Possibility of direct $E2$ capture in ^{21}Ne

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It is shown that the partial cross section for the $E2$ transition at 6415 keV in ^{21}Ne emitted following thermal neutron capture is extremely large if it is a result of resonance capture. The possibility of a direct $E2$ contribution is explored. The cross section can be accounted for by direct capture if the effective charge appropriate for low energy electric quadrupole transitions is used.

A general theory describing radiative capture reactions has been developed by Lane and Lynn.[1] These authors use perturbation techniques to obtain the collision matrix element connecting the particle entrance channel to the photon exit channel. The distinctive feature of the radiative capture cross section which results from this formulation is that it contains contributions corresponding to the configuration space beyond the channel radius. These contributions are expected to be particularly large for transitions to final states exhibiting a well-defined single-particle character. A simplified closed form expression for the most important case of $E1$ radiation thermal neutron capture was also calculated by Lane and Lynn.[2] A detailed comparison between theory and experiment for specific cases in the mass regions around $A=40$ and $A=140$ has been discussed by Mughabghab.[3] In the latest neutron cross-section compilation[4] this author has applied the theory to calculate the direct capture cross sections of several light nuclei. In this Brief Report, the consequences of these calculations for the specific case of ^{20}Ne are explored.

The experimental value of the ^{20}Ne radiation capture cross section is given as 37 ± 4 mb.[4] The value calculated for the direct capture cross section is 38.2 mb, while the contribution from known resonances is 1900 μb.[4] The major component of the latter is the 1765 μb arising from the $l=0$ resonance at 473 keV. Bellmann[5] observes a gamma ray at 6415.4 keV for which the final state is the $J^\pi=\frac{5}{2}^+$ first excited state of ^{21}Ne at 350.5 keV.[6] Since the initial state following s-wave capture is $J^\pi=\frac{1}{2}^+$, this transition is $E2$ in nature. The reported intensity[5] for this transition is $0.8\pm0.4\%$. The partial cross section for this line is therefore 300 μb. If this line is a result of resonance capture, then it would correspond to approximately 16% of the total resonance radiation. This intensity is surprisingly large. For example, in the nearby nucleus ^{26}Mg, Selin and Hardell[7] report an 11 MeV $E2$ transition with an intensity of 0.1% of the total resonance radiation.

The magnitude of the branching ratio for the 6415.4 keV transition in ^{21}Ne only appears anomalous if the assumption is made that there is no direct $E2$ transition. In this case, however, the final state exhibits well-developed single-particle $d^{\nu}_{5/2}$ state characteristics, with an $l_n=2$ spectroscopic factor of 4.3 for the (d,p) reaction.[8] It was decided, therefore, to extend the calculation of direct capture cross sections to the case of $E2$ radiation in order to investigate the possibility of a direct contribution.

Following Lane and Lynn,[1] the partial radiative capture cross section may be written

$$\sigma^J_{n\gamma_f}=\frac{g_J\pi}{k_n^2}|U^J_{n\gamma_f}|^2 \ , \tag{1}$$

where J is the spin of the initial state, g_J is the statistical factor, k_n is the neutron wave number, and $U^J_{n\gamma_f}$ is the collision matrix element connecting the entrance neutron channel to the exit channel consisting of a photon populating the final state f. From perturbation theory

$$|U^J_{n\gamma_f}|^2=\frac{4\pi\bar{e}^2}{75}\frac{k_\gamma^5}{\hbar}\frac{(i||E^{(2)}||f)^2}{2J+1} \ . \tag{2}$$

In Eq. (2), $\bar{e}$ is the effective neutron charge, k_γ is the photon wave number, and the reduced matrix element is taken between initial and final states for the quadrupole operator

$$E^{(2)}=r^2Y^{(2)} \ , \tag{3}$$

where r is the distance from the center of mass and $Y^{(2)}$ represents the spherical tensor associated with the spherical harmonics of second order.

In the channel spin representation, the spin of the target, J_t, is coupled to the neutron intrinsic spin to form the channel spin S. This is, in turn, coupled to the relative orbital angular momentum l to produce the initial state of spin J. Similarly, the final state may be considered as characterized by spin J_f resulting from the coupling of angular momentum l_f to the same channel spin. Defining the channel radius to be R, then, in the region $r>R$ each wave function may be written in the form

$$\psi=u_l\phi_{lsJ\mu} \ , \tag{4}$$

where

$$\phi_{lsJ\mu}=\frac{i^l}{r}\sum_{m_lm_s}(lm_lSm_s|J\mu)\,Y^{(l)}_{m_l}\chi_{Sm_s} \ . \tag{5}$$

In Eq. (5), χ_{Sm_s} represents the intrinsic wave function of the neutron and target with spins coupled so as to produce channel spin S with projection m_s.

The external contribution to the reduced matrix element in Eq. (2) may be written

$$(i||E^{(2)}||f)=M_{if}I_{if} \ , \tag{6}$$

where

$$M_{if}=(r\phi_f||Y^{(2)}||r\phi_i)/\sqrt{2J+1} \tag{7}$$

represents the integration over the angular coordinates and

$$I_{if}=\left|\int_R^\infty u_l^*r^2u_{l_f}dr\right| \tag{8}$$

represents the radial integral.

From Edmonds,[9] the matrix element in Eq. (7) may be reduced to

$$M_{if}=(-1)^{S+J}\sqrt{2J_f+1}\left[\frac{5}{4\pi}(2l+1)(2l_f+1)\right]^{1/2}\times\begin{Bmatrix} l_f & J_f & S\\ J & l & 2\end{Bmatrix}\begin{pmatrix} l_f & 2 & l\\ 0 & 0 & 0\end{pmatrix} . \tag{9}$$

If consideration is restricted to thermal capture, then $l=0$, $S=J$, and $l_f=2$. In this case Eq. (9) simplifies to

$$M_{if}=\frac{1}{\sqrt{4\pi}}\left[\frac{2J_f+1\delta}{2J+1}\right]^{1/2} . \tag{10}$$

Substituting in Eq. (1) leads to the expression

$$\sigma^J_{n\gamma_f}=\frac{g_{J_f}\pi}{k_n^2}\frac{\bar{e}}{75}\frac{k_\gamma^5}{\hbar}I_{if}^2 . \tag{11}$$

The reduced radial wave function in the initial unbound state may be written

$$u_0=\frac{1}{\sqrt{v_n}}(e^{-ik_nr}-U^J_{nn}e^{ik_nr}) , \tag{12}$$

where the collision matrix element for neutron scattering is given by

$$U^J_{nn}=e^{-2ik_nR}\left[1+i\sum_\lambda\frac{\Gamma_{n_\lambda}}{E_\lambda-E-i\Gamma_\lambda/2}\right] , \tag{13}$$

and v_n is the neutron velocity. For thermal capture, the exponents in (12) and (13) may be taken as very much less than one over the range of distances for which the integrand in (8) is significant. In addition, it is assumed that the condition

$$E_\lambda>>E+\frac{i}{2}\Gamma_\lambda \tag{14}$$

is fulfilled. Equation (14) expresses the normal situation in which the resonances are narrow and well removed from the thermal region. In this approximation, the function in Eq. (12) becomes

$$u_0=\frac{2ik_n}{\sqrt{v_n}}\left[R-r-\frac{1}{2k_n}\sum_\lambda\frac{\Gamma_{\lambda n}}{E_\lambda}\right] . \tag{15}$$

For the final state, the wave function may be written

$$u_2=C_2(k_f^2r^2+3k_fr+3)\frac{e^{-k_fr}}{r^2} , \tag{16}$$

where the constant C_2 is determined by the relationship between the reduced neutron width and the value of the wave function at the channel radius

$$\gamma_n=\left[\frac{\hbar^2}{2MR}\right]^{1/2}u_2(R) . \tag{17}$$

Defining the dimensionless reduced width by

$$\theta_n=\gamma_n\left[\frac{\hbar^2}{MR^2}\right]^{-1/2} , \tag{18}$$

this condition becomes

$$\theta_n=\left[\frac{R}{2}\right]^{1/2}u_2(R) . \tag{19}$$

Evaluation of Eq. (8) by use of the wave functions in Eqs. (15) and (16) gives

$$I_{if}=\frac{2k_n}{\sqrt{v_n}}\left[\frac{2}{R^3}\right]^{1/2}\theta_n\frac{y^3}{k_f^5}\times\left[\frac{y^2+7y+15}{y^2+3y+3}+\frac{k_f}{2k_n}\sum_\lambda\frac{\Gamma_\lambda}{E_\lambda}\frac{y^2+5y+8}{y^2+3y+3}\right] . \tag{20}$$

In the above equation, the dimensionless quantity $y=k_fR$ has been introduced. The final state parameter k_f is determined by the binding energy of the state, which equals the transition energy E_γ in the center of mass. This results in the relationship

$$k_\gamma=\frac{\hbar k_f^2}{2Mc}=\frac{\lambda\!\!\!^{-}_c}{2}k_f^2\left[\frac{A+1}{A}\right] , \tag{21}$$

where $\lambda\!\!\!^{-}_c$ is the Compton wavelength of the neutron and A is the target mass number. Substitution of the results in Eqs. (20) and (21) into Eq. (11) leads to the direct cross section

$$\sigma^J_{n\gamma_f}=\frac{(2J_f+1)\theta_n^2}{2(2J_t+1)}\frac{\pi\bar{e}^2}{300\hbar v_n}\left[\frac{A+1}{A}\right]^5\frac{\lambda\!\!\!^{-}{}_c^5}{R^3}\times y^6\left[\frac{y^2+7y+15}{y^2+3y+3}+\frac{k_f}{2k_n}\sum_\lambda\frac{\Gamma_{\lambda_n}}{E_\lambda}\frac{y^2+5y+8}{y^2+3y+3}\right]^2 . \tag{22}$$

In Eq. (22), the term $(2J_f+1)\theta_n^2$ is the $l=2$ spectroscopic factor for the (d,p) reaction populating the final state. The channel radius is taken to be $R=1.35A^{1/3}$ fm. The cross section which results from the first term in the parentheses is the hard-sphere scattering contribution. The contribution from resonances given by the second term may be evaluated by making use of the relation

$$\frac{1}{2k_n}\sum_\lambda\frac{\Gamma_{\lambda_n}}{E_\lambda}=R'-A_{coh} , \tag{23}$$

where R' is the potential scattering length and A_{coh} is the coherent scattering length.[10]

Using the parameters listed in Table I, evaluation of Eq. (22) yields 1224 ξ^2 μb, where $\xi=\bar{e}/e$ is the effective neutron charge factor. Approximately 90% of this is due to the hard-sphere term. The uncertainty introduced by the errors in experimental parameters is negligible compared with that in the value for the observed $E2$ partial cross section.

In order to proceed further it is necessary to introduce an effective charge for $E2$ transitions. The situation in this case is less clear than for electric dipole transitions. In the latter, the factor $\xi=Z/A$ results directly from the evaluation of the electric dipole operator in the center-of-mass sys-

TABLE I. Parameter values for ^{20}Ne.

Radius	$R=3.66$ fm
Potential scattering length[a]	$R'=4.8\pm0.1$ fm
Coherent scattering length[a]	$A_{coh}=4.42\pm0.03$ fm
Binding energy[b]	$E_\gamma=6415.4\pm0.5$ keV
Spectroscopic factor[c]	$(2J+1)\theta_n^2=4.3$

[a]From Ref. 4.
[b]From Ref. 5.
[c]From Ref. 8.

tem.[1] Center-of-mass effects for the electric quadrupole operator lead to a negligible factor of Z/A^2.[11] However, Wilkinson[12] has shown that for quadrupole transitions in some light nuclei, a value in the neighborhood of 0.5 must be used. In particular, for the transition deexciting the first excited state of ^{19}Ne, $\xi = 0.6$.[12] If this value is accepted for the case under consideration, the predicted partial cross section would be 440 μb, which agrees with the experimental results within error. Alternatively, one might state that the partial cross section observed implies $\xi = 0.49 \pm 0.25$, if it is to be attributed to direct capture.

The use of such an effective charge for direct capture, as distinct from transitions between bound states, is an assumption which deserves further study. For the bound states, the transition probability is determined from both interior and exterior contributions. For the case of ^{19}Ne a simple model using a square well potential with radius $1.35A^{1/3}$ was investigated. The depth was chosen to reproduce the $2s_{1/2}$ binding energy. For the $E2$ transition connecting the $1d_{5/2}$ and $2s_{1/2}$ states, approximately $\frac{1}{2}$ of the transition probability is contributed from the region extending beyond the well boundary. It is therefore possible that even for bound states a significant contribution from the exterior region occurs. This may imply that the concept of an effective charge for $E2$ transitions is applicable to direct capture.

It is concluded that the $E2$ transition strength reported (5) for the thermal radiative neutron capture on ^{20}Ne may be explicable within the context of direct capture. Such an interpretation would require associating an effective charge of approximately $0.5e$ with the neutron in this case. This value is consistent with values observed for $E2$ transitions between bound states in this mass region.[12] Whether or not the use of such a large effective charge in the case of direct capture is justified is a matter requiring detailed investigation.

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