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Parity mixing of elastic scattering resonances: General theory and application to ^{14}N

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We consider the elastic scattering of nucleons from spin $\frac{1}{2}$ targets via parity-mixed resonances and derive expressions for the longitudinal parity nonconserving analyzing power. We neglect any direct parity nonconserving processes—the parity nonconservation being assumed to be due solely to the resonances. We then consider a particularly interesting example—the scattering of protons by ^{13}C over 0^+ and 0^- $I=1$ resonances. We show that in this case longitudinal analyzing power measurements yield, with little theoretical uncertainty, the parity nonconserving matrix element between the 0^+ and 0^- levels. The large width of the $0^-(2s_{1/2})$ resonance compared to that of the $0^+(1p_{1/2})$ resonance produces a significant enhancement of the predicted effect. Because of the large size of elastic scattering cross sections a longitudinal analyzing power experiment can achieve small statistical errors. The ^{14}N resonances are especially interesting because the mixing is essentially pure $\Delta I=0$.

I. INTRODUCTION

The parity-nonconserving (PNC) NN interaction is expected to have a complicated spin and isospin structure. For example, to characterize the low-energy limit of the interaction one needs to know the amplitude of five $s\leftrightarrow p$ transitions: one $^3S_1\leftrightarrow{}^1P_1$ ($\Delta I=0$) amplitude, one $^3S_1\leftrightarrow{}^3P_1$ ($\Delta I=1$) amplitude, and three $^1S_0\leftrightarrow{}^3P_0$ ($\Delta I=0$, 1, and 2) amplitudes. In principle, one could determine the strengths of all five of these $s\leftrightarrow p$ transitions with five or more independent PNC experiments in the NN system. However, the predicted PNC effects in the NN system are so small ($\sim 10^{-7}$—10^{-8}) that experiments of the required sensitivity are extremely difficult and in spite of concerted efforts[1-4] definite effects have only been observed for one observable, the longitudinal analyzing power, A_L, in p+p scattering.[1,2]

The five $s\leftrightarrow p$ transitions can also be probed by studying parity mixing of nuclear levels. In certain favorable circumstances the nuclear structure can lead to large enhancements (by factors of 10 or 100 or even more) of the PNC observable. Because of this quite a few nuclear parity mixing experiments have succeeded in detecting positive effects (see Ref. 5 for a review). Usually such enhancements are offset by correspondingly large theoretical uncertainties in the extraction of the PNC NN parameters from the experimental data. The same circumstances which generate the enhanced effect (typically a highly retarded parity-allowed transition) make it difficult to compute the required nuclear matrix element with sufficient reliability. However, in a few exceptional cases in light nuclei there occur closely spaced doublets of same-spin opposite-parity levels. In these cases the parity impurities are well approximated by simple two-state mixing. This greatly simplifies the analysis and isolates specific isospin components of the PNC interaction (see, for example, Ref. 6). All existing experiments probing the parity admixtures in these "two-level" nuclei have detected pseudoscalar observables in the γ decay of one member of the parity-mixed doublet (forward-backward anisotropies of γ rays emitted by polarized nuclei or circular polarizations of γ rays emitted by unpolarized states). This is a powerful technique because the electromagnetic (EM) interaction is well understood and so does not contribute to the theoretical uncertainty. Unfortunately, the proper combination of favorable nuclear structure and favorable experimental circumstances occurs so rarely that, to date, only three two-level systems (see Ref. 6) have been studied by these methods.

In this paper we discuss a new method for studying two-level systems which is applicable in cases where the levels are unbound to nucleon decays and thus could not be studied using the conventional γ-ray observables. We consider the case where the parity-mixed levels of a doublet are populated as well-defined elastic scattering resonances. We first develop a formalism for describing elastic scattering via parity-mixed resonances. We then show with a specific example, the 9 MeV 0^+ and 0^- $I=1$ resonances in ^{13}C+p, how in favorable circumstances the parity-mixing matrix element can be extracted from the longitudinal analyzing power (or equivalently the circular polarization of protons emitted by an unpolarized state) almost as reliably as from the γ-ray circular polarization. The potential value of the longitudinal analyzing power technique is also illustrated by our example in ^{13}C+p. To a good approximation (see Sec. III) the parity mixing of the ^{14}N 0^+ and 0^- resonances probes only the $\Delta I=0$ component of the PNC NN force. The only other pure $\Delta I=0$ parity admixture, the α decay[7] of ^{16}O(2^-), cannot be readily interpreted because of large uncertainties due to complicated nuclear structure. The ^{14}N system, on the other hand, is much simpler and the structure theory is expected to be quite reliable.

II. THEORY OF RESONANCE PARITY MIXING

In this section we shall outline the theoretical treatment of proton scattering via two closely spaced nuclear resonances of equal angular momentum but opposite parities in the presence of a parity-violating Hamiltonian. Explicit expressions for the cross section will be obtained for the case of polarized protons on a spin one-half unpolarized target and undetected final polarizations. The formulae can be straightforwardly generalized to include targets of arbitrary spin.

Resonance scattering, for our purposes, is most conveniently described in the language of Bloch's general formulation of reaction theory.[8] The R matrix theories of Wigner and Eisenbud,[9] and of Kapur and Peierls,[10] are recoverable as special cases of the theory. Bloch's formulation has the advantage in that one can directly obtain the S matrix without calculating the R matrix as an intermediate step.

Bloch's method essentially consists of formally solving the boundary value problem by construction of an appropriate Green's function. This Green's function comes from the inversion of an operator which is the sum of the Hamiltonian H and a "boundary condition operator" L. This latter object is a differential operator which builds in the correct boundary conditions for scattering. The S matrix is then constructed from the formal solution to the Green's function. The reader is referred to the original work of Bloch[8] for the notation and concepts used in the following.

A. S matrix

We shall take the total (projectile + target) Hamiltonian governing the system to be the sum of the strong nuclear Hamiltonian H_0 and the parity violating part of the weak Hamiltonian H_{PNC}

$$H = H_0 + H_{\text{PNC}} \; . \tag{1}$$

In the theory of resonance reactions, one distinguishes between the "interior" and "exterior" regions separated by a "channel surface," these regions being regions of a configuration space of $3A$ dimensions. For the complete set of basis vectors in the interior region, we shall make the choice of the normalized eigenstates of H_0,

$$H_0 \,|\, s\rangle = \epsilon_s \,|\, s\rangle \; . \tag{2}$$

The S matrix can be expressed in terms of the inverse operator $(\epsilon - K)^{-1}$ in this basis,

$$S = e^{i\xi}\left[1 - i\sum_{st}\langle s \,|\, (\epsilon-K)^{-1} \,|\, t\rangle \sqrt{\Gamma_s}\times\sqrt{\Gamma_t}\right] e^{i\xi} \; , \tag{3}$$

where

$$\begin{aligned} K &\equiv H + L \\ &= H_0 + H_{\text{PNC}} + L \; , \end{aligned} \tag{4}$$

L is the boundary condition operator which annihilates outgoing waves,[8] and $e^{i\xi}$ is a diagonal matrix in the space of open channels whose elements are $e^{i\xi_\lambda}$ (λ denotes the channel quantum numbers). The quantity $\sqrt{\Gamma_s}\times\sqrt{\Gamma_t}$ is the direct product of two width matrices, the elements of each matrix being of the form $(\Gamma_s^{(\lambda)})^{1/2}$. These latter quantities are proportional to the incomplete overlap[8] between the basis states $|\, s\rangle$ and channels $|\, \lambda)$

$$\begin{aligned} (\Gamma_s^{(\lambda)})^{1/2} &= (2P_\lambda)^{1/2}\left[\frac{\hbar^2 a_\lambda}{2\mu_\lambda}\right]^{1/2} (\lambda \,|\, s\rangle \\ &\equiv (2P_\lambda)^{1/2}\left[\frac{\hbar^2 a_\lambda}{2\mu_\lambda}\right]^{1/2} \phi_s^{(\lambda)}(a_\lambda) \; . \end{aligned} \tag{5}$$

In Eq. (5), a_λ is the channel radius, μ_λ is the reduced proton mass, and P_λ is the penetrability defined by

$$P_\lambda = \frac{k a_\lambda}{(G_l^2 + F_l^2)} \; . \tag{6}$$

F_l and G_l are the regular and irregular Coulomb functions, respectively.

We are now in a position to specialize the above formulae for the case of two opposite parity, equal angular momentum, closely separated resonances. To do so, we shall need to make a number of simplifying assumptions. First, we expand the operator $(\epsilon - K)^{-1}$ to first order in H_{PNC}. This should certainly be an excellent approximation. Second, we assume that the projectile and target are parity eigenstates. Third, we ignore any parity violation arising from the direct scattering and keep only effects related to the closeness of the two resonances. Fourth, we make the single-level approximation for the parity-conserving sector of the S matrix in Eq. (3). This means that two levels of the same J^π are never so close in energy that their mutual interference must be treated exactly.

With the above approximations, the parity-violating part of the S matrix becomes

$$S^J_{l_1 s_1, l_2 s_2} = -i e^{i(\xi_{l_1}+\xi_{l_2})} \frac{(\Gamma^J_{l_1 s_1}\Gamma^J_{l_2 s_2})^{1/2}\langle (l_1 s_1)J \,|\, H_{\text{PNC}} \,|\, (l_2 s_2)J\rangle}{\left[\epsilon - \epsilon^{J,\pi_1} + \frac{i}{2}\Gamma^J_{l_1 s_1}\right]\left[\epsilon - \epsilon^{J,\pi_2} + \frac{i}{2}\Gamma^J_{l_2 s_2}\right]} \; , \tag{7}$$

while the parity conserving part has the standard form

$$S^J_{l_1 s_1, l_2 s_2} = e^{i(\xi_{l_1}+\xi_{l_2})}\left[\delta_{s_1 s_2}\delta_{l_1 l_2} - i\frac{(\Gamma^J_{l_1 s_1}\Gamma^J_{l_2 s_2})^{1/2}}{\epsilon - \epsilon^{J,\pi} + \frac{i}{2}\Gamma^J}\right] \; . \tag{8}$$

The notation in Eqs. (7) and (8) is standard: l and s are the channel orbital and spin quantum numbers which combine to give the resonance J. The channel parity is given by $(-1)^l$ times the parity of the target nucleus. The resonance energies $\epsilon^{J,\pi}$ and partial widths Γ_{ls}^J are experimentally determined parameters and the total width Γ^J is the sum of all partial widths of the same J,π. The phase shift ξ_l is given by

$$e^{2i\xi_l}=\frac{G_l(a)-iF_l(a)}{G_l(a)+iF_l(a)}e^{2i(\sigma_l-\sigma_0)}, \quad (9)$$

where

$$\sigma_l-\sigma_0=\sum_{s=1}^{l}\tan^{-1}\eta/s \quad (10)$$

and

$$\eta=\frac{Z_1Z_1e^2}{\hbar v}. \quad (11)$$

Finally, the states are normalized such that under the time reversal operation they transform according to

$$T\,|\,j,m\,\rangle=(-1)^{j-m}\,|\,j,-m\,\rangle . \quad (12)$$

With this convention, the parity-violating matrix element in Eq. (7) is purely real.

B. Differential cross sections

From simple geometric considerations it is clear that the most general form of the proton-nucleus scattering cross section with proton polarization $\vec{\sigma}$, and unpolarized target and undetected final polarizations, will be of the form

$$\frac{d\sigma}{d\Omega}=K_U(\theta,E)+K_L(\theta,E)\vec{\sigma}\cdot\hat{p}_{\rm in}+K_T(\theta,E)\vec{\sigma}\cdot\hat{n}, \quad (13)$$

where $\vec{n}$ is the vector normal to the scattering plane,

$$\vec{n}=\vec{p}_{\rm in}\times\vec{p}_{\rm out}, \quad (14)$$

K_U is evidently the unpolarized cross section. K_U and K_T are coefficients of spatial scalars and $A_T\equiv K_T/K_U$ is the normal (parity-conserving) analyzing power for a transversely polarized beam. On the other hand, K_L multiplies a pseudoscalar and is thus a direct measure of the parity-violating scattering. The longitudinal (PNC) analyzing power is $A_L=K_L/K_U$.

Given the S matrix elements Eqs. (7) and (8), one can derive formulae for the cross section by examining the asymptotic wave function in the usual way as, for example, in Lane and Thomas[11] where unpolarized cross sections are obtained. For the case of longitudinal initial proton polarization μ_i, one needs to work out a sum of the form

$$\left[\frac{d\sigma}{d\Omega}\right]^{\mu_i}=\tfrac{1}{2}\sum_{m_im_f\mu_f}|\langle m_f\mu_f\,|\,T\,|\,m_i\mu_i\rangle|^2 . \quad (15)$$

For the case of transverse initial proton polarization [parallel $(+)$ or antiparallel $(-)$ to n] a similar quantity must be calculated,

$$\left[\frac{d\sigma}{d\Omega}\right]_{\pm}=\tfrac{1}{2}\sum_{m_im_f\mu_f}|\langle m_f\mu_f\,|\,T\,|\,m_i\pm\rangle|^2 , \quad (16)$$

where

$$|\pm\rangle=\sqrt{1/2}(\,|\tfrac{1}{2}\rangle\pm|-\tfrac{1}{2}\rangle) . \quad (17)$$

To work out the sums in Eqs. (15) and (16), one may use the T matrices worked out in Lane and Thomas,[11] after making a simple unitary transformation from the channel spin representation to the $\{\mu,m\}$ representation. Straightforward, but lengthy, calculations then yield the desired expressions for K_U, K_L, and K_T. Since the amplitude contains the sum of Coulomb and resonance contributions, its square contains pure Coulomb (CC), Coulomb-resonance (CR), and resonance-resonance (RR) terms. The results are summarized below:

$$K_U=K_U^{\rm CC}+K_U^{\rm CR}+K_U^{\rm RR}, \quad (18)$$

$$K_L=K_L^{\rm CR}+K_L^{\rm RR}, \quad (19)$$

$$K_T=K_T^{\rm CR}+K_T^{\rm RR}. \quad (20)$$

Each of the above terms is defined in terms of certain common factors Γ and Δ, and the Coulomb quantity $C(\theta)$. For the unpolarized terms

$$K_U^{\rm CC}(\theta,E)=\frac{|C(\theta)|^2}{k^2}, \quad (21)$$

$$K_U^{\rm CR}(\theta,E)={\rm Re}\left\{i\sum_{lj}\Gamma(\theta,E,j,l,l)[l]^{-1}P_l(\cos\theta)\right\}, \quad (22)$$

$$K_U^{\rm RR}(\theta,E)=(1/\sqrt{2})\sum_{L,J}\Delta(E,L,0,J)\begin{bmatrix}L&0&J\\0&0&0\end{bmatrix}P_L(\cos\theta), \quad (23)$$

for the longitudinal terms

$$K_L^{\rm CR}(\theta,E)={\rm Re}\left\{i\sum_{l_1l_2j}\Gamma(\theta,E,j,l_1,l_2)\begin{bmatrix}j&\tfrac{1}{2}&l_1\\-\tfrac{1}{2}&\tfrac{1}{2}&0\end{bmatrix}\begin{bmatrix}j&\tfrac{1}{2}&l_2\\-\tfrac{1}{2}&\tfrac{1}{2}&0\end{bmatrix}[1-(-1)^{l_1+l_2}]P_{l_1}(\cos\theta)\right\}, \quad (24)$$

$$K_L^{\rm RR}(\theta,E)=(1/\sqrt{6})\sum_{L,J}\Delta(E,L,1,J)\begin{bmatrix}L&1&J\\0&0&0\end{bmatrix}P_L(\cos\theta), \quad (25)$$

and for the transverse terms,

$$K_T^{\rm CR}(\theta,E)={\rm Re}\left\{\sum_{l_1 l_2 j}\Gamma(\theta,E,j,l_1,l_2)\begin{bmatrix} j & \frac{1}{2} & l_1 \\ -\frac{1}{2} & -\frac{1}{2} & 1\end{bmatrix}\begin{bmatrix} j & \frac{1}{2} & l_2 \\ -\frac{1}{2} & \frac{1}{2} & 0\end{bmatrix}[1+(-1)^{l_1+l_2}][l_1(l_1+1)]^{-1/2}P_{l_1}^{(1)}(\cos\theta)\right\}, \quad (26)$$

$$K_T^{\rm RR}(\theta,E)=(1/\sqrt{6})\ {\rm Re}\left\{\sum_{LSJ}\Delta(E,L,1,J)\begin{bmatrix} L & 1 & J \\ 1 & -1 & 0\end{bmatrix}[L(L+1)]^{-1/2}P_L^{(1)}(\cos\theta)\right\}. \quad (27)$$

We now define the remaining quantities needed to complete Eqs. (21)—(27):

$$C(\theta)=(\eta/2)\ \csc^2(\theta/2)\exp[-2i\eta\ \ln(\sin\theta/2)]\ , \quad (28)$$

$$\Gamma(\theta,E,j,l_1,l_2)=\frac{1}{4k^2}C(\theta)\sum_{s_1 s_2 J}(-1)^{s_1+s_2+l_1+l_2}[s_1]^{1/2}[s_2]^{1/2}[l_1]^{1/2}[l_2]^{1/2}[J][j]\begin{Bmatrix} j & \frac{1}{2} & l_2 \\ \frac{1}{2} & \frac{1}{2} & s_2 \\ l_1 & s_1 & J\end{Bmatrix}T^{J*}_{l_1 s_1, l_2 s_2}\ , \quad (29)$$

$$\Delta(E,L,S,j)=\frac{1}{8k^2}\sum_{\substack{ll'l_1l_2\\ ss_1s_2JJ'}}(-1)^{s+s_2+l_1+J+L}[l]^{1/2}[l']^{1/2}[l_1]^{1/2}[l_2]^{1/2}[s_1]^{1/2}[s_2]^{1/2}[J][J'][L][S][j]$$

$$\times\begin{bmatrix} l_1 & l_2 & j \\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix} l & l' & L \\ 0 & 0 & 0\end{bmatrix}W(JlJ'l';sL)W(\tfrac{1}{2}\tfrac{1}{2}s_1s_2;S\tfrac{1}{2})\begin{Bmatrix} l_1 & l_2 & j \\ s_1 & s_2 & S \\ J & J' & L\end{Bmatrix}T^{J}_{ls,l_1 s_1}T^{J'*}_{l's,l_2 s_2}\ . \quad (30)$$

The T matrices in Eqs. (29) and (30) are the usual[11] ones,

$$T^J_{l_1 s_1, l_2 s_2}=e^{2i(\sigma_{l_1}-\sigma_0)}\delta_{l_1 l_2}\delta_{s_1 s_2}-S^J_{l_1 s_1, l_2 s_2}\ , \quad (31)$$

where S^J is defined for parity violating transitions by Eq. (7) and for parity conserving transitions by Eq. (8). Finally, the [ ] notation is defined by

$$[l]=2l+1\ . \quad (32)$$

This completes our specification of the theoretical cross sections.

C. A special case

Given the rather complicated nature of the expressions derived in the preceding section, it is useful to consider a special case which includes much of the essential physics of the full problem. Consider, therefore, a simple two-level intermediate nucleus with $J=0$ and with opposite parities for the two levels. The channel spin is $S=0,1$ and combines with the orbital angular momentum $l=0,1$ to give $J=0$. The simplification of the cross section expression is dramatic, and we have

$$K_U^{J=0}(\theta,E)=\frac{1}{k^2}\,|\,C(\theta)\,|^2 - \frac{1}{4k^2}{\rm Re}[iC(\theta)(T^{J=0}_{0,0}+T^{J=0}_{1,1})] + \frac{1}{16k^2}(\,|\,T_{0,0}\,|^2+|\,T_{1,1}\,|^2) \quad (33)$$

and

$$K_L^{J=0}(\theta,E)=\frac{1}{4k^2}(1+\cos\theta)C(\theta){\rm Re}(iT_{0,1}) - \frac{1}{4k^2}{\rm Re}[T_{0,1}(T^*_{0,0}+T^*_{1,1})]\ , \quad (34)$$

(notation: $T_{0,1}\equiv T^{J=0}_{l_1=s_1=0,\ l_2=s_2=1}$, etc.) .

A number of features are readily apparent now. First, the resonance-resonance term is isotropic and dominates the Coulomb-resonance term for large angles. Second, the parity violation necessarily involves a change of l by one unit. This is also evident from Eq. (26) for the Coulomb-resonance term and from Eq. (25) for the resonance-resonance term.

Let us now simplify even further by supposing the incident projectile to be a neutron. Denote the resonance energies and widths by ϵ_1,Γ_1 and ϵ_2,Γ_2, respectively, and assume that, for example, the first resonance is narrow while the second is relatively broad. Let the incident neutron energy be equal to the energy of the first resonance, i.e., $E=\epsilon_1$. Then,

$$A_L\equiv\left.\frac{K_L^{J=0}}{K_U^{J=0}}\right|_{E=\epsilon_1}\sim\left[\frac{\Gamma_2}{\Gamma_1}\right]^{1/2}\frac{\langle 1\,|\,H_{\rm PNC}\,|\,2\rangle}{\epsilon_1-\epsilon_2+\frac{i}{2}\Gamma_2}\ . \quad (35)$$

As expected the parity-violating effect is amplified if the admixed levels are near each other. Now make the additional assumption that $|\,\epsilon_1-\epsilon_2\,|\gg\Gamma_2/2$. In this case the expression for K_L becomes very similar to that for the

PNC circular polarization of a two-level nucleus such as ^{18}F (see Ref. 6). The factor $\sqrt{\Gamma_2/\Gamma_1}$ in Eq. (35) plays the role of the electromagnetic amplification factor $\langle M1\rangle/\langle E1\rangle$ which occurs in the expression for the PNC circular polarization.

III. APPLICATION TO THE 9 MeV 0^+ AND 0^- $T=1$ STATES IN ^{14}N

In this section we discuss a particularly interesting application of A_L measurements over parity-mixed elastic scattering resonances. In the self-conjugate nucleus ^{14}N there is a closely spaced doublet of $J=0, T=1$ levels—a 0^+ state at $E_x=8.618$ MeV and a 0^- level at $E_x=8.79$ MeV (see Ref. 12). These two resonances have only one open particle decay channel— ^{13}C + p. The 0^+ level corresponds to a $1p_{1/2}$ resonance with $E_p^{\text{lab}}=1150$ keV ($\Gamma^{\text{c.m.}}=7\pm1$ keV). The 0^- level corresponds to a $2s_{1/2}$ resonance with $E_p^{\text{lab}}=1340$ keV and $\Gamma^{\text{c.m.}}\sim460$ keV.

The parity mixing between these two levels is of particular interest because:

(1) The mixing is sensitive primarily to the $\Delta I=0$ component of H_{PNC}. Although the isospin structure of the problem also allows $\Delta I=2$ mixing, the $\Delta I=2$ matrix element will be small compared to the $\Delta I=0$ matrix element because there cannot be any coherent buildup of $\Delta I=2$ contributions (i.e., there is no $\Delta I=2$ component of an effective one-body PNC potential). There is at present only one experiment[7] which is sensitive only to the $\Delta I=0$ component of the PNC NN force—the α decay of the 8.8 MeV 2^- state in ^{16}O. However, the interpretation of this highly precise experimental result is clouded by nuclear structure uncertainties. It is not a case of simple two-level mixing. To interpret the result one needs shell model calculations which include up to $4\hbar\omega$ configurations. As a result there is a large and somewhat poorly defined theoretical uncertainty.

(2) The observable provides a very sensitive way to measure H_{PNC}. The anomaly in A_L is magnified by nuclear structure effects and the achievable statistical precision is high because elastic scattering cross sections are very large. The magnification arises because of the very large width of the 0^- resonance compared to that of the 0^+ resonance. As can be seen from Eq. (35), if the beam energy is selected to lie on top of the narrow (0^+) resonance the longitudinal analyzing power is enhanced by a factor which is roughly $[\Gamma(0^-)/\Gamma(0^+)]^{1/2}=8.1$.

(3) The longitudinal analyzing power on the 8.618 MeV 0^+ resonance should be well described by two-level mixing. The nearby broad 0^- resonance has nearly the full $2s_{1/2}$ single-particle strength (see Sec. IV). Therefore, there cannot be any other 0^- resonances with larger $2s_{1/2}$ decay width whose admixtures could contribute appreciably to A_L.

(4) There is only one open strong interaction channel (elastic scattering). Furthermore, the parity-mixed resonances are $J=0$ and hence have unique values of l and S. These circumstances lead to considerable simplifications in the reaction theory and therefore in the analysis of the experimental results.

(5) The theoretical models of the 0^+ and 0^- levels in ^{14}N are reasonably good. In the following section we show that even a very simple shell model gives a decent account of many of the properties of the 0^+ and 0^- levels. Furthermore, the $A=14$ system has sufficiently few particles that more sophisticated calculations which include all excitations up to $2\hbar\omega$ are feasible. Therefore we expect that the analyses of the experimental results in ^{14}N in terms of the $\Delta I=0$ PNC NN interaction will be relatively free from nuclear structure uncertainties.

IV. SHELL MODEL PREDICTIONS FOR PARITY MIXING IN ^{14}N

In order to determine the practicality of an experiment to measure A_L over the second 0^+; $T=1$ (0_2^+) state of ^{14}N we have made a shell model estimate of $\langle 0_1^-\,|H_{\text{PNC}}|\,0_2^+\rangle$. The estimate is based on the "best value" PNC NN potential[13] of Desplanques, Donoghue, and Holstein which has been quite successful[6] in accounting for PNC effects in p+p, ^{18}F, ^{19}F, and ^{21}Ne. The calculation was performed using the Oxford shell model codes as described by Brown, Richter, and Godwin.[14] The wave functions were diagonalized in a model space in which the $1s_{1/2}$ and $1p_{3/2}$ orbits were filled and the active (valence) particles were restricted to $1p_{1/2}$, $2s_{1/2}$, and $1d_{5/2}$ orbits. The single-particle energies and residual matrix elements were taken from Ref. 15. The predicted energy levels of ^{14}N are compared with experiment in Fig. 1.

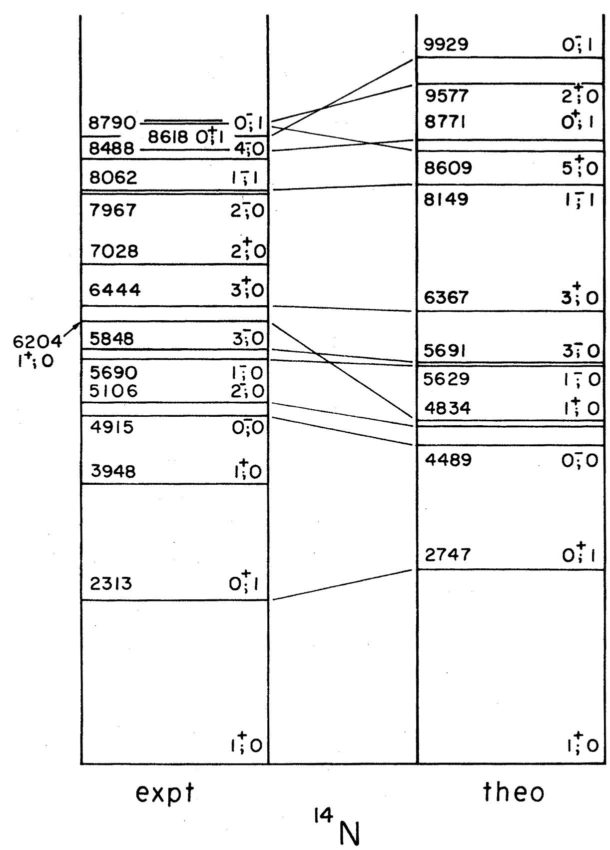

FIG. 1. Comparison of the observed spectrum of levels in ^{14}N with the results of the simplified shell-model calculation described in the text.

The simple model gives a rather good account of the data. Its main shortcomings are the failure to reproduce the 3948, 7028, and 7967 keV levels which presumably have large amplitudes for configurations containing holes in the $1p_{3/2}$ subshell.

Since $H_{\rm PNC}$ is a short-range two-body operator we need to use correlated wave functions in order to calculate PNC matrix elements. Correlations were included by multiplying the harmonic-oscillator wave functions (with $\hbar\omega=14$ MeV) by a factor given by Miller and Spencer.[16] This procedure gives results which are consistent with more elaborate treatments of the correlations. The predicted PNC matrix element is

$$|\langle 0^- | H_{\rm PNC} | 0^+ \rangle| = (1.49+0-0.10)\ {\rm eV}\ , \qquad (36)$$

where the three numbers are the contributions of the $\Delta I=0$, 1, and 2 components of $H_{\rm PNC}$.

The phase of this matrix element is arbitrary since it depends on algorithms used in diagonalizing the wave functions. However, the experimental observable depends on the product $\sqrt{\Gamma_+\Gamma_-}\langle - | H_{\rm PNC} | + \rangle$, the sign of which is not arbitrary. Our shell model wave functions are normalized without the i^l factor used in the scattering formalism of Sec. II and with the single-particle radial functions chosen to be positive at small r. The shell model PNC matrix element is related to that employed in Sec. II by

$$\langle (l_1 s_1)J | H_{\rm PNC} | (l_2 s_2)J \rangle_{\rm scattering} = i^{(l_2-l_1)} \langle (l_1 s_1)J | H_{\rm PNC} | (l_2 s_2)J \rangle_{\rm shell\ model}\ . \qquad (37)$$

The width factors are given by

$$(\Gamma^\alpha)^{1/2} = (2P)^{1/2} \left[\frac{\hbar^2 a}{2\mu} \right]^{1/2} C_\alpha \phi_\alpha(a)\ , \qquad (38)$$

where C_α is a fractional parentage coefficient whose square is equal to the familiar quantity $({\rm CG})^2 S$, where CG is an isospin Clebsch-Gordan coefficient and S is a single-nucleon spectroscopic factor which, in our case, connects the $A=13$ ground state and the $A=14$ state labeled α. The label α stands for the quantum numbers J, π, and T as well as an index specifying the particular state (if there is more than one) with these quantum numbers. In terms of a doubly reduced spin-isospin matrix element

$$C_\alpha = \frac{1}{\sqrt{6}} \langle A=14(\alpha) ||| a^\dagger ||| A=13({\rm g.s.}) \rangle\ . \qquad (39)$$

The quantity $\phi_\alpha(a)$ is the projectile wave function at the channel surface. If we assume $\phi_\alpha(r)$ to be roughly constant for $r<a$ then the condition $\int_0^a dr\, r^2 \phi^2 = 1$ gives $\phi^2 \approx 3/a^3$. When this value of ϕ is inserted into Eq. (38) it provides a crude estimate of $(\Gamma^\alpha)^{1/2}$. However, we need only rely on theory to give the sign of $\sqrt{\Gamma_+\Gamma_-}$ since the magnitudes of $\sqrt{\Gamma_+}$ and $\sqrt{\Gamma_-}$ are known from experiment. The shell model predicts

$$C_{0^-} = +\frac{1.00}{\sqrt{2}} \qquad (40)$$

and

$$C_{0^+} = +\frac{0.299}{\sqrt{2}}\ .$$

It also requires that $\phi_{0^-}(a) = \phi(2s_{1/2}, a)$ and $\phi_{0^+}(a) = \phi(1p_{1/2}, a)$ have opposite signs at the channel radius a. With these conditions we obtain the relation

$$(\Gamma^0_{l=0;s=0})^{1/2} (\Gamma^0_{l=1;s=1})^{1/2} \langle (l=0;s=0)J=0 | H_{\rm PNC} | (l=1;s=1)J=0 \rangle = |[\Gamma_{\rm expt}(0^-)\Gamma_{\rm expt}(0^+)]^{1/2}| (-1.39\ {\rm eV})\ . \qquad (41)$$

How much credence should we place on the simplified shell model predictions for matrix elements of $H_{\rm PNC}$? One can get some idea from the accuracy of the predictions for parity-conserving quantities. The 0^+; $T=1$, 0^-; $T=1$, 1^+; $T=0$, and 1^-; $T=0$ levels obtained in this restricted model space are shown in Table I. Considering the severity of the truncation the predicted excitation energies are in reasonable agreement with experiment.

TABLE I. Comparison of theory and experiment for some $J=0$ and 1 levels in ^{14}N.

State	$E_x^{\rm theory}$ (MeV)	$E_x^{\rm expt}$ (MeV)
0^+ $T=1$	2.747	2.313
	8.711	8.618
	13.058	
0^- $T=1$	9.929	8.79
1^+ $T=0$	0	0
	a	3.948
	4.834	6.204
	13.162	
1^- $T=0$	5.629	5.690

[a]The observed 1^+ $I=0$ level at 3.948 MeV is not contained in this model space because its wave function has large amplitudes for configurations with holes in the $p_{3/2}$ subshell.

How well does the simplified shell model account for the proton widths of the 0^+ and 0^- levels? Since Eq. (38) with $\phi^2=3/a^3$ gives only a crude estimate of the width we used a more sophisticated procedure to estimate $\Gamma(0^+)$ and $\Gamma(0^-)$ from the shell model C's. We first estimated the single-particle (s.p.) widths from a procedure[17] based on the relation

$$\Gamma_{\rm s.p.} \approx 2 \left[\frac{\partial \delta}{\partial E} \right]^{-1}_{\delta=\pi/2}\ , \qquad (42)$$

where $\delta(E)$ is the scattering phase shift as a function of projectile energy. The derivative was evaluated using optical model wave functions. The Woods-Saxon nuclear potential well had a standard geometry and the depth was adjusted to produce a resonance at the observed energy. Then we estimated Γ from the relation

$$\Gamma = \Gamma_{\rm s.p.} C^2\ , \qquad (43)$$

where C was defined in Eq. (39). This procedure gave

TABLE II. Comparison of theory and experiment for γ-ray decays of the 9 MeV $J=0$ doublet in ^{14}N.

Transition: E_i(MeV)$\rightarrow E_f$(MeV) Expt	Theory	Reduced transition strength $B_{\rm expt}$	$B_{\rm theory}$ [c]
8.618$\rightarrow$0.000	8.771$\rightarrow$0.000	$B(M1)^{a}=0.11\pm0.04\ \mu_N^2$	$B(M1)=0.32\ \mu_N^2$
$\rightarrow$3.948		$B(M1)^{a}=1.3\pm0.4\ \mu_N^2$	
$\rightarrow$5.690	$\rightarrow$5.629	$B(E1)^{a}=0.022\pm0.007\ e^2{\rm fm}^2$	$B(E1)=0.16\ e^2{\rm fm}^2$
$\rightarrow$6.204	$\rightarrow$4.834	$B(M1)^{a}=14\pm5\ \mu_N^2$	$B(M1)=12.7\ \mu_N^2$
8.79 $\rightarrow$0.000	9.929$\rightarrow$0.000	$B(E1)^{b}=0.065\pm0.017\ e^2{\rm fm}^2$	$B(E1)=0.086\ e^2{\rm fm}^2$

[a]P. M. Endt, At. Data Nucl. Data Tables **23**, 3 (1979).
[b]Reference 12.
[c]See the text.

good results for the "single-particle" $2s_{1/2}$ resonance in ^{12}C+p scattering at $E_p=461$ keV. The predicted width of 39 keV (assuming $C=1$) agrees well with the observed value of 34 keV. The predictions for the 0^+ and 0^- levels of ^{14}N using the shell model C's are 6.9 and 1045 keV, respectively. The width $\Gamma(0^+)$ is in excellent agreement with the measured value of 7 ± 1 keV, but $\Gamma(0^-)$ is considerably larger than the observed value of $\sim$460 keV. This may be a result of the pure $1p_{1/2}2s_{1/2}$ configuration assigned by the simplified shell model or it may reflect the fact that the approximation in Eq. (42) is not valid when the width becomes very large.

Gamma-ray transition rates provide a more demanding test of the wave functions. The measured γ-ray transition rates between these levels are compared with our predictions in Table II. The agreement is again quite satisfactory considering the restricted model space. However, it has been shown experimentally[6] that in the $A=18$ and 19 systems simple shell model calculations, such as we report here, overestimate the matrix elements of $H_{\rm PNC}$ and of the closely related $\Delta J=0$ first-forbidden β-decay operator by factors of $\sim$3. To get agreement between the first-forbidden decay rates and theory one needs to employ a much larger model space.[6] Because our calculation was restricted to a small model space the predicted matrix element of $H_{\rm PNC}$ will only be a crude estimate. However, it is adequate for our purpose, namely to show the utility of longitudinal analyzing power measurements over elastic scattering resonances.

We have computed the ^{13}C + p cross sections and analyzing powers predicted by Eqs. (13)—(31) using the resonance energies, widths, and J^π values given in the latest compilation[12] plus the shell model PNC quantities given in Eq. (41). The results are shown in Figs. 2 and 3. In Fig. 2 we compare our calculation to the unpolarized data of Latorre and Armstrong.[18] The agreement between our calculation (which included only the resonances at $E_p^{\rm lab}=551$, 1150, 1340, 1462, and 1540 keV) and the data is quite good. In Fig. 3, we show, on an expanded horizontal scale, some quantities relevant to an experiment designed to measure $\langle 0^+|H_{\rm PNC}|0^-\rangle$ by detecting A_L over the narrow 0^+ resonance. We postulate an apparatus consisting of two annular proton counters—a back counter subtending values of θ between 125° and 175°, and a front counter subtending polar angles between 25° and 35°. These two counters view a ^{13}C target with a thickness sufficient to give a 10 keV energy loss to the proton beam. We display the predicted counting rates in these counters plus the energy-and-angle-averaged longitudinal and transverse analyzing powers. The experimental signal

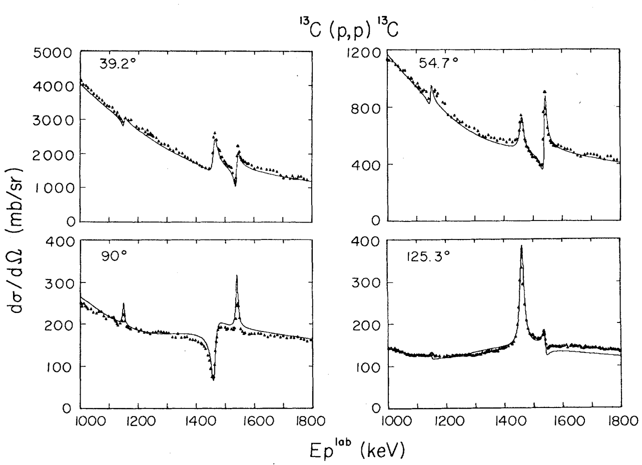

FIG. 2. Comparison of the unpolarized scattering cross sections of Ref. 18 with calculations using the formalism discussed in the text and resonance parameters taken from Ref. 12. The narrow structure at $E_p\sim1150$ keV is the 0^+ resonance. The 0^- resonance is too broad to show up as a distinct structure.

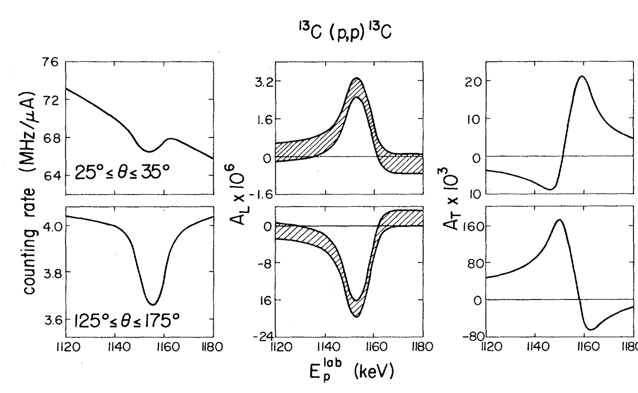

FIG. 3. Predicted counting rates and analyzing powers for an experiment (see the text) to study parity mixing of the 0^+ and 0^- $T=1$ levels of ^{14}N. The cross hatched area on the A_L plot indicates a $\pm1\sigma$ band centered on the predicted A_L, where σ is the statistical standard deviation expected after an integrated beam charge of 1 μA d.

is the ratio of A_L in the back counter to A_L in the front counter. It can be seen that the statistical power of the helicity measurement is quite good. After 1 μA d of integrated beam charge the statistical standard deviation is 10% of the predicted PNC anomaly. This example demonstrates the utility of the proposed approach. An experiment along these lines is currently being prepared at the University of Washington Nuclear Physics Laboratory.

V. CONCLUSION

We have shown that the longitudinal analyzing power associated with parity-mixed scattering resonances can provide a sensitive and interpretable measurement of the PNC matrix element connecting a parity-mixed two-level system. Implementation of these ideas will expand the number of two-level systems which are accessible experimentally. In particular, the parity mixing in ^{14}N, because of its virtually pure $\Delta I = 0$ character, will provide a constraint on the PNC NN interaction which complements those available from the other two-level nuclei ^{18}F, ^{19}F, and ^{21}Ne.

Longitudinal analyzing powers have already been studied in neutron scattering[19] and in (p,α) reactions.[20] However, these examples can not be analyzed in terms of two-level mixing and therefore are subject to large uncertainties from nuclear structure. A longitudinal analyzing power measurement in the ^{4}He(^{6}Li,γ) reaction has been proposed by Bizzetti and Perego.[21] This would be a case suitable for a two-level analysis but unfortunately it is not very favorable on experimental grounds.[22]

Note added. W. C. Haxton[23] has calculated the PNC matrix element in ^{14}N using a complete $2\hbar\omega$ model with c.m. projection and obtains $|\langle 0^- | H_{\rm PNC} | 0^+ \rangle| = 1.04$ eV. Our cruder result agrees fairly well with this prediction.

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