Diffractive, diffusive, and statistical aspects of deep inelastic heavy-ion collisions

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Deep inelastic collisions between "light" heavy ions are considered in a formalism containing diffractive, diffusive, and statistical aspects. A closed-form diffractive cross section is derived, with the deflection function being parametrized in a classically-motivated way. A statistical argument is used to demonstrate how the observed double differential cross section is built up from different diffractive contributions each with its own weight. The form of the weighting function is derived. The observed forward-peaked exponentially-decaying form of deep inelastic collision angular distributions is accounted for. The possibility of exciting a nonzero spin state is explicitly included, enabling the same formalism to be used to explain the spin polarization occurring in deep inelastic collisions. This quantity is much more sensitive than the angular distribution to the choice of deflection function parametrization. Reasonable fits to data are obtained with only one free parameter, apart from an overall normalization factor.

I. INTRODUCTION

Deep inelastic collisions (DIC's) have been studied extensively in recent years, both from experimental and theoretical viewpoints.[1] In the latter case, calculations using Newtonian mechanics, with the inclusion of both frictional forces and deformation degrees of freedom,[2] reproduce the average trends of the energy loss-scattering angle correlation, typically illustrated by a Wilczynski plot.[3] Element distributions are described fairly well using a different approach, namely the Fokker-Planck equation.[4] However, DIC angular distributions have not been systematically described in such models, and, in particular, little work has been done for relatively high energy collisions and/or "light" heavy ions, where the modified Sommerfeld parameter

$$\eta' = Z_1 Z_2 e^2/\hbar v' \lesssim 100\text{–}200 \ .$$

Here v' is the relative velocity of the two ions at the barrier radius. For large energy losses the angular distributions of such reactions are generally forward peaked and fall off roughly exponentially with angle.[5] In one study Gelbke *et al.*[6] fitted 315 MeV $^{16}O+^{208}Pb$ energy-loss integrated data with the form

$$\frac{d\sigma}{d\theta} \sim e^{-\theta/\tau}, \tag{1}$$

where τ was an adjustable parameter. Elsewhere[7] a similar form has been used for the *double differential* cross section for 88 MeV $^{16-18}O+^{27}Al$. The parameter τ reflects the degree of inelasticity of the process and increases with energy loss and/or number of transferred nucleons. It is straightforward to show that Eq. (1) arises classically in collisions involving orbiting, but it is important to note that the classical *double differential* cross section $d^2\sigma/dE\,d\Omega$ has a spike at the classical scattering angle, since in this case a given Q value corresponds to a unique scattering angle. Thus it is not clear, *a priori,* why the observed exponential shape, with forward peaking, persists for all fully damped reactions in this energy domain.

Recently Tamura *et al.*[8] used the distorted-wave Born approximation (DWBA) to describe DIC angular distributions. Presumably statistical fluctuations in macroscopic degrees of freedom, such as the energy loss, which arise from the coupling between the relative motion of the two ions and their intrinsic degrees of freedom, are included in this formalism if a large enough number of channels and steps are considered, but for practical reasons, calculations are limited to one- and two-step processes.

Hartmann[9] approaches the problem in a similar way to that presented here, but ignores any spin transfer between the two heavy ions, which is known to be large.[10,11] By including the possibility of spin transfer our model gives both angular distributions and polarization data in a consistent manner. In addition, the form of the weighting function that we use to describe statistical fluctuations has some justification, unlike that used elsewhere.[9,12]

The plan of the paper is as follows. In Sec. II we derive a closed-form expression for the diffractive cross section. In Sec. III we justify the form of the weighting function used to describe the diffusive part of the cross section. The concept of diffusion is explained in this section also, and we show how the statistical randomness of the intrinsic states is used in our formalism. These results are combined in Sec. IV to describe the angular distributions of the $^{58}Ni(^{16}O,X)$ reactions at 100 MeV.[13] The same formalism is used to describe polarization data for the same reactions[10,11] in Sec. V, and we end with a summary.

II. THE DIFFRACTIVE CROSS SECTION

In DIC's, nuclear identities are approximately maintained and so we assume that direct reaction theory is applicable. The transition amplitude may then be written in the form[14]

$$\beta_{IM}(\theta)=\frac{4\pi}{kk'}\sum_{ll'} i^{l-l'-I}(2l'+1)^{1/2}\langle l'I00\,|\,l0\rangle\langle l'I-MM\,|\,l0\rangle I^I_{ll'}Y_{l',-M}(\theta,0)\ , \tag{2}$$

where we have assumed that both nuclei have zero spin initially and that the final state has spin (I,M). The initial and final wave numbers are k and k', respectively, the quantization axis is along the incoming beam direction, $\hat{z}=\hat{k}$, and $\cos\theta=\hat{k}\times\hat{k}'$. The transfer form factor $I^I_{l'l}$ depends on the number of nucleons transferred, and the number of steps in this process. We parametrize this quantity as[15]

$$I^I_{l'l}=G_Q(I)f(l'-L')g(l-l')\ , \tag{3}$$

particular forms of which have been used elsewhere.[16] Here $G_Q(I)$, which is related to the level density, describes the probability of angular momentum transfer I at a given Q value, $g(l-l')$ describes the kinematic matching of the reaction, and $f(l'-L')$ contains the diffractive part of the process, L' being the grazing angular momentum in the final channel. Brink's matching condition[17] implies that $g(l-l')$ is peaked at $l-l'\approx L-L'\equiv\xi$, where L is the equivalent of L' for the initial channel and ξ is the mismatch of the reaction.

Using Eq. (3) in Eq. (4) and utilizing the asymptotic forms of the Clebsch-Gordan coefficients[18] we obtain

$$\beta_{IM}(\theta)=\frac{4\pi}{kk'}G_Q(I)\sum_{\nu} i^{\nu-I}d^I_{0\nu}(\tfrac{1}{2}\pi)d^I_{M\nu}(\tfrac{1}{2}\pi)g(\nu)\sum_{l'}(2l'+1)^{1/2}f(l'-L')Y_{l',-M}(\theta,0)\ , \tag{4}$$

where $\nu\equiv l-l'$. We now replace the spherical harmonics by their asymptotic forms[19] and rotate the axes to a more useful system, where the quantization axis is perpendicular to the reaction plane, $\vec{z}=\hat{k}\times\hat{k}'$, using the transformation[18]

$$\tilde{\beta}_{IM}(\theta)=\sum_{M'}\beta_{IM'}(\theta)\mathscr{D}^{I*}_{M'M}(-\tfrac{1}{2}\pi,-\tfrac{1}{2}\pi,0) \tag{5}$$

so that

$$\tilde{\beta}_{IM}(\theta)=\frac{2i^{I+M}}{kk'\sqrt{\sin\theta'}}d^I_{0M}(\tfrac{1}{2}\pi)G_Q(I)\times[g(M)F^+(\theta)+g(-M)F^-(\theta)]\ , \tag{6}$$

with

$$F^{\pm}(\theta)\equiv\sum_{l'}(2l'+1)^{1/2}f(l'-L')\times\exp\{\pm i[(l'+\tfrac{1}{2})\theta-\tfrac{1}{4}\pi]\}\ . \tag{7}$$

Equations (6) and (7) show the roles of the matching function g and the diffraction function f. The quantities $F^{\pm}$ are the farside and nearside amplitudes, respectively,[20,21] and these require $l-l'=\pm M$, respectively, in the present coordinate system.

A Gaussian form for $f(l'-L')$ has been used in the past,[16] but we chose instead to work with the parametrization

$$f(x)=e^{2i\delta(x)}\,\mathrm{sech}\left[\frac{x}{2\Delta}+\tfrac{1}{2}i\gamma\right]\ . \tag{8}$$

Here $\delta(x)$ is the real part of the phase shift, Δ is the width of the angular momentum window, and γ can be used to describe the asymmetry of the transition form factor. In principle, Δ and γ are functions of the Q value, but they are held constant in the present discussion.

The form of Eq. (8) is motivated by the relationship between orbital angular momentum and the energy loss in DIC's (Ref. 2) which gives an l-window effect. Large values of $|Q|$ arise when there is considerable overlap between the two nuclei, that is, at low angular momentum, but for $l'\sim L'$ the overlap, and hence $|Q|$, is smaller. Large angular momentum values do not contribute because of the enhanced angular momentum barrier, which prevents a significant overlap of the two ions. More quantitatively, work on one- and two-step processes by Chu *et al.*[22] gives some justification for the form of Eq. (8).

With Eq. (8), we can evaluate $F^{\pm}(\theta)$ analytically using the Poisson summation formula:[21]

$$F^{\pm}(\theta)\propto e^{\pm i[\Lambda'\theta-(1/4)\pi]}\sum_{n=-\infty}^{\infty}\frac{\exp[2\pi i n(\Lambda'-\frac{1}{2})+\gamma\Delta(2\pi n+\Theta\pm\theta)]}{\cosh[\pi\Delta(2\pi n+\Theta\pm\theta)]}\ , \tag{9}$$

with $\Lambda'=L'+\frac{1}{2}$. The approximation used to obtain Eq. (9) is that Δ is sufficiently small and the phase shift δ can be expanded linearly around the classical deflection angle. The physics behind the different Poisson terms has been extensively discussed in the literature.[21,23]

The deflection function Θ is defined to be

$$\Theta \equiv \frac{d}{dl'}(2\delta)\,|_{l'=L'} \tag{10}$$

and describes the "classical trajectory" of the heavy ions in their mean field, which is characterized by the phase shifts δ. We have neglected the change in Θ across the angular momentum window, and so have ignored the so-called dynamic fluctuations[9,12] in the amplitude. For light heavy ions this is a reasonable approximation.

The advantage of a Gaussian parametrization of $f(x)$ is that dynamic fluctuations can be readily included, and, indeed, they are essential ingredients in a description of DIC's between very heavy ions.[9] For lighter systems they become important only for the very highest energy losses. Inclusion of these fluctuations prevents us from being able to write a closed-form expression for the different Poisson amplitudes contained in Eq. (9), if we use the sech parametrization. However, studies of DIC's using classical transport equations[24] show that a nonsymmetric l window is necessary to reproduce data, and such a window can be very easily introduced (through the parameter γ) using the sech parametrization. In this respect the Gaussian form is less suitable. Thus for light heavy systems we feel that Eq. (8) is a more useful parametrization of the DIC angular momentum window.

To account for dynamic fluctuations in an approximate way with the sech parametrization, we could allow Δ to depend on the Q value. A suitable dependence, motivated by results[9] from the Gaussian form of $f(x)$, is

$$\Delta(Q) = \frac{\Delta_0}{(1+\frac{1}{4}\Delta_0^4\Theta'^2)^{1/2}}\ , \tag{11}$$

where Θ' is the derivative of the deflection function with respect to angular momentum evaluated at the grazing angular momentum and Δ_0 is some constant. Such a variation has not been included in the present discussion.

Since the level density, which is related to $G_Q(I)$, is high for DIC's, we assume that the measured cross section is an incoherent sum over all the different spin states which can possibly be excited in the collision. Thus the double differential cross section can be written, using Eq. (6), as

$$\frac{d^2\sigma}{dE_c d\Omega} = \sum_{IM} |\,\tilde{\beta}_{IM}\,|^2 \propto \left\{\sum_{IM} |\,G_Q(I)g(M)d^I_{0M}(\tfrac{1}{2}\pi)\,|^2\right\}[\,|\,F^+(\theta)\,|^2 + |\,F^-(\theta)\,|^2]\ , \tag{12}$$

$$\frac{d^2\sigma}{dE_c d\Omega} \equiv A(Q)D(\theta,\Theta)\ , \tag{13}$$

where $A(Q)$ and $D(\theta,\Theta)$ are equal to the parts of Eq. (12) in curly and square brackets, respectively. Spectroscopic factors and the kinematic matching conditions are contained in $A(Q)$, while $D(\theta,\Theta)$ describes the diffraction around the deflection angle Θ, which is Q dependent. The quantity E_c is the final state energy. We have neglected the interference term in writing Eq. (13), because the kinematic matching condition dictates that $g(M)$ is only important for $M \sim \xi \gg 0$. Consequently, as the product $g(M)g^*(-M)$ occurs in the interference term, this contribution can be neglected, which is consistent with the fact that DIC differential cross sections show no oscillatory structure.

Having derived an expression for the diffractive component of the DIC double differential cross section we now proceed to introduce the effects of statistical fluctuations or diffusion.

III. STATISTICAL FLUCTUATIONS AND RANDOMNESS

In Sec. II we have derived an expression [Eq. (6)] for the scattering amplitude, $\tilde{\beta}_{IM}$, which describes a transition between definite initial and final states. As many final states contribute to the *measured* cross section, however, we should, in principle, allow the quantities Λ', Δ, γ, and Θ appearing implicitly in Eq. (6) to be different for each of these final states. This is, of course, impractical.

The approach taken elsewhere[9,12] is to assume that the values of these parameters can be described by distribution functions, with their mean values being dependent only on the Q value. Both δ function and Gaussian distributions have been used in the past:[9,12] use of the former is equivalent to neglecting statistical fluctuations.

Our approach is different since we assume that the effect of statistical fluctuations can be introduced by applying a Q-dependent distribution function directly to the cross section of Eq. (13). This means that the fluctuations in Λ', Δ, γ, and Θ are implicit, and that Eq. (13) describes the cross section associated with the mean values of these parameters. We now justify this procedure by deriving an expression for this distribution function directly from formal scattering theory, rather than introducing some arbitrary form for it.

We consider two heavy ions moving in their combined mean field, which is independent of the intrinsic states, and assume that the relative kinetic energy is transferred into internal excitation energy through multistep multinucleon transfer processes. In each step Brink's rule[17] governs the kinematic matching of the angular momentum, while the (radial) matrix elements appearing in the transition amplitude have a random nature, similar to that pictured in the microscopic theory of DIC's given by Agassi *et al.*[25]

Let $|\chi_{E_\beta}(R)\rangle$ and $|\phi_\beta(\zeta_\beta)\rangle$ be the wave functions of relative motion in the mean field (with energy E_β) and of the intrinsic nuclear states (with excitation energy ϵ_β), respectively. Here ζ_β represents all the internal coordinates, $E_\beta+\epsilon_\beta=E_\alpha$, the total c.m. energy, and $\epsilon_\alpha=0$. Let V be the residual interaction and $G_0^+(E)$ the Green's function of the mean-field Hamiltonian describing the two-body heavy-ion system.

Then the full stationary-state wave function $|\Psi_\alpha^+\rangle$ is

given by the Lippmann-Schwinger equation[14]

$$|\Psi_\alpha^+\rangle=(1-G_0^+V)^{-1}|\chi_{E_\alpha}^+\phi_\alpha\rangle\ , \tag{14}$$

(where we have dropped the ζ_β for notational simplicity), and the transition amplitude, $T_{\beta\alpha}$, between the initial and final states is

$$T_{\beta\alpha}\equiv\langle\chi_{E_\beta}^-\phi_\beta|\,V\,|\Psi_\alpha^+\rangle \tag{15a}$$

$$=\langle\chi_{E_\beta}^-\phi_\beta|\,V\,|\chi_{E_\alpha}^+\phi_\alpha\rangle+\langle\chi_{E_\beta}^-\phi_\beta|\,VG_0^+V\,|\Psi_\alpha^+\rangle \tag{15b}$$

using Eq. (14). Note here that the mean-field Green's function G_0^+ describes the relative motion of the two heavy ions after losing a certain energy in intrinsic excitations, and is therefore to be evaluated at $E=E_\beta$. We now make the assumption that, because of the large energy losses involved, deep inelastic collisions are not one-step processes, and so the first term in Eq. (15b) can be neglected with respect to the second. Then, as we do not measure a precise final state, the double differential cross section may be written as

$$\frac{d^2\sigma}{dE_f d\Omega}=\sum_\beta\rho_\beta(E_\beta)\,|\,T_{\beta\alpha}\,|^2$$

$$\approx\sum_\beta\rho_\beta(E_\beta)\,|\,\langle\chi_{E_\beta}^-\phi_\beta|\,VG_0^+V\,|\Psi_\alpha^+\rangle\,|^2\ , \tag{16}$$

where ρ_β is the level density of the intrinsic states β, and dE_f is the energy element, centered at $E_\beta\approx E_f$, over which we perform the "coarse graining." We write the Green's function as

$$G_0^+(E_\beta)=\sum_\gamma\int dE_\gamma\frac{\rho_\gamma(E_\gamma)\,|\chi_{E_\gamma}^+\phi_\gamma\rangle\langle\chi_{E_\gamma}^-\phi_\gamma|}{E_\beta-E_\gamma+\frac{1}{2}i\Gamma_\gamma}\ , \tag{17}$$

where we can regard the Γ_γ as the decay widths of the resonance states which we take as our "doorways." Alternatively, in a classical language, Eq. (17) shows that G_0^+ acts as a "wave guide" by restricting the heavy ions to something close to their classical path.

Assuming that the matrix elements $\langle\chi_{E_\beta}^-\phi_\beta|\,V\,|\chi_{E_\gamma}^+\phi_\gamma\rangle$ have phases which fluctuate randomly, we are then able to write

$$\frac{d^2\sigma}{dE_f d\Omega}=\sum_{\beta\gamma}\rho_\beta(E_\beta)\int dE_\gamma\rho_\gamma(E_\gamma)\frac{|\,\langle\chi_{E_\gamma}^-\phi_\gamma|\,V\,|\Psi_\alpha^+\rangle\,|^2\,|\,\langle\chi_{E_\beta}^-\phi_\beta|\,V\,|\chi_{E_\gamma}^+\phi_\gamma\rangle\,|^2}{(E_\beta-E_\gamma)^2+\frac{1}{4}\Gamma_\gamma^2}\ . \tag{18}$$

Similar assumptions about phases of matrix elements have been used elsewhere[8,25] and are justified since we sum over a very large number of intrinsic states in evaluating the double differential cross section.

We can replace E_β in the denominator of Eq. (18) by E_f, as noted above, and then perform the sum over the intrinsic states β, which gives the total transition rate from the state $|\chi_{E_\gamma}^+\phi_\gamma\rangle$ to all states $|\chi_{E_\beta}^-\phi_\beta\rangle$. This is simply the width of the former,

$$\sum_\beta\rho_\beta(E_\beta)\,|\,\langle\chi_{E_\beta}^-\phi_\beta|\,V\,|\chi_{E_\gamma}^+\phi_\gamma\rangle\,|^2=\frac{\Gamma_\gamma}{2\pi}\ . \tag{19}$$

In addition, the sum over the intrinsic states γ in Eq. (18) gives the coarse-grained double differential cross section for a transition to a final state of mean relative kinetic energy E_γ,

$$\sum_\gamma\rho_\gamma(E_\gamma)\,|\,\langle\chi_{E_\gamma}^-\phi_\gamma|\,V\,|\Psi_\alpha^+\rangle\,|^2=\frac{d^2\sigma}{dE_\gamma d\Omega}\ . \tag{20}$$

We have assumed that the widths Γ_γ are only functions of the energies of the resonance states and do not depend on their precise structure. We identify Eq. (20) with the cross section given by Eq. (13), and so, replacing E_γ by E_c we obtain

$$\frac{d^2\sigma}{dE_f d\Omega}=\int dE_c\frac{\Gamma_c/2\pi}{(E_f-E_c)^2+\frac{1}{4}\Gamma_c^2}\frac{d^2\sigma}{dE_c d\Omega}\ , \tag{21}$$

where the subscript on Γ_c indicates that this quantity is, in general, a function of E_c.

The interpretation of this equation is clear: the cross section is built up from a diffusion of diffractive contributions about the energy E_f. The energy dependence of the diffractive cross sections gives the fluctuations in Λ', Δ, γ, and Θ, and we see that each such diffractive contribution is weighted by a factor equivalent to the distribution function introduced elsewhere,[9,12] but stress, however, that this *Lorentzian* weighting factor has some theoretical justification.

Classically we can picture this diffusion in terms of different trajectories, each making a contribution to the coarse-grained cross section which is weighted by a distribution function. We now apply the results of Eqs. (13) and (21) to experimental data.

IV. ANALYSIS OF ANGULAR DISTRIBUTIONS

We consider the reaction ^{58}Ni(^{16}O,X) at $E_{\rm lab}=100$ MeV and use the data of Harris and his co-workers.[13] Double differential cross sections for various products were measured, but for definiteness we concentrate on those where the light ejectile was boron or carbon. The data were taken in Q bins 8 MeV wide ($\langle Q\rangle-4\le Q_f\le\langle Q\rangle+4$ MeV), and so we note here that in our analysis we have evaluated

$$\frac{d\sigma}{d\Omega}=\int_{\langle Q\rangle-4}^{\langle Q\rangle+4}dQ_f\frac{d^2\sigma}{dE_f d\Omega}\quad E_f+Q_f=E_i\ , \tag{22}$$

where $d^2\sigma/dE_f d\Omega$ is taken from Eq. (21). The cross section obtained in Eq. (22) is effectively the product of the double differential cross section, $d^2\sigma/dQ\,d\Omega$, and the Q-bin width, provided that the Q-bin width is small.

Figure 1 shows a Wilczynski plot of the carbon data. Superimposed on the figure is the classical trajectory, which follows the ridge of the Wilczynski plot, parametrized by

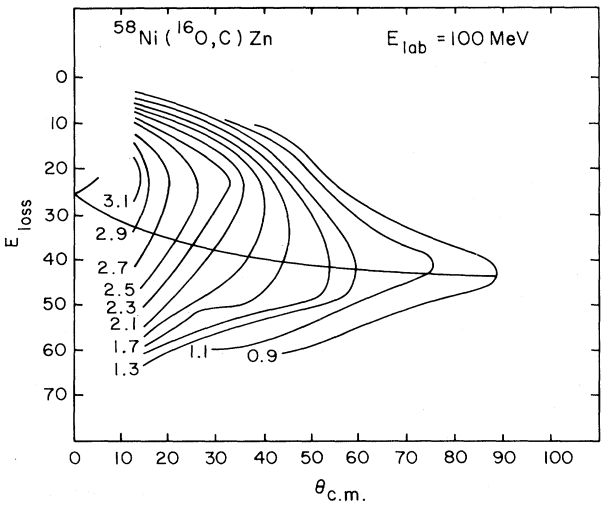

FIG. 1. A Wilczynski plot of the data of Harris *et al.* (Ref. 13). Also shown is the "classical trajectory" given by Eq. (23). The double differential cross section is in units of mb sr^{-1} MeV^{-1}.

$$Q_f = Q_m + \beta e^{\alpha\Theta} , \tag{23}$$

where $\alpha = 2.22$ rad^{-1}, $\beta = 18$ MeV, and $Q_m = -44$ MeV. The boron data are very similar[13] and the corresponding classical trajectory has parameters $\alpha = 2.22$ rad^{-1}, $\beta = 18$ MeV, and $Q_m = -48$ MeV. Classically, $\Theta = -\theta$ on the fully damped branch of a Wilczynski plot, and the maximum Q value, Q_m, is limited by the centrifugal, Coulomb, and deformation energies of the final system. The trajectories given by Eq. (23) are representative of those resulting from classical friction-model calculations,[2,3,13] although there is some difficulty in unambiguously specifying the trajectory at intermediate angles, a point to which we will return in Sec. V.

We identify the Θ of Eq. (23) with that appearing in Eq. (13), which is equivalent to assuming that the classical trajectory given above corresponds to the dynamics of the energy dissipation mechanism. In principle, Θ can be found from the mean field of the two nuclei, as shown by Eq. (10), but this is not a trivial problem. The effect of angular momentum transfer is partially taken into account by fitting Eq. (23) to the Wilczynski plot phenomenologically.

Once Θ is known, the diffraction function $D(\theta,\Theta)$ of Eq. (13) can be evaluated, as reasonable values can be assigned to the parameters γ and Δ.[21] For our calculations we choose $\gamma = 1.0$ and $\Delta = 1.25$ and hold these values fixed as a function of Q value, since any variation is likely to be fairly slow. [See, however, the discussion preceding Eq. (11).] In addition, we take only two terms of the Poisson sum: $n = 0, +1$ and $n = 0, -1$ for F^+ and F^-, respectively, since no other terms are important.[21] Furthermore, we neglect interference between the different Poisson terms which makes no discernible difference to our calculation, and also removes the necessity of evaluating Λ' as a function of Q value.

The remaining unknown quantity in Eq. (13) is $A(Q)$. We found that the data could be fitted with the parametrization

$$A(Q) = A_0 e^{Q/T} \quad (Q < 0) , \tag{24}$$

where A_0 is essentially independent of Q, and T is a constant. Clearly $A(Q)$ depends on the spectroscopic details of the interacting nuclei as well as details of the kinematic matching [see Eq. (13)], but our lack of knowledge of these quantities precludes direct evaluation of $A(Q)$. This form does, however, have some justification since, if the two nuclei reach thermal equilibrium in the collision, the parameter T can be associated with a temperature (of around 2—3 MeV).[26] This works well for collisions between heavy nuclei,[26] but for lighter systems, such as that considered here, it has been noted[7] that the thermal equilibration is less complete. In this case we can regard T as a fitting parameter.

A highly simplified model illustrates how Eq. (24) might come about. We suppose that both the nucleon and the total energy transfer proceed in n steps, the latter of energy ΔE each. The probability of each step occurring is taken to be a constant, p, and so $n = -Q/\Delta E$ and $A(Q) \propto p^n$. Noting that $p < 1$, we then obtain

$$\begin{aligned} A(Q) &\propto \exp(-n \ln 1/p) \\ &\propto \exp[Q(\ln 1/p)/\Delta E] , \end{aligned} \tag{25}$$

which is of the form of Eq. (24) if we define $T \equiv \Delta E / \ln(1/p)$.

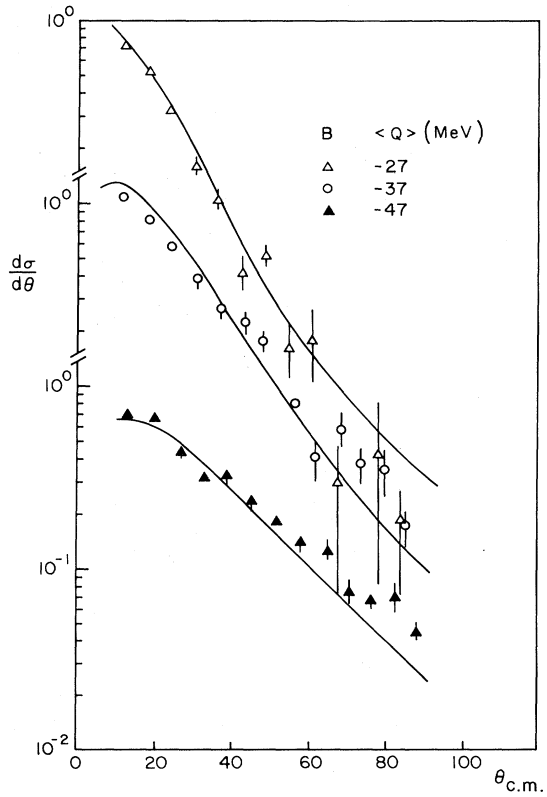

FIG. 2. The angular distributions for the reaction $^{58}Ni(^{16}O,B)Ga$ for various Q values (Ref. 13). The solid lines are the cross sections (in mb rad^{-1}) given by Eqs. (13) and (21).

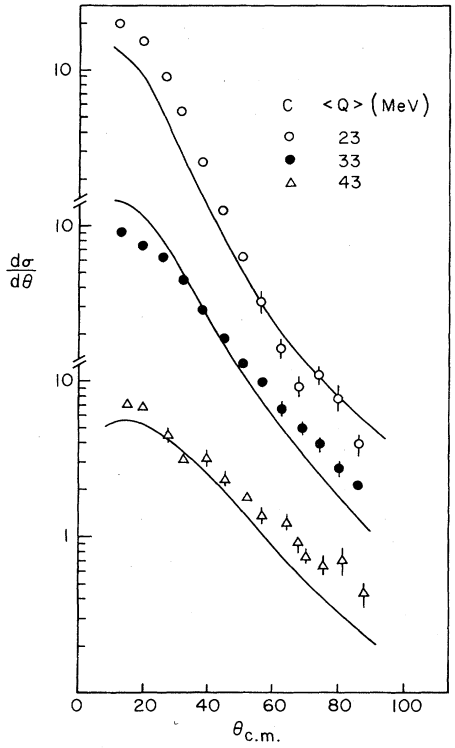

FIG. 3. The same as Fig. 2, but for ^{58}Ni(^{16}O,C)Zn (Ref. 13).

Finally, we note that Γ_c in Eq. (21) increases with E_c, and it is reasonable to choose

$$\Gamma_c=\Gamma_0+\Gamma_1\frac{Q_c}{Q_m}\ , \tag{26}$$

where $E_c+Q_c=E_i$, $\Gamma_0=1$ MeV, and $\Gamma_1=20$ MeV.

The results of our calculations are shown in Figs. 2 and 3 for boron and carbon, respectively. The values of $\langle Q\rangle$ are given in the figures. The only quantity which was varied in fitting all sets of data, apart from the overall normalization A_0, is T, and we found that $T=8.5$ MeV was a suitable value. The values of A_0 (given in the figures) are seen to be of the order of 100–200, irrespective of the case considered.

We do not claim that we have fitted the data perfectly, indeed, better fits can be obtained if, for example, γ and Δ are also allowed to vary, particularly if Δ is reduced slightly with increasing energy loss.[27] Such a reduction would be expected from the inclusion of dynamic fluctuations, as explained in Sec. II [see Eq. (11)].

It is pleasing, though, that we get a reasonable fit and obtain normalization factors that do not vary drastically from one data set to another.

We see that forward peaked and approximately exponentially decaying angular distributions are obtained with our formalism, and stress that diffractive, diffusive, and statistical features are all indispensable in producing such distributions. Thus the form of the double differential cross section used elsewhere[7] has justification. A further test of our formalism is provided by a study of polarization data.

V. POLARIZATIONS IN DIC'S

The possibility of exciting a nonzero spin state in a DIC has been explicitly included in our formalism, and so we are able to evaluate the polarization of the nuclear spin after the collision, which is not possible in other formalisms.[9] Since a considerable part of the orbital angular momentum is converted into intrinsic spins of the fragments through tangential friction,[10,11] inclusion of such spin effects is very desirable. A similar approach has been worked out by Dünnweber and Hartmann.[28]

It has been noted[10,11,29] that the extraction of spin-polarization data from continuum γ-ray polarizations is uncertain when there is the possibility of both nuclei being excited. Indeed, spin polarizations so extracted may have the opposite sign to that predicted from frictional models, and do not reflect the transition from negative- to positive-angle scattering at some Q value. However, γ-ray multiplicities suggest that in the case of interest here (^{16}O+^{58}Ni at 100 MeV), the circular polarizations measured experimentally are essentially those of the γ radiation whose source is the *targetlike* fragment.[10,11] Therefore, in the following, we assume that we excite only the target.

In terms of the $\tilde{\beta}_{IM}$ of Eq. (6), the polarization from the mean diffractive contribution is given by

$$P(\theta)\equiv\frac{\sum_{IM} M\,|\tilde{\beta}_{IM}(\theta)|^2}{\sum_{IM} I\,|\tilde{\beta}_{IM}(\theta)|^2}\ , \tag{27}$$

where the Q dependence of P and $\tilde{\beta}_{IM}$ is not explicitly indicated. Using Eq. (6) this can be written as

$$P(\theta)=B(Q)\left[\frac{|F^+(\theta)|^2-|F^-(\theta)|^2}{|F^+(\theta)|^2+|F^-(\theta)|^2}\right], \tag{28}$$

where we have defined

$$B(Q)\equiv\frac{\sum_{IM} M\,|G_Q(I)g(M)d^I_{0M}(\frac{1}{2}\pi)|^2}{\sum_{IM} I\,|G_Q(I)g(M)d^I_{0M}(\frac{1}{2}\pi)|^2}\ , \tag{29}$$

and ignored the interference between F^+ and F^- as explained in Sec. II. Since $g(M)$ is peaked at $M\approx\xi\gg 0$, $B(Q)$ is positive definite. The *sign* of the polarization is given by the relative importance of F^+ and F^- in Eq. (28), which depends on the scattering angle as well as the Q value, but its *magnitude* depends also on the form of $B(Q)$, and it is this that we now consider.

If we assume that $|g(M)|^2$ is a peaked function (such as a Gaussian[16]) centered on $M=\xi$, then, as a first approximation, we may write

$$\sum_M M\,|g(M)d^I_{0M}(\tfrac{1}{2}\pi)|^2 \approx \min[I,\xi]\sum_M |g(M)d^I_{0M}(\tfrac{1}{2}\pi)|^2\ , \tag{30}$$

and so if we only excite spins $I<\xi$, $B(Q)\approx 1$. For the ^{16}O+^{58}Ni reaction studied in Sec. IV, $\xi\sim 2\hbar$, $\sim 4\hbar$, and $\sim 6\hbar$ at $Q=-10$, -20, and -30 MeV, respectively,[11] while the data suggest[10] I of the order of $10\hbar$–$15\hbar$. Thus

$\min[I,\xi]$ is, in this case, equal to ξ, but the important point is that $B(Q)$ increases slowly with increasing energy loss. In the Appendix we derive an analytic expression for $B(Q)$ which further illustrates this point.

Since an exclusively near or far sided process gives $|P(\theta)| = B(Q)$, we may identify $B(Q)$ with P_γ^0, the maximum γ-ray polarization, used elsewhere.[11]

To incorporate diffusion into the polarization calculation, we must consider each substrate cross section $|\tilde{\beta}_{IM}(\theta)|^2$ separately. Noting that $B(Q)$ varies slowly with Q, we then obtain

$$\langle P\rangle = B(Q)\langle \tilde{P}\rangle \ , \tag{31}$$

where

$$\langle \tilde{P}\rangle = \frac{\int dE_c\, W(E_c)(|F^+|^2 - |F^-|^2)}{\int dE_c\, W(E_c)(|F^+|^2 + |F^-|^2)} \tag{32}$$

is the "reduced polarization," and $W(E_c)$ is the Lorentzian weighting factor appearing in Eq. (21).

In Fig. 4 we display $\langle \tilde{P}\rangle$ as a function of θ and Q value for the case where carbon is the light fragment in the $^{16}O+^{58}Ni$ reaction. To obtain $\langle P\rangle$ the curves should be multiplied by $B(Q)$, which we estimate in the Appendix.

Polarization data for this reaction exist[10,11] and so we can compare our results with experiment. Trautmann *et al.*[10] found polarizations of about 0.55 ± 0.20, 0.70 ± 0.15, and $0.90^{+0.10}_{-0.15}$ for Q values around -23, -33, and -43 MeV, respectively, and a c.m. scattering angle of about 47°. Figure 4 shows that for a grazing angle of 5° (see below), $\langle \tilde{P}\rangle$ is 0.92, 0.98, and 0.99 for Q values of -23, -33, and -43 MeV, and so $\langle P\rangle$ is 0.58, 0.74, and 0.81, respectively. (See the Appendix.) These values are in agreement with experiment, and illustrate the increasing importance of negative-angle scattering for large energy losses.[2,3,30]

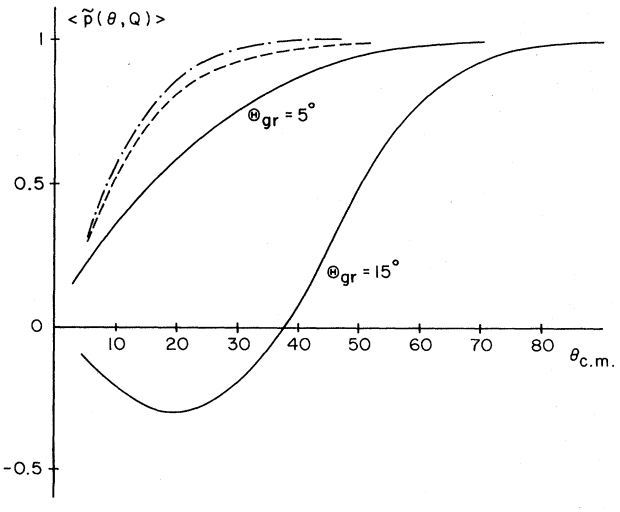

FIG. 4. The "reduced polarization," as defined in the text, is shown as a function of angle for the reaction $^{58}Ni(^{16}O,C)Zn$ for $\langle Q\rangle = -23$ MeV (solid), -33 MeV (dashed), and -43 MeV (dotted-dashed). For a full discussion, see the text.

Finally, we note that the calculation of $\langle \tilde{P}\rangle$ is more sensitive than that of the angular distributions to the choice of classical trajectory. This is illustrated in Fig. 4 where, for $Q = -23$ MeV, we show the effect of altering the grazing angle. This quantity is not well determined from the data[13] and so there is an uncertainty in the parametrization of Eq. (23) and in the limits of integration in Eqs. (21) and (32), since we (numerically) integrate over Θ rather than E_c:

$$\int_0^{E_i} \cdots dE_c \rightarrow \int_{-\pi}^{\Theta_{gr}} \cdots d\Theta \ , \tag{33}$$

where Θ_{gr} corresponds to a Q value of zero. The angular distributions can be fitted with different values of Θ_{gr} if different "temperatures" are used, but this quantity does not appear in the expression for $\langle P\rangle$. In Fig. 4 we show the $Q = -23$ MeV polarization for Θ_{gr} equal to 5° and 15°, and it is clear that $\langle \tilde{P}\rangle$ is very sensitive to this choice. We conclude, therefore, that polarization data for DIC's give a better determination of the classical trajectory than is possible with a Wilczynski plot alone.

VI. SUMMARY

By including the effects of diffraction and diffusion, and utilizing statistical arguments because of the high level densities involved, we are able to account for the forward-peaked, approximately exponentially decaying nature of the angular distributions observed in DIC's between light heavy ions at relatively high energies. With a simple parametrization of the transfer form factor we obtain an expression for the diffractive cross section which involves the deflection function Θ. We choose this to fit the ridge of the Wilczynski plot, which is equivalent to a classical trajectory.[2,3] Diffusion around this classical trajectory is introduced, and the form of the weighting function is derived. This approach is different from that used elsewhere,[9,12] and we are able to justify the Lorentzian form of $W(E_c)$.

All these effects are necessary if angular distributions are to be reproduced, and with one free parameter we obtain reasonable fits to data. Better fits could be obtained if other parameters were allowed to vary. In particular, a Δ which decreases with increasing energy loss accounts for dynamic fluctuations.

A big advantage of the present work over that of other authors[9,12,28] is that polarization phenomena can be treated correctly, since the excitation of nonzero spin states is explicitly included. The agreement with experimental data is good, but we found that $\langle \tilde{P}\rangle$ is much more sensitive than the angular distribution to the choice of classical trajectory. Further measurements of the polarization produced in a DIC are therefore desirable.

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APPENDIX

We demonstrate with a simple model that the function $B(Q)$ appearing in the DIC polarization formula, and defined by Eq. (29), has a slow Q dependence. We assume that

$$|g(M)|^2=\exp\left[-\frac{|M-\xi|}{\Delta}\right] \tag{A1}$$

and

$$|G_Q(I)|^2\propto\theta(\xi-I)\ , \tag{A2}$$

implying that all states with $I\leq\xi(Q)$ can be excited. Furthermore, we approximate the reduced rotation matrix elements in Eq. (29) by their asymptotic forms[19]

$$|d^I_{0M}(\tfrac{1}{2}\pi)|^2\approx\frac{2}{\pi I}[1+(-1)^{I+M}] \tag{A3}$$

(which is clearly not as good for small I) and replace the sums over I and M in Eq. (29) by integrals. The M sum in the numerator becomes

$$\int_{-I}^{I}dM\,Me^{-|M-\xi|/\Delta}[1+(1-)^{I+M}]\approx\tfrac{1}{2}\int_{-I}^{I}dM\,Me^{-|M-\xi|/\Delta}=\Delta e^{-\xi/\Delta}(I\cosh I/\Delta-\Delta\sinh I/\Delta)\ , \tag{A4}$$

and so the numerator of Eq. (29) becomes, on performing the I integral,

$$\Delta^2e^{-\xi/\Delta}[\sinh\xi/\Delta-\mathrm{shi}(\xi/\Delta)]\ ,$$

where[19]

$$\mathrm{shi}(y)\equiv\int_0^y\frac{\sinh x}{x}dx\ . \tag{A5}$$

Performing the denominator integrations also, we obtain

$$B(Q)\approx\frac{\sinh(y)-\mathrm{shi}(y)}{\cosh(y)-1} \tag{A6}$$

with $y\equiv\xi/\Delta$. Calculations[2] show that there is an approximately linear relationship between ξ and Q, and so we write[9]

$$\xi=(L-L_{\mathrm{crit}})Q/Q_m\ , \tag{A7}$$

where L is the initial channel grazing angular momentum and L_{crit} is the angular momentum below which fusion occurs, both in units of $\hbar$. We take $L=50$ and $L_{\mathrm{crit}}=41$ for the O + Ni reaction (Ref. 11) and can therefore estimate $\xi(Q)$. Using $\Delta=1.25$ as in the text, we obtain $B(Q)\approx0.63$, 0.76, and 0.82 for $Q=-23$, -33, and -43 MeV, respectively. [This illustrates, albeit with a very simple model, the slow dependence of $B(Q)$ on the Q value.] The value of P_γ^0 for this reaction, which can be identified with $B(Q)$ as discussed in Sec. V, is estimated to be 0.72 (Ref. 11), in good agreement with the above figures.

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