fruitful for $\pi\vec{d}\to\pi\vec{d}$ as it was for $\vec{p}p\to\vec{d}\pi$. This is due in part to the large number of partial waves needed for a realistic description; while for $\vec{p}p\to\vec{d}\pi$ there were nine amplitudes contributing for J^π values up to 4^+, the corresponding number for $\pi\vec{d}\to\pi\vec{d}$ is 17. The other reason is that, as it turns out, very similar combinations of amplitudes contribute to every observable and no observable stands out as being indispensible for learning about particular partial waves. We shall therefore not present tables (like Table II) but instead simply give results of our calculations using the few-body model.

We feel that, at this stage, the most interesting aspect of investigating $\pi\vec{d}\to\pi\vec{d}$ observables is the above-mentioned possibility of obtaining information about the pole and nonpole pieces of the P_{11} interaction. We would therefore like to test the sensitivity of the polarization observables to different models of the P_{11} interaction, and for this purpose we employ the *M*1, *P*6, and *BO*8 choices introduced in Sec. III A. Moreover, we should emphasize that our π-d elastic amplitudes come from the very same calculation that generated the 567 MeV $pp\to d\pi^+$ results discussed previously; the corresponding kinetic energy in the π-d laboratory system is 140 MeV. In Fig. 6 we show the

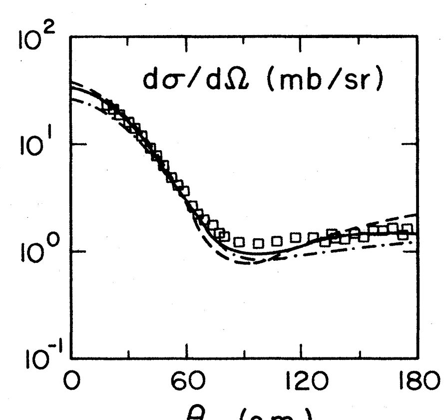

FIG. 6. Model dependence of the center of mass differential cross section for the reaction $d(\pi,\pi)d$ at the pion laboratory kinetic energy of 140 MeV. The three curves are results from few-body calculations using the P_{11} interactions *M*1 (solid curve) (Ref. 29), *P*6 (dash curve) (Ref. 29), and *BO*8 (dash-dot curve) (Ref. 7). The experimental points are from Ref. 43.

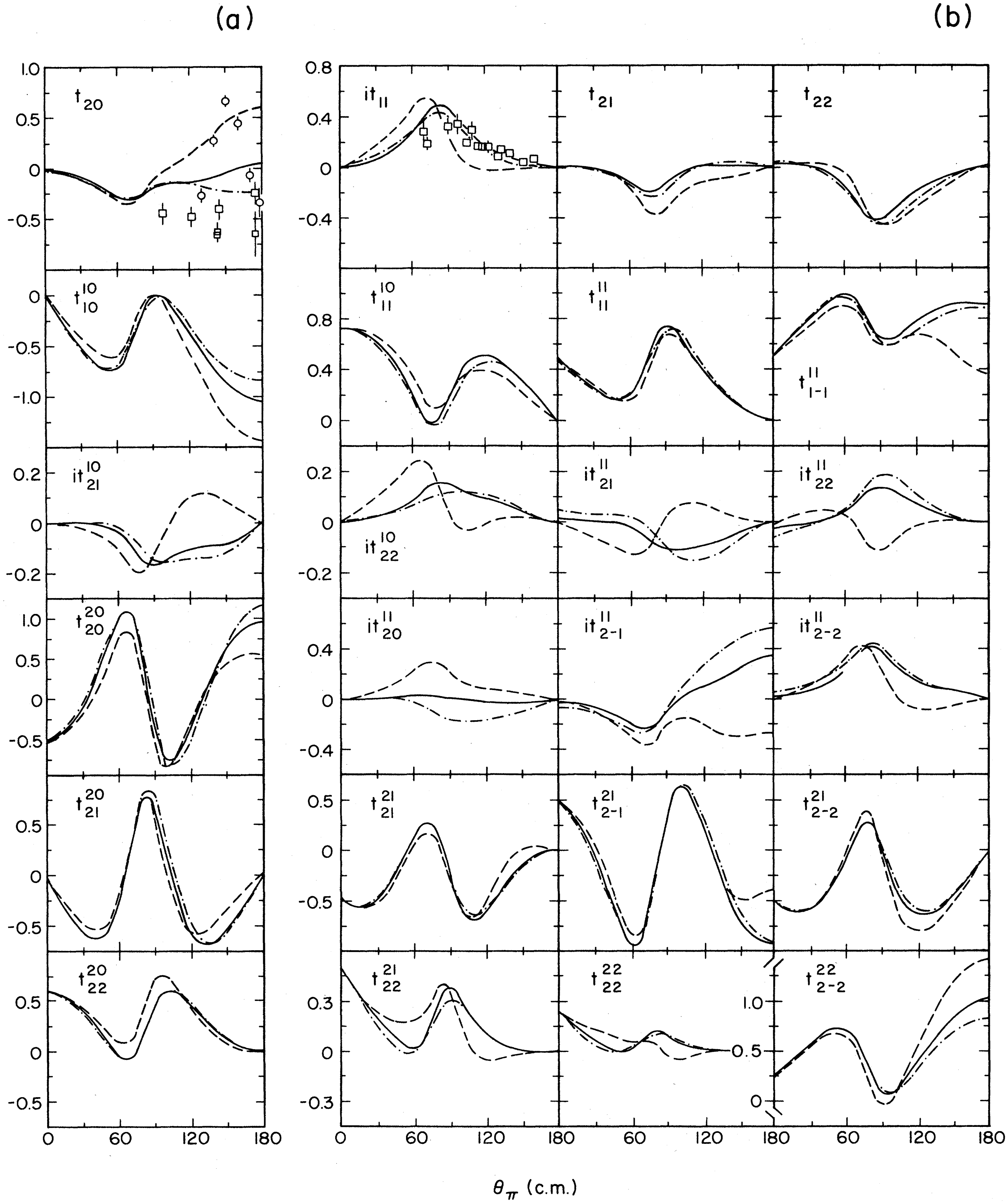

FIG. 7. Model dependence of the laboratory-system polarization transfer observables for the reaction $\vec{d}(\pi,\pi)\vec{d}$ at the pion laboratory kinetic energy of 140 MeV. The three curves are results from few-body calculations using the P_{11} interactions $M1$ (solid curve) (Ref. 29), $P6$ (dash curve) (Ref. 29), and $BO8$ (dash-dot curve) (Ref. 7). The experimental points are from Refs. 32, 33, and 44.

π-d differential cross section in the center of mass for each choice of P_{11} interaction. Although the interaction $M1$ provides the best fit in this case, it in fact provides the worst fit for the corresponding pp$\to$dπ^+ differential cross section. Thus there really is no "best" choice amongst the three P_{11} interactions. The spin dependent observables are presented in Fig. 7. Many of the observables display large sensitivity to our three choices of P_{11} model. It is significant that one of the most sensitive such observables is t_{20}. Not only is this quantity measurable with present technology, but the behavior of other observables, like t_{10}^{10}, it_{2-1}^{11}, and t_{2-2}^{22} appears so similar that we suspect that they all carry much of the same information. As t_{20} must be much less difficult to measure than the "transfer" observables, this should provide even more incentive for an extensive program to measure t_{20}. Experimentally the most tractable spin transfer observables would be the vector to vector ones. Of these the least difficult would probably be the ones involving polarizations perpendicular to the deuteron's momenta, namely t_{11}^{11} and t_{1-1}^{11}. In this respect it is extremely interesting that one of these, t_{1-1}^{11}, displays a sensitivity to the P_{11} interactions that appears rather different to that displayed by t_{20} (naturally there is a closer relation to it_{11}); in addition, the size of t_{1-1}^{11} can be quite large so that this observable should probably be the logical choice for the next generation of polarization experiments of π-d elastic scattering.

IV. SUMMARY

In this paper we have investigated the spin observables of the reactions $\vec{p}\vec{p}\to\pi^+$d, $\vec{p}$p$\to\vec{d}\pi^+$, and $\pi\vec{d}\to\pi\vec{d}$. To calculate all the observables we have specifically chosen a procedure that relates partial wave amplitudes directly to observables. We feel that an investigation of such direct relationships is important as it relates as closely as possible theoretical predictions that use a partial wave decomposition to an experimentally measurable quantity. Our procedure is similar to the one used by Mandl and Regge[20] for the reaction $\vec{p}$p$\to\pi^+$d and involves expressing an observable in terms of an orthogonal polynomial expansion as given by Eqs. (1) and (2). The factors $C_L^\nu(I,I')$ determine the contribution of the partial wave amplitude product $a_I a_{I'}^*$ to the observable in question and depend only on angular momentum recoupling coefficients. In Eqs. (23)—(28) we have given specific expressions for these factors for the case of $\vec{p}\vec{p}\to\pi^+$d spin-correlation observables. We have also derived an expression for the general polarization transfer reaction $\vec{a}+b\to\vec{c}+d$—given by Eq. (18). In Table II we gave a tabulation of these factors for the spin-correlation observables of $\vec{p}\vec{p}\to\pi^+$d and the tensor polarizations of pp$\to\vec{d}\pi^+$. An exhaustive tabulation for all observables was not given both because of space limitations and because Table II suffices to demonstrate the way in which one might use the information contained in such a tabulation. We gave two examples of such uses. The first was to use a theoretically and/or experimentally based prejudice regarding the size of some amplitude to estimate the sensitivity of an observable to other less known amplitudes. This was illustrated by the well established assumption that the dominant amplitude in pp$\to\pi^+$d is the 2^+ amplitude a_2. Then by looking for products of a_2 with other amplitudes (Table III) we deduced that it_{11} should be the only observable highly sensitive to the smaller amplitudes a_0, a_7, and a_8. Similarly we found that $\vec{p}$p$\to\vec{d}\pi^+$ observables involving a longitudinally polarized proton beam should be sensitive to the 2^- amplitudes a_4 and a_5. The second example that we gave for exploiting the tabulation consisted of looking for linear combinations of observables that result in a cancellation of some of the partial wave contributions. Such linear combinations could bring an even more useful connection between theory and experiment. For $\vec{p}\vec{p}\to\pi^+$d we found that the combinations $\sigma_{00}(1+A_{zz})$, $\sigma_{00}(1+A_{xx})$, and $\sigma_{00}(1+A_{yy})$ depend only on triplet amplitudes, $\sigma_{00}(1-A_{zz}-A_{xx}-A_{yy})$ depends only on singlet amplitudes, and $\sigma_{00}(1-A_{zz}+A_{xx}+A_{yy})$ depends (at medium energies) only on the two 2^- amplitudes a_4 and a_5. Our examples were mainly for illustrative purposes and undoubtedly do not exhaust all the possibilities that may be obtained from the tabulation of the $C_L^\nu(I,I')$.

We have also provided numerical calculations of all the observables for the above reactions. Many of the observables involving polarization transfer have not, as far as we know, been previously calculated. To generate the amplitudes we used a unitary few-body model of the NN-πNN system.[7,29] This has the feature that our results for both pp$\to\pi^+$d and πd$\to\pi$d come from solving the one set of coupled equations. The energy dependence of the pp$\to\pi^+$d observables was investigated (Figs. 2 and 3), and a rather interesting behavior of the tensor polarizations t_{20}, t_{21}, and t_{22} was uncovered. The tensors were found to have only mild energy dependence—their shapes being well described by assuming a dominant a_2 amplitude. Indeed throughout our investigation we have found the tensors very insensitive to the choice of energy or model. The further (experimental) specification of A_{yy} leads, via Eq. (23), to highly constrained values of t_{20} and t_{22}. This may be of practical value in calibrating a deuteron polarimeter or in monitoring a tensor deuteron target in π-$\vec{d}$ experiments.

As a major source of uncertainty in the unitary models is the form taken by the pole and nonpole pieces of the P_{11} interaction, we have tested the model dependence of our results by using three different descriptions of the P_{11} channel. Two of these ($M1$ and $P6$ of Ref. 29) implement the propagator and vertex dressing as prescribed by the unitary theory.[6] The other ($BO8$ of Ref. 7) includes the dressings only in an effective way. The three resulting descriptions gave very different results for the differential cross sections of pp$\to\pi^+$d (Fig. 4) and πd$\to\pi$d (Fig. 6); however, not all the spin-dependent observables were substantially different for the three cases. Of the observables in pp$\to\pi^+$d that do not involve tensor components, we found that A_{y0}, A_{zx}, it_{11}, t_{11}^{11}, t_{10}^{10}, and t_{10}^{11} are the most sensitive to our choice of model (Fig. 5). For πd$\to\pi$d many of the observables involving spin transfer displayed a strong sensitivity to the choice of the P_{11} model (Fig. 7). Unfortunately most of these observables would be difficult or impossible to measure with present technology. One possibility, however, could be t_{1-1}^{11} which appears to display a strong sensitivity at backward angles that is rather different from that of t_{20}.

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