Proximity potential for deformed, oriented collisions and its application to $^{238}U+^{238}U$

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The proximity potential for the collision of two deformed and oriented nuclei is rederived in a completely simple and alternative method. Our method is applicable to axially symmetric nuclei lying in the same plane and we get an analytical expression for the shortest distance between two colliding surfaces. The method is applied to the calculation of $^{238}U+^{238}U$ interaction potential.

I. INTRODUCTION

Collisions involving deformed nuclei have been studied experimentally and the deformation effects in fusion cross sections have been established.[1] A deformed nucleus, however, can have many orientations in the ground state. In view of this, many theoretical studies have been carried out[2–9] which are concerned with the effect of deformation and orientation of the colliding nuclei on the interaction potential.

For the proximity potential, we know from Blocki *et al.*[10] that the essential point is the determination of the shortest distance, s_0, between the colliding surfaces. Baltz and Bayman[2] were the first to give an iterative procedure to calculate this distance s_0 for interaction between two deformed nuclei with orientation degrees of freedom included. Recently, Seiwert *et al.*[3] used a different iterative procedure for determining this minimum distance s_0 between two oriented nuclear surfaces. In this paper, we give a completely alternative, simple derivation for this shortest distance s_0 between two deformed, oriented colliding nuclei.

We have applied our method to calculate the proximity potential of the $^{238}U+^{238}U$ reaction, based on the "pocket formula" of Blocki *et al.*[10] The proximity potential for this system is also calculated by Seiwert *et al.*[3] by using their numerical iterative procedure in the pocket formula of Blocki *et al.*[10] and in the Bass-type (Ref. 11) potential, with the principal radii of curvature calculated differently. The deformation and orientation dependent potentials for this system are also calculated by Münchow *et al.*[4] and by Rhoades-Brown *et al.*[5] by using the double-folding model. The interest in choosing this system lies in the possibility, suggested by Greiner,[12] of their forming a very long-lived (lifetime of the order of 10^{-20} sec) giant $^{476}184$ composite system, at an appropriate bombarding energy. This necessitates the knowledge of the barrier height and the presence of a deep pocket for the system to stay together for a longer time and form a nuclear molecule.[13] The possibility of a deep minima in the interaction potential and the lowering of its barrier height can arise due to the inclusion of the deformation and orientation effects of the colliding partners. We look for such effects in this paper.

In Sec. II, we give our derivation of the shortest distance s_0 and the method of calculating the proximity potential between two deformed, oriented nuclei. Application of our method to the $^{238}U+^{238}U$ system and the comparison of our results with earlier calculations[3–5] is carried out in Sec. III. A summary and discussion of our results is given in Sec. IV.

II. PROXIMITY POTENTIAL BETWEEN TWO DEFORMED ORIENTED NUCLEI

As already stated in the Introduction, the essential quantity required for the evaluation of a proximity potential is the shortest distance s_0 between the colliding nuclear surfaces. Then, according to the "pocket formula" of Blocki *et al.*,[10] the proximity nuclear potential is given as

$$V_N = 4\pi\gamma \bar{R} b \Phi(\zeta_0) , \tag{1}$$

where the surface energy coefficient $\gamma = 0.9517[1 - 1.7826(N-Z)^2/A^2]$ MeV fm^{-1}; the surface thickness $b \approx 1$ fm; $\bar{R}$ is the mean curvature radius characterizing the gap (defined later); and $\Phi(\zeta_0)$, the universal function, is of the form

$$\Phi(\zeta_0) = -\tfrac{1}{2}(\zeta_0 - 2.54)^2 - 0.0852(\zeta_0 - 2.54)^3 \quad \text{for } \zeta_0 \le 1.2511$$
$$= -3.437\exp(-\zeta_0/0.75) \quad \text{for } \zeta_0 \ge 1.2511 . \tag{2}$$

Here $\zeta_0 = s_0/b$, i.e., s_0 is in units of b. This function is defined[10] for negative (the overlap region), zero (touching configuration), and positive values of s_0. However, for negative s_0, the definition becomes somewhat arbitrary, as will be discussed in Sec. III (see also Ref. 3).

Figure 1 gives the geometry for determining s_0, under

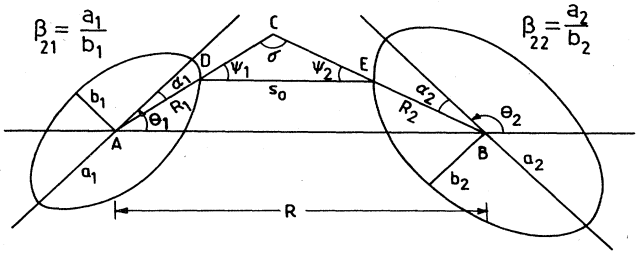

FIG. 1. A schematic configuration of two axially symmetric deformed, oriented nuclei, lying in the same plane. This defines the orientation angles θ_1 and θ_2, the shortest distance s_0, and the related geometry.

the simplifying assumptions of the colliding nuclei being axially symmetric and lying in the same plane. The nuclear radius parameter for each nucleus is then defined in the usual way, as

$$R_i(\alpha_i)=R_{0i}[1+\beta_{2i}P_2(\cos\alpha_i)] \quad (i=1,2) , \tag{3}$$

with $R_{0i}=r_0A_i^{1/3}$, $r_0=1.15$ fm.

For a given relative separation R and the orientations θ_1 and θ_2, we can write from the triangles ABC and CDE in Fig. 1

$$\psi_1+\psi_2=(\theta_1-\alpha_1)+(180-\theta_2-\alpha_2) \tag{4}$$

or

$$\theta_1-\theta_2+180=\psi_1+\psi_2+\alpha_1+\alpha_2 . \tag{5}$$

For the special case of s_0 being parallel to R, condition (4) splits into two equations:

$$\psi_1=\theta_1-\alpha_1 , \tag{6a}$$

$$\psi_2=180-\theta_2-\alpha_2 . \tag{6b}$$

Also, for s_0 to be the shortest distance, the line DE must be perpendicular to both of the nuclear surfaces. Taking DE as the perpendiculars at both points D and E (the normal vectors), we can write

$$\tan\psi_1=-\frac{R_1'(\alpha_1)}{R_1(\alpha_1)} \quad \text{and} \quad \tan\psi_2=-\frac{R_2'(\alpha_2)}{R_2(\alpha_2)} , \tag{7}$$

where $R'(\alpha)$ is the derivative of $R(\alpha)$ with respect to α. An additional condition of s_0 being a minimum distance is obtained from

$$\frac{\partial s_0}{\partial\alpha_1}=0=\frac{\partial s_0}{\partial\alpha_2} . \tag{8}$$

Satisfying conditions (5), (7), and (8) [or (6) and (7) for the special case] iteratively, we can determine α_1 and α_2. The iterative procedure is to choose α_1 and α_2, calculate ψ_1 and ψ_2 from (7), and check if (5) and (8) [or (6) for the special case] is satisfied. Vary the initial set until the process converges. Obtaining α_1 and α_2, we can determine angle σ:

$$\sigma=180-\psi_1-\psi_2=\alpha_1+\alpha_2-\theta_1+\theta_2 . \tag{9}$$

Next, from the same two triangles ABC and CDE, we can write

$$\frac{\text{CE}}{\sin\psi_1}=\frac{\text{CD}}{\sin\psi_2}=\frac{s_0}{\sin\sigma} \tag{10}$$

and

$$\frac{\text{CE}+R_2(\alpha_2)}{\sin(\theta_1-\alpha_1)}=\frac{\text{CD}+R_1(\alpha_1)}{\sin(\theta_2+\alpha_2)}=\frac{R}{\sin\sigma} . \tag{11}$$

Solving equations (10) and (11), we get

$$s_0=\frac{R\sin(\theta_1-\alpha_1)-R_2(\alpha_2)\sin\sigma}{\sin\psi_1}$$
$$=\frac{R\sin(\theta_2+\alpha_2)-R_1(\alpha_1)\sin\sigma}{\sin\psi_2} . \tag{12}$$

For the special case of s_0 being parallel to R, Eq. (12) simplifies to

$$s_0=R-R_1(\alpha_1)\cos\psi_1-R_2(\alpha_2)\cos\psi_2 , \tag{13}$$

which also follows directly from the geometry of Fig. 1.

The mean curvature radius $\bar{R}$ in proximity potential (1) is obtained[10] for two deformed nuclei as

$$\frac{1}{\bar{R}^2}=\frac{1}{R_{11}R_{12}}+\frac{1}{R_{21}R_{22}}+\left[\frac{1}{R_{11}R_{21}}+\frac{1}{R_{12}R_{22}}\right]\sin^2\varphi$$
$$+\left[\frac{1}{R_{11}R_{22}}+\frac{1}{R_{21}R_{12}}\right]\cos^2\varphi , \tag{14}$$

where φ is the azimuthal angle between the principal planes of curvature of nucleus 1 and nucleus 2 (the angle $\varphi=0°$ for the present case of coplanar nuclei); and the four principal radii of curvature at the points D and E of minimum s_0 are given by Baltz and Bayman,[2]

$$R_{i1}=\left|\frac{\{R_i^2(\alpha_i)+[R_i'(\alpha_i)]^2\}^{3/2}}{R_i''(\alpha_i)R_i(\alpha_i)-2[R_i'(\alpha_i)]^2-R_i^2(\alpha_i)}\right| , \tag{15a}$$

$$R_{i2}=\left|\frac{R_i(\alpha_i)\sin\alpha_i[R_i^2(\alpha_i)+(R_i'(\alpha_i))^2]^{1/2}}{R_i'(\alpha_i)\cos\alpha_i-R_i(\alpha_i)\sin\alpha_i}\right| , \tag{15b}$$

with $i=1,2$ for the two interacting nuclei, respectively. Thus all the quantities required to evaluate the nuclear proximity potential V_N are defined. However, to calculate the heavy-ion interaction potential $V(R)$, we also need to determine the Coulomb interaction V_C between the two deformed nuclei, such that

$$V(R)=V_N+V_C . \tag{16}$$

Various prescriptions have been used[4,5,8,14] for the Coulomb interaction between two deformed and oriented nuclei. Here we use the expression of Wong,[14] obtained by following Alder and Winther,[15] for two nonoverlapping charge distributions,

$$V_C=\frac{Z_1Z_2e^2}{R}+\left[\frac{9}{20\pi}\right]^{1/2}\left[\frac{Z_1Z_2e^2}{R^3}\right]\sum_{i=1}^{2}R_{0i}^2\beta_{2i}P_2(\cos\alpha_i)+\left[\frac{3}{7\pi}\right]\left[\frac{Z_1Z_2e^2}{R^3}\right]\sum_{i=1}^{2}R_{0i}^2[\beta_{2i}P_2(\cos\alpha_i)]^2 , \tag{17}$$

where α_i is the angle between the radius vector and the symmetry axis of the ith nucleus (Fig. 1). The quadrupole-quadrupole interaction term, which is proportional to $\beta_{21}\beta_{22}$, is neglected because of its short range character. It is important to mention here that once again we do not have a proper method to be able to calculate the Coulomb potential for two overlapping (negative s_0) deformed nuclei. Within the double-folding model, Rhoades-Brown *et al.*[5] have, in principle, given a method for calculating the Coulomb potential for all values of s_0,

but for a low energy, this model is unrealistic in the overlap region since the overlapping nuclear densities get compressed over the static equilibrium value.[4,5]

III. CALCULATION OF $^{238}U + ^{238}U$ POTENTIAL

Figure 2 gives the scattering potential $V(R)$ for the $^{238}U + ^{238}U$ system, calculated for four different orientations and using quadrupole deformations $\beta_{21}=\beta_{22}=0.261$ (from electron scattering data[16]), for the simple case of the shortest distance s_0 parallel to the relative distance R. Seiwert *et al.*[3] have also used this simplification of s_0 being parallel to R in the overlap regions. The energy scale in Fig. 2 is normalized to the binding energy E_B (=3604.2 MeV) of the $^{238}U + ^{238}U$ system at $R=\infty$. The arrows in this figure refer to the touching point ($s_0=0$) for each configuration. Since the proximity model becomes less and less accurate as the overlap increases (negative s_0), we have studied here three different possibilities for the overlap regions: (i) The proximity potential (1) is used as such for the negative s_0. The solid curves beyond the arrow marks represent this calculation. (ii) The curves beyond the touching points (arrow marks) are all extrapolated to the ground state liquid drop energy E_{LD} of the compound system ($E_{LD}+E_B=-2772.85+3604.2=831.35$ MeV), as is also done by Münchow *et al.*[4] This is shown as dashed lines. (iii) The $^{238}U + ^{238}U$ system is considered to make a single necked system and the corresponding expression (A6) for V_N (derived in the Appendix) is used instead of Eq. (1). The result of this calculation is shown as a dot-dashed line for

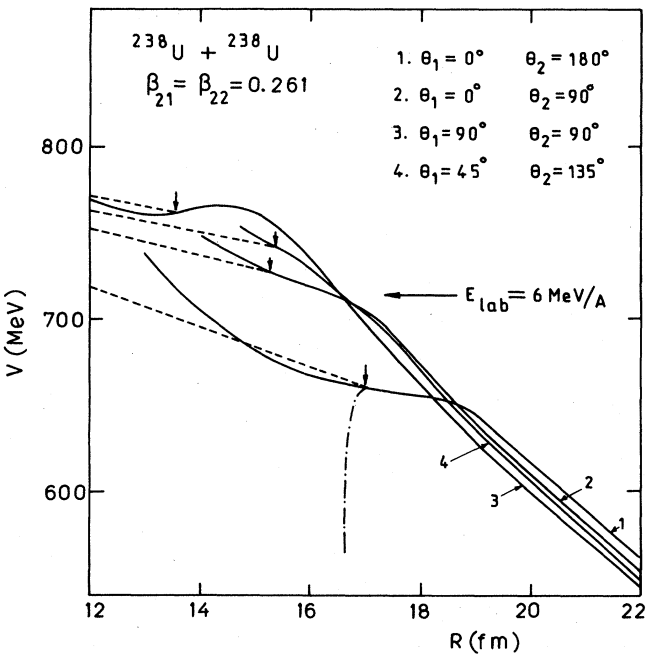

FIG. 2. The nuclear proximity plus Coulomb potential of $^{238}U + ^{238}U$ for four different relative orientations θ_1 and θ_2 with quadrupole deformation $\beta_{21}=\beta_{22}=0.261$. For the overlapping region (to the left of the touching configurations marked by arrows) three different methods have been used (see the text).

$\theta_1=0°$, $\theta_2=180°$ only, using the nuclear shapes calculated by Zohni *et al.*[17] on the asymmetric two-center shell model.

We observe that the calculated potentials in Fig. 2 show the effect of orientations in shifting the relative barrier heights by about 110 MeV and the positions by about 4 fm, which is in general agreement with the earlier calculations of Münchow *et al.*[4] and of Rhoades-Brown *et al.*,[5] both using the double-folding model, and that of Seiwert *et al.*[3] using the proximity function of a Bass-type potential[11] whose parameters are determined by fitting the double-folding potential for spherical nuclei. The barrier for $\theta_1=0°$, $\theta_2=180°$ is shown to be the lowest and to be at nearly the same position in all the calculations. As the orientation increases, the barrier height increases but its position decreases systematically. This result has important consequences for the choice of incident bombarding energy when other dynamical effects of friction and viscosity are included.[18]

On the absolute energy scale, however, our calculated proximity potential barriers are 10—15 MeV lower than those of Rhoades-Brown *et al.*,[5] whereas Münchow *et al.*[4] obtain their calculated barriers about 75 MeV higher than these authors.[5] The above-mentioned proximity potentials of Seiwert *et al.*[3] were also lower by about 20 MeV, as compared to those of Rhoades-Brown *et al.*[5] In other words, the proximity model predicts barriers that are lower than the ones calculated on the double-folding model, and the double-folding model calculations of Münchow *et al.*[4] give barriers that are higher than those obtained in similar double-folding model calculations of Rhoades-Brown *et al.*[5]

In Fig. 2, we have also indicated an experimental beam energy of 6 MeV/nucleon ($E_{c.m.}=714$ MeV). It is evident that for experiments with laboratory energy, E_{lab}, of the order of 6 MeV/nucleon, only the lowest interaction barrier with $\theta_1=0°$, $\theta_2=180°$ can be overcome to form a long-lived giant compound system. Higher beam energies are required at other orientations. The presence of "pockets" is indicated in our calculations (see curve 1 with a dot-dashed line beyond the arrow mark and curve 3, solid line); this presence apparently should depend on the nature of the nuclear interaction and the proper determination of the Coulomb potential for the overlap regions.

IV. SUMMARY AND DISCUSSION OF RESULTS

We have derived the proximity potential between two axially symmetric deformed and oriented nuclei, lying in the same plane, by using an altogether different procedure for estimating the shortest distance between the colliding surfaces. As a first application of our method the pocket formula of Blocki *et al.*[10] is used. The expressions for the principal radii of curvature are taken from the work of Baltz and Bayman.[2] Only the quadrupole deformations are considered. Of course, our method can equally well be extended to include hexadecapole deformations and be extended to other forms of the proximity model—like the modified proximity formula of Blocki and Swiatecki[19] and the Bass[11]-type potential.

We have used our method to calculate the $^{238}U + ^{238}U$

interaction potential. We find that for this system, our proximity method yields results which are quite similar to those obtained with the double-folding models.[4,5] The barrier is shown to be the lowest for $\theta_1=0°$ and $\theta_2=180°$. In all the calculations the value taken for the quadrupole deformation is $\beta_2=0.261$ for ^{238}U, though in one of the calculations[5] a small hexadecapole deformation $\beta_4=0.087$ is also included. In complete analogy with our work, Seiwert *et al.*[3] have also used the proximity method of Blocki *et al.*[10] but with principal radii calculated differently from those of Baltz and Bayman.[2] For the $^{238}\text{U}+^{238}\text{U}$ system, using a larger quadrupole deformation ($\beta_2=0.264$) and with larger hexadecapole deformation ($\beta_4=0.106$), they[3] obtained the minima in potential energy surfaces corresponding to the orientations $\theta_1=\theta_2=55°$ and $\theta_1=115°$, $\theta_2=65°$, instead of $\theta_1=0°$ and $\theta_2=180°$. This difference has, perhaps, to do with the use of different expressions for the principal radii of curvature and the different deformation parameters. These authors have already shown[3] that for a fixed orientation ($\theta_1=\theta_2=50°$), the use of different deformation parameters lead to not only the radial shift of the minima (by about 1 fm) but also its height changes by up to 50 MeV. At this orientation, the increase in hexadecapole deformation alone (from 0.087 to 0.106) lowers the minima by more than 20 MeV and shifts it radially by about 1 fm. Effects of similar orders are observed[5] at other orientations also. It is also relevant to remind the reader here that the two double-folding model calculations,[4,5] using the same two-body interaction (the Gaussian type), result in barriers differing by about 75 MeV.

We, therefore, conclude that further investigation is required for us to say when the proximity and the double-folding potentials deviate in the case of two deformed nuclei. Our present calculations using the pocket formula of Blocki *et al.*[10] and another different calculation of Seiwert *et al.*[3] support the idea that there is good similarity between these proximity potentials and the double-folding model calculations of Rhoades-Brown *et al.*[5]

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APPENDIX: PROXIMITY POTENTIAL FOR A NECKED SYSTEM FORMED BY TWO EQUAL COLLIDING NUCLEI

As the two colliding nuclei start to overlap, they form a crevice ($s_0=0$) and in an adiabatic approximation the system would adjust its shape parameters [the deformations β_{2i} and the neck ϵ; see Fig. 3(a)] such that it has a minimum of energy. We assume that for collisions between two identical nuclei $\beta_{21}=\beta_{22}$, as is shown[17] to be the case for $^{238}\text{U}+^{238}\text{U}$. The two nuclei would thus form a single indented body in the form of a single hyperboloid of one sheet with a hyperboloidal crevice [Fig. 3(b)]. For such a necked system, Blocki *et al.*[10] obtained the proximity potential to be of the form

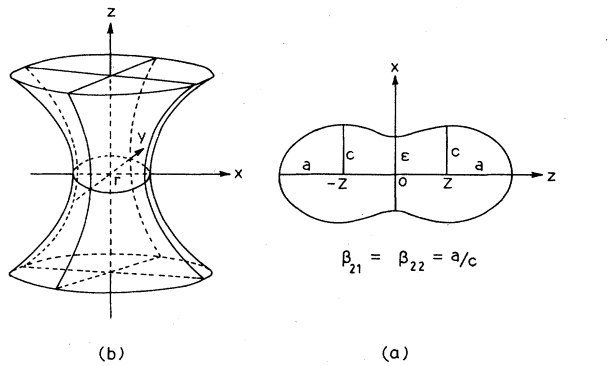

FIG. 3. (a) A sample nuclear shape formed in the two-center shell model during the collision of two identical nuclei. (See Ref. 17 for the actual shapes of the $^{238}\text{U}+^{238}\text{U}$ system.) (b) A schematic representation of a hyperboloid of revolution in one sheet.

$$V_N=4\pi\gamma b^2\frac{B^2}{4C^2}\Phi_1(\zeta_0=0)\ , \tag{A1}$$

where $\Phi_1(\zeta_0)$ is the first moment of the universal function (1) and for $\zeta_0=0$, $\Phi_1(\zeta_0=0)=-2.0306$ (from Table 1 of Ref. 10). B and C are the semiaxes of the hyperboloid with C along the line of centers [Fig. 3(b)], and all other quantities are as defined earlier.

The equation of a hyperboloid of revolution in one sheet [Fig. 3(b)] is

$$\frac{x^2}{C^2}+\frac{y^2}{C^2}-\frac{z^2}{B^2}=1 \tag{A2}$$

or

$$x^2+y^2=C^2\left[1+\frac{z^2}{B^2}\right]\equiv r^2\ , \tag{A3}$$

the radius of the circle. For the geometry relevant to our problem [Fig. 3(a)], $r=\epsilon$ (the neck parameter) at $z=0$ and $r=c$ at $z=\pm Z$. Then, it follows from (A3):

$$C^2=\epsilon^2 \tag{A4}$$

and

$$B^2=\frac{C^2Z^2}{c^2-C^2}\ . \tag{A5}$$

Substituting for B^2 and C^2 in (A1), we get

$$V_N=\pi\gamma b^2\frac{Z^2}{c^2-\epsilon^2}\Phi_1(\zeta_0=0)\ . \tag{A6}$$

Knowing the shape parameters Z, ϵ, and c, we can use (A6) to calculate the proximity potential for the overlap region (negative s_0) forming a necked shape during the collision of two equal nuclei. This formula is valid until the neck disappears, i.e., for $\epsilon<c$. One could also use (A6) for the symmetric fission of a fissioning nucleus.

[1]R. G. Stokstad, Y. Eisen, S. Kaplanis, D. Pelte, U. Smilanski, and I. Tserruya, S. Afr. J. Phys. **1**, 171 (1978); R. G. Stokstad and E. E. Gross, Phys. Rev. C **23**, 281 (1981); B. B. Back, R. R. Betts, W. Henning, K. L. Wolf, A. C. Mignerey, and J. M. Lebowitz, Phys. Rev. Lett. **45**, 1230 (1980).

[2]A. J. Baltz and B. F. Bayman, Phys. Rev. C **26**, 1969 (1982).

[3]M. Seiwert, W. Greiner, V. Oberacker, and M. J. Rhoades-Brown, Phys. Rev. C **29**, 477 (1984).

[4]M. Münchow, D. Hahn, and W. Scheid, Nucl. Phys. **A388**, 381 (1982).

[5]M. J. Rhoades-Brown, V. E. Oberacker, M. Seiwert, and W. Greiner, Z. Phys. A **310**, 287 (1983).

[6]R. K. Gupta, N. Malhotra, and R. Aroumougame, International Centre for Theoretical Physics, Trieste (Italy) Report No. IC/81/63, 1981.

[7]H. Tricoire, H. Flocard, and D. Vautherin, Phys. Lett. **100B**, 106 (1981).

[8]R. Aroumougame and R. K. Gupta, J. Phys. G **6**, L155 (1980).

[9]J. Randrup and J. S. Vaagen, Phys. Lett. **77B**, 170 (1978).

[10]J. Blocki, J. Randrup, W. J. Swiatecki, and C. F. Tsang, Ann. Phys. (N.Y.) **105**, 427 (1977).

[11]R. Bass, Phys. Lett. **B47**, 139 (1973).

[12]W. Greiner, International Advanced Course on Quantum Electrodynamics of Strong Fields, Lahnstein, 1981; M. Seiwert, N. Abul-Naga, V. Oberacker, J. A. Maruhn, and W. Greiner, Gesellschaft für Schwerionenforschung Annual Report, 1981.

[13]T. Morović and W. Greiner, Z. Naturforsch. **31a**, 327 (1976).

[14]C. Y. Wong, Phys. Rev. Lett. **31**, 766 (1973).

[15]K. Alder and A. Winther, Nucl. Phys. **A132**, 1 (1969).

[16]T. Cooper, W. Bertozzi, J. Heisenberg, S. Kowalski, W. Turchintz, C. Williamson, L. Cardman, S. Fivozinsky, J. Lightbody, Jr., and S. Penner, Phys. Rev. C **13**, 1083 (1976).

[17]O. Zohni, J. Maruhn, W. Scheid, and W. Greiner, Z. Phys. A **275**, 235 (1975); O. Zohni, Nucleonika **21**, 801 (1976).

[18]M. Münchow, R. K. Gupta, and W. Scheid, J. Phys. (Paris) Colloq. (to be published).

[19]J. Blocki and W. J. Swiatecki, Ann. Phys. (N.Y.) **132**, 53 (1981).