# Off-shell behavior of a $\Lambda$-$\alpha$ interaction

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The off-shell properties of a recently proposed $\Lambda$-$\alpha$ interaction is studied with particular emphasis on the action of the different regions of the potential in producing the on- and off-shell amplitudes. To that end we have made use of a combined variable phase off-shell scattering theory. It is found that the potential has desirable off-shell properties to be useful in hypernuclear few-body problems.

In dealing with problems of constructing $\Lambda$-nucleus effective potentials from realistic $\Lambda$-N interactions, Kurihara *et al.*[1] have observed that the repulsive core of the $\Lambda$-N interaction simulates a central repulsion in the $\Lambda$-nucleus potential, the strength of which decreases with the increase of the rms radius of the nucleus presumably due to averaging out of the core effect of the two-body interaction. Recently,[2] they have used the Dalitz $\Lambda$-N potential[3] to parametrize a two-range Gaussian form:

$$U_{\Lambda\alpha}(r)=V_R\exp[-(r/b_R)^2]-V_A\exp[-(r/b_A)^2] \ , \tag{1}$$

with $V_R=450.43$ MeV, $b_R=1.25$ fm, $V_A=404.88$ MeV, and $b_A=1.41$ fm for the $\Lambda$-$\alpha$ potential. This potential is normalized to produce the binding energy $B_\Lambda(=3.10$ MeV) of ${}^5_\Lambda$He. It has a central repulsion due to the compactness of the $\alpha$ particle and is very different from that used hitherto on the studies of light hypernuclei. The potential in Eq. (1) is expected to play a role in the theories of hypernuclear few-body systems.[4]

In this report we use the algorithms of a combined variable phase off-shell scattering theory[5] to compute the half-off-shell $t$ matrix and the off-shell extension function[6] for the potential in Eq. (1) and study their off-shell behavior. The generalized variable phase method which we use here not only yields reliable numbers for the off-shell amplitudes but also helps acquire a physical feeling with regard to the action of the different regions of the potential in producing these quantities.

The $s$-wave interpolating on-shell and half-off-shell $t$ matrix functions $t(k,k,k^2;r)$ and $t(k,q,k^2;r)$ (for a reduced potential $V(r)$ satisfy the equations

$$\frac{dt(k,k,k^2;r)}{dr}=\frac{2}{\pi k^2}V(r)\left[\sin kr-\frac{\pi k}{2}e^{ikr}t(k,k,k^2;r)\right]^2 \ , \tag{2}$$

and

$$\frac{dt(k,q,k^2;r)}{dr}=\frac{2}{\pi kq}V(r)\left[\sin kr-\frac{\pi k}{2}e^{ikr}t(k,k,k^2;r)\right]\left[\sin qr-\frac{\pi q}{2}e^{ikr}t(k,q,k^2;r)\right] \ , \tag{3}$$

with boundry conditions $t(k,k,k^2;0)=0$ and $t(k,k,k^2;\infty)=t(k,k,k^2)$, the on-shell $t$ matrix and $t(k,q,k^2;0)=0$ and $t(k,q,k^2;\infty)=t(k,q,k^2)$, the half-off-shell $t$ matrix. Here $k$ and $q$ are on- and off-shell momenta. The on-shell normalization of the $t$ matrix function is

$$t(k,k,k^2;r)=-\frac{2}{\pi k}e^{i\delta(k,r)}\sin\delta(k,r) \ . \tag{4}$$

The phase function $\delta(k,r)$ satisfies the nonlinear differential equation[7]

$$\frac{d\delta(k,r)}{dr}=-k^{-1}V(r)\sin^2[kr+\delta(k,r)] \ , \tag{5}$$

with $\delta(k,0)=0$ and $\delta(k,\infty)=\delta(k)$, the phase shift.

The interpolating function $f(k,q,r)$ for the off-shell extension function $f(k,q)[=f(k,q,\infty)]$ is given by[8]

$$f(k,q,r)=\frac{\Delta(k,q,r)}{\sin\delta(k,r)} \ . \tag{6}$$

The quasiphase function $\Delta(k,q,r)$ satisfies the linear differential equation

$$\frac{d\Delta(k,q,r)}{dr}=-V(r)\sin[kr+\delta(k,r)]\{q^{-1}\sin qr+k^{-1}\Delta(k,q,r)\cos[kr+\delta(k,r)]\} \ . \tag{7}$$

We have solved Eqs. (2)–(7) for the potential in Eq. (1) by using the Runge-Kutta method with an appropriate stability check at $E_{\rm lab}=40$ MeV to compute the half-off-shell $t$ matrix and off-shell extension functions. The results for the interpolating on-shell $t$ matrix function $t(k,k,k^2;r)$ as a function of $r$ are shown in Fig. 1. To see how the potential acts in producing $t(k,k,k^2)$ we have also included in this figure the reduced potential $V(r)$ corresponding to $U_{\Lambda\alpha}(r)$ as a function of $r$. This is shown by the curve A. The potential $V(r)$ is repulsive for $r<0.88$ fm and attractive elsewhere. The curves B and C give the variation of ${\rm Re}\,t(k,k,k^2;r)$ and ${\rm Im}\,t(k,k,k^2;r)$ with $r$. For $r<0.88$ fm

 

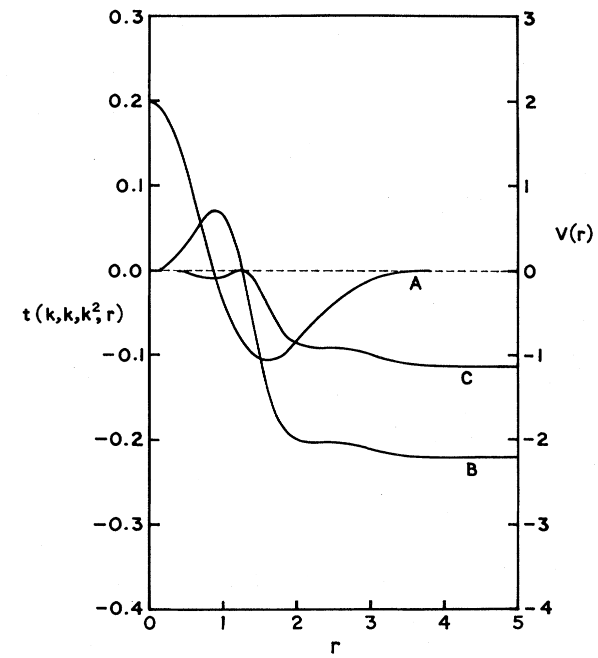


FIG. 1. $\mathrm{Re}\,t(k,k,k^2;r)$, $\mathrm{Im}\,t(k,k,k^2;r)$, and $V(r)$ as a function of $r$.

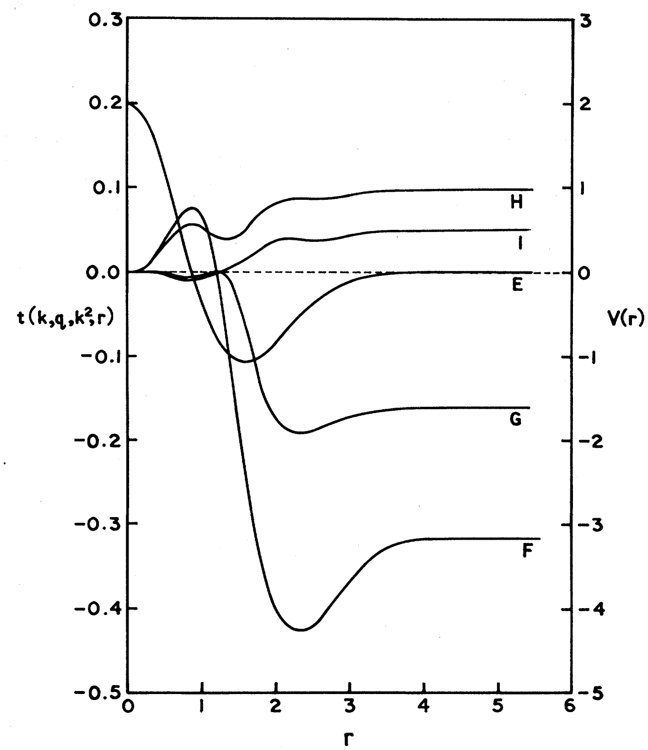


FIG. 2. $\mathrm{Re}\,t(k,q,k^2;r)$, $\mathrm{Im}\,t(k,q,k^2;r)$, and $V(r)$ as a function of $r$ for $q=k/2$ and $q=2k$.

$\mathrm{Re}\,t(k,k,k^2;r)$ takes up postive values and $\mathrm{Im}\,t(k,k,k^2;r)$ takes up negative values. This implies that the repulsive part of the potential gives negative contribution to the phase function in the process of building up the phase shift. The maximum and minimum values of $\mathrm{Re}\,t(k,k,k^2;r)$ and $\mathrm{Im}\,t(k,k,k^2;r)$ occur at $r=0.88$ fm where the potential changes sign. The saturation values of $\mathrm{Re}\,t(k,k,k^2;r)$ and $\mathrm{Im}\,t(k,k,k^2;r)$ occur after 3.5 fm. From these curves the phase shift

$$\delta(k)=\arctan\left[\frac{\mathrm{Im}\,t(k,k,k^2)}{\mathrm{Re}\,t(k,k,k^2)}\right]$$

at $E_{\mathrm{lab}}=40$ MeV comes out to be 0.4694 rad.

In Fig. 2 we plot $\mathrm{Re}\,t(k,q,k^2;r)$ and $\mathrm{Im}\,t(k,q,k^2;r)$ for the off-shell momenta $q=k/2$ and $q=2k$ as a function of $r$. Here also we have plotted $V(r)$ as a function of $r$. The curve for $V(r)$ has been denoted by E. The curves F and G give the variation of $\mathrm{Re}\,t(k,k/2,k^2;r)$ and $\mathrm{Im}\,t(k,k/2,k^2;r)$ with $r$ while H and I show the similar variation for $\mathrm{Re}\,t(k,2k,k^2;r)$ and $\mathrm{Im}\,t(k,2k,k^2;r)$. The curves F and G closely resemble the on-shell curves except that they go

TABLE I. Half-off-shell $t$ matrix $t(k,q,k^2)$ and off-shell extension function as a function of the off-shell momentum $q$ at $E_{\mathrm{lab}}=40$ MeV. The on-shell momentum $k=1.1649\ \mathrm{fm}^{-1}$.

| Off-shell momentum $q$ ($\mathrm{fm}^{-1}$) | $\mathrm{Re}\,t(k,qk^2)$ ($\mathrm{fm}^2$) | $\mathrm{Im}\,t(k,q,k^2)$ ($\mathrm{fm}^2$) | Off-shell extension function $f(k,q)$ |
|---|---|---|---|
| 0.2912 | −0.3165 | −0.1604 | 1.4250 |
| 0.5825 | −0.3073 | −0.1558 | 1.3838 |
| 0.8737 | −0.2786 | −0.1413 | 1.2550 |
| 1.1649 | −0.2219 | −0.1125 | 1.0001 |
| 1.4562 | −0.1398 | −0.0708 | 0.6300 |
| 1.7474 | −0.0464 | −0.0234 | 0.2094 |
| 2.0386 | 0.0380 | 0.0195 | −0.1711 |
| 2.3299 | 0.0967 | 0.0494 | −0.4358 |
| 2.6211 | 0.1227 | 0.0626 | −0.5530 |
| 2.9123 | 0.1196 | 0.0610 | −0.5391 |
| 3.2036 | 0.0978 | 0.0498 | −0.4406 |
| 3.4948 | 0.0688 | 0.0351 | −0.3102 |
| 3.7860 | 0.0417 | 0.0213 | −0.1880 |
| 4.0773 | 0.0210 | 0.0108 | −0.0949 |
| 4.3685 | 0.0077 | 0.0040 | −0.0351 |
| 4.6597 | 0.0005 | 0.0004 | −0.0029 |
| 4.9510 | −0.0024 | −0.0011 | 0.0106 |

through minimum before reaching the saturation values. In contrast to this the nature of the curves I and H are quite different. This means that the action of the potential in producing a half-off-shell $t$ matrix $t(k,q,k^2)$ depends also on $q$. The phase of the half-off-shell $t$ matrix is the phase shift. Both set of curves (G,F) and (I,H) satisfy this criterion.

In Table I we have presented results for the half-off-shell $t$ matrix and off-shell extension function as a function of off-shell momenta $q$ for $E_{\rm lab}=40$ MeV. These numbers show that both $\mathrm{Re}\,t(k,q,k^2)$ and $\mathrm{Im}\,t(k,q,k^2)$ oscillates but approaches zero as $q$ becomes large. The function $f(k,q)$ also tends to zero as $q$ increaes. These indicate that the off-shell behavior of the potential in Eq. (1) is quite satisfactory.

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