Alpha transfer process in $^{16}O+^{24}Mg$ elastic scattering

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In elastic scattering between $4N$ nuclei, a rising oscillatory structure often appears in the differential cross section at backward angles. This anomalous phenomenon is explained by an α transfer process between the nuclei. For $^{16}O+^{24}Mg$ scattering, ^{24}Mg is considered to be composed of an ^{16}O core with two alpha particles orbiting around it. These valence particles may be transferred between the cores during the scattering. The nuclear molecular orbital model is applied. A natural agreement with the experimental data can be reached without any adjustment of the parameters involved.

I. INTRODUCTION

In the elastic scattering between $4N$ nuclei, a rising oscillatory structure often appears in the differential cross section at backward angles.[1,12–15] This anomalous phenomenon becomes a very interesting research subject, because it contains information about the reaction mechanism and nuclear structure.

Recently, Paul and collaborators have measured the differential cross section for $^{16}O+^{24}Mg$ elastic scattering from 3°–180°.[1] A strongly rising oscillatory structure appeared in the large angle region. They have made a conventional optical model calculation, and it apparently cannot describe the oscillatory structure. This severe discrepancy hints that there is a different reaction mechanism responsible for the backward angle rise.

Landowne and Wolter have studied the interference of the two-step α transfer process with the direct elastic and inelastic scattering of $^{16}O+^{24}Mg$.[11] They found that the two-step α transfer cannot explain a reported anomaly in the inelastic scattering distribution. At that time the experimental result for an all angle angular distribution of $^{16}O+^{24}Mg$ elastic scattering was not available, however, their published elastic cross section fails to describe the data of Paul *et al.*

It is well known that a $4N$ nucleus like ^{24}Mg may have an α-cluster structure. There is a significant probability of forming an α cluster at its nuclear surface. Khanna and Shabma have calculated the energy spectrum of ^{24}Mg by assuming that ^{24}Mg is composed of an ^{16}O core with two alpha particles orbiting around it.[2] Recent investigation of the spectrum of ^{24}Mg based on the cluster model also supports the existence of $^{16}O+{}^{8}Be$ or $^{16}O+2\alpha$ cluster structure.[3]

Because of these considerations, the elastic scattering of $^{16}O+^{24}Mg$ may take place by two channels simultaneously: a direct process ^{24}Mg $(^{16}O,^{16}O)^{24}Mg$ and a $^{24}Mg(^{16}O,^{24}Mg)^{16}O$ transfer process. The transfer process may still have two mechanisms: either a single transfer with ^{8}Be as the valence cluster or a double valence 2α transfer. Because one cannot distinguish experimentally between the direct and transfer processes, they interfere, and the interference superposition of these two processes may give rise to the anomalous rising structure in the backward angle region.

In our previous paper we employed the nuclear molecular orbital model to calculate the differential cross section for this $^{16}O+^{24}Mg$ elastic scattering in the framework of a ^{8}Be cluster transfer. Fonseca and Shanley[5] have shown that for a three-body system involving two heavy particles and a light one, the Born-Oppenheimer approximation yields remarkably good results for the binding energies and wave functions, even when the mass ratio M/m between the heavy and light particles is not large. It seems that the nuclear molecular orbital model may be applicable to the ^{8}Be transfer process in $^{16}O+^{24}Mg$ scattering. This initial calculation did show the backward angle rising oscillatory structure, but the results did not agree very well with the data.[4]

In this paper we have recalculated the differential cross section of $^{16}O+^{24}Mg$ by considering that ^{24}Mg is composed of an ^{16}O core and the two α valence clusters. A two α transfer mechanism is assumed in the nuclear molecular orbital model calculation. A significant improvement has been gained as compared with the result of the ^{8}Be transfer mechanism.

Section II gives a brief description of the nuclear molecular orbital theory. Section III discusses the exchange potential problem. In Sec. IV we show a comparison between theory and experiment for the $^{16}O+^{24}Mg$ scattering cross section. The conclusions are drawn in Sec. V.

II. THEORY

The Hamiltonian for a system composed of two spinless identical cores with two identical valence particles a_1 and a_2 may be written as

$$H=T_R+2E_A+V_C+V_N+\hat{H}(\bar{R})\ , \tag{1}$$

where T_R is the relative kinetic energy operator for the two cores, and $2E_A$ their approaching energy, V_C and V_N are the Coulomb and nuclear potential between them, and $H(\bar{R})$ is the Hamiltonian for the two valence particles.

$$\hat{H}(\bar{R})=\hat{H}_1(\bar{R})+\hat{H}_2(\bar{R})+V(r_0)\ , \tag{2}$$

where

$$\hat{H}_1(\bar{R})=T_{r_{11}}+V_{A_1a_1}(r_{11})+V_{A_2a_2}(r_{12}) ,$$

$$\hat{H}_2(\bar{R})=T_{r_{22}}+V_{A_aa_2}(r_{22})+V_{A_1a_2}(r_{21}) ,$$

are Hamiltonians for the valence particles a_1 and a_2, separately. Figure 1 shows the corresponding configuration of this system. If the residual interaction $V(r_0)$ between the valence particles is small compared with that between valence particle and core V_{Aa} (such as the two alpha particles in ^{8}Be), $V(r_0)$ may be neglected; then

$$\hat{H}=\hat{H}_1+\hat{H}_2 .$$

The wave function for the whole system satisfies

$$H\Psi=E\Psi . \tag{3}$$

In the nuclear molecular orbital model, Ψ may be expressed as a linear combination of the molecular state wave functions $\Phi_{\mu\mu'}(1,2,\bar{R})$ of the valence particle,

$$\Psi(1,2,\bar{R})=\sum_{\mu\mu'}C_{\mu\mu'}F_{\mu\mu'}(\bar{R})\Phi_{\mu\mu'}(1,2,\bar{R}) , \tag{4}$$

where $F_{\mu\mu'}(\bar{R})$ describes the relative motion between two cores. As the two identical cores commute with each other,

$$F_{\mu\mu'}(-\bar{R})=pp'F_{\mu\mu'}(\bar{R}) .$$

The eigenvalue of the commuting operator $p(p')$ may take either $+1$ or -1. Therefore the partial wave expansion $F_{\mu\mu'}(\bar{R})$ contains only even partial waves for $p=p'$, while for $p\neq p'$ only odd partial waves appear. The expansion coefficients $C_{\mu\mu'}$ may be determined by the approaching behavior of the incident particle.

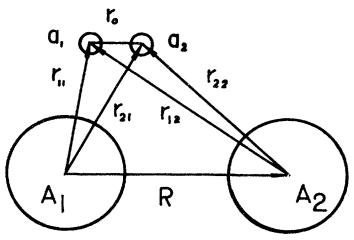

FIG. 1. A configuration for the system with two alpha particles orbiting around two core nuclei.

The molecular state wave functions are the eigenfunctions for the valence particles in Eq. (2),

$$\hat{H}\Phi_{\mu\mu'}=\epsilon_{\mu\mu'}\Phi_{\mu\mu'} ,$$

where $\epsilon_{\mu\mu'}$ is the molecular energy of the two valence particles. If the scattering process does not involve excited states, each valence particle may be in either of the following two states:

$$\Phi_\mu=\Phi^m_{A1}(i)+p\Phi^m_{A2}(i) ,$$

where p is $+1$ or -1 and $i=1,2$ and $\Phi^m_{Aj}(i)$ is the nuclear orbital wave function for the ith valence particle bound to core A_j on the m state. Then the molecular state wave function for the two valence particles will be

$$\begin{aligned}\Phi_{\mu\mu'}(1,2,\bar{R})&=\left[\frac{1}{\sqrt{2}N(R)}\right]^2[\Phi^m_{A1}(1)+p\Phi^m_{A2}(1)][\Phi^{m'}_{A1}(2)+p'\Phi^{m'}_{A2}(2)]\\&=\frac{1}{2}\frac{1}{1+p\Delta(R)}[\Phi^m_{A1}(1)\Phi^{m'}_{A1}(2)+pp'\Phi^m_{A2}(1)\Phi^{m'}_{A2}(2)]_{\text{ion}}\\&\quad+\frac{1}{2}\frac{p}{1+p'\Delta(R)}[\Phi^m_{A2}(1)\Phi^{m'}_{A1}(2)+pp'\Phi^m_{A1}(1)\Phi^{m'}_{A2}(2)]_{\text{cov}} .\end{aligned} \tag{5}$$

The first term on the right-hand side of Eq. (5) is the ion configuration state. In this state valence particles a_1 and a_2 are both bound to the same core A_1 or A_2. The second term is the covalent configuration state. Valence particles are bound to different core nuclei, respectively.

For the case when the initial scattering system is in the ion configuration state, the magnitude of the covalent channel is small. For instance, in the $^{16}O+^{24}Mg$ system, even though the correlation between two alpha valence particles is weak, the Q value for the covalent channel $^{24}Mg(^{16}O,^{20}Ne)^{20}Ne$ is a large negative value, $Q=-4.58$ MeV. We have neglected the correlation between ion and covalent configuration states. We consider only the case in which both the incident and outgoing channels are in ion configuration states.

Neglecting the residual interaction between the two valence particles under the two-state approximation, one substitutes Eq. (4) into Eq. (3) and gets the decoupling equation

$$\left[-\frac{\hbar^2}{M}\nabla^2_{\vec{R}}+V_{\text{opt}}+2p\delta pp'J(R)-E\right]F_{\mu\mu'}(\bar{R})=0 . \tag{6}$$

Expand $F_{\mu\mu'}(\bar{R})$ into partial waves and get the radial equation

$$\left\{\frac{d^2}{dR^2}-\frac{L(L+1)}{R^2}-\frac{M}{\hbar^2}[V_{\text{opt}}(R)+2p\delta pp'J(R)]+k^2\right\}u_L(R)=0 , \tag{7}$$

where M is the reduced mass, k the relative motion wave number, and $J(R)$ the exchange integral or exchange potential,

$$J(R)=\langle \Phi_{A_1a_1}(\bar{r}_{11}) \mid V_{A_1a_1}(r_{11}) \mid \Phi_{A_2a_1}(\bar{r}_{11}-\bar{R})\rangle \ . \tag{8}$$

The factor $p\delta_{pp'}$ comes from the exchange symmetry of A_1 and A_2. It makes one set of the even partial waves ($p=p'=+1$) scattered from both the optical and exchange potential, another set of the even partial waves ($p=p'=-1$) scattered from the optical potential minus the exchange potential, and the odd partial waves scattered from the optical potential only.

By solving Eq. (7), four amplitudes f^{++}, f^{--}, f^{+-}, and f^{-+} are obtained. $f^{+-}=f^{-+}$ for valence particles in the same state. For a double α transfer under the ion configuration states, the differential cross section is made by the four scattering amplitudes as

$$\frac{d\sigma}{d\Omega}=\left|\tfrac{1}{2}(f^{++}+f^{--})+f^{+-}\right|^2 . \tag{9}$$

Von Oertzen has discussed some of these derivations in detail.[6,7]

III. EXCHANGE POTENTIAL

The exchange integral is commonly evaluated numerically. In order to save computer time, an approximation for large R is used.[8]

$$J(R)\approx[(SN)^2E_B/\alpha^3]\exp(-\alpha R)/\alpha R \ , \tag{10}$$

where

$$\alpha=\sqrt{2m_aE_B}/\hbar \ ,$$

E_B and m_a are the bound energy and mass for the valence particle, S^2 is the spectrum factor for the valence particle bound to core nucleus, and N is the normalization constant. In Eq. (10) the (SN') value is usually taken as an adjustable parameter.

To get more detailed information about how the exchange potential depends upon the valence particle wave function and the interaction between the core and valence particle, we have used a specific wave function for the alpha particle as the nuclear orbiting wave function outside ^{16}O,

$$\Phi_{Aa}(r)=\frac{1}{\sqrt{8}}(\pi a^2)^{-3/4}\left[(2-\sqrt{6})+\sqrt{(8/3)}\left[\frac{r}{a}\right]^2\right]$$

$$\times\exp(-r^2/2a^2) \ , \tag{11}$$

which has been derived from an α-cluster model of $4N$ nuclei.[9] The parameter $a=1.5$ fm is determined from the experimental mean square radius of ^{24}Mg.[10]

The interaction between the valence particle and the core nucleus is taken to be the double Gauss potential given by Khanna and Shabma:[2]

$$V_{C\alpha}(r)=V^R_{C\alpha}e^{-\mu_Rr^2}+V^A_{C\alpha}e^{-\mu_Ar^2} \ . \tag{12}$$

They calculated the interaction energy of α and ^{16}O this way and added it to the energy spectrum of the ^{16}O core.

TABLE I. Parameters of the α-^{16}O interaction potential.

$V^R_{C\alpha}$ (MeV)	$V^A_{C\alpha}$ (MeV)	μ_R (fm^{-2})	μ_R (fm^{-2})
389.0	−325.0	0.342	0.290

The net energy spectrum agreed very well with the experimental spectrum of ^{24}Mg. The adjustable parameters thus determined are given in Table I.

In our calculation the same set of values for these parameters is used. This potential has a soft core with 64 MeV height as shown in Fig. 2. It agrees with the potential which was derived from the alpha particle density in an α-cluster model calculation.[9] Substituting Eqs. (11) and (12) into Eq. (8), the integration can be easily carried out and an analytic form for $J(R)$ is obtained as follows:

$$J(R)=Z_R(A_0+A_2R^2+A_4R^4)e^{-\alpha R^2}$$

$$+Z_A(B_0+B_2R^2+B_4R^4)e^{-\beta R^2} \ , \tag{13}$$

where all the coefficients are functions of known parameters a, $V^R_{C\alpha}$, $V^R_{C\alpha}$, μ_R, and μ_A as shown in the Appendix. $J(R)$ has been plotted, and it decreases rapidly with R as shown in Fig. 2.

IV. COMPARISON WITH EXPERIMENT

In the process of calculating the elastic scattering differential cross section, the Woods-Saxon type of optical potential of Von Oertzen and Bohlen for ^{16}O + ^{16}O was adopted with their optical parameters given in Table II. For the exchange potential Eq. (13) was used.

Leaving all the parameters fixed to fit all the other requirements, respectively, our theoretical results agree very well with the experimental curve as shown in Fig. 3. In

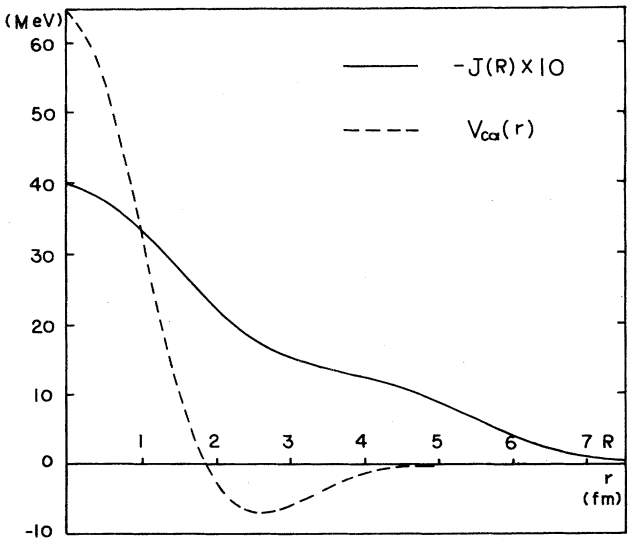

FIG. 2. The soft core double Gauss potential $V_{C\alpha}(r)$ between α and the core nucleus and the exchange potential $J(R)$ obtained from an α-cluster model.

TABLE II. Optical potential parameters.

Type	V (MeV)	r_V (fm)	a_V (fm)	W (MeV)	r_w (fm)	a_w (fm)
$^{16}O + ^{24}Mg$ ANLIA potential	37	1.35	0.404	78	1.29	0.174
$^{16}O+^{16}O$ Shallow potential	17	1.35	0.49	4	1.35	0.49

the whole calculation no adjustment of any of these parameters has been made. It naturally explains the oscillatory structure in the large angle region.

For the purpose of comparing the 2α transfer and one ^{8}Be transfer mechanism in the $^{16}O+^{24}Mg$ scattering process, we have also used the large R approximation to evaluate the exchange integral for the 2α transfer process, as we did for the ^{8}Be transfer process.[4] All the other parameters were kept the same, except the (SN) value was varied.

The (SN) value reflects the strength of the exchange potential or relates to the chance of an exchange process. When the (SN) value increases gradually, an oscillatory structure starts to appear and becomes more prominent as (SN) becomes large. When $(SN)=32$, the position and amplitudes of the oscillatory structure peaks agree well with the experimental data. As can be seen from Fig. 4, the 2α transfer mechanism gives better agreement than that of one ^{8}Be transfer. Particularly, the (SN) value required for a 2α transfer mechanism is small and much more reasonable.

V. CONCLUSION

Using the nuclear molecular orbital model, we have calculated the differential cross section for $^{16}O+^{24}Mg$ elastic scattering in the framework of the 2α transfer. Without any adjustment of the parameters involved, a nearly complete agreement with the experimental result is obtained.

Our investigation shows that the cause of the rising oscillatory structure appearing in the backward angle region of $4N$ nuclei scattering may be due to an α-cluster transfer mechanism. The two-state approximation calculation of the nuclear molecular orbital model is sufficient for $^{16}O + ^{24}Mg$ scattering in the energy range considered. The success of this calculation is very encouraging and further work is warranted. Even though the large R approximation evaluation for the exchange integral can

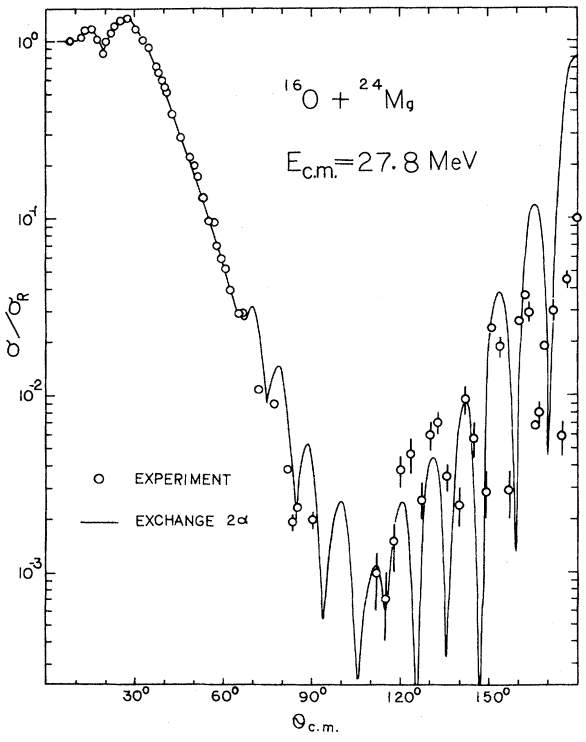

FIG. 3. Differential cross sections for $^{16}O+^{24}Mg$ in comparison with a 2α transfer molecular orbital model prediction.

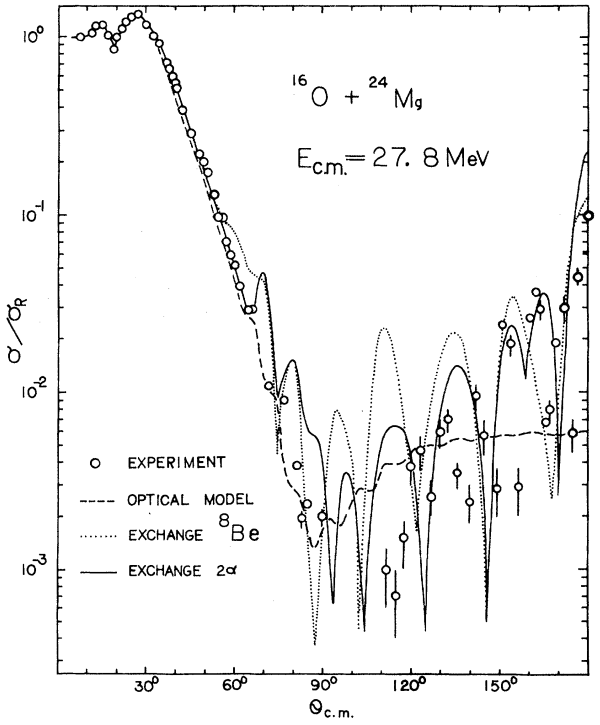

FIG. 4. Differential cross sections for $^{16}O+^{24}Mg$ in comparison with optical model calculations (ANLIA potential) and the molecular orbital model predictions with either transfer of ^{8}Be or of 2α in the large R approximation.

roughly explain the oscillatory structure and give the right number of oscillatory peaks, more information and better fitting can be obtained by considering more detailed structure of the α and nucleus core system. In addition, the coupling effect of the ion and covalent channels can be considered, and the recoil of the core nuclei taken into account.

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APPENDIX

Let

$$\mu'_A=\mu_A^2+1 ,$$

$$\mu'_R=\mu_R^{a^2}+1 ,$$

$$u_A=\mu_A+1/a^2 ,$$

$$u_R=\mu_R+1/a^2 ;$$

then

$$A_0=0.03788a^2-0.4129/u_R+1.875/(\mu_R^a+1/a)^2 ,$$

$$B_0=0.03788a^2-0.4129/u_A+1.875/(\mu_A^a+1/a)^2 ,$$

$$A_2=-0.1376+0.8876/\mu'_R-1.3188/\mu'^2_R+0.625/\mu'^3_R ,$$

$$B_2=-0.1376+0.8876/\mu'_A-1.3188/\mu'^2_A+0.625/\mu'^3_A ,$$

$$A_4=0.125\mu_R/\mu'^3_R+0.03125/(a^2\mu'^4_R) ,$$

$$B_4=0.125\mu_A/\mu'^3_A+0.03125/(a^2\mu'^4_A) ,$$

$$\alpha=\tfrac{1}{2}(\mu_R+1/2a^2)/\mu'_R ,$$

$$\beta=\tfrac{1}{2}(\mu_A+1/2a^2)/\mu'_A ,$$

$$Z_R=0.6667V^R_{C\alpha}/(a^5u_R^{3/2}) ,$$

$$Z_A=0.6667V^A_{C\alpha}/(a^5u_A^{3/2}) ,$$

where a is the harmonic parameter of the wave function (11), $V^R_{C\alpha}$ ($V^A_{C\alpha}$) is the strength of the repulsive (attractive) potential in Eq. (12), and μ_R (μ_A) is the decay factor of the repulsive (attractive) potential. All of the other coefficients are functions of known parameters a, $V^R_{C\alpha}$, $V^A_{C\alpha}$, μ_R, and μ_A.

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