Theory of two-nucleon transfer reactions

N. Austern
Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, Pennsylvania 15260 and Department of Physics, Kyushu University, Fukuoka, Japan

M. Kawai
Department of Physics, Kyushu University, Fukuoka, Japan
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The two-step matrix element for two-nucleon transfer is evaluated by use of closure for the continuum intermediate states of relative motion of the transferred particles. Special assumptions that the intermediate system is a deuteron, or is in a particular spin state, are avoided.

I. INTRODUCTION

Direct reaction theories of two-nucleon transfer reactions with light ions remain incomplete, despite considerable effort to develop suitable approaches.[1-8] To understand the previous work and to introduce our new procedure, we consider a schematic (p,t) reaction

$$(A,1,2)+\mathrm{p} \xrightarrow{\text{one step}} A+(\mathrm{p},1,2)\ .$$
$$\searrow \text{ two step } \nearrow$$
$$(A,1)+\mathrm{p}+2$$

Here the notation $(A,1,2)$ represents the target nucleus state, with neutrons 1,2 bound to nucleus A; (p,1,2) represents the product triton bound state. The reaction may proceed either by the simultaneous transfer of both neutrons, as indicated by the first line above, or by successive transfer of the neutrons one at a time, as indicated in the second line. In the two-step contribution, the system formed by the transfer of the first neutron is understood to contain one or more states of an intermediate bound nucleus $(A,1)$. Difficulties arise in the description of the motion of the unbound particles p + 2 in the intermediate system.

The relative motion of particles p + 2 is most frequently taken to be that of the deuteron ground state,[2-6] so the intermediate system consists of the bound nucleus $(A,1)$ plus an intermediate bound deuteron (p,2). In this approach the transition through the intermediate system is treated by two perturbative steps (two-step DWBA), using a Green's function for the motion of the deuteron center of mass. One defect of this approach is that the deuteron wave function is a very restricted representation of the possible relative motion of p + 2. It is also unclear what distorting potential should be used in the intermediate Green's function. As a more extended approach one could introduce the resonant singlet state of the deuteron, in addition to the ground state.[7] Unfortunately this extension has a further handicap, that the radial form factor to be associated with the singlet state is unclear.

Another extended treatment of the intermediate system supplements the deuteron ground state with a set of excited, breakup states.[8] In principle this allows a complete description of the intermediate state of the p + 2 system. However, thus far such additional states have been used only in a two-step DWBA procedure, therefore this extended treatment still suffers from the inadequacies of the DWBA. In particular it is known from nonperturbative three-body treatments of the deuteron-nucleus system that coupling between breakup states is strong[9,10] and it significantly affects the three-body continuum. Although this difficulty can be met by a discretized coupled channels treatment of the intermediate three-body system,[9,11] it will be some while before such a complicated approach can even be attempted numerically.

In the present paper we apply closure (the adiabatic method) to the excited states of the intermediate deuteron. By this means we deal with nearly all the problems mentioned above: We include the three-body continuum, we include the effect of coupling among breakup states, and we have a completely definite treatment of singlet intermediate states. Coupled equation methods are not required. The potential in the intermediate Green's function is not fitted to any asymptotic scattering channel. The new approach can be applied to a variety of other transfer reactions, as well as (p,t). We note, however, that the closure theory has no bearing on the recent disagreements[7,12] about phase relations in the interference between one-step and two-step contributions.

II. THEORY

A careful direct reaction analysis begins with a choice of a model space that describes the active degrees of freedom of the dynamical system. For simplicity let us assume first that the intermediate system contains only one state of $(A,1)$, with neutron 1 in a bound shell-model orbital $\varphi(1)$. We then define the model space in terms of a wave function for the interacting system, of the form of a linear combination

$$\Psi=\chi_{\mathrm{p}}^{(+)}(\mathbf{r}_{\mathrm{p}})\varphi_{\mathrm{p}}(1,2)+\varphi(1)\eta(\mathrm{p},2)+\chi_{\mathrm{t}}^{(+)}(\mathbf{R}_{\mathrm{t}})\varphi_{\mathrm{t}}(\mathrm{p},1,2)\ , \tag{1}$$

with an associated Hamiltonian

$$H=K_{\rm p}+K_1+K_2+U_{\rm p}(r_{\rm p})+U_1(r_1)+U_2(r_2)$$
$$+V_{\rm p1}(r_{\rm p1})+V_{\rm p2}(r_{\rm p2})+V_{12}(r_{12})\ . \quad (2)$$

Extensions of the model to include many states of $(A,1)$ and antisymmetry with respect to the neutrons are considered at the end of this section. In the present discussion the target nucleus is understood to be structureless and fixed at the origin. The K_i are single-particle kinetic energy operators, the U_i are shell-model potentials or optical potentials, and the V_{ij} are nucleon-nucleon interactions. The bound wave functions in Eq. (1) are $\varphi_{\rm p}(1,2)$ for the $(A,1,2)$ bound target nucleus in the proton channel, $\varphi(1)$ for the $(A,1)$ state, and $\varphi_{\rm t}$ for the triton internal wave function.

We assume $\chi_{\rm p}^{(+)}(\mathbf{r}_{\rm p})$ to be a known function $\hat{\chi}_{\rm p}^{(+)}$, governed by some empirical proton elastic optical potential $\hat{U}_{\rm p}(r_{\rm p})$, due to nucleus A. We now proceed to determine the intermediate wave function $\eta({\rm p},2)$ and the exit channel wave function $\chi_{\rm t}^{(+)}$.

The $\chi_{\rm t}^{(+)}$ function is determined by the projection

$$0=(\varphi_{\rm t}\,|\,E-H\,|\,\Psi\rangle\ , \quad (3)$$

where as usual the parenthesis (| indicates integration over only the coordinates of the wave function it encloses. Equation (3) yields

$$[E-\epsilon_{\rm t}-K_{\rm t}-\overline{U}(R_{\rm t})]\chi_{\rm t}^{(+)}(\mathbf{R}_{\rm t})$$
$$=(\varphi_{\rm t}\,|\,V_{\rm p1}+V_{\rm p2}\,|\,\varphi_{\rm p}\hat{\chi}_{\rm p}^{(+)}(\mathbf{r}_{\rm p})\rangle$$
$$+(\varphi_{\rm t}\,|\,H-E\,|\,\varphi(1)\eta({\rm p},2)\rangle\ , \quad (4)$$

with

$$\overline{U}(R_{\rm t})=(\varphi_{\rm t}\,|\,U_{\rm p}+U_1+U_2\,|\,\varphi_{\rm t})\ . \quad (5)$$

In Eq. (4) $\epsilon_{\rm t}$ is the internal energy of $\varphi_{\rm t}$ and $K_{\rm t}$ is the kinetic energy operator for the triton center of mass. The interaction operator in the first term on the right-hand side (rhs) of Eq. (4) is obtained by allowing $H-E$ to operate to the right on $\varphi_{\rm p}\hat{\chi}_{\rm p}^{(+)}$. A term $U_{\rm p}-\hat{U}_{\rm p}$ is omitted from this operator. The (p,t) amplitude is now expressed in terms of the standard, time-reversed, normalized, homogeneous solutions $\hat{\chi}_{\rm t}^{(-)}$ of the left-hand side (lhs) of Eq. (4) as

$$T({\rm p,t})=\langle\hat{\chi}_{\rm t}^{(-)}(\mathbf{R}_{\rm t})\varphi_{\rm t}\,|\,V_{\rm p1}+V_{\rm p2}\,|\,\varphi_{\rm p}\hat{\chi}_{\rm p}^{(+)}\rangle$$
$$+\langle\hat{\chi}_{\rm t}^{(-)}(\mathbf{R}_{\rm t})\varphi_{\rm t}\,|\,\Delta U\,|\,\varphi(1)\eta({\rm p},2)\rangle\ , \quad (6)$$
$$\equiv T^{(1)}+T^{(2)}\ . \quad (7)$$

Here $T^{(1)}$ is the familiar one-step amplitude for the reaction, and $T^{(2)}$ is a kind of two-step amplitude, defined in terms of the intermediate three-body wave function $\eta({\rm p},2)$. The interaction

$$\Delta U\equiv U_{\rm p}+U_1+U_2-\overline{U}(R_{\rm t}) \quad (8)$$

in $T^{(2)}$ is obtained by allowing $H-E$ to operate to the left on $\hat{\chi}_{\rm t}^{(-)}\varphi_{\rm t}$.

A modified version of $T^{(2)}$ can be constructed by introducing the projection operators

$$P_{\rm t}=|\,\varphi_{\rm t})(\varphi_{\rm t}\,|\ ,\quad Q_{\rm t}=1-P_{\rm t}\ , \quad (9)$$

in Eq. (4) to yield the modified equation

$$(E-\epsilon_{\rm t}-K_{\rm t}-\overline{U})[\chi_{\rm t}+(\varphi_{\rm t}\,|\,\varphi\eta)]$$
$$=(\varphi_{\rm t}\,|\,V_{\rm p1}+V_{\rm p2}\,|\,\varphi_{\rm p}\hat{\chi}_{\rm p}^{(+)}\rangle+(\varphi_{\rm t}\,|\,H-E\,|\,Q_{\rm t}\varphi\eta\rangle\ . \quad (4')$$

In this equation the lack of orthogonality between $\varphi\eta$ and the exit channel is recognized, and the projection of $\varphi\eta$ on the exit channel is incorporated on the lhs as part of the (undetermined) exit channel relative wave function. This modification is not a "correction" for nonorthogonality; all linearly-independent nonorthogonal bases are "correct," even if they may be clumsy. Rather, at the cost of some complication in the calculation of $\chi_{\rm t}$, the modification introduced in Eq. (4') may reduce the inaccuracy introduced by our subsequent approximate calculation of $\eta({\rm p},2)$. This modification is not pursued further in the present paper.

The intermediate three-body wave function $\eta({\rm p},2)$ can be derived from the projection equation

$$0=(\varphi(1)\,|\,E-H\,|\,\Psi\rangle\ ,$$

from which we have

$$[E-\epsilon-K_2-K_{\rm p}-V_{\rm p2}-W(\mathbf{r}_2,\mathbf{r}_{\rm p})]\eta(p,2)$$
$$=(\varphi(1)\,|\,V_{\rm p2}\,|\,\varphi_{\rm p}\hat{\chi}_{\rm p}^{(+)}\rangle\ , \quad (10)$$

with the distorting potential

$$W(\mathbf{r}_2,\mathbf{r}_{\rm p})=\overline{U}_2+\overline{U}_{\rm p}\ ,$$

where

$$\overline{U}_i=U_i+(\varphi(1)\,|\,V_{i1}\,|\,\varphi(1))\ , \quad (11)$$

for $i=2$ and p are the optical potentials due to the intermediate nucleus $(A,1)$. For large A, it seems reasonable to assume

$$U_{\rm p}\approx\overline{U}_{\rm p}\approx\hat{U}_{\rm p}\ \text{and}\ U_2\approx\overline{U}_2 \quad (12)$$

in the matrix element. Equation (10) adopts a DWBA approximation, in that a small term due to coupling to $\chi_{\rm t}\varphi_{\rm t}$ has been omitted. Also, a small term

$$(\varphi(1)\,|\,V_{\rm p1}+U_{\rm p}-\hat{U}_{\rm p}\,|\,\varphi_{\rm p}\chi_{\rm p}^{(+)})$$

is neglected; this term would be proportional to $\overline{U}_{\rm p}-\hat{U}_{\rm p}$ if $\varphi_{\rm p}$ were to have a factorized form, say, $\varphi_{\rm p}=\varphi(1)\varphi(2)$. The eigenenergy ϵ of the bound function $\varphi(1)$ appears on the lhs of Eq. (10).

The two-step amplitude $T^{(2)}$ of Eq. (6) is obtained formally by substituting the Green's function solution of Eq. (10) for $\eta({\rm p},2)$ in Eq. (6). We have

$$T^{(2)}=\langle\hat{\chi}_{\rm t}^{(-)}\,|\,\Delta U\,|\,\varphi(1))$$
$$\times[E^{(+)}-\epsilon-K_2-K_{\rm p}-V_{\rm p2}-W(\mathbf{r}_2,\mathbf{r}_{\rm p})]^{-1}$$
$$\times(\varphi(1)\,|\,V_{\rm p2}\,|\,\varphi_{\rm p}\hat{\chi}_{\rm p}^{(+)}\rangle\ . \quad (13)$$

Let us transform the kinetic energy operator in the Green's function to relative and center of mass coordinates, $\mathbf{r}_{\rm p2}$ and $\mathbf{R}$, respectively, so that

$$K_2+K_{\rm p}+V_{\rm p2}=K_R+h_{\rm p2}\ , \quad (14a)$$

$$h_{\rm p2}=K_{\rm p2}+V_{\rm p2}\ . \quad (14b)$$

Previous approximations[2-8] of Eq. (13) simplify the three-body Green's function by projecting it on various chosen intermediate eigenstates of the relative Hamiltonian. However, as was noted in the Introduction, this is an inadequate procedure.

In the present work we carry *all* the eigenstates of the relative Hamiltonian h_{p2} in Eq. (13), using a closure approximation in which the associated eigenenergies are treated as having a single, common value $\bar{\epsilon}$. To insert this approximation in Eq. (13) we merely replace h_{p2} by $\bar{\epsilon}$. Under this closure approximation the Green's function becomes a local function of $\mathbf{r}_{p2}$, as if $\mathbf{r}_{p2}$ were a classical parameter, whose value remains constant as the remainder of the Green's function is computed. Although such an approximation is not valid throughout the entire configuration space, it only deteriorates gradually in matrix elements like Eq. (13), in which the effect of nonlocality of the Green's function is fairly limited. It is probably a much more plausible approximation for Eq. (13) than any restricted choice of intermediate eigenstates.

A further simplification to make the Green's function easier to handle is to omit the parametric dependence on $\mathbf{r}_{p2}$ from the distorting potential $W(\mathbf{r}_2,\mathbf{r}_p)$. At $\mathbf{r}_{p2}=0$ this potential reduces to the Johnson-Soper potential[13]

$$W(\mathbf{r}_2,\mathbf{r}_p) \underset{r_{p2}=0}{\rightarrow} U_{JS}(R) \tag{15}$$

for the p + 2 system.

The two-step amplitude has now been reduced to

$$T^{(2)} \approx \langle \hat{\chi}_t^{(-)} \varphi_t \mid \Delta U \mid \varphi(1) \rangle [E^{(+)} - \epsilon - \bar{\epsilon} - K_R - U_{JS}(R)]^{-1} \langle \varphi(1) \mid V_{p2} \mid \varphi_p \hat{\chi}_p^{(+)} \rangle \ . \tag{16}$$

Equation (16) is not more difficult to evaluate than previous two-step DWBA expressions. All the dependence on $\mathbf{r}_{p2}$ is in the interactions and the channel wave functions at the two ends of the above expression. Since the normalized triton wave function is more compact than the deuteron ground state wave function, its magnitude within the range of the potentials in Eq. (16) must be larger, and therefore it is plausible that the magnitude of $T^{(2)}$ from Eq. (16) will be larger than from previous works. We also note the presence of the rather strong, but weakly-absorbing U_{JS} distorting potential in the Green's function in Eq. (16).

The relation to previous two-step theories is easy to see. For example, the intermediate deuteron model[2-7] is obtained if the Green's function in Eq. (16) is multiplied by a deuteron projection operator $|\phi_d(\mathbf{r}_{p2})\rangle\langle\phi_d(\mathbf{r}_{p2})|$ and $U_{JS}(R)$ is replaced by a deuteron elastic scattering optical potential.

The value of the closure energy $\bar{\epsilon}$ can be estimated by examining the matrix elements of h_{p2} as it operates to the left and right in Eq. (13). In general $\bar{\epsilon}$ must depend on the triton wave function $\varphi_t(p,1,2)$ and on the target nucleus bound state $\varphi_p(1,2)$. To operate on φ_t, h_{p2} only encounters the rather smooth optical potentials in ΔU; however, to operate on φ_p it must encounter the short-ranged potential V_{p2}. It seems plausible that $\bar{\epsilon}$ is controlled by $\varphi_t \Delta U$, and that it should have a value a little greater than the average two-body energy in φ_t. We estimate

$$\bar{\epsilon} \approx -2 \text{ MeV} \ , \tag{17}$$

independent of the properties of φ_p. It might be desirable to use different values for $\bar{\epsilon}$ for singlet and triplet intermediate spin states.

Some generalization of the model wave function can be considered. Let us allow a *set* of intermediate bound states, so that the linear combination becomes

$$\Psi = \chi_p^{(+)} \varphi_p + \sum_i \varphi_i(1) \eta_i(p,2) + \chi_t^{(+)} \varphi_t \ . \tag{1'}$$

New effects could occur if we would allow the operator W in Eq. (10) to couple different φ_i states with each other. However, the inclusion of such couplings would be inconsistent with our other approximations of W; therefore we disregard them. On this basis the generalized intermediate state in Eq. (1') gives a very simple generalization of the two-step amplitude

$$T^{(2)} \approx \sum_i \langle \hat{\chi}_t^{(-)} \varphi_t \mid \Delta U \mid \varphi_i(1) \rangle \bar{G}_i^{(+)} \langle \varphi_i(1) \mid V_{p2} \mid \varphi_p \hat{\chi}_p^{(+)} \rangle \ , \tag{16'}$$

where

$$\bar{G}_i^{(+)} = [E^{(+)} - \epsilon_i - \bar{\epsilon} - K_R - U_{JS}(R)]^{-1} \ .$$

Equation (16') would be necessary, for example, if $\varphi_p(1,2)$ would contain pairing correlations.

We also need to pay attention to the Pauli principle. The model Hamiltonian is already symmetric in the coordinates of the two neutrons, and the bound wave functions $\varphi_p(1,2)$ and $\varphi_t(p,1,2)$ are antisymmetric. It is only necessary to modify Ψ by antisymmetrizing the intermediate state wave function of the two-step process:

$$\Psi = \chi_p^{(+)} \varphi_p + \sum_i [\varphi_i(1) \eta_i(p,2) - \varphi_i(2) \eta_i(p,1)] + \varphi_t \chi_t^{(+)} \ . \tag{1''}$$

This modification of Ψ causes an interesting modification of Eq. (10), which now becomes

$$[E-\epsilon_i-\bar{\epsilon}-K_R-U_{\rm JS}(R)]\left[\eta_i(\mathrm{p},2)-\sum_j \varphi_j(2)(\varphi_i(1)\,|\,\eta_j(\mathrm{p},1))\right]=(\varphi_i(1)\,|\,V_{\rm p1}+V_{\rm p2}+U_{\rm p}-\hat{U}_{\rm p}\,|\,\varphi_{\rm p}\hat{\chi}^{(+)}_{\rm p}\rangle$$
$$\approx(\varphi_i(1)\,|\,V_{\rm p2}\,|\,\varphi_{\rm p}\hat{\chi}^{(+)}_{\rm p}\rangle\ , \tag{10''}$$

where the approximations related to closure are inserted already on the lhs, and approximation Eq. (12) is used again on the rhs. Clearly the exchange term in Eq. (10'') interferes with the easy solution of Eq. (10'') for $\eta_i(\mathrm{p},2)$. Iterative procedures are required.

Fortunately the overlaps of neutron bound state $\varphi_i(1)$ with the continuum functions $\eta_j(\mathrm{p},1)$ are likely to be weak. If we disregard the exchange term in Eq. (10''), then the Pauli modification of the two-step amplitudes Eq. (16) or Eq. (16') is simple. It is only necessary to symmetrize them for the intermediate neutron bound state, so that

$$T^{(2)}\approx\sum_i\langle\hat{\chi}^{(-)}_{\rm t}\varphi_{\rm t}\,|\,\Delta U\,|\,[\varphi_i(1))\bar{G}^{(+)}_i(\varphi_i(1)\,|\,V_{\rm p2}+\varphi_i(2))\bar{G}^{(+)}_i(\varphi_i(2)\,|\,V_{\rm p1}]\,|\,\varphi_{\rm p}\chi^{(+)}_{\rm p}\rangle\ . \tag{16''}$$

Equation (16'') is now a matrix element of symmetric operators taken between antisymmetric wave functions. The rhs is exactly equal to $2\times$[rhs of (16')], as it should be, because now there are two routes of equal weight with bound $\sum_i\varphi_i(1)$ and $\sum_i\varphi_i(2)$ functions, instead of one with $\sum_i\varphi_i(1)$ only.

III. PRIOR-PRIOR FORM FOR PRACTICAL USE

We have derived a new theory for the two-step term of the (p,t) reaction, using a method that is applicable to other multistep direct reactions. Although the bound target nucleus is pictured as going through a definite sequence of energy eigenstates, the associated intermediate state dynamical behavior of the projectile fragments in the continuum is allowed to be arbitrary. We use closure to sum up all possible intermediate motions of the projectile fragments.

Equation (16'') summarizes the new results. It has the structure post-prior, which is known to avoid nonorthogonality terms. On the other hand, the post interaction ΔU (in this case) consists of long-range potentials, as seen in Eq. (8), and it is not convenient for practical calculations. It may be worthwhile to produce an alternative expression in which ΔU is replaced by short-range potentials, even if there are accompanying nonorthogonality terms.

The interaction ΔU in Eq. (16) first appeared in Eq. (6), when $H-E$ was allowed to operate to the left on $\chi^{(-)}_{\rm t}\varphi_{\rm t}$. We may instead replace ΔU by $H-E$ in Eq. (16) and operate to the right. Consistency of approximation between $H-E$ and the Green's function leads to

$$T^{(2)}\approx\langle\hat{\chi}^{(-)}_{\rm t}\varphi_{\rm t}\,|\,V_{\rm p1}+V_{21}\,|\,\varphi(1))\bar{G}^{(+)}(\varphi(1)\,|\,V_{\rm p2}\,|\,\varphi_p\hat{\chi}^{(+)}_{\rm p}\rangle-\langle\hat{\chi}^{(-)}_{\rm t}\varphi_{\rm t}\,|\,\varphi(1))(\varphi(1)\,|\,V_{\rm p2}\,|\,\varphi_p\hat{\chi}^{(+)}_{\rm p}\rangle\ , \tag{18}$$

in which approximation (12) is used.

Equation (18) has the expected form of a matrix element with short range interactions, corrected by a nonorthogonality term.[3,14] A corresponding version of Eq. (16'') is

$$T^{(2)}\approx\sum_i\langle\hat{\chi}^{(-)}_{\rm t}\varphi_{\rm t}\,|\,[(V_{21}+V_{\rm p1})\,|\,\varphi_i(1))\bar{G}^{(+)}_i(\varphi_i(1)\,|\,V_{\rm p2}+(V_{12}+V_{\rm p2})\,|\,\varphi_i(2))\bar{G}^{(+)}_i(\varphi_i(2)\,|\,V_{\rm p1}]\,|\,\varphi_{\rm p}\hat{\chi}^{(+)}_{\rm p}\rangle$$
$$-\sum_i\langle\hat{\chi}^{(-)}_{\rm t}\varphi_{\rm t}\,|\,[\varphi_i(1))(\varphi_i(1)\,|\,V_{\rm p2}+\varphi_i(2))(\varphi_i(2)\,|\,V_{\rm p1}]\,|\,\varphi_{\rm p}\hat{\chi}^{(+)}_{\rm p}\rangle\ , \tag{19}$$

which is exactly twice $T^{(2)}$ given by Eq. (18). It should be feasible to perform practical calculations with Eq. (19).

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