p-^{11}B spectroscopic factor from the interference of potential scattering and elastic transfer at low energies

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The elastic scattering in the $^{11}B + {}^{12}C$ system was measured at c.m. energies of 5.43, 6.47, and 7.60 MeV in the angular range from 13° to 160° c.m. The analysis, taking into account the interference between potential scattering and direct elastic transfer, led to the value of the asymptotic magnitude of the $1p_{3/2}$ proton wave function in ^{12}C, $\overline{C}=12.9\pm1.0$. A proton spectroscopic factor equal to 2.75 ± 0.4 is obtained for a binding potential with geometrical parameters $r_0=1.25$ fm and $a=0.65$ fm.

I. INTRODUCTION

The collision of heavy nuclei offers the possibility of studying the exchange of large fragments of nuclear matter between the colliding particles. However, it appears that even the apparently uncomplicated process of a one-nucleon transfer is still not fully understood. The DWBA theory, usually providing a good description of such reactions, fails in some cases to adequately reproduce the experimental data.[1] Contributions either from some multistep or from molecular processes have been suggested to explain these discrepancies.

Recently, an extensive study of the one-proton transfer in the elastic scattering of heavy ions at energies above the Coulomb barrier was undertaken.[2] In most cases, the DWBA approach gave an adequate description of the experimental data of elastic scattering at backward angles with proton spectroscopic factors close to those known from the theory or other heavy ion experiments. For the $^{11}B+{}^{12}C$ system, however, a remarkable discrepancy appeared. While the shape of the angular distribution measured at a c.m. energy of 14.6 MeV is well reproduced by the DWBA theory for angles larger than 140°, the value of the spectroscopic factor necessary to fit the experimental absolute cross sections exceeds the theoretical Cohen-Kurath prediction[3] by a factor of approximately 2 and lies even 40% above the highest, for theoretical reasons, acceptable value. Similarly, from data measured at energies somewhat smaller than 14.6 MeV (c.m.) and analyzed in the no-recoil DWBA (Ref. 4) or by means of the molecular wave function method,[5] large values of the proton spectroscopic factor, again considerably exceeding the theoretical calculation,[3] were also found. An estimate from an experiment performed at a much higher energy (41.6 MeV c.m.),[6] however, is in agreement with the Cohen-Kurath prediction. From light particle reactions, as, e.g., (^{3}He,d), values similar to or lower than the theoretical prediction were extracted.

The aim of the present work is to determine in a first step the asymptotic strength of the $1p_{3/2}$ proton wave function in ^{12}C from a study of the elastic transfer in $^{12}C+{}^{11}B$ scattering at very low energies. It was shown[7] that, using low energy data, the asymptotic magnitude of the wave function of the transferred bound particle can be obtained in a model independent way from the interference between the potential scattering and the direct elastic transfer. In a second step, the spectroscopic factor, which depends on the p-^{11}B interaction potential, is calculated.

II. EXPERIMENTAL PROCEDURE AND RESULTS

The measurements were performed at the tandem Van de Graaff accelerator of the Eidgenössische Technische Hochschule (ETH) in Zürich using ^{11}B beams with energies of 10.4, 12.4, and 14.6 MeV (lab) and an intensity of approximately 100 nA. The target was an approximately 50 μg/cm^2 thick ^{12}C foil. Telescopes consisting of an ionization chamber (ΔE counter) and a semiconductor detector (E counter) were used for particle detection and identification. The differential cross sections were determined in the range of c.m. angles from 13° to 160° by measuring either the scattered ^{11}B particles or the recoiling ^{12}C nuclei in the forward direction.

The beam was monitored by two semiconductor detectors placed symmetrically at 24.6° on both sides of the beam direction. With this arrangement the correct position of the beam spot on the target could be checked continuously. For the absolute normalization of the measured cross sections, the particles scattered from a thin layer of gold evaporated onto the target were detected simultaneously. The relative thickness of both layers was determined from the observation of pure Rutherford scattering at a sufficiently reduced beam energy.

The results of the performed measurements are shown in Fig. 1. The error bars attached to the individual points include the statistical uncertainty as well as some nonstatistical contributions due to inaccuracies in the background subtraction, separation of the different particles in the mass spectra, and determination of the solid angles of the detectors. The overall error in the absolute normalization is estimated to be around 5%.

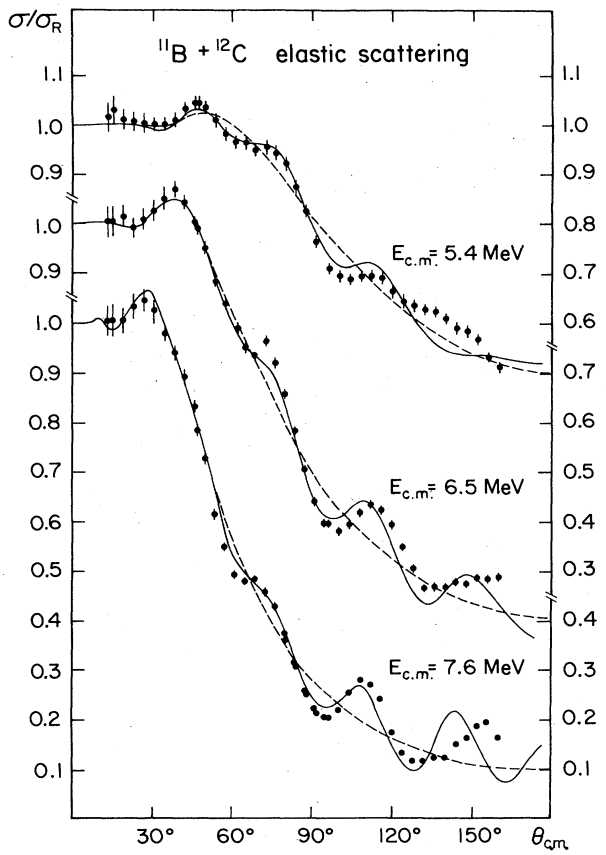

FIG. 1. Cross section for elastic scattering of ^{12}C on ^{11}B measured at $E_{lab}=10.4$, 12.4, and 14.57 MeV. The broken curves are the result of the optical model calculation, whereas in the solid lines the interference with the elastic proton transfer is included.

III. OPTICAL MODEL ANALYSIS

The potential part of elastic scattering is described as usual by an appropriate optical model. First, an optical potential with global parameters fitting ^{11}B+^{12}C elastic scattering data in the forward region for a broad energy range was searched. The following parametrization of the optical model (OM) potential was assumed:

$$U_{\mathrm{OM}}=-(V_0+V_1E_{\mathrm{c.m.}})\left[1+\exp\frac{r-R_v}{a_v}\right]^{-1} -i(W_0+W_1E_{\mathrm{c.m.}})\left[1+\exp\frac{r-R_w}{a_w}\right]^{-1}, \quad R_v=r_v(A_1^{1/3}+A_2^{1/3}),\ R_w=r_w(A_1^{1/3}+A_2^{1/3}), \tag{1}$$

with a linear dependence on energy of the depths of the real and imaginary parts. Existing experimental cross sections,[2,4,5,8] as well as the measurements of the present work for c.m. energies ranging from 5.4 up to 26.1 MeV, were used for this analysis. The results are shown in Fig. 2 and the parameter values obtained under the assumption $r_v=r_w$ and $a_v=a_w$ are listed in Table I. In the neighborhood of the Coulomb barrier, the strongly reduced number of open reaction channels leads to a decreased depth of the imaginary part of the potential. For the lowest energy (5.4 MeV) we used the best fit value $W=13.3$ MeV. Figure 2 demonstrates clearly that the structures present in the theoretical curves at higher energies tend to disappear when the energy is decreased. In the region of the three lowest energies under investigation, the calculations approximately average the oscillations of the experimental angular distributions (Fig. 1). This indicates that the observed structures are really due to the interference between the potential scattering amplitude and that for the direct transfer process leading to the same exit channel.

TABLE I. Parameters of the global optical potential.

V_0 (MeV)	V_1	W_0 (MeV)	W_1	$r_v=r_w$ (fm)	$a_v=a_w$ (fm)
53.5	1.25	23.3	1.028	1.108	0.545

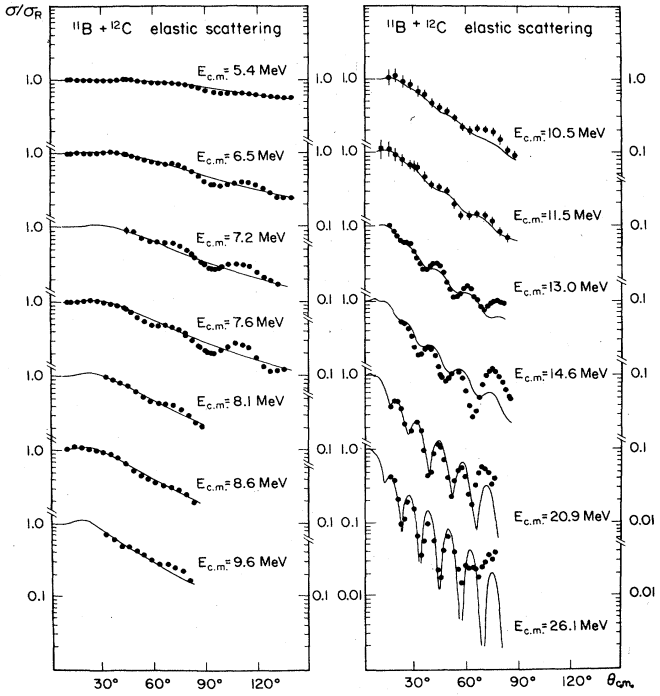

FIG. 2. Optical model fits to elastic scattering of ^{12}C on ^{11}B at various energies from a global parameter search.

IV. THE ELASTIC TRANSFER PROCESS

The observed oscillations in the angular distributions are characteristic of interference phenomena between potential scattering and elastic transfer. In order to include the latter process explicitly in the theoretical calculations, the potential scattering amplitude $f^{\mathrm{pot}}(\theta)$ was supplemented by the elastic transfer amplitudes $f^{\mathrm{tr}}_{lm}(\pi-\theta)$ corresponding to the angular momentum transfer l and its projection m. The differential cross section is then expressed by[7,9]

$$\frac{d\sigma}{d\Omega}(\theta)=|f^{\mathrm{pot}}(\theta)+f^{\mathrm{tr}}_{00}(\pi-\theta)|^2+\sum_{\substack{l\neq 0\\ m}}|f^{\mathrm{tr}}_{lm}(\pi-\theta)|^2. \tag{2}$$

Since potential scattering with an interaction defined by Eq. (1) does not change angular momentum, its amplitude interferes coherently with the transfer amplitude with $l=0$ only. According to Ref. 10, the spectroscopic information for the configuration $p+^{11}B$ in ^{12}C is contained in the asymptotic strength $\bar{C}$ of the bound state wave function, which scales the magnitude of the transfer amplitude. This quantity can be determined essentially in a model independent way by comparing the results of the calculation according to formula (2) with the experimental data. $\bar{C}$ is connected to the spectroscopic factor S by the relation[7]

$$\bar{C}^2=\frac{1}{2\kappa}SN^2 , \tag{3}$$

where N is the asymptotic normalization of the proton bound state wave function to the appropriate Whittaker function and $\kappa=(2\mu\,|\,E_B\,|\,/\hbar^2)^{1/2}$ with binding energy E_B and reduced mass μ.

The calculation of the transfer amplitude in Eq. (2) was performed using the SATURN2-MARS1 code of Tamura and Low.[11] The distorted waves in the entrance and exit channels were generated by means of the optical model potential discussed in Sec. III which describes well the smooth part of the angular distribution for the scattering. The bound state wave function of the $1p$-shell proton with respect to the ^{11}B core was calculated from a Woods-Saxon-well—type potential with the commonly accepted[12] values of the geometrical parameters $R=1.25A^{1/3}$ fm and $a=0.65$ fm, supplemented with a Thomas-type spin-orbit term of 6 MeV strength. The depth of the central part of this potential was adjusted to obtain the appropriate binding energy of the $1p_{3/2}$ proton in the ^{12}C nucleus.

The result of the search for the asymptotic strength best fitting the experimental data is presented in Fig. 3 for the three energies of the present experiment. From these plots one reads the values 11.0 ± 1.1, 12.5 ± 0.4, and 13.3 ± 0.3 with errors determined from variations which increase the minimum χ^2/n by the factor $1+1/n$ (n denotes the number of degrees of freedom). The weighted mean value is then

$$\bar{C}=12.9\pm1.0 .$$

Here, the quoted error is given by the external variance and also includes, therefore, nonstatistical contributions.

It can be expected that the deduction of $\bar{C}$ is stable for variations of the geometrical parameters of the proton binding potential, since for energies below the Coulomb barrier only the asymptotic behavior of the bound state wave function of the proton is important. And, indeed, it could be demonstrated that variations of the radius r_0 or the diffuseness a of this potential by $\pm10\%$ gave no noticeable change (greater than 2%) of the best fit value of $\bar{C}$. Due to the well-known ambiguity in the determination of the optical model potential, it is possible to reproduce the smooth part of the elastic scattering angular distribution with many different sets of parameters. We have checked that it is possible to choose the depth of the real and of the imaginary potential independently between 10 and 100 MeV (adjusting the geometrical parameters) without affecting the quality of the fit to the average angular distribution. It is important to notice that for all of these different sets of optical model parameters, the calculation of the interference pattern with the transfer amplitude, using the same value of $\bar{C}$, leads to an indistinguishably good description of the experimental data.

To convert the $\bar{C}$ value into a spectroscopic factor S, according to formula (3), the normalization factor N, which is dependent on the geometrical parameters of the bound state potential, has to be known. Thus the value of the spectroscopic factor can be found only if the parameters of the binding potential can be fixed independently, e.g., from the knowledge of the charge distribution of the nucleus. The geometry of the proton binding potential used in the present work leads to a charge distribution radius of the ^{12}C nucleus of 2.46 fm (Ref. 12), in agreement with the rms radius 2.46 ± 0.02 fm found in high energy electron scattering experiments.[13] The proton spectro-

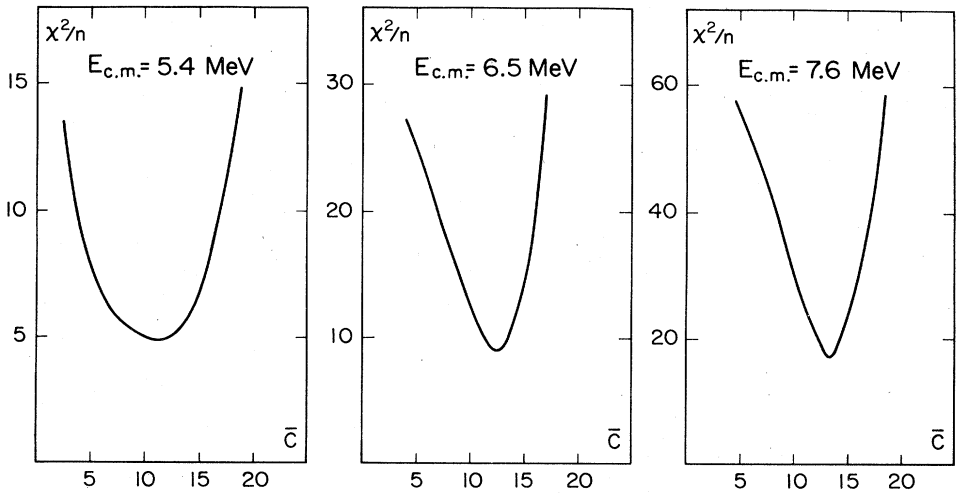

FIG. 3. Determination of the asymptotic strength $\bar{C}$. The values of χ^2/n for fits to the measurements at the energies 10.4, 12.4, and 14.57 MeV are plotted as a function of $\bar{C}$.

scopic factor of ^{12}C obtained from our experimental value of the asymptotic coupling constant $\overline{C}$ and from the mentioned geometry of the proton binding potential is equal to

$$S=2.75\pm0.4\ .$$

V. DISCUSSION

In this paper we presented new measurements of the elastic scattering of ^{11}B on ^{12}C nuclei for energies of 5.4, 6.5, and 7.6 MeV in the c.m. system. The experiment was undertaken in order to extract the asymptotic strength and the spectroscopic factor of the $1p_{3/2}$ proton in ^{12}C. This information is of special interest since the proton transfer reaction (^{11}B,^{12}C) is frequently used in spectroscopic studies of other nuclei[12,14] and since vastly different values for the spectroscopic factor have been published in the past.

The elastic transfer reaction ^{11}B(^{12}C,^{11}B)^{12}C was chosen since the *same* vertex appears *twice* in the reaction graph and since no other spectroscopic factor of a different reaction has to be known. Especially at low energies, where the interference of the proton transfer with the potential scattering enhances the sensitivity of the method, this reaction can provide an accurate determination of the asymptotic strength $\overline{C}$. We obtained for this quantity the value $\overline{C}=12.9\pm1.0$. Using a geometry for the proton binding potential consistent with the charge distribution obtained in high energy electron scattering, we obtained for the $1p_{3/2}$ proton spectroscopic factor a value of $S=2.75\pm0.4$.

A comparison with results from other experiments and with the theoretical prediction is summarized in Table II. The value determined in the present experiment lies very close to the spectroscopic factor calculated by Cohen-Kurath. The results of light particle stripping and (p,2p) reactions are as a whole slightly lower but still in reasonable agreement. With regard to the measurements of the elastic scattering of ^{11}B on ^{12}C one finds agreement with the value reported at 87 MeV, but on the other hand, values obtained at energies between 15 and 28 MeV exceed the theoretical prediction considerably. Since in these papers[2,4–6] different evaluation methods and optical potentials have been used, a reanalysis of the measurements reported in the literature was made. Each energy was analyzed separately and in all cases the same finite range DWBA program was used. Optical model parameters were determined from fits to the forward angle part of the angular distributions as described in Sec. II, and the proton binding potential in ^{12}C was always that used in the present work. The results are shown as full circles and triangles in Fig. 4 (together with values from light particle reactions copied from Table II). The "error bars" give an idea of the range for which a description of the measured cross sections still could be accepted. The values obtained in this way vary now in a regular manner, but above the Coulomb barrier they are still too large, exceeding at some energies even the theoretical limit, $S_{max}=4$. One could,

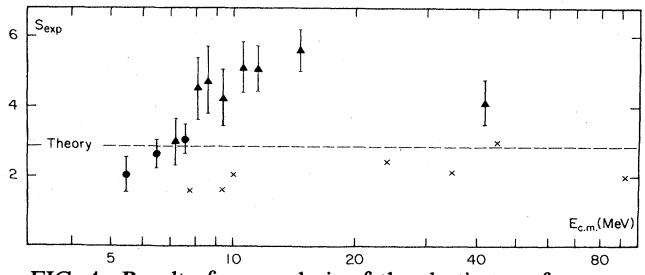

FIG. 4. Result of a reanalysis of the elastic transfer process in ^{11}B+^{12}C elastic scattering based on the present experiment (full circles) and published cross sections (triangles). For comparison, values deduced by different authors from light particle reactions are also inserted (crosses).

TABLE II. Asymptotic strength $\overline{C}$ and spectroscopic factor S given in the literature. (ZR) denotes zero range; EFR denotes exact finite range; DWIA denotes distorted wave impulse approximation; NR denotes no recoil; and MWFM denotes molecular wave function method.

Reaction	Energy (MeV)	Method of analysis	Geometry of proton binding potential r_0 (fm)	Geometry of proton binding potential a (fm)	$\overline{C}$	S	Reference
^{12}C(p,2p)^{11}B	100	DWIA				2.0	15
^{12}C(d,^{3}He)^{11}B	28	ZR DWBA	1.25	0.65	12.1	2.44	16
^{12}C(d,^{3}He)^{11}B	52	ZR DWBA	1.25	0.65	13.4	2.98	17
^{11}B(^{3}He,d)^{12}C	10,12	ZR DWBA	1.40	0.70	9.8	1.6	18
^{11}B(^{3}He,d)^{12}C	44	ZR DWBA	1.2	0.7	11.3	2.125	19
^{11}B(d,n)^{12}C	11.8	ZR DWBA	1.32	0.57	11.1	2.09	20
^{12}C(^{11}B,^{12}C)^{11}B	10.4,12.4, 14.57	EFR DWBA	1.25	0.65	12.9	2.75	Present work
^{11}B(^{12}C,^{11}B)^{12}C	15,17 20,24	NR DWBA	1.25	0.65	18.3	5.6	4
^{11}B(^{12}C,^{11}B)^{12}C	16,18 22,24	MWFM	1.25	0.65	14.7	3.6	5
^{12}C(^{11}B,^{12}C)^{11}B	28	ZR DWBA	1.25	0.65	21.2	7.5	4
^{12}C(^{11}B,^{12}C)^{11}B	28	EFR DWBA	1.25	0.65	18.3	5.6	2
^{11}B(^{12}C,^{11}B)^{12}C	87	EFR DWBA	1.25	0.65	12.5	2.6	5
Theory						2.85	3

of course, suspect that the observed discrepancies are due to the onset of a different reaction mechanism, as, e.g., a two-step transfer becoming important in this energy region. However, this supposition was shown to be invalid.[2] One has, however, to bear in mind that at higher energies the DWBA analysis might be more dependent on the choice of the optical and binding potential and therefore the low energy results should give the most reliable value. In order to shed more light on this unsettled discrepancy, additional measurements of the elastic transfer reaction $^{11}B+^{12}C$ above the Coulomb barrier, in the gap between 28 and 87 MeV (lab), would be desirable.

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