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Single-particle levels in the doubly magic ^{132}Sn and ^{100}Sn nuclei

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The observed single-particle energies in ^{132}Sn are compared with three current models. For each of the three models the deviations from experiment in ^{132}Sn are found to be correlated with the corresponding deviations in ^{208}Pb. Assuming a similar relation between ^{100}Sn and ^{56}Ni, the proton binding energy of ^{100}Sn is predicted to be about 3 MeV.

Until recent years, $^{208}_{82}Pb_{126}$ was the only heavy doubly magic nucleus where the binding energies of the valence single-particle states were known from experiment. However, studies of neutron-rich fission products during the last decade are now beginning to give a fairly extensive picture of the single-particle spectrum in doubly magic $^{132}_{50}Sn_{82}$. Following the recent discovery[1] of the $d_{5/2}$ level in ^{131}Sn, which completes the neutron single-particle spectrum anticipated in the major shell below $N=82$, we undertake in this Rapid Communication to compare the available data around ^{132}Sn with some standard Hartree-Fock and independent-particle models.

Each of these models has a set of phenomenological parameters which could probably be improved, but nonetheless represents the best of what is currently available for a universal description of single-particle properties throughout the chart of the nuclides. Empirical single-particle energies in ^{132}Sn were not taken into account in fitting any of the parameter sets. The goal of the present work is *not* to see which model is "best," though obviously Table I can be used to see which model is best for some specific purpose.[2] The present work aims rather at general conclusions about the ability of one-body potentials to describe and predict single-particle energies.

On the opposite, proton-rich side of the valley of stability, the doubly magic nucleus $^{100}_{50}Sn_{50}$ has not yet been reached. Recently, however, its doubly odd neighbor $^{100}_{49}In_{51}$ was created as a product of heavy-ion bombardment,[3] indicating the future feasibility of spectroscopy in this region. Theoretical results for ^{100}Sn are also given below.

The three models are: (i) Hartree-Fock with the Skyrme III interaction introduced in Ref. 4. The parameters were selected to reproduce total binding energies and charge radii of spherical nuclei in all mass regions, as well as single-particle energies. (ii) The folded Yukawa single-particle potential, used to obtain the microscopic energy in a recent mass formula.[5] Two parameter sets were originally fitted to Nilsson levels in deformed rare-earth and actinide nuclei,[6] respectively, and a smooth extrapolation from these two sets can be made[5] to other mass regions all the way down to ^{16}O. (iii) The Woods-Saxon single-particle potential of Ref. 7 with the "universal" parameter set from Ref. 8. This set was originally obtained by simplifying Rost's parameter set[9] for single-particle levels in ^{208}Pb, and is currently being used by a Warsaw-Lund collaboration for calculations ranging from the $A \sim 80$ region to the actinides.

In the two latter models the numerical convergence of the present calculations is limited by the fact that diagonalization is carried out in a harmonic oscillator basis truncated to major shells with $N \leq 14$. The errors can be several tens of keV for single-particle levels very near the top of the potential well.

The comparison between theory and experiment will focus on two rather independent quantities. One is the average binding energy $\bar{B}$ of all the known proton or neutron levels. The other is the relative spacing of the levels, i.e., the rms deviation σ between theory and experiment after the respective $\bar{B}$'s have been subtracted out. The latter comparison is similar in spirit to an analysis by Blomqvist,[10] who compared observed ^{132}Sn levels with a Woods-Saxon calculation, using ^{208}Pb parameters except for the central well depths for neutrons and protons which were adjusted *ad hoc* to the data in ^{132}Sn. Blomqvist noted a correspondence between ^{132}Sn and ^{208}Pb: for each level (nlj) in ^{132}Sn there is a corresponding level $(n\,l+1\,j+1)$ in ^{208}Pb, and differences between theory and experiment for

TABLE I. Calculated and experimental single-particle binding energies in MeV. The column headings stand for Woods-Saxon, folded Yukawa, Skyrme III, and experimental.

	WS	FY	SIII	Expt.
^{56}Ni		Neutrons		
$s_{1/2}$	21.60	20.92	22.20	
$d_{3/2}$	21.14	20.71	22.84	
$f_{7/2}$	16.87	15.39	16.12	
$p_{3/2}$	12.36	10.56	10.25	10.3
$f_{5/2}$	10.00	8.69	8.23	9.55
$p_{1/2}$	10.04	8.00	8.47	9.25
$g_{9/2}$	7.94	5.65	4.53	
$d_{5/2}$	3.71	1.02	. . .	
$s_{1/2}$	2.23	. . .	. . .	
$d_{3/2}$	0.39	. . .	. . .	
^{100}Sn				
$f_{7/2}$	25.11	24.49	26.84	
$f_{5/2}$	20.88	20.38	21.99	
$p_{3/2}$	20.92	20.18	20.55	
$p_{1/2}$	19.32	18.45	18.91	
$g_{9/2}$	17.87	16.55	16.98	
$g_{7/2}$	11.51	10.20	9.61	
$d_{5/2}$	12.96	11.29	10.08	
$s_{1/2}$	10.80	8.83	7.54	
$h_{11/2}$	10.19	8.16	6.84	
$d_{3/2}$	10.10	8.07	7.64	
$f_{7/2}$	5.11	2.57	. . .	
$p_{3/2}$	3.13	0.62	. . .	
$p_{1/2}$	1.85	. . .	. . .	
$h_{9/2}$	1.59	. . .	. . .	
$f_{5/2}$	1.28	. . .	. . .	
$i_{13/2}$	2.14	. . .	. . .	
^{132}Sn				
$g_{7/2}$	9.03	10.70	11.30	9.72
$d_{5/2}$	9.28	10.70	10.69	8.95
$s_{1/2}$	7.63	8.61	8.22	7.62
$h_{11/2}$	6.83	8.04	8.20	7.53
$d_{3/2}$	7.38	8.33	8.24	7.29
$f_{7/2}$	2.69	3.05	1.69	2.63
$p_{3/2}$	1.18	1.12	. . .	
$p_{1/2}$	0.35	0.02	. . .	
$h_{9/2}$	1.01	1.52	. . .	
$f_{5/2}$	0.15	. . .	. . .	
$i_{13/2}$	0.06	0.45	. . .	

	WS	FY	SIII	Expt.
^{208}Pb		Neutrons		
$h_{9/2}$	10.62	11.60	12.64	10.85
$f_{7/2}$	10.35	11.18	11.21	9.72
$i_{13/2}$	9.03	9.72	10.18	9.01
$p_{3/2}$	8.29	8.64	8.13	8.27
$f_{5/2}$	8.17	8.46	8.42	7.95
$p_{1/2}$	7.46	7.51	7.10	7.38
$g_{9/2}$	4.30	4.28	2.94	3.94
$i_{11/2}$	3.52	3.53	2.74	3.16
$j_{15/2}$	2.93	2.76	1.90	2.51
$d_{5/2}$	2.28	1.76	0.35	2.37
$s_{1/2}$	1.44	0.82	. . .	1.90
$g_{7/2}$	1.47	0.51	. . .	1.44
$d_{3/2}$	1.06	0.12	. . .	1.40
^{56}Ni		Protons		
$s_{1/2}$	9.71	11.27	12.68	
$d_{3/2}$	9.29	11.39	13.47	
$f_{7/2}$	5.00	6.18	6.99	7.20
$p_{3/2}$	1.04	1.44	1.21	
^{100}Sn				
$f_{7/2}$	8.38	10.57	12.71	
$f_{5/2}$	4.22	6.48	7.89	
$p_{3/2}$	4.07	5.84	6.19	
$p_{1/2}$	2.43	4.19	4.65	
$g_{9/2}$	1.17	3.02	3.17	
^{132}Sn				
$f_{7/2}$	22.04	21.82	24.26	
$f_{5/2}$	18.71	18.43	20.53	
$p_{3/2}$	17.54	17.25	17.89	
$p_{1/2}$	16.14	15.82	16.54	
$g_{9/2}$	15.06	14.84	15.37	15.38
$g_{7/2}$	9.79	9.57	9.56	9.68
$d_{5/2}$	9.74	9.13	7.89	8.72
$s_{1/2}$	6.95	6.43	4.94	
$d_{3/2}$	7.22	6.36	5.77	
$h_{11/2}$	7.63	7.34	6.02	6.89
^{208}Pb				
$g_{7/2}$	11.99	12.13	13.59	11.51
$d_{5/2}$	10.20	10.55	10.28	9.70
$h_{11/2}$	9.22	9.88	9.65	9.37
$d_{3/2}$	8.43	8.58	8.51	8.38
$s_{1/2}$	7.94	8.10	7.36	8.03
$h_{9/2}$	4.50	4.59	4.24	3.80
$f_{7/2}$	3.62	3.66	1.66	2.91
$i_{13/2}$	2.86	3.52	1.54	2.20
$f_{5/2}$	0.67	0.46	. . .	0.99
$p_{3/2}$	0.73	0.37	. . .	0.69

the ^{132}Sn levels are correlated to the differences obtained for the corresponding levels in ^{208}Pb. A main point of the present note will be to confirm this finding in all three of the models studied. For the nucleus ^{100}Sn the corresponding structural analog is expected[11] to be ^{56}Ni.

Table I lists the "raw" single-particle energies in ^{100,132}Sn and their respective analogs, ^{56}Ni and ^{208}Pb. Theoretical values are given in most cases for the entire major shell below the shell closure and up to the highest bound state. Experimental values for ^{56}Ni and ^{208}Pb are taken from the compilation of Bohr and Mottelson.[12] For ^{132}Sn, the five experimental masses needed to determine single-particle en-

ergies above and below the shell gaps have now all been measured. The mass excess in MeV is −68.50(25) for ^{131}In,[13] −77.37(10) for ^{131}Sn,[14,15] −76.59(8) for ^{132}Sn,[16] −71.15(25) for ^{133}Sn,[17] and −78.98(21) for ^{133}Sb.[18] The binding energy of the last filled orbit is obtained from these numbers by the relation

$$S_p = B(Z,N) - B(Z-1,N)$$
$$= \mathrm{ME}(H) - \mathrm{ME}(Z,N) + \mathrm{ME}(Z-1,N) \ ,$$

or the corresponding relation for neutrons. The experimental uncertainty is seen to be about 0.2 MeV. The size of the $Z=50$ and $N=82$ gaps in the single-particle spectrum deduced from the masses is perfectly consistent with two-proton and two-neutron excitation energies observed[19] in ^{132}Sn (c.f. Ref. 10, where the mass excesses of ^{131}Sn and ^{133}Sn were correctly predicted from the two-particle excitation energies). The relative energies of states assumed to be pure single-particle states above and below the shell gaps can be determined to high accuracy by gamma ray spectroscopy. Level schemes have been established for two odd-A neighbors of ^{132}Sn, namely, $^{133}Sb_{82}$ (Refs. 17, 20, and 21) and $^{131}Sn_{81}$,[1,13] whence several proton states above the $Z=50$ gap and all the neutron states between the $N=50$ and 82 gaps are known (Table I).

The rms deviations σ of the *relative* level energies for neutrons and protons in ^{132}Sn are shown in Table II. Values of σ are also given for ^{208}Pb, computed *only* from levels $(n\,l+1\,j+1)$ corresponding to observed levels (nlj) in ^{132}Sn. Table II is divided into boxes I–IV for easy reference. Box I shows σ for the selected ^{208}Pb levels. These levels were more or less heavily weighted into the various model parameter fits. Box II shows σ in ^{132}Sn, which was not considered in determining model parameters. There is not a uniform deterioration in going to ^{132}Sn: the more accurate ^{208}Pb fits get worse but the less accurate ones improve. Box III shows results for ^{132}Sn after the theoretical single-particle energies ϵ have been corrected by the empirical relative errors in ^{208}Pb according to the prescription[22]

$$\epsilon^{\mathrm{theor}}_{nlj}(^{132}\mathrm{Sn}) \rightarrow \epsilon^{\mathrm{theor}}_{nlj}(^{132}\mathrm{Sn}) + [\epsilon^{\mathrm{expt}}_{n\,l+1\,j+1}(^{208}\mathrm{Pb}) - \bar{B}^{\mathrm{expt}}(^{208}\mathrm{Pb})] - [\epsilon^{\mathrm{theor}}_{n\,l+1\,j+1}(^{208}\mathrm{Pb}) - \bar{B}^{\mathrm{theor}}(^{208}\mathrm{Pb})] \ .$$

This gives a significant decrease of σ. Even for Skyrme III protons, which have a small σ in box II and do not improve much in box III, all four empirical corrections from ^{208}Pb actually have the correct sign though they are too large and

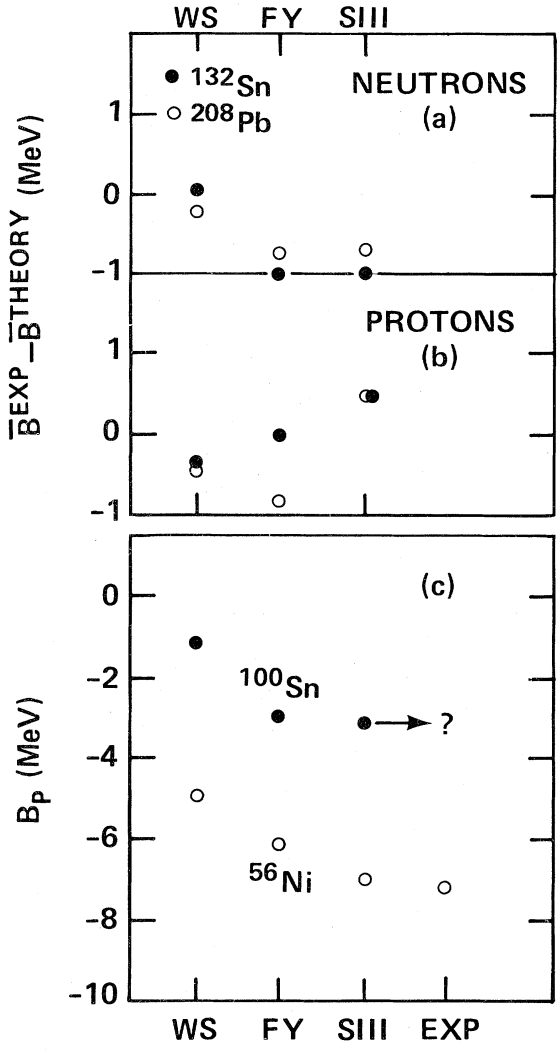

FIG. 1. The upper parts (a) and (b) show how the center of gravity of a set of single-particle levels deviates from experiment in the Woods-Saxon, folded Yukawa, and Skyrme III models. Clearly, the deviations in ^{132}Sn and ^{208}Pb mostly go together. The lower part (c) shows the binding energy of the proton $g_{9/2}$ shell in ^{100}Sn and $f_{7/2}$ shell in ^{56}Ni, including a point for experiment. If the deviations from experiment go together in these two nuclei, ^{100}Sn would be inside the proton drip line, with a proton binding energy of about 3 MeV.

TABLE II. The rms deviation σ between theories and experiment for the relative energies of known single-particle levels in ^{132}Sn, and the corresponding subset of levels in ^{208}Pb. The column headings stand for Woods-Saxon, folded Yukawa, Skyrme III, and ^{208}Pb experimental levels. Conclusions are discussed in the text.

σ (MeV)	WS	FY	SIII	WS	FY (Corrected by ^{208}Pb errors)	SIII (Corrected by ^{208}Pb errors)	Expt.
		I					
^{208}Pb Neutrons	0.28	0.38	0.97	0	0	0	0
Protons	0.36	0.30	0.69	0	0	0	0
^{132}Sn Neutrons	0.40	0.43	0.87	0.25	0.27	0.41	0.33
Protons	0.53	0.41	0.39	0.33	0.27	0.36	0.53
		II			III		IV

overcompensate the errors in ^{132}Sn. It may be mentioned that a recent "SkM" version of the Skyrme force[23] gives more accurate relative levels in ^{208}Pb than Skyrme III, and the smaller ^{208}Pb corrections in the SkM case do give a significant improvement of the ^{132}Sn relative levels for both neutrons and protons. The conclusion is that the errors in a one-body potential are partly of a systematic nature, and thus it is worthwhile to try to understand them in terms of many-body residual interactions.

Box IV shows the error in simply using ^{208}Pb experimental relative energies to predict ^{132}Sn. The empirical corrections as in box III are obviously incorporated; nevertheless, the values in box IV are larger than five of the six corresponding values in box III. This indicates that changes of the nuclear potential well near the Fermi level in going from ^{208}Pb to ^{132}Sn are reproduced by the theoretical single-particle models.

A systematic trend in average binding energy $\bar{B}$ can be seen in Fig. 1 where the difference between theory and experiment is shown for ^{132}Sn and the corresponding subset of levels in ^{208}Pb, for neutrons in Fig. 1(a) and for protons in Fig. 1(b). The errors are about equal for both nuclei in five out of the six cases shown in Figs. 1(a) and 1(b). Thus, if a model overbinds in ^{208}Pb, it also overbinds in ^{132}Sn and vice versa.

If the same holds true for the analog pair ^{56}Ni and ^{100}Sn, it should be possible to use the known proton binding energy in ^{56}Ni to estimate the proton binding in ^{100}Sn, a number that is crucial for the formation cross section of ^{100}Sn in heavy-ion reactions. The last filled shells in these two nuclei are the analog states $1f_{7/2}$ and $1g_{9/2}$, respectively, and Fig. 1(c) shows the model systematics of their binding energies. Taking the empirical trend from ^{56}Ni, all the models point at the relatively optimistic value of about 3 MeV for the proton binding in ^{100}Sn.

In summary, the single-particle levels in ^{132}Sn are similar to those in ^{208}Pb. A part of the difference between the two nuclei is accounted for by standard single-particle models (box III versus box IV in Table II). A part of the discrepancy between theory and experiment is the same in both nuclei, both for individual levels (box III versus box II) and the overall binding (Fig. 1). On the basis of the ^{132}Sn experience, the proton binding energy in ^{100}Sn is estimated and found to be relatively favorable for synthesis of this doubly magic nuclide, although, for example, neighboring ^{101}Sb would be proton unstable.

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