Difficulties of the thermodynamical model approach to pion production in relativistic ion collisions

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(Received 17 November 1983)

Thermodynamical models with various forms of partial transparency of nuclear matter are considered. It is shown that the introduction of transparency, however, significantly improves agreement with pion data concerning multiplicities and transverse momenta leads to a serious discrepancy with average rapidities of pions. Qualitative arguments are given that difficulties of the thermodynamical approach can be overcome if one assumes hydrodynamical expansion in the first stage of nuclear interactions.

Thermodynamics models based on various assumptions concerning the geometry and thermodynamics of collisions are widely used in describing data from relativistic ion collisions.[1] The models reproduce reasonably the shapes of the spectra of produced particles, but essential problems arise when the total number of pions produced is considered. Thermodynamics models predict about twice as many pions in comparison with experimental data.

In the present paper we concentrate on reproducing, in thermodynamical models, the average multiplicity of negative pions $\langle n \rangle^{\text{inel}}$ produced in inelastic collisions of ^{4}He and ^{12}C with various nuclear targets (Li ... Pb) at a 4.5 GeV/c momentum per incident nucleon. We compare results of model calculations with experimental average transverse momenta $\langle p_T \rangle$ of π^- mesons and average rapidities $\langle y \rangle$.

Two thermodynamics models, firestreak[2,3] and firetube,[4] have been tested. The models differ only in geometrical assumptions concerning the dynamics of collisions. Because the firestreak model is widely described in the literature[1–3] we briefly present geometrical assumptions of the firetube model only. Diffuse surface density distributions are used in this model as in the firestreak one. Interactions between collinear tubes of nucleons (with geometrical cross sections $\sigma = \sigma_{NN}^{\text{tot}}$) are assumed to occur independently. The probability of finding n nucleons in a projectile or target tube centered at $\bar{b}$ in the plane perpendicular to the collision axis z is

$$P_{p,t}^{n}(\bar{b}) = \binom{A_{p,t}}{n} \left(\frac{\sigma T_{p,t}(\bar{b})}{A_{p,t}} \right)^{n} \left(1 - \frac{\sigma T_{p,t}(\bar{b})}{A_{p,t}} \right)^{A_{p,t}-n} ,$$

where $A_{p,t}$ is the projectile (target) mass number and the "thickness function," $T_{p,t}(b)$ is[5]

$$T_{p,t}(\bar{b}) = \int_{-\infty}^{+\infty} dz\, \rho_{p,t}(\bar{b},z) .$$

The cross section for projectile-target interaction is

$$\sigma_{A_pA_t} = \int d^2B\, P^{\text{int}}(\bar{B}) ,$$

where

$$P^{\text{int}}(\bar{B}) = 1 - \exp\left[\int \frac{d^2b}{\sigma} \ln P^0(\bar{b},\bar{B}) \right] ,$$

$$P^0(\bar{b},\bar{B}) = P_p^0(\bar{b}) + P_t^0(\bar{B}-\bar{b}) - P_p^0(\bar{b}) P_t^0(\bar{B}-\bar{b}) .$$

A geometrical part of the firetube model imitates the one of the collective tube model.[6] An advantage of this geometrical approach is that the absolute values of cross sections are determined without additional assumptions (as opposed to the firestreak model). We have found reasonable agreement (differences less than 15%) of calculated and experimental[7] total inelastic cross sections. So, in the remaining part of our paper we shall concentrate on the firetube model. However, if the experimental values of the cross sections are used for normalization in the firestreak model, results of this model for $\langle n \rangle^{\text{inel}}$ become close to those of the firetube one. Other results of the firestreak model being independent of normalization are very similar to those given by the firetube model.

In the two tested models the nuclear thermodynamics of Ref. 3 has been used, where besides pions and deltas, light nuclei and resonances have been considered. We have taken a sharp Δ mass and, instead of $\eta_{\text{min, max}}$ $[\eta = a_p/(a_p + a_t)$ with $a_{p,t}$ the number of nucleons from the projectile (target) tube], we used a cutoff for 939 MeV mass per baryon of fireobject (see Ref. 4). We fixed the ratio of the charge to the baryon number equal to that of the whole system. When thermal equilibrium is assumed to be achieved, the system expands and at some critical density ρ_c decays as an ideal gas. The parameters of particles distributions (temperatures and chemical potentials) are fixed on the assumption of chemical equilibrium, charge, baryon number, and energy conservations.

Colliding objects (streaks or tubes) are usually assumed to stop in their c.m. system; i.e., they lose their momenta completely and the total kinetic energy undergoes thermalization (full thermalization case). In Fig. 1 there are presented average multiplicities of mesons produced in collisions of ^{4}He and ^{12}C nuclei with nuclear targets from ^{6}Li to ^{207}Pb.[7–9] The dash-dotted lines correspond to the assumption of full thermalization. Results of the model differ from experi-

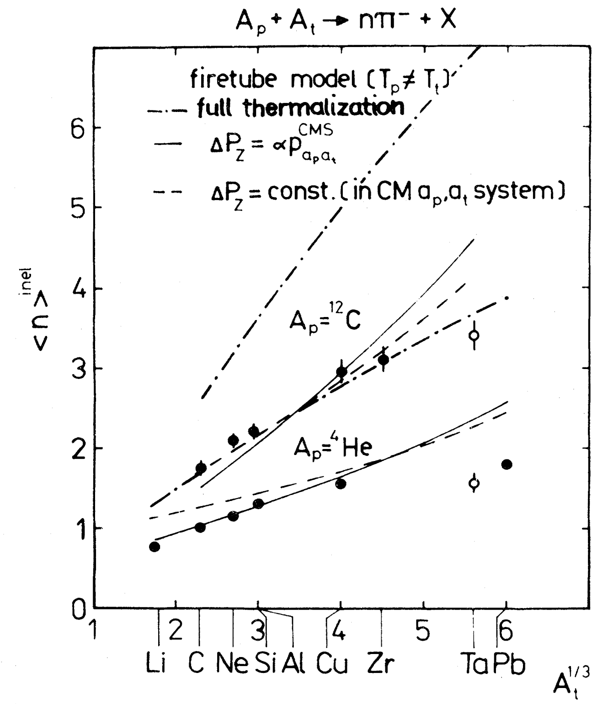

FIG. 1. Average multiplicities $\langle n \rangle^{\text{inel}}$ of negative pions (normalized to σ^{inel}) vs the target mass number $A_t^{1/3}$. Experimental points (●) from Ref. 7 and (○) from Ref. 9.

mental values of $\langle n \rangle^{\text{inel}}$ by factor of about 2 and exhibit a too steep rise with target mass number. Let us discuss the sensitivity of this result to the assumptions of the model. In our calculations we have used the value of critical density $\rho_c = 0.12$ fm^{-3} as in Ref. 3. The increase of ρ_c slightly decreases the number of produced pions as discussed in Ref. 10. The number of pions obtained from the calculations is sensitive to the choice of particles and resonances taken into account in the thermalization process. Taking into account nucleon resonances affects the number of pions only insignificantly.[10] Some decrease of $\langle n \rangle^{\text{inel}}$ values can be obtained if one takes into account strange particles production. This leads to the change of the results by less than 10% in the collisions studied.

Trying to explain the anisotropy of angular distributions of particles produced in central ^{12}C-^{12}C collisions, Das Gupta has assumed[11] that nuclear matter is in part transparent; i.e., the colliding objects do not stop completely but lose only some fraction of their momenta. The colliding parts of projectile and target independently undergo thermalization and finally decay separately. The temperatures of colliding fire objects from projectile and target are of course different ($T_p \neq T_t$).

Following the Das Gupta idea, we introduce the transparency to describe pion multiplicities.

The momentum loss of the colliding objects in thermodynamics models can take the form

$$\Delta P_z = f(a_p, a_t, P^{\text{c.m.}}) \ .$$

$P^{\text{c.m.}}$ is the center of mass incident momentum. If the interaction of the objects is assumed to be coherent, it would be appropriate to study the momentum loss in the c.m. of the objects. If, on the other hand, the momentum loss is assumed to occur in separate nucleon-nucleon interactions, it is better to consider the momentum loss in the nucleon-nucleon c.m. system.

We have examined five one-free-parameter α forms of the function $f(a_p, a_t, P^{\text{c.m.}})$.

(i) The nucleons undergo multiple scattering losing in each interaction a constant amount $\alpha P_{\text{NN}}^{\text{c.m.}}$ of their momenta,

$$\Delta P_z = \alpha P_{\text{NN}}^{\text{c.m.}} a_p a_t \ .$$

(ii) The nucleons undergo multiple scattering losing in each interaction a constant fraction α of their momenta,

$$\Delta P_z = P_{a_p a_t}^{\text{c.m.}} \{1 - \exp[-\alpha(a_p + a_t)]\} \ .$$

(iii) The nucleons from target and projectile tubes interact coherently. During the collision the tubes lose a constant amount of their momenta (dashed lines in Fig. 1),

$$\Delta P_z = \text{const} \quad (\text{in c.m. } a_p, a_t \text{ system}) \ .$$

(iv) The tubes interact coherently and lose a constant fraction of their momenta (solid lines in Fig. 1),

$$\Delta P_z = \alpha P_{a_p a_t}^{\text{c.m.}} \ .$$

(v) The nucleon passing through a nucleus can interact only once with one of the nucleons of the target nucleus (dashed lines in Fig. 2),

$$\Delta P_z = \alpha P_{\text{NN}}^{\text{c.m.}} \min(a_p, a_t) \ .$$

In cases (i) and (ii) (not presented in the figures) we have obtained the reduction of the value of $\langle n \rangle^{\text{inel}}$ by fitting parameter α. However, the increase of multiplicity with target mass number is as strong as for the full thermalization case. A weaker dependence on A_t has been found for cases (iii) and (iv), but the agreement with experimental data is

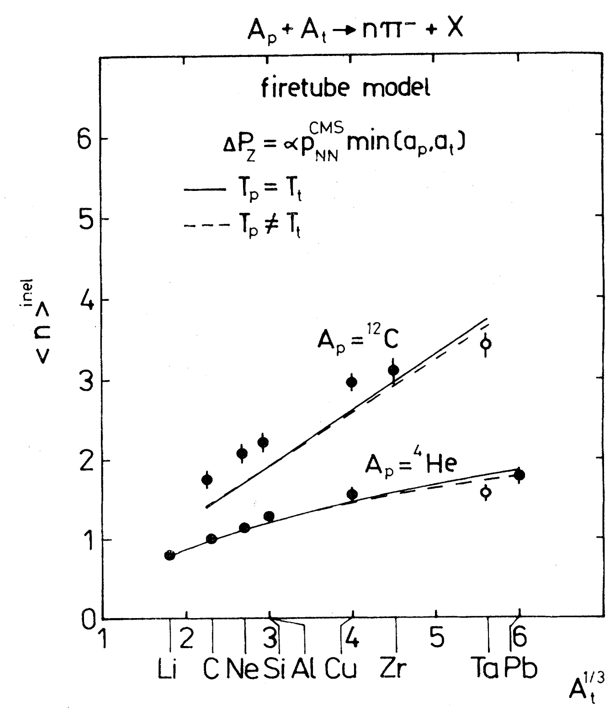

FIG. 2. Caption as for Fig. 1.

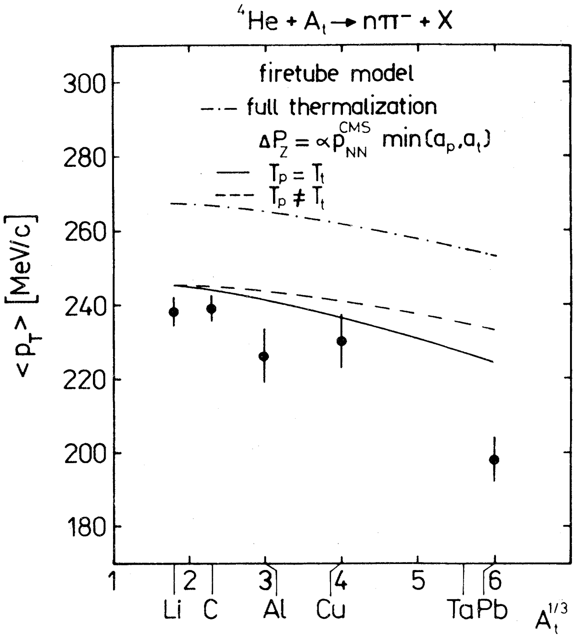

FIG. 3. Average transverse momenta $\langle p_T \rangle$ of negative pions vs the target mass number $A_t^{1/3}$. Experimental points from Ref. 8.

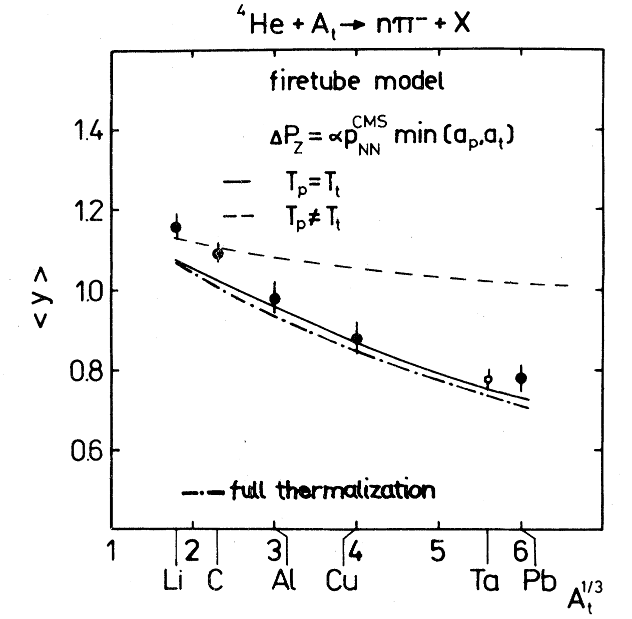

FIG. 4. Average rapidity $\langle y \rangle$ of negative pions vs target mass number $A_t^{1/3}$. Experimental points (●) from Ref. 8 and (○) Ref. 12.

not satisfactory (see Fig. 1). We have found a reasonable description of multiplicity only for case (v) ($\alpha = 0.47$, Fig. 2, dashed lines, $T_p \neq T_t$), where momentum loss is roughly independent of the target mass. In Figs. 3 and 4 we compare results of calculations, performed with such a momentum loss function (dashed lines, $T_p \neq T_t$), with data[8,12] concerning $\langle p_T \rangle$ and $\langle y \rangle$ of negative pions. A significant *disagreement* has been found in the rapidity case. This descrepancy can be removed on the unphysical assumption that, despite the existence of transparency, the temperatures of the projectile and target fire objects are equal: $T_p = T_t$. On such an assumption a good agreement with $\langle y \rangle$ values has been found, whereas the model predictions concerning $\langle n \rangle^{\text{inel}}$ and $\langle p_T \rangle$ have been slightly changed only (solid lines in Figs. 2, 3, and 4).

We have also found an agreement of the experimental values of $\langle y \rangle$ with those calculated under assumption of full thermalization (see Fig. 4). This fact means that the effective center of mass of produced pions is equal to the center of mass of colliding objects. So, if we assume that pions are emitted by two sources, as in the concept of partial transparency, the temperatures of the fire objects have to be *equal.*[13] Because the transparency leads to different temperatures of the fire objects from target and projectile (independent of the form of the function Δp_z), this idea is in *contradiction* with data concerning $\langle y \rangle$ of pions.

What is really needed for description of various experimental data in the frame of the thermodynamical models is the longitudinal collective motion of excited nuclear matter in the own center of mass. Such a motion introduces the anisotropy of radiated particles and reduces the energy which undergoes thermalization. The longitudinal collective motion naturally arises in the hydrodynamical approach to nuclear collisions.[14] In this approach the first stage of interaction is hydrodynamical expansion (along the beam axis) which goes into a thermodynamical one at some critical temperature. Because the hydrodynamical expansion is symmetric (or slightly shifted to the backward hemisphere[15]) in the c.m. of colliding objects and the average temperatures of parts of matter expanding forward and backward are equal, the average rapidity of produced pions should be zero in the c.m. of colliding objects. So, an agreement with experimental data has to be obtained.

We conclude with the following statements. (1) The idea of transparency of nuclear matter is in contradiction with experimental data. (2) The difficulties which arise at the thermodynamical description of pion production without transparency (too weak anisotropy of produced particles, overestimation of pion multiplicities and too high transverse momenta) can be overcome if one assumes that the first stage of nuclear collisions at high energy is governed by hydrodynamics.

The authors are grateful to Dr. P. Danielewicz for permission to use his computing programs. We wish to thank Professor E. Skrzypczak and Professor P. Zielinski and Dr. H. Bialkowska, Dr. V. V. Avdeichikov, and Dr. K. Lang for fruitful discussions.

[1]S. Das Gupta and A. Z. Mekijan, Phys. Rep. 72, 131 (1981).
[2]W. D. Myers, Nucl. Phys. A296, 177 (1978).
[3]J. Gosset, J. I. Kapusta, and G. D. Westfall, Phys. Rev. C 18, 844 (1978).
[4]P. Danielewicz and J. M. Namyslowski, Acta Phys. Pol. B 12, 676 (1981).
[5]In our calculations we have the following density distribution function:

$$\rho(r) = \rho_0 \begin{cases} \left[1 - \left(1 + \frac{R}{a}\right) \frac{sh(r/a)}{r/a} \exp\left(\frac{-R}{a}\right)\right]; & r \leq R \ , \\ \left[\frac{R}{a} ch\left(\frac{R}{a}\right) - sh\left(\frac{R}{a}\right)\right] \frac{\exp(-r/a)}{r/a}; & r \geq R \ , \end{cases}$$

where $R = 1.18\ A^{1/3}$ fm, $\rho_0 = 0.145\ \text{fm}^{-3}$, and $a = \sqrt{1/2}$.

[6]Y. Afek, G. Berlad, G. Eilam, and A. Dar, Nuovo Cimento A43, 485 (1978).

[7]C. D. Aksinenko *et al.*, Nucl. Phys. A348, 518 (1980).

[8]A. Kh. Abdurakhimov *et al.*, Nucl. Phys. A362, 379 (1981).

[9]E. O. Abdurkhmanov *et al.*, Z. Phys. C 56, 1 (1980).

[10]J. I. Kapusta, Phys. Rev. C 16, 1493 (1977).

[11]S. Das Gupta, Phys. Rev. Lett. 41, 1450 (1978); S. Das Gupta and G. S. Lam, Phys. Rev. C 20, 1192 (1979).

[12]H. H. Agakishiev *et al.*, Yad. Fiz. 34, 1517 (1981) [Sov. J. Nucl. Phys. 34, 842 (1981)].

[13]The number of pions emitted from the source is a function of its temperature $f(T)$ and is proportional to the mass M of the source. So, the average rapidity of pions radiated from two sources can be described by the formula

$$\langle y \rangle = \frac{f(T_1)M_1y_1 + f(T_2)M_2y_2}{f(T_1)M_1 + f(T_2)M_2} ,$$

where y_i is the rapidity of an ith source. If we require $\langle y \rangle = 0$ in the c.m. of the sources, where $M_1y_1 \cong -M_2y_2$ (good approximation for $M_1 - M_2$ or $|y_1| + |y_2|$ not too high), we get $f(T_1) = f(T_2)$. So, $T_1 = T_2$.

[14]L. D. Landau, Izv. Acad. Nauk SSSR Ser. Fiz. 17, 51 (1953); *Collected Papers of L. D. Landau,* edited by D. Ter Haar (Gordon and Breach, New York, 1965).

[15]Y. A. Tarasov, Yad. Fiz. 26, 770 (1977) [Sov. J. Nucl. Phys. 26, 405 (1977)].