Deuteron forward photodisintegration

J. L. Friar and B. F. Gibson
Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545

G. L. Payne
Department of Physics and Astronomy, University of Iowa, Iowa City, Iowa 52242
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The deuteron photodisintegration with forwardgoing proton is calculated using a new formalism for organizing matrix elements. Relativistic effects are thoroughly discussed. Different ways of calculating retarded electric multipoles are investigated. Meson exchange currents are included in various approximations. The spin-orbit dipole operator is found to dominate the relativistic corrections.

I. INTRODUCTION

The remarkable measurement by Hughes *et al.* of deuteron photodisintegration[1] with the proton forwardgoing (0°) was not explicable by conventional theoretical calculations.[2–5] Although this experiment was extremely difficult, it has been confirmed by two independent[6,7] measurements. Conventional wisdom held that deuteron photodisintegration was well understood and relatively uninteresting; consequently, little theoretical work was performed in the time span between Refs. 1 and 2. The fact that theoretical calculations of electric-dipole-dominated photodisintegration were 20% higher than experiment was stunning.

In order to understand the significance of this result, the essence of the physics must be understood. Consider Fig. 1. The photon (wavy lines) impinges on the deuteron. The electric field of the photon is orthogonal to the wave vector $\vec{q}$, and the electric force is therefore in the same direction. Classically, the electric force cannot drive the proton forward; the most probable event, classical or quantum, is for the proton to move at 90° to the photon. The lower figure depicts the unit angular momentum (helicity) of the photon and the zero *orbital* angular momentum of the outgoing nucleons along the photon direction. Thus, unless the nucleon spins participate, the process along the beam direction is forbidden by angular momentum conservation.

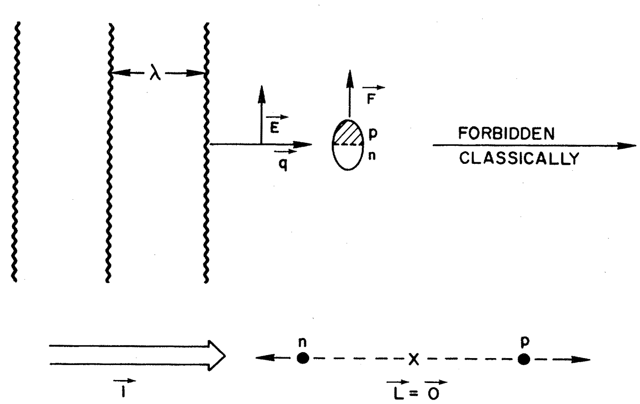

FIG. 1. Classical description of deuteron forward photodisintegration.

The spins do participate, of course, but not in a first approximation for the electric dipole part of the process. The Siegert electric dipole operator is given by

$$\vec{D}_0 = \sum_i \frac{\tau_i^3}{2}\vec{r}'_i = \frac{\Delta\tau}{2}\frac{\vec{r}}{2} , \tag{1}$$

where $\vec{r}'_i$ is the relative coordinate of nucleon i with respect to the nuclear center-of-mass $\vec{R}$ $(\vec{r}_i = \vec{r}'_i + \vec{R})$, $\vec{\tau}_i$ is the (Pauli) isospin operator of nucleon i, $\vec{r}$ is the deuteron relative coordinate, $(\vec{r}'_1 - \vec{r}'_2)$, and $\Delta\tau = \tau_1^3 - \tau_2^3$. This spin-independent operator leads only to triplet final states, and involves the nuclear spins only through the deuteron D state, which couples orbital and spin angular momenta, and through noncentral forces in the unbound system, which lead to different radial wave functions for the various partial waves of the excited states. Thus, the forward photodisintegration probes

(1) spin-dependent transition operators;
(2) the deuteron D state;
(3) noncentral forces in the excited state;
(4) possible exotic (non-nucleonic) phenomena.

The classical argument given above leads to a large suppression of 0° photodisintegration relative to 90° photodisintegration for photon energies between 20 and 100 MeV.

The theoretical attempts to understand the discrepancy between theory and experiment have centered on various relativistic phenomena.[7] Primary among these is the electromagnetic spin-orbit interaction:[8,9]

$$H_{\rm so} = -\vec{E}\cdot\Delta\vec{D}_{\rm so} , \tag{2}$$

where

$$\Delta\vec{D}_{\rm so} = -\frac{1}{4m^2c^2}\sum_i (2\mu_i - e_i)\vec{\sigma}_i \times \vec{\pi}_i , \tag{3}$$

and $\vec{E} \sim i\omega\vec{A}$ is the photon electric field; ω is the photon energy; e_i, μ_i, and m are the charge, magnetic moment, and mass of nucleon i; $\vec{\sigma}_i$ is its spin; and $\vec{\pi}_i$ is the momentum of nucleon i relative to the center-of-mass, whose (frame-dependent) total momentum is $\vec{P}$ $[\vec{p}_i = \vec{\pi}_i + (\vec{P}/A)]$. Operator (3) is spin dependent and can contribute substantially at 0°, where the usual nonrelativistic $\vec{D}_0$ contribution is suppressed. The surprisingly large size of the spin-orbit operator was first noted in the original calculations of Cambi, Mosconi, and Ricci,[11] and will be verified later.

The electromagnetic spin-orbit interaction is responsible for the spin-orbit splitting in the hydrogen atom and its use is not new in nuclear physics.[8,9] In the former case the electric field produced by the nucleus transforms into a magnetic field component which interacts with the electron's magnetic moment, μ. The Thomas precession accounts for the e term in Eq. (3). Spin-orbit interactions are the quintessential relativistic corrections. The same spin-orbit interaction generates a small charge density for neutrons in a nucleus, and is a major component of *observed* isotopic charge density[12] *differences*. In addition, the proper use of the spin-orbit interaction, and spin-dependent kinematic relativistic corrections, are needed to prove the spin-dependent form of the low-energy theorem for Compton scattering and the Drell-Hearn-Gerasimov sum rule.[13]

A simple calculation provides a quick estimate of the size of this effect for forward photodisintegration. Keeping only spin-dependent impulse approximation terms in the $E1$ interaction, the transverse (to $\vec{q}$) current[10] is given by

$$\vec{J}_\perp(\vec{q}) \cong i[H_0, \vec{D}_0 + \Delta\vec{D}_{so}] + \frac{q^2\vec{N}_s}{6}, \tag{4}$$

with

$$\vec{N}_s = -\frac{3}{2m}\sum_i \mu_i \vec{\sigma}_i \times \vec{r}_i' . \tag{5}$$

Replacing the commutator of the nuclear Hamiltonian, H_0, by ω, equating q and ω, and writing

$$\vec{p}_i = im[H_0, \vec{r}_i'] \cong i\omega m \vec{r}_i' ,$$

we obtain

$$\vec{J}_\perp(\vec{q}) \cong i\omega\vec{D}_0 + \frac{\omega^2}{4m}\sum_i (2\mu_i - e_i)(\vec{\sigma}_i \times \vec{r}_i') - \frac{\omega^2}{4m}\sum_i \mu_i(\vec{\sigma}_i \times \vec{r}_i') . \tag{6}$$

An important aspect of the problem is that the ${}^3P_J$-${}^3F_J$ two-body final states correspond to isospin 1, and the transition has $\Delta T = 1$; the corresponding magnetic moment is $\mu_v = \mu_p - \mu_n = 4.7\mu_N$, which is very large. Neglecting e_i in the second term in Eq. (6) compared to μ_v, we see that the spin-orbit interaction contribution is *twice* that of the spin magnetization current, with the *opposite* sign. From the early work of Partovi[2] we know that the latter is roughly $+10\%$ of the total cross section for $\omega \sim 100$ MeV, implying that the former is roughly -20%. This is a good estimate, as seen in Fig. 2, which depicts the impulse approximation for the Paris potential,[14] with and without the spin-orbit interaction. The latter is in reasonably good agreement with the data. This estimate could have been made 20 years ago.

The previous rough argument neglects the noncommutativity of the nucleon-nucleon potential with spin and isospin factors, and glosses over possible complications with recoil and center-of-mass motion. In the atomic physics case, where such problems do not arise to lowest order, neglecting the small anomalous magnetic moment of the electron produces $\mu_i \cong e_i$, and the two terms cancel.[15] Spin-flip electric dipole processes proceed through spin impurities in the wave functions induced by the (electronic) spin-orbit potential. The ${}^3P_1 \to {}^1S_0$ transition in Helium-type ions is a good example.[15]

Another type of process which has been much calculated, if not studied, is the contribution of the one-pion exchange (OPE) dipole operator, $\Delta\vec{D}_\pi$.[13,16–20] The analogous process in an atom would be a one-photon-exchange dipole operator. The latter depends on the gauge chosen for photon exchange, and vanishes in Coulomb gauge.[15] Several points should be made about the OPE charge operator:

(1) it is of order $(1/c^2)$, i.e., a relativistic correction;[16]

(2) it is spin dependent;[16]

(3) it is different for ordinary pseudoscalar (PS) and pseudovector (PV) pion-nucleon couplings;[21]

(4) it is nonlocal, i.e., momentum dependent;[16]

(5) it depends on the method *chosen* to perform the calculation.[22]

Most calculations use PS-coupling Born terms; this is inconsistent with current algebra and with experimental threshold pion photoproduction[21] and exaggerates the effect. Most calculations also drop the momentum-dependent terms;[5,11,17,18,20] this underestimates the effect. More serious is comment (5). Different methods of calculation lead to different transition operators, which are nevertheless members of a family of unitarily equivalent

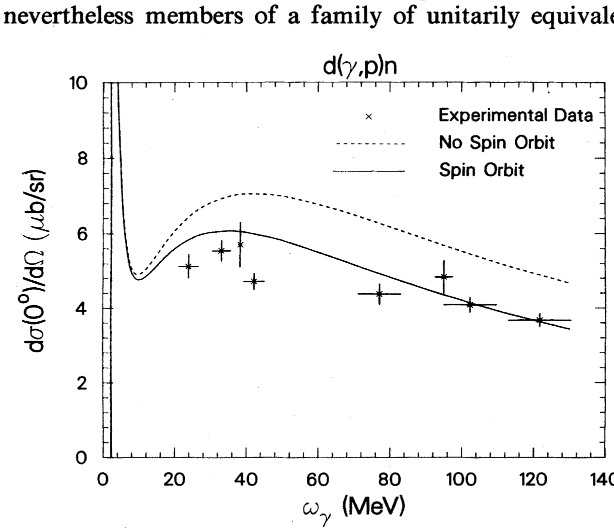

FIG. 2. Deuteron forward photodisintegration for the Paris potential with and without the spin-orbit dipole operator.

operators.[16,22] In order to obtain an unambiguous matrix element, corresponding wave functions from corresponding nucleon-nucleon potentials must be chosen from the family. This cannot be done with common "realistic" potentials, because such potentials do not correspond to *any* of the representations mentioned above. The difficulty lies in the complicated spin dependencies, which leads to many different types of term, not all of which can be eliminated by "dialing" the unitary transformation. The OPE isoscalar dipole operator is unique, because the isoscalar (Siegert) dipole operator vanishes, unlike the isovector one. The most common method of calculating the OPE charge operator *maximizes* the effect. Although we have not performed an unambiguous calculation either, our results for various unitary representations suggest that the OPE contribution is *probably* no more than 40–50% of the spin-orbit effect and the most likely representations will lower the cross section. A detailed discussion is given in Sec. V.

Several other contributions of relativistic order arise from recoil, or nuclear motion. These are discussed in Sec. IV. Lorentz invariance specifies relations between various multipoles in different reference frames. Typically these corrections are of order ω^2/m_t^2 or ω/m_t times the $M1$ or $E2$ contributions, where m_t is the deuteron mass, and are obviously unimportant.

The remaining contribution to the electromagnetic interaction of order $(1/c^2)$ is the Darwin-Foldy term.[9,39] We show that this term is proportional to $(\vec{q}^{\,2}-\omega^2)$ in any Lorentz gauge and hence vanishes for real photons. This, however, points out another problem: Should a nucleon form factor be included in the current operators, as it is in treatments of electron scattering? The Darwin-Foldy term generates a component of the nucleon charge form factor which vanishes for real photons. This does *not* imply that the identical situation holds for all parts of the nucleon form factor. Arguments based on Lorentz invariance have been made[23] which show that other "intrinsic" components of the nucleon densities have an approximate argument $\bar{q}^{\,2}=\vec{q}^{\,2}-\omega^2+\omega_{fi}^2$, where ω_{fi} is the intrinsic energy (mass) difference of final and initial nuclear states. Neglecting recoil produces $\bar{q}^{\,2}\cong\vec{q}^{\,2}$, which is the naive result. The less naive replacement in form factors of $\vec{q}^{\,2}\to q^2\equiv 0$ does not *necessarily* hold, because the nucleons in the nucleus are off shell and depend on more than one variable (e.g., ω_{fi}^2, in addition to q^2). Indeed, the purely nuclear part of photodisintegration obviously cannot depend solely on q^2. The (ambiguous) inclusion of a nucleon form factor is shown to be completely unimportant at low energies and relatively unimportant at higher energies. The effect is a relativistic correction.

A potentially more serious problem concerns the various ways of computing retardation in the (dominant) electric multipoles. The use of Siegert's form of the electric dipole operator is the backbone of the photonuclear field. Any electric multipole field must have this property in the long wavelength limit. If the model nuclear current is not conserved, different forms generate different retarded electric multipoles. The necessity of incorporating explicit exchange currents in order to achieve a conserved current ensures different numerical results. Recently[10] it has been shown that a unique extension of Siegert's result exists for arbitrary wavelengths, and the use of this new form versus older forms is studied in Sec. III. Ultimately, the effect is not large, but the new $E1$ form has considerably smaller spin-independent retardation corrections than the standard form.

The Reid soft core (RSC) potential model, as originally published,[24] does not define an interaction for total two-body angular momenta, J, greater than 2. These potentials were developed by Reid and have been published by Day.[25] We have included these interactions in our RSC calculations. The effect is small, but visible. In order to convince the reader that the results for various realistic potential models are commensurate, we have plotted in Sec. V cross sections for eight such models,[14,24,27–32] and have also scaled these results to a common value of A_D $(\equiv A_S\eta)$, the deuteron asymptotic D-wave normalization, as suggested by Schulze, Saylor, and Galoskie.[26] The latter results all lie in a narrow band, except for the Hamada-Johnston[27] and RSC potentials. The former lies low, presumably because it has defective deuteron binding,[28] while the latter has poor low-energy p-wave phase shifts.

Our basic approach to calculating the amplitudes is new and systematizes the nucleon spin-photon polarization-angular dependence into twelve invariants: three singlet and nine triplet, similar to the relativistic approach of Ref. 33 and analogous to that of Ref. 34. This expansion is elegant and is particularly simple for 0° photodisintegration. We evaluate multipoles through $L=2$, which is sufficient for discussing the physics.[2] This is discussed in Sec. II.

Finally, we note that there has been recent controversy concerning the deuteron data. An excellent discussion of the older data has been given by Fuller,[35] while new polarized photon data have been generated in Ref. 36. We greatly encourage renewed study of the deuteron, which continues to produce surprises.

II. ANGULAR DISTRIBUTION

Deuteron photodisintegration is complicated by the large number of possible spin combinations of incoming deuteron and photon and outgoing nucleons. Nonrelativistically there are two independent vectors together with spin directions which determine the geometry. A matrix element of the form $\langle f\vec{P}_f;\vec{k}\,|\,i\vec{P}_i;\vec{q}\,\rangle$ specifies a deuteron with internal quantum numbers i and momentum $\vec{P}_i$ interacting with an incoming photon with momentum $\vec{q}$ to form two (identical) nucleons with *total* momentum $\vec{P}_f$, internal quantum numbers f, and relative momentum $\vec{k}$ $[\vec{k}=(\vec{p}_1-\vec{p}_2)/2;\ \vec{P}_f=\vec{p}_1+\vec{p}_2]$. Clearly

$$\vec{P}_i+\vec{q}=\vec{P}_f\ , \tag{7}$$

which specifies that we can define two independent combinations of total momentum vectors;

$$\vec{S}=\vec{P}_f+\vec{P}_i \tag{8a}$$

and

$$\vec{q}=\vec{P}_f-\vec{P}_i\ . \tag{8b}$$

Nonrelativistically, $\vec{q}$ is frame independent while $\vec{S}$ is manifestly frame dependent, although the form of the nucleon current's dependence on the latter is trivial ($\sim\vec{S}$) and vanishes in transverse gauges and frames for which $\vec{S}\sim\vec{q}$. These specific frames have been designated q-congruent frames,[8] and include the center-of-mass and laboratory frames for our process. Relativistic corrections will (must) depend on $\vec{S}$. We choose to work in the center-of-mass frame ($\vec{S}\equiv\vec{q}$). Thus $\hat{k}$ and $\hat{q}$ specify those geometrical aspects not connected with spin.

Matrix elements of the current operator are conventionally and conveniently broken into magnetic and electric contributions. Both can be schematized in the form[10]

$$\vec{\epsilon}\cdot\vec{J}(\vec{q})=A_L\cdot\mathscr{M}_L \quad (\text{magnetic } L \text{ pole}), \tag{9a}$$

$$\vec{\epsilon}\cdot\vec{J}(\vec{q})=B_L\cdot\mathscr{E}_L \quad (\text{electric } L \text{ pole}), \tag{9b}$$

with

$$A_{LM}=\sqrt{4\pi}[\lambda_1\otimes Y_{L-1}(\hat{q})]_{LM}, \tag{10a}$$

$$B_{LM}=\sqrt{4\pi}[\epsilon_1\otimes Y_{L-1}(\hat{q})]_{LM}, \tag{10b}$$

where $\vec{\lambda}=\hat{q}\times\vec{\epsilon}$, $\vec{\epsilon}$ is the photon polarization vector and $\mathscr{E}_L$ and $\mathscr{M}_L$ are the nuclear electric and magnetic multipole operators.

The remaining angular dependence is contained in the final state two-nucleon wave function. We ignore tensor coupling of waves for demonstration purposes and write that quantity in the form

$$\psi_{\vec{k}}^{Sm_s}=4\pi\sum_{\substack{l,J\\ m_l,M_J}} i^l Y_{lm_l}(\hat{k})\langle lm_l Sm_s\,|\,JM_J\rangle \chi_{J,M_J}^{l,S} R_{l,J}^S(k;r), \tag{11}$$

where we have coupled the spin and orbital angular momenta. It is convenient to introduce a unit polarization vector $\vec{e}$ for a deuteron with a wave function, d_m, corresponding to magnetic quantum number m, and sum over m to form

$$\vec{e}\cdot\vec{d}\equiv\sum_\lambda e_\lambda^* d_\lambda . \tag{12}$$

Choosing $\lambda=m$ and using $e_\lambda^*(m)=\delta_{\lambda,m}$ reduces $\vec{e}\cdot\vec{d}$ to d_m. We use the same device (vector $\vec{e}\,'$) for triplet final spin states.

Taking matrix elements of the operator in Eq. (9a), multiplying by $e_{m_s}^{*\prime}e_m^*$ and summing over m and m_s, we obtain for the Lth electric multipole

$$\langle \vec{k}m_s\,|\,\vec{\epsilon}\cdot\vec{J}_L(\vec{q})\,|\,dm\rangle\to\sum_{l,J}(-i)^l\langle lJk\,||\mathscr{E}_L||\,d\rangle\sqrt{4\pi}T_J^{lL} \tag{13a}$$

and

$$\langle \vec{k}0\,|\,\vec{\epsilon}\cdot\vec{J}_L(\vec{q})\,|\,dm\rangle\to\sum_{l}(-i)^l\langle llk\,||\mathscr{E}_L||\,d\rangle\sqrt{4\pi}S_l^{lL}, \tag{13b}$$

where the singlet and triplet functions are

$$S_l^{lL}=\sqrt{4\pi}\frac{(B_L\otimes e)_l\cdot Y_l(\hat{k})}{(2l+1)^{1/2}} \tag{13c}$$

and

$$T_J^{lL}=\sqrt{4\pi}\frac{(B_L\otimes e)_J\cdot[Y_L(\hat{k})\otimes e'^*]_J}{(2J+1)^{1/2}} . \tag{13d}$$

A corresponding result holds for the magnetic multipoles (9a), with $B_L\to A_L$. The utility of this scheme is that all the angular and spin information is contained in S and T; all the purely nuclear information is contained in the reduced matrix elements. The radial final-state wave functions are complex, containing phase (shift) factors of $e^{-i\delta}$, while $\mathscr{E}_L$ and $\mathscr{M}_L$ contain an explicit factor of i^L. Allowing a tensor force in the final state produces the same forms we have discussed above, with the radial wave functions in the reduced matrix elements appropriately modified.

The only problem associated with this scheme is that each new multipole or partial wave introduces a new spin-angular function. These functions are reducible; it is possible to express each one in terms of an irreducible set of twelve invariants.[33] For each initial state with a given parity, the photon has two possible polarizations and the deuteron three possible spin projections, each of which can produce either a triplet or singlet final state. Thus twelve invariants ($2\times3\times2$) are possible. Such a set are the three singlet and nine triplet invariants listed in Table I. We have chosen to work in transverse gauge, so that no factors of $\vec{\epsilon}\cdot\hat{q}$ ($\equiv0$) result. Moreover, a variety of isotropic Cartesian tensor identities exist which allow other forms to be reduced to the 12 in Table I. For example, $\vec{\epsilon}\cdot\hat{k}\times\hat{q}\,\vec{e}\cdot\hat{k}$ is identical to $I_2+I_3-\hat{k}\cdot\hat{q}I_1$ and $\vec{\epsilon}\cdot\hat{k}\times\hat{q}\,\vec{e}\cdot\hat{q}$ is equivalent to $\hat{k}\cdot\hat{q}I_2-I_1$, while double cross products, etc., can also be reduced. We note that under a time reversal transformation all spin-polarization factors and momenta are odd, so that both I_N and J_N are invariant if we treat $\vec{\epsilon}$, $\vec{e}$, and $\vec{e}\,'$ as real. Under a parity transformation, $\vec{\epsilon}$ and the momenta are odd while $\vec{e}$ and $\vec{e}\,'$ are even. Thus each invariant requires an odd number of momenta for invariance. The mapping of S or T to I_N or J_N can produce expansion coefficients which depend on $\hat{k}\cdot\hat{q}\equiv x$.

The type of scheme introduced above is particularly common in particle physics, but less so in nuclear physics.

TABLE I. Spin invariants for singlet (I_N) and triplet (J_N) transitions.

I_1: $i\vec{\epsilon}\times\vec{e}\cdot\hat{k}$	J_1: $\vec{\epsilon}\cdot\vec{e}\,\vec{e}\,'^*\cdot\hat{k}$
I_2: $i\vec{\epsilon}\times\vec{e}\cdot\hat{q}$	J_2: $\vec{\epsilon}\cdot\vec{e}\,\vec{e}\,'^*\cdot\hat{q}$
I_3: $i\vec{\epsilon}\cdot\hat{k}\vec{e}\cdot\hat{k}\times\hat{q}$	J_3: $\vec{\epsilon}\cdot\vec{e}\,'^*\vec{e}\cdot\hat{k}$
	J_4: $\vec{\epsilon}\cdot\vec{e}\,'^*\vec{e}\cdot\hat{q}$
	J_5: $\vec{\epsilon}\cdot\hat{k}\vec{e}\cdot\vec{e}\,'^*$
	J_6: $\vec{\epsilon}\cdot\hat{k}\vec{e}\cdot\hat{k}\vec{e}\,'^*\cdot\hat{k}$
	J_7: $\vec{\epsilon}\cdot\hat{k}\vec{e}\cdot\hat{q}\vec{e}\,'^*\cdot\hat{k}$
	J_8: $\vec{\epsilon}\cdot\hat{k}\vec{e}\cdot\hat{k}\vec{e}\,'^*\cdot\hat{q}$
	J_9: $\vec{\epsilon}\cdot\hat{k}\vec{e}\cdot\hat{q}\vec{e}\,'^*\cdot\hat{q}$

It has a number of attractive attributes that accrue from a separation of spins from internal variables:

(1) it is an elegant and efficient organization of the problem;
(2) it allows easy incorporation of symmetries;
(3) it permits easier discussions of unitarity;
(4) it allows easier counting of the number of independent experiments needed to determine all or a selected subset of the invariants.

We will see below that certain kinematic situations greatly reduce the number of nonvanishing invariants.

Photodisintegration cross sections are calculated in the usual way by averaging over photon and deuteron polarizations and summing over final state spins. Expanding the matrix elements[33] in the form

$$\langle \vec{k} \,|\, \vec{\epsilon}\cdot\vec{J} \,|\, d \rangle = \sqrt{4\pi}\left[\sum_{N=1}^{9} J_N a_N + \sum_{N=1}^{3} I_N b_N\right] , \tag{14}$$

the cross section in the c.m. frame becomes

$$\frac{d\sigma}{d\Omega} = \frac{\alpha E_k k}{6\omega_\gamma(1+\omega_\gamma/E_D)}\left[\sum_{NM}^{9} J_{MN} A_{MN} + \sum_{NM}^{3} I_{MN} B_{MN}\right] , \tag{15}$$

where k is the proton c.m. momentum in the final state, $E_k=(m^2+k^2)^{1/2}$, $E_D=(\omega_\gamma^2+m_D^2)^{1/2}$, while α, m, and m_D are the fine structure constant and the nucleon and deuteron masses. In addition

$$A_{NM}=\mathrm{Re}(a_N a_M^*) , \tag{16a}$$

$$B_{NM}=\mathrm{Re}(b_N b_M^*) , \tag{16b}$$

and

$$J_{MN}=\sum_{\substack{\text{spins}\\ \gamma}} (J_M J_N^*)=J_{NM} . \tag{16c}$$

The I_{MN} are defined analogously and both are listed in Table II. All relations follow from photon and spin sums:

$$\sum \epsilon^i \epsilon^{j*} = \delta^{ij} - \hat{q}^i \hat{q}^j ;$$

and

$$\sum e^i e^{j*} = \delta^{ij} ,$$

respectively. In Table II we identify $s^2=1-x^2$ for unpolarized photons. For linearly polarized photons we use $s^2=(1-x^2)(1+\sum_l \cos 2\phi)$, adopting Partovi's notation[2] and geometry. We note that the cross section for a given multipole L is a polynomial in x of maximum order $2L$.

The form of the tabulated functions is particularly simple for $\theta=0$ or π, where I_3 and J_5–J_9 vanish. Moreover, at these angles $I_1=xI_2$, $J_1=xJ_2$, and $J_3=xJ_4$. Thus there are only three independent invariants, which makes theoretical analysis particularly simple.

Finally, as we noted in Ref. 37, it is consistent with the relativistic corrections formalism we will introduce in Sec. IV to use relativistic kinematics in determining the relationships between various energies and momenta in the problem.

The relationships we need in the c.m. frame are

$$q \equiv \omega_\gamma = \frac{\omega_L}{(1+2\omega_L/M_D)^{1/2}} , \tag{17a}$$

$$k^2 = \frac{2\omega_\gamma^2 + 2\omega_\gamma E_d + m_D^2 - 4m^2}{4} , \tag{17b}$$

and

$$\omega_N = 2E_d - M_D , \tag{17c}$$

where $m_D=2m-\epsilon_d$, ω_L is the photon laboratory frame energy, and the deuteron binding energy, ϵ_d, is determined from the nuclear Hamiltonian in the usual fashion.

III. RETARDATION

In the previous section we set up the calculation of the radial matrix elements in terms of the operators $\mathscr{E}_{Lm}$ and $\mathscr{M}_{Lm}$. The latter is given by

TABLE II. Angular distribution coefficients from spin-summed invariants, $J_M J_N^*$ and $I_M I_N^*$. We have defined $x \equiv \hat{k}\cdot\hat{q}$, and $s^2=1-x^2$ holds for unpolarized photons.

	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9
J_1	2	$2x$	s^2	0	s^2	s^2	0	xs^2	0
J_2		2	0	0	0	xs^2	0	s^2	0
J_3			2	$2x$	s^2	s^2	xs^2	0	0
J_4				2	0	xs^2	s^2	0	0
J_5					$3s^2$	s^2	xs^2	xs^2	s^2
J_6						s^2	xs^2	xs^2	x^2s^2
J_7							s^2	x^2s^2	xs^2
J_8								s^2	xs^2
J_9									s^2

	I_1	I_2	I_3
I_1	$2-s^2$	$2x$	$-xs^2$
I_2		2	$-s^2$
I_3			$s^2(1-x^2)$

$$\mathscr{M}_{Lm}=\frac{2i^L(2L+1)\sqrt{4\pi}}{(L+1)}\int d^3x\left[\frac{j_L(qx)}{qx}\right]\times[Y_{L-1}(\hat{x})\otimes\vec{\mu}_1(\vec{x})]_{Lm}\ , \tag{18a}$$

where the magnetic density operator is obtained from the nuclear current, $\vec{J}$:

$$\vec{\mu}(\vec{x})=\tfrac{1}{2}\vec{x}\times\vec{J}(\vec{x})\ . \tag{18b}$$

Several forms exist for the electric multipoles. The crudest do not reduce to Siegert's theorem in the long wavelength limit and are completely unsatisfactory; large exchange current contributions combine with the "classical" parts of the current to produce Siegert's result. Therefore most calculations use a form which, for $L=1$, is given by

$$\vec{\mathscr{E}}'_1=i\left[H_0,\int d^3x\ \vec{x}\rho(\vec{x})\bar{g}_1(qx)\right]+\frac{q^2}{2}\int d^3x\ \vec{x}\cdot\vec{J}(\vec{x})\vec{x}\bar{h}_1(qx)\ , \tag{19a}$$

with

$$\bar{g}_1(z)=\frac{3}{2z}\left[j_1(z)+z\frac{d}{dz}j_1(z)\right]\cong 1-z^2/5+\cdots \tag{19b}$$

and

$$\bar{h}_1(z)=3j_1(z)/z\cong 1-z^2/10+\cdots\ , \tag{19c}$$

which clearly has the correct long wavelength behavior. The obvious question is whether the q^2 term in Eq. (19a) (i.e., the second term) has a form which properly incorporates the necessary exchange currents to ensure a conserved current. The answer to this question is no;[10] moreover, the $\vec{x}\cdot\vec{J}$ form generates $r(\partial/\partial r)$ radial derivatives, which are unpleasant in numerical calculations. The form of the electric multipole operator which does possess the optimal properties[10] is

$$\mathscr{E}_{Lm}=\frac{q^{L-1}i^L\sqrt{4\pi}}{(2L+1)!!}\left[\frac{(2L+1)}{L}\right]^{1/2}\left[H_0,\int d^3x\ x^LY_{Lm}g_L(qx)\rho(\vec{x})\right]-\frac{2q^2}{(L+2)}\left[\frac{L}{L+1}\right]^{1/2}\int d^3x\ x^L[Y_L\otimes\vec{\mu}_1(\vec{x})]_Lh_L(qx)\ , \tag{20}$$

which produces

$$\vec{\mathscr{E}}_1=i\left[H_0,\int d^3x\ \vec{x}g_1(qx)\rho(\vec{x})\right]+\frac{q^2}{3}\int d^3x\ h_1(qx)[\vec{x}\times\vec{\mu}(\vec{x})]\ , \tag{21a}$$

with

$$g_1(z)=\frac{3\,\mathrm{Si}(z)}{2z}-\frac{3j_1(z)}{2z}\simeq 1-\frac{z^2}{30}+\cdots\ , \tag{21b}$$

$$h_1(z)=-\frac{9j_0(z)}{z^2}+\frac{9\,\mathrm{Si}(z)}{z^3}\simeq 1-\frac{3z^2}{50}+\cdots\ , \tag{21c}$$

expressed in terms of Bessel and sine integral functions. The forms of g_L and h_L for $L\neq 1$ can be found in Ref. 10. For small q^2 both (19) and (21) can be written in the form

$$\vec{\mathscr{E}}_1\simeq i\left[H_0,\int d^3x\ \rho(\vec{x})(\vec{x}+\vec{O})\right]+\frac{q^2}{6}\vec{N}\ . \tag{22}$$

The quantity $\vec{O}$ which determines retardation of the Siegert term is six times smaller in the preferred form (21) than in the standard form (19). Thus ordinary retardation effects will be much smaller. Similarly, we expect the $\vec{N}$ corresponding to the convection current to be smaller, also. Only the contribution of purely solenoidal currents such as the spin magnetization current ($\vec{N}_s$) and isobar current ($\vec{N}_\Delta$) is identical in the two representations. Only if the nuclear current model is conserved will Eqs. (19) and (21) give identical results. We find for the deuteron in the preferred form (21):

$$\vec{N}_s=-\frac{3}{2m}\sum_i\mu_i\vec{\sigma}_i\times\vec{r}'_i\to\frac{3}{16m}\vec{r}\times(\mu_v\Delta\tau\vec{\sigma}+2\mu_s\Delta\vec{\sigma})\ , \tag{23a}$$

$$\vec{N}_o=\frac{1}{2m}\sum_i e_i\{\vec{r}'_i,\times\vec{L}_i\}\to\frac{\Delta\tau}{16m}\{\vec{r},\times\vec{L}\}\ , \tag{23b}$$

$$\vec{N}_{SG}=\frac{f_0^2}{6\mu^2}t_{12}[r^2h'_0(\hat{r}S_{12}-\vec{T}_{12})]\ , \tag{23c}$$

$$\vec{N}_{ex}=\frac{f_0^2}{3\mu^2}t_{12}[\vec{T}_{12}h'_1+\hat{r}S_{12}(rh''_1-h'_1)+\hat{r}\vec{\sigma}_1\cdot\vec{\sigma}_2(rh''_1+6h'_1)]\ , \tag{23d}$$

$$\vec{N}_\Delta=-\frac{\xi f_0^2}{\mu^4}[t_{12}(rh''_0-h'_0)(-\vec{T}_{12}+\hat{r}S_{12})+3h'_0\Delta\tau(\vec{\sigma}\times\hat{r})]\ , \tag{23e}$$

where

$$f_0^2=0.079\ ,\quad \vec{\sigma}=\vec{\sigma}_1+\vec{\sigma}_2\ ,\quad \Delta\vec{\sigma}=\vec{\sigma}_1-\vec{\sigma}_2\ ,$$

$$\vec{L}=\vec{r}\times\vec{p}\ ,\quad t_{12}=(\vec{\tau}_1\times\vec{\tau}_2)_3\ ,\quad \Delta\tau=\tau_1^3-\tau_2^3\ ,$$

$$\vec{T}_{12}=3(\vec{\sigma}_1\vec{\sigma}_2\cdot\hat{r}+\vec{\sigma}_2\vec{\sigma}_1\cdot\hat{r}-2\vec{\sigma}_1\cdot\vec{\sigma}_2\hat{r}/3)/2\ ,$$

$$\xi=4\mu_v\mu^2/(25\Delta m_\Delta m)\ ,$$

and

$$h_N(r)=4\pi\int\frac{d^3qF_{\pi NN}^2(\vec{q}^{\,2})e^{i\vec{q}\cdot\vec{r}}}{(2\pi)^3(\vec{q}^{\,2}+\mu^2)^{1+N}}\ . \tag{24}$$

The pion exchange current contribution, $\vec{N}_\pi$, has been broken down into the seagull or pair term, $\vec{N}_{SG}$; the true exchange part, $\vec{N}_{ex}$; and the isobar contribution, $\vec{N}_\Delta$. In these expressions Δm_Δ is the mass difference of the Δ and the nucleon, while μ is the pion mass. The pion-nucleon form factor $F_{\pi NN}(\vec{q}^{\,2})$ is taken to be a monopole form[38] with mass $\Lambda=\beta\mu$: $(\Lambda^2-\mu^2)/(\vec{q}^{\,2}+\Lambda^2)$. From our experience with the pion-exchange currents we expect $\vec{N}_\pi$ to be roughly 10–20 % of the spin contribution, $\vec{N}_s$, and therefore quite unimportant for the deuteron problem. In Sec. VI this expectation is confirmed.

IV. RELATIVISTIC CORRECTIONS

There are three primary categories of relativistic corrections in nuclei:

(1) single-particle kinematic corrections, usually obtained from a Foldy-Wouthuysen[39] reduction;
(2) kinematic recoil corrections, which depend on m_t, the total nuclear mass;
(3) potential-dependent or interaction corrections from meson currents, quark substructure, nucleon excited states, etc.

These categories are not arbitrary. They must mesh together so that the constraints of "boosting" a system from one reference frame to another are maintained[23,40–42] and so that overall current conservation is maintained. Although these constraints are powerful, they obviously do not determine the electromagnetic interaction completely. We will deal with categories (1) and (2) in this section, but relegate category (3) to Sec. V.

A Foldy-Wouthuysen transformation, or its equivalent, applied to the Dirac equation for a single nucleon (or electron) produces relativistic corrections to the electromagnetic Hamiltonian of the form[9,39]

$$\Delta H_{\rm em}=-\frac{(2\mu-e)}{8m^2}\vec{\nabla}\cdot\vec{E}+\frac{(2\mu-e)}{8m^2}\vec{\sigma}\cdot\{\vec{p}\times\vec{E}\}+O(1/m^3)+\cdots \tag{25}$$

where μ, e, m, $\vec{p}$, and $\vec{\sigma}$ are the particle's magnetic moment, charge, mass, momentum, and (Pauli) spin, while $\vec{E}$ is the external electric field. The first term is the Darwin-Foldy term,[39] while the second is the spin-orbit interaction. The third class of $(1/m^3)$ terms are not explicitly needed for our purposes here.

The Darwin-Foldy term is best known for its contribution to the nucleon charge form factor.[39] In any Lorentz gauge we have

$$H_0+\Delta H_{\rm DF}\sim e\phi\left[F_1-\frac{(2F_2+F_1)q^2}{8m^2}\right]\cong e\phi G_E(1-q^2/8m^2)\ , \tag{26a}$$

where

$$G_E=F_1-\kappa q^2F_2/4m^2\ , \tag{26b}$$

and $q^2=\vec{q}^{\,2}-q_0^2$. In Eq. (26) we have substituted $e\to eF_1$ and $\kappa\to\kappa F_2$ with $\mu=e(1+\kappa)$. The combination of Dirac (F_1) and Pauli (F_2) form factors in Eq. (26b) is the (Sachs) electric form factor. Note that for *photons*, $q^2=0$, and the Darwin-Foldy term vanishes identically. Physically, this arises because the $\vec{\nabla}$ in $\vec{\nabla}\cdot\vec{E}$ lies along the Poynting vector while the electric field is transverse to that vector. This result contradicts Ref. 11, which is in error.

The spin-orbit term generates both a current and a closely related charge density.[12] The latter is

$$\rho_{\rm so}(\vec{x})=-\sum_i\left[\frac{2\mu_i-e_i}{8m^2}\right]\{\vec{\sigma}_i\times\vec{p}_i\ ,\cdot\vec{\nabla}_i\delta^3(\vec{x}-\vec{x}_i)\}\ , \tag{27a}$$

with a corresponding dipole operator

$$\Delta\vec{D}_{\rm so}=\int d^3x\,\vec{x}\rho_{\rm so}(\vec{x})=-\sum_i\left[\frac{2\mu_i-e_i}{4m^2}\right]\vec{\sigma}_i\times(\vec{\pi}_i+m\vec{P}/m_t)\ , \tag{27b}$$

where $\vec{p}_i$ is the momentum of the ith nucleon, and we have written $\vec{p}_i=\vec{\pi}_i+m_i\vec{P}/m_t$ in terms of the total momentum, $\vec{P}$, and mass, m_t. The $\vec{P}$ term causes problems; it gives a "dipole" operator whose internal variables have the wrong parity.

In order to cure this problem, we must investigate the dependence of the current and charge operators on the overall motion of the nucleus; that is, on factors of $\vec{P}$. This dependence is clearly determined by special relativity, since it will determine the frame dependence of matrix elements. Moreover, in addition to finding such factors in operators we must also look at the wave functions. Galilean invariance states succinctly that the wave function of a moving system is simply the product of the internal, center-of-mass (c.m.) wave function, $\phi_{\rm c.m.}$, and a plane wave, $\exp(i\vec{P}\cdot\vec{R})$, determining the motion of the center-of-mass in terms of $\vec{P}$ and $\vec{R}$, the *usual* center-of-mass coordinate: $\sum_i m_i\vec{r}_i/m_t$. This structure cannot be correct when $(v/c)^2$ corrections are allowed, because such phenomena as Lorentz contraction and the Thomas precession alter the c.m. wave function. The complete wave function which is a solution of the Foldy-Wouthuysen Hamiltonian, $H_{\rm FW}$ [including $(v/c)^2$ corrections], has the form

$$\psi_{\vec{P}}=[1-i\chi(\vec{P})]\phi_{\rm c.m.}e^{i\vec{P}\cdot\vec{R}}\ , \tag{28}$$

where $\chi(0)=0$ and $\chi^\dagger=\chi$. Note that this has the form of a unitary transformation applied to a nonrelativistic wave function. The matrix elements of the Foldy-Wouthuysen four-current $J^\mu_{\rm FW}$, $\langle f\vec{P}_f\,|\,J^\mu_{\rm FW}(\vec{S},\vec{q})\,|\,i\vec{P}_i\rangle$, determine the transition. Because it is easier to deal with matrix elements involving only c.m. wave functions and because the

$\vec{P}$ factors we wish to discuss are located both in J^μ and in ψ, we undo the unitary transformation of order $(v/c)^2$:

$$\langle f\vec{P}_f \,|\, J^\mu_{\rm FW} \,|\, \vec{P}_i\rangle \equiv \langle f \,|\, J^\mu(\vec{S},\vec{q}) \,|\, i\rangle \ , \tag{29a}$$

where

$$J^\mu(\vec{S},\vec{q}) \cong J^\mu_{\rm FW}(\vec{S},\vec{q}) + i[\chi(\vec{P}_f)J^\mu_{\rm FW} - J^\mu_{\rm FW}\chi(\vec{P}_i)] \ . \tag{29b}$$

In addition, the transformed nuclear Hamiltonian has the form

$$H \cong H_{\rm FW} + i[\chi, H_{\rm FW}] \cong (H^2_{\rm c.m.} + \vec{P}^2)^{1/2}$$
$$\cong m_t + h_0 + \frac{\vec{P}^2}{2m_t} - \frac{\vec{P}^4}{8m_t^3} - \frac{h_0\vec{P}^2}{2m_t^2} \ , \tag{29c}$$

where $H_{\rm c.m.}$ includes the rest masses of the nucleons, and is given by $H_{\rm FW}(\vec{P}=0) \equiv m_t + h_0$.

An exceptionally tedious calculation[37] using explicit χ's produces the following results:

$$\rho(\vec{S},\vec{q}) \cong \rho_0(\vec{q}) + \Delta\rho(\vec{q}) + \frac{i}{8m_t^2}\{\vec{S}\times\vec{q}\cdot\hat{J}, \rho_0(\vec{q})\}$$
$$+ \hat{L}\rho_0(\vec{q}) + \left[\frac{\vec{S}\cdot\vec{J}_0(\vec{q})}{2m_t}\right] \ , \tag{30a}$$

where

$$\hat{L}\rho_0 = -\left[\frac{1}{4m_t}(\omega_R + 2\omega_{fi})\vec{S}\cdot\vec{\nabla}_q + \frac{\vec{q}^{\,4}}{4m_t^2}\frac{\partial}{\partial\vec{q}^{\,2}}\right]\rho_0(\vec{q}) \ . \tag{30b}$$

We have written $\omega_{fi} = \epsilon_f - \epsilon_i$ for the eigenvalue differences of h_0; the recoil energy is $\omega_R = \vec{S}\cdot\vec{q}/2m_t$, $\hat{J}$ is the internal (c.m.) angular momentum operator, and $\Delta\rho(\vec{q})$ is the $\vec{S}$-independent part of the $(v/c)^2$ correction to ρ. In addition, we find

$$\vec{J}(\vec{S},\vec{q}) = \vec{J}_0(\vec{q})[1 - (\vec{P}_f^2 + \vec{P}_i^2)/4m_t^2] + \bar{\rho}\vec{V} + \frac{\vec{S}\vec{S}\cdot\vec{J}_0}{8m_t^2}$$
$$+ i\frac{\{\vec{S}\times\vec{q}\cdot\hat{J}, \vec{J}_0\}}{8m_t^2} + \Delta\vec{J}(\vec{q}) + \hat{L}\vec{J}_0(\vec{q}) \ , \tag{30c}$$

where $\vec{J}_0$ is the internal (c.m.) part of the *nonrelativistic* current operator, $\Delta\vec{J}(\vec{q})$ is the $\vec{S}$-independent part of the $(v/c)^2$ correction to $\vec{J}_0$, $\bar{\rho}$ is all but the bracketed (last) term in Eq. (30a), and $\vec{V}$ is given by

$$\vec{V} = \frac{\vec{S}}{2m_t}\left[\left[1 - \frac{\vec{P}_f^2 + \vec{P}_i^2}{4m_t^2}\right] - \left[\frac{\epsilon_f + \epsilon_i}{2m_t}\right]\right] . \tag{30d}$$

Equations (30) are replete with physical content, and subsume all the mechanisms to be discussed subsequently. The terms involving $\hat{J}$ generate the Wigner rotation, a classical phenomenon, while the $\hat{L}$ term manifests the Lorentz contraction and renders the arguments of form factors Lorentz invariant. In q-congruent frames, where $\vec{S}$ and $\vec{q}$ are collinear, we can write for both $\hat{L}\rho_0(\vec{q})$ and $\hat{L}\vec{J}_0(\vec{q})$:

$$\hat{L}F(\vec{q}) \cong F(\bar{q}\hat{q}) \ , \tag{30e}$$

where

$$\bar{q}^{\,2} = \vec{q}^{\,2} - q_0^2 + \omega_{fi}^2 - \vec{q}^{\,4}/4m_t^2 \tag{30f}$$

and

$$q_0 = \omega_{fi} + \omega_R \ . \tag{30g}$$

The combination $\vec{q}^{\,2} - q_0^2$ defines q^2, the squared four-momentum transfer, and the $\vec{q}^{\,4}$ term "Lorentz contracts" the form factor. Note also that the photon case corresponds to $\bar{q}^{\,2} \cong \omega_{fi}^2$ ($\cong \vec{q}^{\,2}$ if we neglect recoil) since $\vec{q}^{\,4}/m_t^2 \sim \omega_{fi}^4/m_t^2$. The latter correction should be quite negligible for our electric dipole transitions since it is a relativistic correction to a retardation correction, and $\omega_{fi} \sim 1/m$, implying an overall $1/m^4$ correction to ω_{fi}^2. Note that to order $(v/c)^2$ the argument $\bar{q}^{\,2}$ is a Lorentz invariant combination of two other Lorentz invariants, q^2 and ω_{fi}.

Additional terms which manifest the effects of a Lorentz transformation are the bracketed term in (30a) and the first three terms in (30c), all determined by the average nuclear velocity, $\vec{V}$. The second term in Eq. (30c) is the generalization of the usual convection current; $\rho\vec{V}$. Each of the indicated terms has a classical analog. We also note that our preferred (c.m.) reference frame is q congruent, with $\vec{P}_f = 0$, $\vec{P}_i = -\vec{q}$, and $\vec{S} = -\vec{q}$, so that in transverse gauge only the first and last two of the six terms in (30c) are nonvanishing. Moreover,

$$(\vec{P}_f^2 + \vec{P}_i^2)/4m_t^2 = \vec{q}^{\,2}/4m_t^2 = \vec{q}^{\,2}/16m^2$$

and is completely negligible for 100 MeV photons. For electric dipole transitions, only $\Delta\vec{J}(\vec{q})$ is important and we deal with it now.

Current conservation states

$$\vec{q}\cdot\vec{J}(\vec{S},\vec{q}) = H(\vec{P}_f^2)\rho(\vec{S},\vec{q}) - \rho(\vec{S},\vec{q})H(\vec{P}_i^2) \ . \tag{31}$$

Taking one derivative with respect to $\vec{q}$ for fixed $\vec{S}$ and setting $\vec{q}$ to zero produces[10]

$$\vec{J}(\vec{S},0) = i[H(\vec{S}/2), \vec{D}_0 + \Delta\vec{D}] + i[h_0, \Delta\vec{D}(\vec{S})]$$
$$+ \frac{Z\vec{S}}{2m_t}\left[1 - \frac{\vec{S}^2}{8m_t^2} - \frac{h_0}{m_t}\right] + \frac{\vec{S}}{2m_t}\Delta\rho(\vec{S},0) \ , \tag{32}$$

where we have written

$$\rho(\vec{S},\vec{q}) = \rho_0(\vec{q}) + \Delta\rho(\vec{q}) + \Delta\rho(\vec{S},\vec{q}) \ ,$$
$$\vec{D}_0 + \Delta\vec{D} = -i\vec{\nabla}_q[\rho(0) + \Delta\rho(0)] \ ,$$

and

$$\Delta\vec{D}(\vec{S}) = -i\vec{\nabla}_q \Delta\rho(\vec{S},0) .$$

Equation (32) can be rewritten in the form

$$\vec{J}(\vec{S},0) \cong i[h_0, \vec{D}_0 + \Delta\vec{D}] + \frac{\vec{S}Z}{2m_t}\left[1 - \frac{\vec{S}^2}{8m_t^2} - \frac{h_0}{m_t}\right] + \frac{\vec{S}}{2m_t}\Delta\rho(\vec{S},0) - i\frac{\vec{S}^2}{8m_t^2}[h_0,\vec{D}_0] + i[h_0, \Delta\vec{D}(\vec{S})] . \tag{33}$$

One of the basic tenets of special relativity is that the *total* charge of a system must be frame independent. This follows in our formalism from $\Delta\rho(0) \equiv 0$ and $\rho_0(0) = Z$ and the cancellation of the bracketed term in (30a) with the ω_{fi} term in $\hat{L}\rho_0$. Thus $\rho(\vec{S},0) \equiv 0$. Implementation of this condition in other formalisms requires a proper normalization. We can also calculate $\Delta D(\vec{S})$ from Eq. (30), which leads to

$$\Delta\vec{D}(\vec{S}) = -\frac{\vec{S}\vec{D}_0\cdot\vec{S}}{8m_t^2} - \frac{Z}{4m_t^2}\vec{S}\times\hat{J} - \frac{i}{4m_t}[h_0, \vec{S}\cdot\overleftrightarrow{Q}] + \frac{\vec{S}\times\vec{\mu}}{4m_t^2} , \tag{34}$$

where we have expanded

$$\rho_0(\vec{q}) \cong i\vec{q}\cdot\vec{D} - \vec{q}\cdot\overleftrightarrow{Q}\cdot\vec{q}/2 + \cdots$$

in terms of the usual dipole and quadrupole operators. This can be compared to an expansion of Eq. (30c) using the appropriate low energy theorem for

$$\vec{J}_0(\vec{q}) \cong i[h_0, \vec{D}_0] - i\vec{q}\times\vec{\mu}/2m_t - [h_0, \vec{q}\cdot\overleftrightarrow{Q}]/2 + \cdots ,$$

and is found to be consistent. If we ignore terms in $\vec{J}$ directed along $\vec{S}$ we find

$$\vec{J}(\vec{S},0) \cong i[h_0, \vec{D}](1 - \vec{S}^2/8m_t^2) + i\left[h_0, \frac{\vec{S}\times\vec{\mu}}{4m_t^2}\right] + \left[h_0, \left[h_0, \frac{\vec{S}\cdot\overleftrightarrow{Q}}{4m_t}\right]\right] . \tag{35}$$

We note that the parity of the last two operators is positive, and that they arise from the effective argument transformation $\vec{q}^{\,2} \to \bar{q}^{\,2}$. Compared to the corresponding terms in $\vec{J}_0(\vec{q})$ they differ by a factor of $\omega_{fi}/2m_t \sim 2\frac{1}{2}\%$ enhancement in the c.m. frame for $\omega_\gamma = 100$ MeV. Since retarded (non-$E1$) terms are approximately 40% of the result for 100 MeV, these corrections are approximately 1% and are therefore quite unimportant.

The remaining task is to evaluate $\Delta\vec{D}$. Writing

$$\chi(\vec{P}) \cong \vec{\chi}\cdot\vec{P} + \text{order } (\vec{P}^2) , \tag{36}$$

we find from Eq. (29b)

$$\rho(\vec{q}) \equiv \rho_{FW}(\vec{q}) + i\frac{\vec{q}}{2}\cdot[\vec{\chi}\rho_0(\vec{q}) + \rho_0(\vec{q})\vec{\chi}]/2 + \text{order } (\vec{q}^{\,2}) , \tag{37}$$

and thus

$$\vec{D} = \vec{D}_{FW} + Z\vec{\chi} . \tag{38}$$

To proceed further requires a knowledge of $\vec{\chi}$. This is obtained for the deuteron from Ref. 16 or the original Ref. 42:

$$\chi = (\vec{\sigma}_1 - \vec{\sigma}_2)\times\vec{p}\cdot\vec{P}/4mm_t - \{\vec{r}\cdot\vec{P}, \vec{p}\cdot\vec{P}\}/4m_t^2 + \chi_\pi , \tag{39a}$$

$$\vec{\chi} = (\vec{\sigma}_1 - \vec{\sigma}_2)\times\vec{p}/4mm_t + \vec{\chi}_\pi , \tag{39b}$$

where $\vec{\chi}_\pi$ is the contribution from the potential-dependent (meson-exchange) parts of χ:χ_π. The complicated center-of-energy terms[42] vanish for two equal mass particles. Putting everything together for the deuteron produces[13]

$$\vec{D} = \left[\frac{\Delta\tau}{2}\right]\vec{r}/2 + \frac{(1-\mu_s)}{4m^2}\Delta\vec{\sigma}\times\vec{p} - \left[\frac{\Delta\tau}{2}\right]\left[\frac{2\mu_v - 1}{8m^2}\right]\vec{\sigma}\times\vec{p} + \Delta\vec{D}_\pi , \tag{40}$$

where $\Delta\vec{D}_\pi$ is the combined dipole moment from mesons. The spin-dependent $\vec{\chi}$ term we found above makes a negligible contribution to $\vec{D}$, since it induces ${}^3S_1$-${}^3D_1 \to {}^1P_1$ transitions. The latter three contributions to $\vec{D}$ arise from $\Delta\vec{J}$ in (30c).

Finally, we remark on the possibility of including the finite size of the nucleon in these calculations. Naive arguments hold that since the nucleon form factors depend on q^2, which vanishes for photons, there is no effect. This argument, if not the conclusion, is spurious, since the nucleons in a nucleus are off shell and the off-shell form factor depends on more than the one parameter, q^2. Alternatively, there will be binding (off-shell) corrections of order $(v/c)^2$ to the nucleon form factor in a nucleus. Indeed, the entire photonuclear process below meson production threshold vanishes without binding. If one were to hold that the latter process depended only on $q^2=0$, ludicrous inconsistencies would result. Our effective momentum transfer variable depends on two such variables. Several years ago one of us argued[23] that the intrinsic part of the charge distribution required careful consideration when incorporating it into the nuclear current in order to preserve Lorentz invariance to order $(v/c)^2$. The argument was made that such parts of the nucleon charge distribution should be functions of $\bar{q}^{\,2}$. That argument was incomplete, since the requisite parts of the nucleon form factors were not, and indeed cannot, be uniquely identified. As an example, we note that the Darwin-Foldy term does not fall in this category; it vanishes for photons. Components of the form factor which arise from direct, virtual hadronic components of the photon (e.g., the vector dominance model) behave similarly. Clearly, other components will behave differently. Nevertheless, in Sec. VI we will investigate numerically the effect of the inclusion of a nucleon form factor.

V. PIONIC CONTRIBUTIONS TO THE DIPOLE OPERATOR

The one-pion-exchange contribution to the nuclear charge density, ρ_π, is extensively discussed elsewhere[16,22] and we will suppress unnecessary details here. The salient points are listed below. A variety of physical processes contribute to $\rho_\pi(\vec{q})$, which is of order $(v/c)^2$. This operator is nonlocal, spin dependent, and model dependent. More seriously, it depends on the method *chosen* to perform the calculation. Different techniques lead to different members of two unitarily equivalent families of operators. In order to calculate unambiguous matrix elements, corresponding Hamiltonians and wave functions must be used with the transition operators. This trivial consistency condition is not simple to implement, since *no* representation corresponds to common (so-called) realistic potential models, and adding terms of relativistic order to the potential requires adjusting the remainder of the potential to keep the deuteron properties and the two-body phase shifts the same. One easy lesson can be learned from this problem: If the short-range unitary transformation U is determined by a parameter μ, then all members of the family

$$\psi' \cong (1-iU)\psi$$

have the same binding energy, asymptotic normalization parameters, phase shifts, etc., independent of μ, because U vanishes for sufficiently large r. The extent to which the various $\Delta\vec{D}_\pi$'s obtained from ρ_π change the cross section for a given wave function is a measure of the extent to which the interior parts of the wave function influence the result, since the two must exactly cancel in a consistent calculation. The nonlocality of ρ_π is reflected in nonlocality of the potential. Studies have repeatedly shown[43] in the past that the use of local potentials constrains observables so that one "tracks" another. Folklore arising from studies utilizing nonlocal separable and one-boson-exchange potentials suggests that nonlocality leads to decoupling of observables. In particular, the deuteron quadrupole moment (exterior sensitive) and the percentage D state (interior sensitive) are less coupled for nonlocal potentials. In numerical calculations associated with the proof[44] that the latter quantity is not an observable (i.e., not measurable), the changes in ψ associated with U produced much greater changes in P_D than in Q.

There are five distinct contributions to $\Delta\vec{D}_\pi$, all obtained in a fairly simple fashion from Ref. 16. From that reference we require Eq. (73a), the seagull term $\rho_\pi^{SG}(\vec{q})$; Eq. (75a), the true exchange term $\rho_\pi^{ex}(\vec{q})$; Eqs. (79)—(81), $\tilde{\rho}_\pi^{\,v}$ and ρ_π^v; Eq. (82), the recoil term $\rho_\pi^{v'}$; and Eq. (88), the motion term $\chi_\pi(\vec{P})$. The only minor change which is needed for completeness is to rewrite $\tilde{\rho}_\pi^{\,v}$ and ρ_π^v to reflect the unitary transformation in Eq. (80) that specifies the retardation representation.[22] We chose to transform to the "soft" representation in Ref. 16 after performing the original calculations in the "standard" representation. We write $\tilde{\rho}_\pi^{\,v}=(1-\nu)\rho_\pi^R+\rho_\pi^v$, where $\nu=0$ corresponds to Eq. (79) of Ref. 16 and the standard case. While $\nu=1$ corresponds to Eq. (81) of that reference and the soft representation in Ref. 22. With this slight change the results of Ref. 16 can be used immediately,

$$\Delta\vec{D}_\pi=\tilde{\vec{D}}_\pi^{\,v}+\vec{D}_\pi^{\,v'}+\vec{D}_\pi^{\,ex}+\vec{D}_\pi^{\,SG}+\vec{D}_\pi^{\,\chi}\,, \tag{41a}$$

$$\tilde{\vec{D}}_\pi^{\,v}=\frac{f_0^2}{8m}\sum_{i\neq j}(\vec{\tau}_i\times\vec{\tau}_j)_3\{(\nu\vec{x}_{ij}+\vec{x}_i'+\vec{x}_j')(\vec{\sigma}_i\cdot\vec{\nabla}_{ij}\vec{\sigma}_j\cdot\vec{\nabla}_{ij})[\vec{x}_{ij}h_0(x_{ij})];\vec{\pi}_j\}$$
$$+(1-\nu)\frac{f_0^2}{16m}\sum_{i\neq j}(\tau_i^3-\tau_j^3)(\vec{\sigma}_i\cdot\vec{\nabla}_{ij}\vec{\sigma}_j\cdot\vec{\nabla}_{ij})[\vec{x}_{ij}h_0(x_{ij})]\,, \tag{41b}$$

$$\vec{D}_\pi^{\,v'}=\frac{f_0^2}{8m}(\mu+1)\sum_{i\neq j}(\vec{\tau}_i\cdot\vec{\tau}_j+\tau_j^3)\vec{\sigma}_i\vec{\sigma}_j\cdot\vec{\nabla}_{ij}h_0(x_{ij})-(\vec{\tau}_i\times\vec{\tau}_j)_3\{\vec{\sigma}_i\cdot\vec{\pi}_i,(\vec{x}_i'+\vec{x}_j')\vec{\sigma}_j\cdot\vec{\nabla}_{ij}h_0(x_{ij})\}\,, \tag{41c}$$

$$\vec{D}_\pi^{\,ex}=\frac{f_0^2}{8m}\sum_{i\neq j}(\vec{\tau}_i\times\vec{\tau}_j)_3[\{(\vec{\pi}_i-\vec{\pi}_j),\vec{\sigma}_i\cdot\vec{\nabla}_{ij}\vec{\sigma}_j\cdot\vec{\nabla}_{ij}h_1(x_{ij})\}-\vec{\sigma}_i\cdot\vec{\nabla}_i\vec{\sigma}_j\cdot\vec{\nabla}_j\{(\vec{\pi}_i+\vec{\pi}_j)^\alpha,\nabla_{ij}^\alpha h_1(x_{ij})(\vec{x}_i'+\vec{x}_j')\}]\,, \tag{41d}$$

$$\vec{D}_\pi^{\,SG}=\frac{f_0^2}{4m}\sum_{i\neq j}(\vec{\tau}_i\times\vec{\tau}_j)_3(\mu+1)\{\vec{\sigma}_i\cdot\vec{\pi}_i,\vec{x}_i'\vec{\sigma}_j\cdot\vec{\nabla}_{ij}h_0(x_{ij})\}+\vec{\sigma}_i\vec{\sigma}_j\cdot\vec{\nabla}_{ij}h_0(x_{ij})[\vec{\tau}_i\cdot\vec{\tau}_j(2\mu_s-1-\mu)+\tau_3(j)(2\mu_v-1-\mu)]\,, \tag{41e}$$

$$\vec{D}_\pi^{\,\chi}=\frac{f_0^2(\mu-1)}{8m}\left[\frac{2Z}{A}\right]\sum_{i\neq j}(\vec{\tau}_i\cdot\vec{\tau}_j)\vec{\sigma}_i\vec{\sigma}_j\cdot\vec{\nabla}_{ij}h_0(x_{ij})\,. \tag{41f}$$

Specializing to the deuteron we find

$$\Delta\vec{\mathrm{D}}_\pi = -\frac{f_0^2}{4m}(\vec{\tau}_1\cdot\vec{\tau}_2)(2\mu_s-1)(\vec{\sigma}_1\times\vec{\sigma}_2)\times\hat{r}h_0'(r) - \frac{f_0^2}{16m}\Delta\tau(4\mu_v-3-\mu+2\nu)h_0'(r)\overleftrightarrow{\mathrm{S}}_{12}\cdot\hat{r}$$
$$+\frac{f_0^2}{8m}\Delta\tau(1-\nu)\hat{r}[\vec{\sigma}_1\cdot\vec{\sigma}_2 h_0' + \vec{\sigma}_1\cdot\hat{r}\vec{\sigma}_2\cdot\hat{r}(rh_0''-h_0')]$$
$$+\frac{f_0^2}{8m}(\vec{\tau}_1\times\vec{\tau}_2)_3((\mu+1-2\nu)S_{12}^{\alpha\beta}\{p^\alpha, r^\beta\hat{r}h_0'\} - 2\nu\{\vec{\mathrm{p}};\vec{\mathrm{r}}[\vec{\sigma}_1\cdot\vec{\sigma}_2 h_0' + \vec{\sigma}_1\cdot\hat{r}\vec{\sigma}_2\cdot\hat{r}(rh_0''-h_0')]\hat{r}\}$$
$$+4\{\vec{\mathrm{p}},\vec{\sigma}_1\cdot\vec{\sigma}_2[h_1'(r)/r] + \vec{\sigma}_1\cdot\hat{r}\vec{\sigma}_2\cdot\hat{r}(h_1''-h_1'/r)\}) \,, \tag{42}$$

where $S_{12}^{\alpha\beta} = \sigma_1^\alpha\sigma_2^\beta + \sigma_1^\beta\sigma_2^\alpha$.

This expression is quite complicated; four of the six terms depend on μ and ν. Noteworthy are the two terms which do not. The last term arises from the true-exchange process. The remaining processes are mixed by the unitary transformations. The first term in (42) is the sum of three μ-dependent terms and is independent of μ. This result is due to the vanishing of the nonrelativistic isoscalar dipole operator; the exchange contribution will always be unambiguous in such cases.[45] Note that the motion term $\vec{\chi}_\pi$ was needed to produce this result. Although the isoscalar dipole operator is unique, the same is not true of the isoscalar charge density; the exchange part vanishes with $\nu=1$ and $\mu=4\mu_s-1$. Also worth remarking is the momentum dependence of three of the six terms.

The Yukawa functions we have detailed contain pion-nucleon form factors. There is no problem if one uses Eq. (89) and follows the discussion on pp. 419 and 420 of Ref. 16. Failure to do so[46] results in operators which fail to satisfy the constraints of Lorentz invariance. We note that these constraints have been verified in Refs. 16, 22, and 47. The objections of Ref. 11 to the inclusion of the form factors are not relevant.

Finally, we note the dependence on the nucleon magnetic moments of the first two terms in Eq. (42). This dependence is characteristic of pseudoscalar (PS) pion-nucleon coupling. Had we chosen pseudovector (PV) coupling we would have arrived at the same result[16] with $\mu_s\to 1$ and $\mu_v\to 1$. In fact, the use of pure uncorrected PS pion photoproduction Born terms is not warranted. Several years ago two of us pointed out (a well-known result) that *neutral* threshold pion photoproduction is very different for PS and PV Born term models and only the latter agrees with experiment.[21] The numerical results are (PS: 1.3 μb/sr), (PV: 0.13 μb/sr), and (experiment: 0.10 ± 0.02 μb/sr). Although many calculations of the elementary amplitudes start with PS-Born terms, current algebra corrections are made which effectively convert to PV coupling.[48] We note that dropping the μ_v ($=4.7\ \mu_N$) factor dramatically reduces the local approximation to $\Delta\vec{\mathrm{D}}_\pi$.

VI. RESULTS AND CORRECTIONS

In order to check our amplitude decomposition formulae, we have made three separate tests. We performed a separate plane wave calculation of the amplitudes and verified our partial wave results. We compared our unretarded partial wave formulae at 0° with the analytic results of Ref. 34 for $E1$, $M1$, and $E2$ amplitudes. We also compared our numerical results with those of Partovi.[2] Our deviations from Partovi were very small (a few parts/thousand) and our formulae agreed with those of Ref. 34 except that our triplet $M1$ results contained slightly different factors involving the nucleon isoscalar magnetic moment [e.g., $(\mu_s-\frac{3}{2})$ rather than $(\mu_s-\frac{1}{2})$].

There are several results by others to which we wish to call attention, even though they are obvious. Several years ago a popular exercise[49] was calculating the 90° photodisintegration cross section and comparing Siegert's form of the current with the classical current, $e\vec{\mathrm{p}}/m$. Needless to say, large differences were found, which follows immediately from the fact that a preponderance of meson exchanges in a nucleus involve charge and generate exchange currents which are *a priori* expected[50] to be the same order of magnitude as the classical current. As pointed out by Arenhövel,[49] the differences are largest for the $E1$ multipole and diminish when explicit exchange currents are added. The latter are noteworthy points.

Unless otherwise stated all figures discussed below refer to the RSC potential model. Figures 3 and 4 show the 90° and 0° cross section for unretarded dipole photodisintegration. The suppression of the electric dipole part of the process occurs for the reason stated in the Introduction. Figure 5 shows the contribution of the four multipoles which we have chosen to calculate. At low energies the singlet $M1$ transition dominates the cross section. At higher energies the $E1$ multipole is most important, although the spin-triplet isotriplet $M2$ transition is quite large. The latter effect results from the large isovector

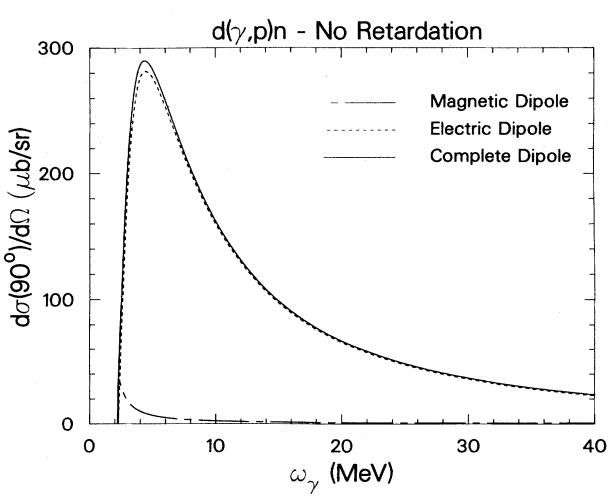

FIG. 3. Deuteron photodisintegration at 90° in the unretarded dipole approximation.

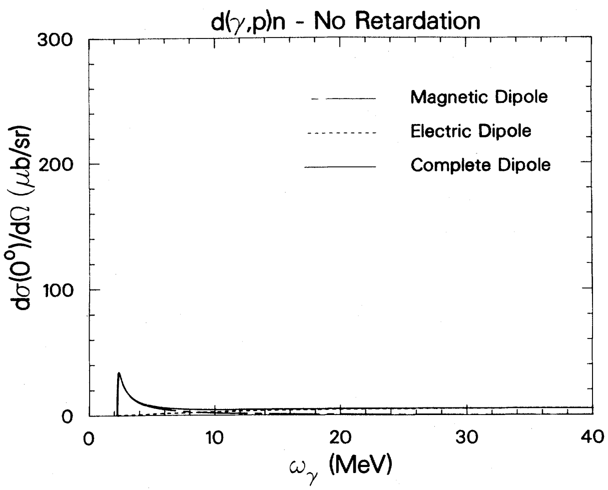

FIG. 4. Deuteron photodisintegration at 0° in the unretarded dipole approximation.

magnetic moment, μ_v; compare with the results of dividing the $M2$ strength by a factor of 5. We expect small, but visible, effects from the $E3$ and (isoscalar, spin triplet) $M3$ multipoles. We have not included these multipoles.

Figure 6 shows the effect of deleting parts of the calculation. The dashed curve is the result of turning off the deuteron D state. Obviously most of the cross section arises from this source. The effect of various components of the RSC force in the final state is shown in the remaining curves. The dotted curve results from deleting all such forces. Were the data less good this would appear to be a satisfactory fit. The RSC potential model as defined in Reid's paper[24] does not include forces for the $J>2$ partial waves; we need the $J=3$ waves, however, for the $L=2$ multipoles. Recently, Day[25] has obtained and slightly modified Reid's unpublished higher partial-wave potentials. We have shown the results of deleting and in-

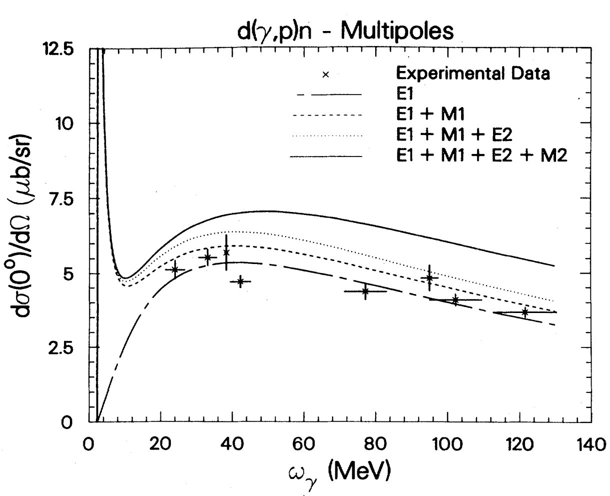

FIG. 5. Deuteron forward photodisintegration decomposed into multipoles.

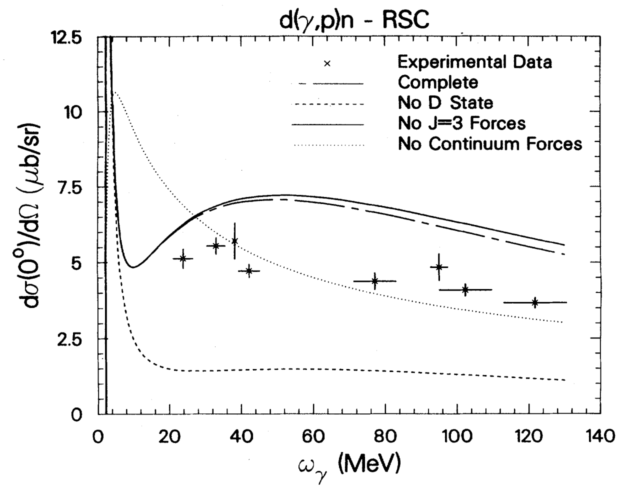

FIG. 6. Deuteron forward photodisintegration in various approximations for the RSC potential.

cluding such forces. The effect is small but not negligible.

Figure 7 illustrates the small effect of retardation in the $E1$ amplitude. The very small effect labeled "old" and the increase labeled "new" are misleading. Figure 8 shows a breakdown of the $E1$ contribution into O and N terms, which result from orbital (O), spin (S), and OPE (ex) currents, as a percentage of the dominant unretarded Siegert term. The spin magnetization contribution to N (N_s) is quite large. The old forms of N_0 and O_0 largely cancel this, however. The much smaller new forms were predicted in Sec. III and allow N_s to dominate. We have computed only the new forms of the pion-exchange amplitude, N_{ex}. It is very small (approximately 2% at 100 MeV), which is gratifying in view of our argument that the new form of the electric multipole amplitudes "used" most of the exchange currents to ensure current conservation. This is also consistent with Arenhövel's remarks.[49]

In order to make a convincing case that the spin-orbit

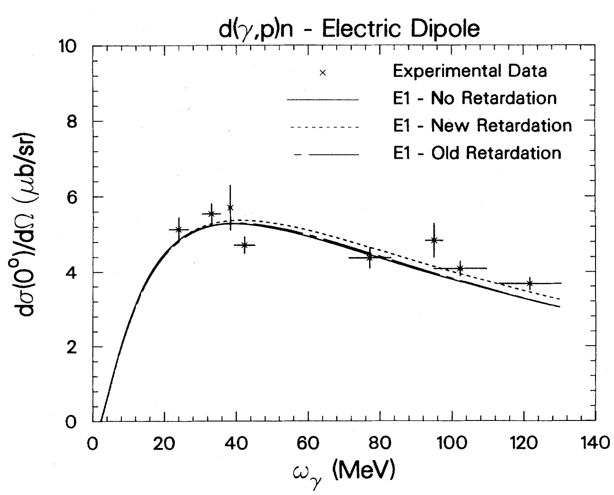

FIG. 7. Deuteron forward photodisintegration in the electric dipole approximation showing two forms of retardation.

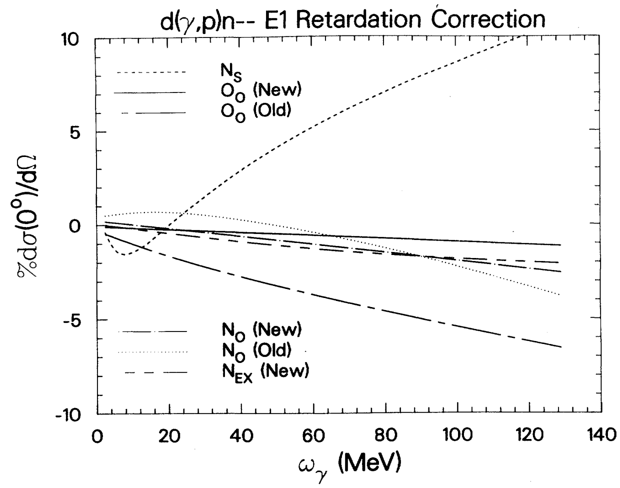

FIG. 8. Percentage retardation correction for various parts of the electric dipole forward photodisintegration in two approximations.

dipole operator is responsible for the discrepancy between the simple theory and experiment, it is necessary to perform identical calculations for a wide variety of potential models. Such a set of calculations is displayed in Fig. 9. Eight models were used.[14,25,27–32] The spread of results is not particularly large, but deserves comment. We noted earlier that the asymptotic D-state normalization, $A_D = \eta A_S$, should determine the cross section, an observation due to Schulze, Saylor, and Goloski.[26] If the cross sections are scaled to the experimental[51] value of A_D, $(0.027)\times(0.885)$, Fig. 10 results. The spread of the curves is greatly compressed, except for the Reid soft core (RSC) and Hamada-Johnston (HJ) cases. The HJ potential has the wrong deuteron binding energy; the Humberston-Wallace (HW) modification (of several parameters in the HJ potential) corrected this defect, with little change in other observables. Presumably this difference accounts for the deviation seen in the figure. The RSC potential curve is notably higher at higher energies. We have traced a large part of that deviation to the poorer p-wave forces in this potential compared to the others. If one calculates the unbound states with the super-soft-core (C) [SSC(C)] potential,[30] the RSC result is lowered. Figure 11 shows the result of including the spin-orbit dipole operator and Fig. 12 is the scaled version. The bands are similar to those of Figs. 9 and 10.

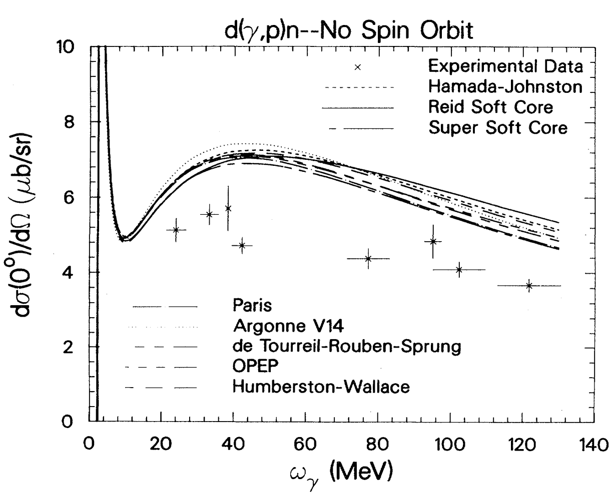

FIG. 9. Deutron forward photodisintegration for eight different potential models without spin-orbit dipole operator.

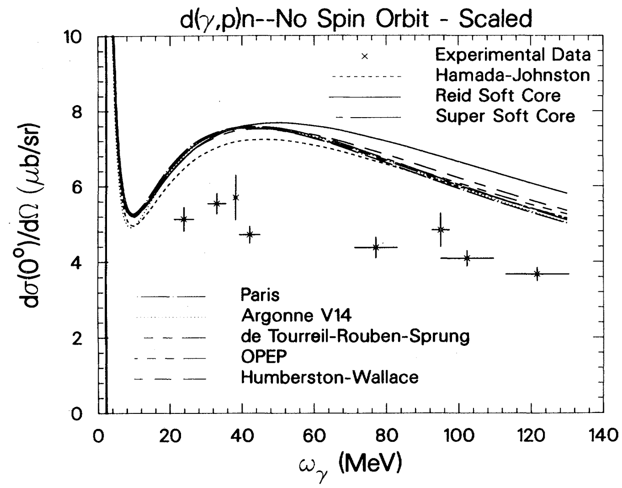

FIG. 10. Deuteron forward photodisintegration as in Fig. 9, scaled to the experimental value of A_D.

Figure 13 shows the results for the RSC potential with and without the spin-orbit contribution. A nucleon dipole form factor has been added to the latter as an overall factor; the effect is not large, but it is not negligible. The remaining corrections in Eq. (30c), including $\hat{L}\,\vec{J}_0$, which generates the effective photon momentum $\bar{q}$, have been added to the former and are negligibly small.

Figure 14 illustrates the effect of adding in $\Delta\vec{D}_\pi$ without retardation in various representations characterized by μ and ν. The comparison curve includes the non-

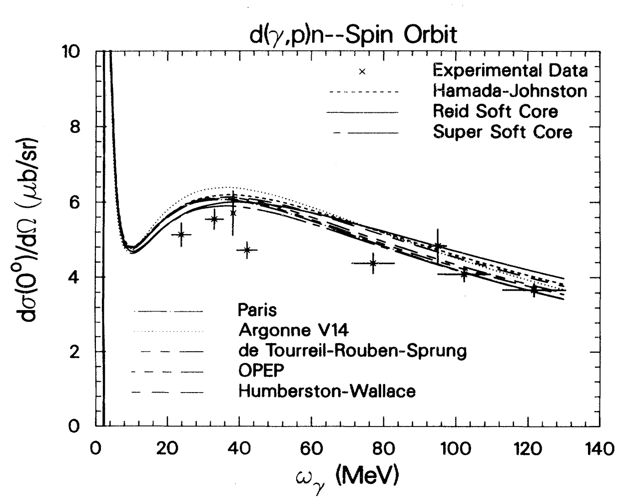

FIG. 11. Deuteron forward photodisintegration for eight different potential models including the spin-orbit dipole operator.

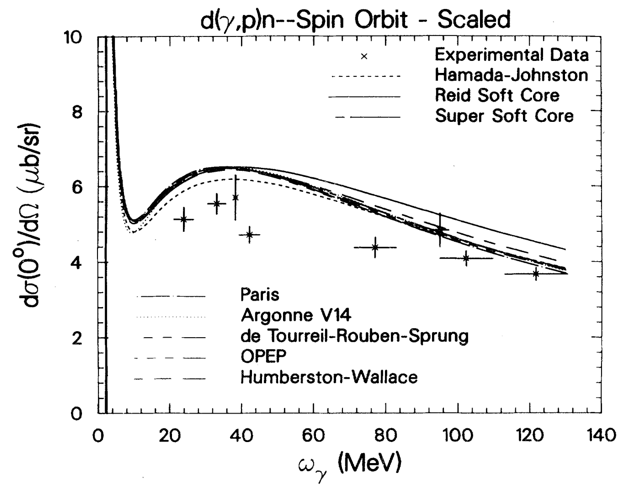

FIG. 12. Deuteron forward photodisintegration as in Fig. 11, scaled to the experimental values of A_D.

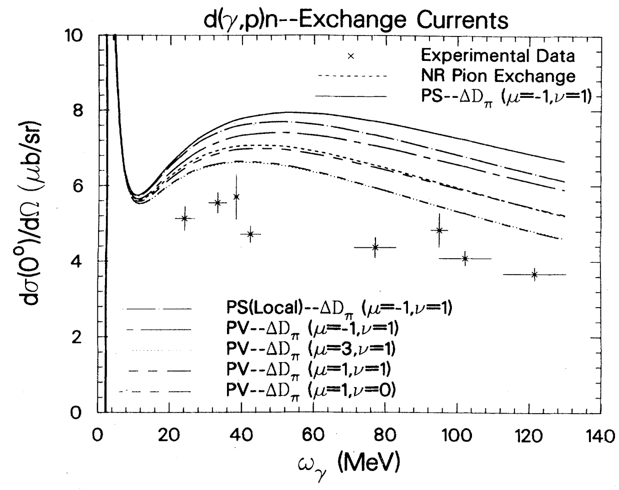

FIG. 14. Deuteron forward photodisintgration including pion exchange currents. The various families of pion-exchange dipole operators labeled by μ and ν are depicted.

relativistic exchange currents, which are very small at higher energies and tend to raise the singlet $M1$ portion of the cross section by 10—15% at low energies. They have been calculated using Eq. (23). The uppermost curve is the PS result in the most common representation; the usual Feynman diagram approach corresponds to $\mu=-1$. The next curve down is the local approximation to the previous PS curve, the most common calculation presented heretofore. The next curve down is the corresponding PV-coupling result; it is considerably smaller than the PS result, as remarked in Sec. VI, and is preferred on physical grounds. Three other PV representations are also listed. One of them ($\mu=1,\nu=1$) differs negligibly from the comparison curve, while the remaining two cases are almost identical. If we denote by X the contribution from $\Delta\vec{\mathbf{D}}_\pi$, all our PV results are reasonably consistent with

$$X=[(\nu-1)+(1-\mu)/2]\Delta(\omega)\ , \tag{43}$$

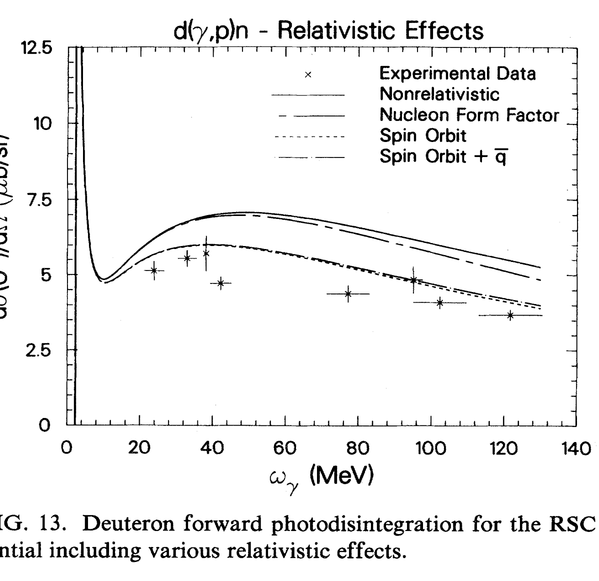

FIG. 13. Deuteron forward photodisintegration for the RSC potential including various relativistic effects.

where Δ is the scale of contribution of the pionic processes. We note that all of the illustrated calculations were performed without a pion-nucleon form factor. Using a representative value of $\beta=6$ (pion masses) we find only slightly smaller results, indicating a dominance by the long-range part of the current. Our discussion in Sec. V centered on using these results as a measure of sensitivity to the interior part of the wave function. The lack of spread in the scaled cross section found earlier may result from their locality; nonlocal potentials may show more variation.

Finally, we have also calculated the c coefficient, which is the difference of forward and backward cross sections.[2] That coefficient is positive, small, and largely unaffected by either $\Delta\vec{\mathbf{D}}_{\rm so}$ or $\Delta\vec{\mathbf{D}}_\pi$. The experimental results of Ref. 6, if they are confirmed, remain a mystery.

In summary, we have calculated the deuteron forward photodisintegration, including a wide variety of physical mechanisms, and have investigated in detail many aspects of the problem. Relativistic effects, retardation effects, and meson-exchange contributions were discussed. We can summarize the set of calculations as follows: (1) The nonrelativistic impulse approximation is roughly 20% above the data; (2) including the spin-orbit dipole operator lowers the theoretical calculations by approximately 20%; (3) most of the remaining effects are small but not entirely negligible; (4) the largest remaining uncertainty is the dipole operator due to meson exchange, for which no internally consistent calculation exists; (5) the representation variation of this effect using *fixed* wave functions may provide a measure of sensitivity to the inner part of the wave functions caused by nonlocalities in the potential.

Finally, we note that two calculations exist[52,53] whose results are at variance with ours and with those of Ref. 11. These calculations are outside the traditional framework and the disagreements are not understood. A pedagogical version of our work was published previously.[54]

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