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Strangeness abundances in $\bar{p}$-nucleus annihilations

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Strange particle abundances in small volumes of hot hadronic gas are determined in the canonical ensemble with exact strangeness and baryon number conservation. Substantial density and baryon number dependence is found. A $\bar{p}d$ experiment is examined and applications to $\bar{p}$-nucleus annihilations are considered.

I. INTRODUCTION

In the annihilation of antiprotons of low kinetic energy on light nuclei, substantial transfer of annihilation energy to spectator nucleons and distribution of the available energy over the transversal degrees of freedom is observed.[1] Furthermore, strange particles produced in hot matter carry information about the dense phase of the annihilation reaction, because strangeness is mostly produced in the hot and dense zone and it is not depleted again during the lifetime of the hadronic fireball. The abundance of strange particles has therefore been suggested to be a signal in the search for quark matter possibly existing in the hot region.[2]

The aim of this paper is to calculate in a statistical model the abundance of strangeness produced in an antiproton annihilation on light nuclei or protons. The conservation of strangeness[3,4] and baryon number in strong interactions is an essential ingredient constraining the particle production in our model.[4] We further apply our model to the currently available $\bar{p}d$ data[5] and show that the substantial high momentum component observed corresponds to the expected distribution from hot hadronic matter. Certain applications for $\bar{p}A$ collisions are also presented.

Our initial hypothesis is that hadronic matter inside the annihilation zone in $\bar{p}$ nucleus reactions can be described by a statistical model, as is, e.g., indicated by the experimentally observed exponential shape of the p spectra from which the temperature parameter can be obtained. Assuming (relative) chemical equilibrium, the mean multiplicity of the different hadronic components produced can then be determined. However, we should not expect perfect chemical or thermal equilibrium to exist for several reasons: (1) the reaction volume is expanding; (2) the temperature decreases due to the expansion and radiation energy losses; and (3) the fireball does not exist long enough to reach the phase space limit of strangeness abundance. But within the strangeness carrying particles a relative chemical equilibrium is possible, as strangeness exchange reactions (e.g., $K^- p \leftrightarrow \Lambda\pi^0$) have relatively large cross sections. Therefore, while absolute multiplicities may be difficult to interpret, ratios of multiplicities of strange particles can be predicted well in our approach.[3] The following assumption is furthermore made: The expansion of the reaction volume and the decrease of temperature are sufficiently slow for the temporary thermal equilibrium and relative chemical equilibrium among strange particles to hold. This picture of the hadronic fireball created in $\bar{p}$ annihilations is directly applicable for arbitrary nuclear target systems, but only for light targets will it be likely that all target baryons participate in the annihilation process in that they share its energy.

II. STRANGENESS CONSERVATION IN THE CANONICAL ENSEMBLE

The statistical description of a small system is not reliable in a grand canonical ensemble, since the assumption of only mean particle number conservation and the use of a corresponding chemical potential cannot be fully justified and substantial deviations from a canonical approach must be expected. Therefore, we describe our system in a canonical ensemble in which the exact conservation of particle numbers is assumed. But in a relativistic system, where particle production and annihilation are possible, the concept of particle number conservation has to be replaced by conservation of quantum numbers. As pair production (at the quark level) is the generating process for strangeness, the following internal conservation laws of hadronic matter have to be taken into account: conservation of electric charge, baryon number, and strangeness. In our calculation we presently ignore electric charge conservation, because in an annihilation a large amount of pions is generated which can be viewed as a reservoir for charge (and isospin). It was shown[6] that isospin conservation in $\bar{p}p$ annihilations leads to certain changes in the statistical distributions, which can be accounted for by changing the volume and/or temperature of the fireball. Hence ratios of strange particle abundances considered here should not be greatly affected.

We obtain for the canonical partition function in volume V for given strangeness S and baryon number A

$$Z_{S,A}(T,V)=\int_0^{2\pi} d\gamma_s d\gamma_b \exp(-i\gamma_s S) \times\exp(-i\gamma_b A)\tilde{Z}(T,V,\gamma_s,\gamma_b)\,, \quad \text{(1a)}$$

where the so-called generating function $\tilde{Z}$ is given by[7]

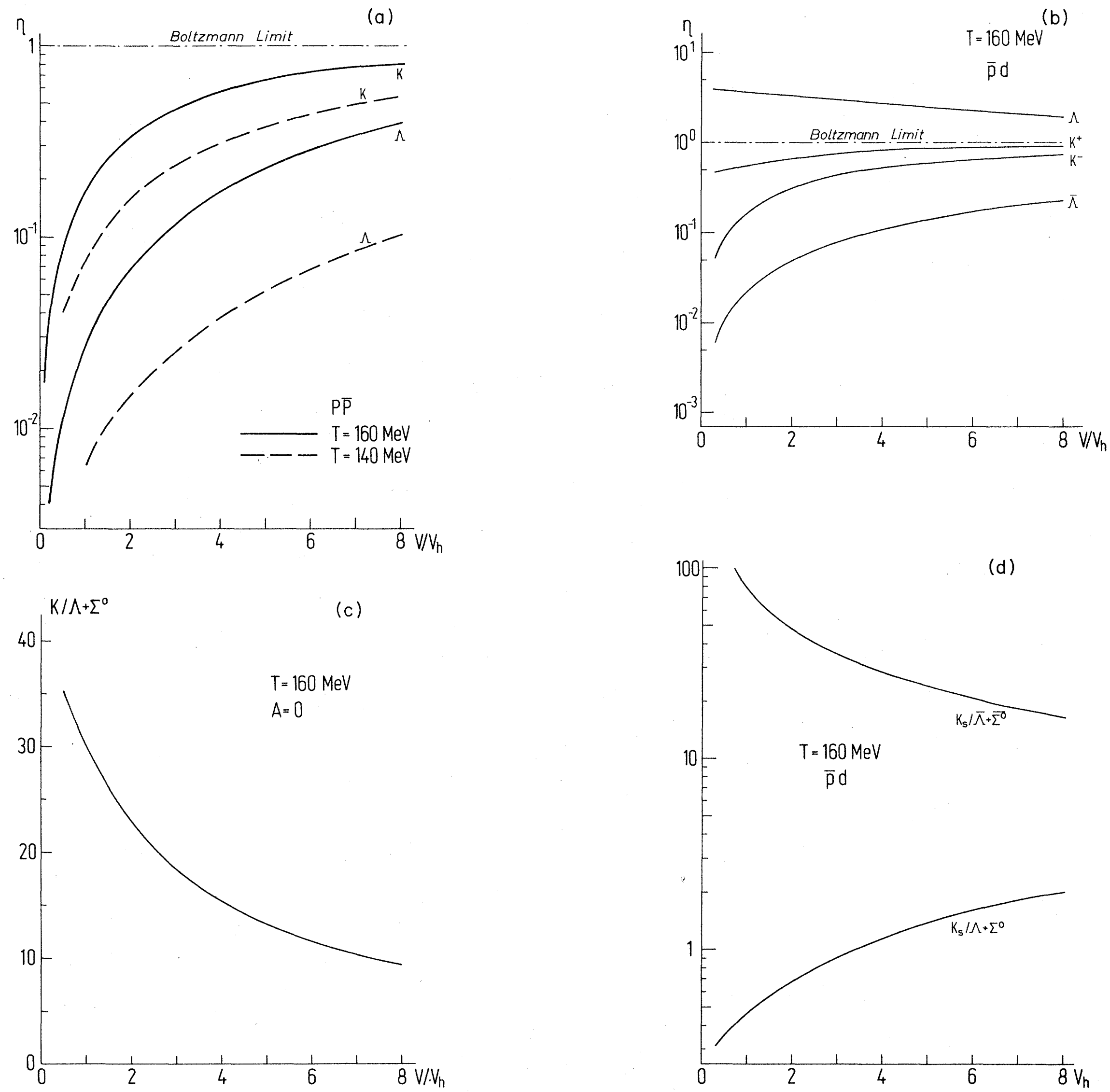

FIG. 1. (a) and (b) The phase space factor for the production of particles as a function of the fireball volume ($V_h = 4\pi$ fm$^3/3$); for (a), $A=0$; for (b), $A=1$. (K represents all particles with $s=1$ and $b=0$, Λ those with $s=1$ and $b=1$. (c) The ratio of the mean multiplicity of K to $(\Lambda+\Sigma^0)$ in the case $A=0$. (d) The ratios of the mean multiplicity of $(K^0+\overline{K}^0)/2$ to $(\Lambda+\Sigma^0)$ and $(\overline{\Lambda}+\overline{\Sigma}^0)$ with $A=1$. $T=160$ MeV in all cases; additionally, $T=140$ MeV in (a).

$$\widetilde{Z}(\beta,V,\gamma_s,\gamma_b) = \exp\left\{\sum_{\alpha}\sum_{j}\exp[-\beta E_j^\alpha + i(\gamma_s s_\alpha + \gamma_b b_\alpha)]\right\} . \qquad (1b)$$

All particles are treated in the Maxwell-Boltzmann (classical) approximation, which is possible since the temperature $T=1/\beta$ is in our case sufficiently large. The indices α and j count different kinds of particles and antiparticles and their energy levels E_j^α. s_α and b_α are particles's strangeness and baryon number, respectively. In our case there is no net strangeness in the system and we get

$$Z_{0,A}(\beta,V) = \int_0^{2\pi} d\gamma_s d\gamma_b \exp(-i\gamma_b A) \times \exp\left[\sum_\alpha \cos(\gamma_s s_\alpha + \gamma_b b_\alpha)\right] Z_1^\alpha(\beta,V), \qquad (2a)$$

with

$$Z_1^\alpha(\beta,V) = \frac{V}{2\pi^2\beta^3}(\beta m_\alpha)^2 K_2(\beta m_\alpha) , \qquad (2b)$$

which is the usual free single particle partition function calculated in the continuum limit. The energy spectrum of particles of the kind "α" can be calculated by a partial differentiation of the logarithm of the canonical partition function with respect to the level's energy; we find

$$\langle n_\alpha(E)\rangle_A = d_\alpha \exp(-\beta E)\eta_\alpha^A(\beta,V)\ , \tag{3}$$

and its phase space integral

$$\langle N_\alpha\rangle_A = d_\alpha \frac{V}{2\pi^2\beta^3}(\beta m_\alpha)^2 K_2(\beta m_\alpha)\eta_\alpha^A(\beta,V)\ . \tag{4}$$

The d_α are the statistical degeneracies of the particles. The temperature and volume dependent influence of the symmetry condition is contained in the function $\eta_\alpha^A(\beta,V)$ which is a ratio of infinite series of Bessel functions and was obtained by an analytical integration following Eqs. (2a):

$$\eta_\alpha^A(\beta,V) = N_\alpha^A / D^A\ , \tag{5a}$$

with

$$N_\alpha^A = \sum_{\nu=-\infty}^{+\infty} I_\nu(Z_1^{\text{nuc}}) I_{\nu+b_\alpha+s_\alpha-A}(Z_1^{\text{kaon}}) \times I_{\nu+b_\alpha-A}(Z_1^{\text{hyp}})\ , \tag{5b}$$

$$D^A = \sum_{\nu=-\infty}^{+\infty} I_\nu(Z_1^{\text{nuc}}) I_{\nu-A}(Z_1^{\text{kaon}}) I_{\nu-A}(Z_1^{\text{hyp}})\ , \tag{5c}$$

and [cf. Eq. (2b)]

$$Z_1^{\text{nuc}}(\beta,V) = Z_1^{\text{N}}(\beta,V) + Z_1^{\Delta}(\beta,V)\ , \tag{6a}$$

$$Z_1^{\text{kaon}}(\beta,V) = Z_1^{\text{K}}(\beta,V) + Z_1^{\text{K}^*}(\beta,V)\ , \tag{6b}$$

$$Z_1^{\text{hyp}}(\beta,V) = Z_1^{\Lambda}(\beta,V) + Z_1^{\Sigma}(\beta,V)\ , \tag{6c}$$

where the relevant particles have been counted. Contributions of heavier baryons are exponentially smaller. Mesons (pions) having no baryon number or strangeness do not directly influence the current considerations. We remark that without exact quantum number conservation, one simply finds $\eta = 1$ also corresponding to the limit for an arbitrarily large fireball.

By numerical summation, which is centered around small indices of the Bessel functions, we get the result shown in Fig. 1(a): The phase space factor η as a function of the fireball volume in units of the hadronic volume $V_h = 4\pi/3$ fm^3 for the proton-antiproton "$\bar{\text{p}}$p" ($A=0$) system at $T=160$ MeV. Relative suppression of Λ against K is obviously a consequence of the fact that K$\bar{\text{K}}$ production is much more likely than the associate KΛ strangeness production in the baryonless small fireball. (K means K^+, K^-, K^0, or $\bar{\text{K}}^0$; Λ is Λ,Σ^0 or $\bar{\Lambda},\bar{\Sigma}^0$). Hence we learn here that exact baryon conservation is a very important ingredient in our considerations as it interplays in the annihilation with strangeness conservation.

In Fig. 1(b) we see that when $\bar{\text{p}}$d annihilation is considered (meaning that there is no spectator), the availability of one extra baryon strongly enhances the probability of Λ production, as the phase space factor of the Λ is now larger than unity. The multiplicities of particles and antiparticles are no more equal for $A \neq 0$.

In Figs. 1(c) and (d) the corresponding relative $\text{K}/(\Lambda+\Sigma^0)$ abundances are shown; they are rather constant [the linear scale in Fig. 1(c)] as a function of the fireball volume, substantiating our conjecture about the insensitivity of ratios to details of the fireball structure.

III. ANTIPROTON ANNIHILATION ON DEUTERON AND LIGHT NUCLEI

Consideration of antiproton-deuteron annihilation with coincident strange particles[5] leads to a test of our model. When analyzing the momentum distribution of protons emerging from a $\bar{\text{p}}\text{d}\to\text{p}+\text{K}\bar{\text{K}}+\pi$'s reaction, spectator behavior below 0.2 GeV/c proton momentum fits expectations based on a standard deuteron wave function well, while a thermal distribution seems to exist above 0.2 GeV/c (data points in Fig. 2).[8] This behavior seems to only occur in the channel containing a kaon pair. In the context of our model this effect could be interpreted as an annihilation reaction involving all three incoming particles. The observed proton is a constituent of the hadronic fireball rather than a "spectator." The required condition of an observed K$\bar{\text{K}}$ pair is accounted for by introducing two auxiliary "quantum numbers" counting kaons and antikaons, respectively. Then the canonical partition function becomes

$$Z_{A=1,\text{K}\bar{\text{K}}} = \frac{1}{(2\pi)^4}\int_0^{2\pi} d\phi_s d\phi_b d\phi_k d\phi_{\bar{k}} \times e^{-i(\phi_b+\phi_k+\phi_{\bar{k}})}\tilde{Z}\ , \tag{7a}$$

with the generating function

$$\tilde{Z} = \exp\{Z_1^{\text{nuc}}(\beta,V)e^{i\phi_b} + Z_1^{\text{hyp}}(\beta,V)e^{i(\phi_b-\phi_s)} + Z_1^{\text{kaon}}(\beta,V)[e^{i(\phi_s+\phi_k)} + e^{i(-\phi_s+\phi_{\bar{k}})}]\}\ . \tag{7b}$$

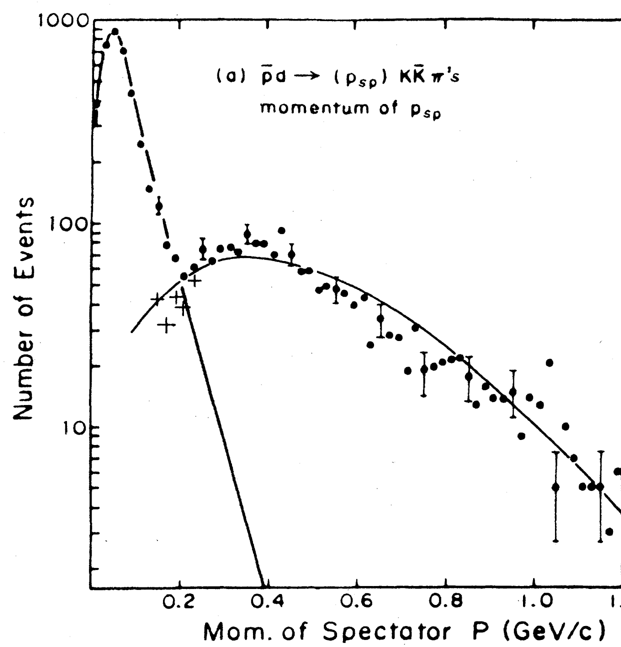

FIG. 2. Spectator proton momentum distribution in the reaction $\bar{\text{p}}\text{d}\to\text{p}\,\text{K}\bar{\text{K}}+\pi$'s, Ref. 5. The line is our theoretical result at $R=1.5$ fm, $T=120$ MeV. See the text for details.

By analytical integration similar to the handling of Eq. (2a) we obtain the phase space factor for protons

$$\eta'(\beta,V)=I_0[Z_1^{\text{nuc}}(\beta,V)]/I_1[Z_1^{\text{nuc}}(\beta,V)] \ . \tag{8}$$

Taking into account the delta resonances which decay with equal rates into protons and neutrons, the approximate proton spectrum becomes

$$\langle p \rangle_{\text{th}}=\frac{V}{2\pi^2}\eta'(\beta,V)\text{p}^2\Delta\text{p}\left(d_{\text{p}}e^{-\beta\sqrt{p^2+m_{\text{p}}^2}} + \tfrac{1}{2}d_{\Delta}e^{-\beta\sqrt{p^2+m_{\Delta}^2}}\right) . \tag{9}$$

Choosing the temperature to be $T=120$ MeV, which is a value derived from $p\bar{p}$ annihilations, the maximum of the distribution is situated about 150 MeV/c too high. To obtain the correct maximum position, the temperature has to be below $T=100$ MeV; in that case the slope of the distribution is far too steep compared to experiment. The correct spectrum is found by the following consideration: The observed particles are emitted from a thin spherical surface layer only. In this layer a statistical equilibrium in two dimensions exists. The spectrum (9a) changes from $\langle p \rangle_{\text{th}} \simeq Vp^2$ to $\langle p \rangle'_{\text{th}} \simeq Sp$, where $S=4\pi R^2$ is the surface of the sphere. The assumption of two-dimensional equilibrium alters the single particle partition function (2b) and therefore the phase space factor, too.

$\langle p \rangle_{\text{th}}$ is almost insensitive to the size of the reaction zone. The existence of a very slight size dependence is due to antinucleon production. The only sensitive parameter in calculating (9a) is the temperature influencing the result proportional to T^{-3} (T^{-2} in the two-dimensional model). The two-dimensional description of particle emission leads to the correct position of the maximum at $T=120$ MeV. For direct comparison to experiment the theoretical distribution has to be normalized. The normalization factor is obtained if we demand that the momentum integral of the spectrum, which can be evaluated analytically, equals the total number of thermal $\bar{\text{K}}\text{K}$ events in the experiment. The low-momentum thermal data points indicated by + are estimated by subtracting the scattering peak from the distribution.

The normalized spectrum is shown in Fig. 2. The data are reproduced well, indicating that our assumption of emission from a surface layer reflects the actual behavior of the reaction. If similar experiments at other energies (different temperatures) were examined, the validity of the phase space factor could be tested.

Finally we present results for $\bar{\text{p}}$ annihilation on light nuclei in Figs. 3(a) and (b). Figure 3(a) gives the ratio of the total strangeness produced per baryon number of the fireball as a function of its baryon number density. This ratio decreases rapidly with increasing density. Mnemonically, one may say here: Baryons crowd out strangeness. The same data are shown in Fig. 3(b), but now the dependence of the total strangeness per baryon number on the total baryon number, with baryon density being a fixed parameter, is displayed. For large values of A the mean strangeness per baryon number becomes independent of A.

Considering our results shown in Figs. 3(a) and (b) we suggest a systematic measurement of the total strangeness yield per baryon number by varying the target size, which also provides a check on the consistency of the statistical model: Given Fig. 3(b) this measurement may be viewed as a determination of the density achieved in the annihilation fireball. Assuming this density to be approximately constant for the hot and dense zone produced by a typical annihilation process, one could even apply our model to the case of heavier target nuclei and use Fig. 3(a) to determine the number of nucleons participating in the annihilation fireball formation inside a larger nucleus.

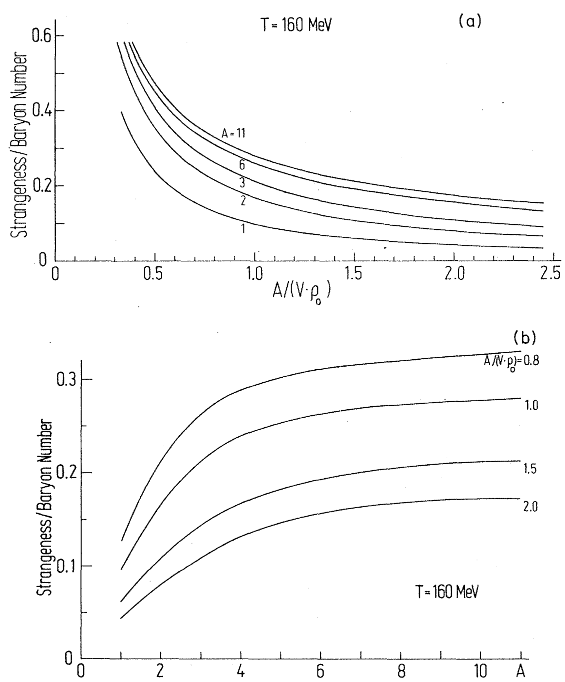

FIG. 3. (a) The total strangeness per baryon number as a function of baryon number density A/V with baryon number A as a parameter. (b) The total strangeness per baryon number as a function of baryon number with baryon number density A/V as a parameter. $T=160$ MeV in all cases and $\rho_0=0.17\ \text{fm}^{-3}$.

IV. CONCLUSIONS

To summarize our results: We showed that the description of annihilations leading to small hadronic fireballs requires imposition of exact conservation of strangeness and baryon number to take due account of the finite (small) size of the system. In particular, strangeness production depends sensitively on the baryon number density in the fireball. We predict that in annihilations on light nuclei the baryon number of the fireball influences strangeness abundance substantially between $A=1$ and $A=6$, *eventually more than doubling the total strangeness per baryon number.*

An examination of a $\bar{\text{p}}$d experiment gives good agreement of a model calculation with reality. An essential feature has been the emission from a spherical surface. In an application to larger systems our approach may help to determine the number of nucleons participating in a typical annihilation event. Finally, we remark that strong enhancements over the statistical estimations in certain experiments may provide a hint that the underlying quark degrees of freedom of the fireball come into play or that particular resonant states have been formed. Thus our calculations can be taken as upper limits on strangeness production in the conventional hadronic picture. Any substantial excess recorded would signal occurrence of exotica, such as quark gluon plasma.

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