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Pionic branches in neutron matter and the problem of pion condensation

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The pion spectrum for charged and neutral pions is investigated in pure neutron matter, by letting the pions interact with a neutron Fermi sea in a self-consistent scheme that renormalizes simultaneously the mesons, considered the source of the interaction, and the nucleons. The possibility of obtaining different kinds of pion condensates is investigated with the result that they cannot be reached even for values of the spin-spin correlation parameter, g', far below the range commonly accepted.

I. INTRODUCTION

In a previous paper[1] the authors developed a many body scheme, based upon the meson exchange model for the N-N interaction, which simultaneously renormalizes the mesons and the nucleons in a neutron medium. Within this scheme the nucleon quasiparticle properties are calculated and the mesons are allowed to interact with the renormalized nucleons, in contrast to most calculations where the pions interact with an uncorrelated Fermi sea. Because of this meson renormalization, the N-N interaction itself is changed and becomes density dependent, establishing a problem of self-consistency which is solved accordingly, in an iterative way, until good convergence is found. In the present paper we use these self-consistent results for the nucleon quasiparticle properties to calculate the pion propagator, then analyze the spectrum of the pions.

One of the reasons for looking at the pion spectrum in the neutron medium is the possibility of finding a pion condensate. This problem has attracted much attention in the last few years with a substantial theoretical effort being invested.[2-8] The problem of pion condensation in a neutron star is more complicated than in symmetric nuclear matter since there are in the former case several modes of accommodating pions in the medium. First of all, a neutron star would also contain a small number of protons in order to provide equilibrium against neutron β decay. Let μ_n be the chemical potential for neutrons, μ_p for protons, and μ_e for electrons; charge neutrality would imply $n_e=n_p$, while β equilibrium would require that $\mu_n-\mu_p=\mu_e$. These conditions would give us the number of protons present. Imagine that we have the spectrum of a π^-, $\omega(q,\rho)$ and at some density ρ_c and momentum q_c we have the relationship $\mu_n-\mu_p=\omega(q_c,\rho_c)$; at this point the medium would allow the reaction $n\rightarrow p+\pi^-$ and this would set the threshold for π^- condensation, the most intuitive kind of pion condensation in the neutron medium. However, there are other possibilities, as pointed out by Migdal.[2] A π^+ could develop a branch with negative energy such that $\omega=-(\mu_n-\mu_p)$. At this point there would be free way for the reaction $p\rightarrow n+\pi^+$, thus introducing positive pions in the medium. There is still another situation when a π^+ with negative energy and a π^- with positive energy would together give zero energy, in which case electrically neutral couples $\pi^+\pi^-$ could be produced from the medium at no energy cost. Migdal results show that, surprisingly, the π^+ condensate would be the first one to appear at densities below ρ_0, the normal nuclear matter density; at higher densities the mode $\pi^+\pi^-$ would also be allowed, while the π^- condensate would by dynamically forbidden. These results are in contrast with other calculations[3,4] where the threshold for π^- condensation is found around two times nuclear matter density.

The use of the results obtained in Ref. 1 for the nucleon quasiparticle quantities will lead us to quite different conclusions, basically ruling out the possibility of the condensate at any density.

II. THE PION SPECTRUM

The pion interacts with the medium primarily via the s- and p-wave π-N interaction. The s-wave part of the interaction can be easily cast in the form of a pion self-energy, which can be written as[1]

$$\Pi_r^{(s)}(q)=4\pi\left[\frac{2\lambda_1}{m_\pi}(\rho_n+\rho_p)-r\frac{\lambda_2}{m_\pi^2}2q^0(\rho_n-\rho_p)\right], \quad (1)$$

where r is the pion isospin index, ρ_n and ρ_p the neutron and proton densities, and λ_1 and λ_2 two parameters which can be related on shell to the π-N s-wave scattering lengths.[9] The mechanism for the p wave is somewhat more involved. The pion can produce particle-hole excitations as well as Δ-h excitations, which, once produced, propagate inside the medium via the p-h, Δ-h interaction that contains more elements than just one pion exchange. A model is used in Refs. 1 and 10 which contains one pion exchange and ρ-meson exchange modulated by a short range correlation function. The mechanism for the p-wave pion renormalization is depicted in Fig. 1.

By using the same nomenclature as in Refs. 1 and 10, the series implicit in Fig. 1 can be summed up to give the following result:

$$\mathscr{D}_r(q)=\frac{1-aU_r-b'\vec{q}^{\,2}U_r}{(q^{02}-\vec{q}^{\,2}-m_\pi^2)(1-aU_r-b'\vec{q}^{\,2}U_r)-\dfrac{f^2(q^2)}{m_\pi^2}\vec{q}^{\,2}U_r}\ , \tag{2}$$

where U_r is the Lindhard function for p-h and Δ-h excitations from a pion of isospin r, which is given in detail in the appendix of Ref. 1; $f(q^2)$ is the πNN coupling constant, containing a monopole form factor; and a and b,

$$b'=b-D_\pi^0(q)\frac{f^2(q^2)}{m_\pi^2}\ ,$$

are two functions of the π and ρ coupling constants, form factors, and correlation function, given as well in Ref. 1. The numerical values for all of these factors taken in the present calculation are the same ones used in Refs. 1 and 10.

The main difference of the present work with respect to standard calculations of the nuclear response to a pion source lies in the fact that here renormalized baryon propagators are used (instead of the free ones of a noninteracting Fermi sea) in the evaluation of the p-h and Δ-h Lindhard functions. A constant shift in the particle or hole energy would not affect the p-h Lindhard function since only the difference between the particle and hole energies is involved in the evaluation of U_r. However, the explicit dependence of the nucleon self-energy on the energy and momentum variables has as a consequence important changes in the Lindhard function, which is somewhat reduced with respect to the standard calculation.[1]

In order to include the s-wave pion self-energy one has to add $\Pi_r^{(s)}$ from (1) to m_π^2 in (2). Altogether, the pion self-energy can be written as

$$\begin{aligned}\Pi_r(q)&=\Pi_r^{(s)}+\Pi_r^{(p)}\\ &=\Pi_r^{(s)}+\frac{f^2(q^2)}{m_\pi^2}\vec{q}^{\,2}U_r(1-aU_r-b'\vec{q}^{\,2}U_r)^{-1}\ .\end{aligned} \tag{3}$$

The numerical values for λ_1 and λ_2 are

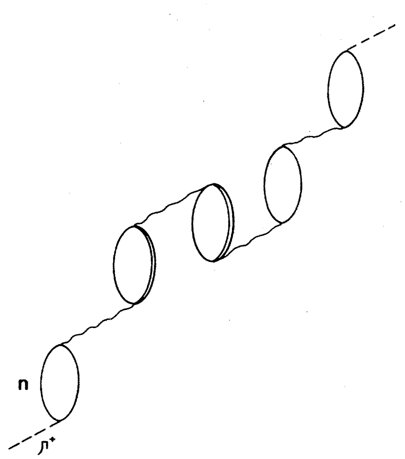

FIG. 1. The mechanism for the p-wave pion renormalization in the neutron medium. The wave lines stand for π and ρ exchange and additional short range correlation.

$$\lambda_1=0.0075,\quad \lambda_2=0.053\ ;$$

thus the s-wave pion self-energy is very small for a π^0, much larger and attractive for a π^+ of positive energy, and repulsive for a π^- of positive energy; the roles are reversed if the energy changes sign. The p wave is essentially attractive in the energy and momentum range where the pion condensates might appear. We can thus envisage the first problem with respect to the possibility of having a π^- condensate in the medium. While now one needs less attraction than in symmetric nuclear matter to produce a pion, since $\epsilon_\pi=\mu_{\rm n}-\mu_{\rm p}$ (and not zero like in symmetric nuclear matter), the s wave for a π^- of this energy is repulsive and one will have to compensate for this extra repulsion.

In order to find the spectrum of the pions in the medium we look at the poles of the pion propagator,

$$D_r^{-1}(q)=q^{02}-\vec{q}^{\,2}-m_\pi^2-\Pi_r(q^0,\vec{q}\,)=0\ . \tag{4}$$

Since $\Pi_r(q^0,\vec{q})$ is in general a complex function, we look at the poles of the real part of (4) in order to define a real pion energy, in the same way as we define the quasiparticle energies of the nucleons in Ref. 1. Thus the equation

$$q^{02}-\vec{q}^{\,2}-m_\pi^2-\mathrm{Re}\Pi_r(q^0,\vec{q}\,)=0 \tag{5}$$

defines q^0 as a function of $\vec{q}$, determining in this way the pion spectrum

$$q^0=\omega(\vec{q})\ . \tag{6}$$

When solving Eq. (5) for a π^+ one normally finds several branches which can correspond to a π^+ or a π^-. The criteria to know to which of the pions the branch corresponds to is the following, according to Migdal:[2] All solutions with

$$2\omega-\partial\Pi_{\pi^+}/\partial\omega>0 \tag{7a}$$

correspond to π^+ mesons, while those with

$$2\omega-\partial\Pi_{\pi^+}/\partial\omega<0 \tag{7b}$$

correspond to π^- mesons after substituting for ω by $-\omega$. Analogously those solutions of (5) for a π^0 correspond to the physical π^0 branch for $\omega>0$.

III. RESULTS AND DISCUSSION

We have looked at the pion spectrum for $\pi^\pm$ and π^0 at different densities, from $0.5\rho_0$ to $4\rho_0$, and have changed the parameter $C_\rho=(f_\rho^2/m_\rho^2)/(f^2/m_\pi^2)$ from 1.1 to 2.3 in order to implement changes in g', the Lorentz-Lorenz parameter. The range most commonly accepted, $g'=0.55$–0.70, would correspond to $C_\rho=1.7$–2.3,[11] which is in good agreement with empirical determinations of the ρ coupling constant.[12] The same ratio is used for the coupling of the ρ meson and pions to Δ isobars.

A typical pion spectrum is shown in Fig. 2 for $\rho=2\rho_0$ and $C_\rho=1.7$. We can appreciate the appearance of several branches in some kinematical domains. The π^+ has a main branch which has developed continuously from the physical free branch $\omega=(q^2+m_\pi^2)^{1/2}$, which we call the "optical" branch. We can appreciate that at $q=0$, $\omega<m_\pi$, which shows the attractive character of the s-wave pion self-energy for π^+ of positive energy. At values of $q\approx 2.5m_\pi$ the branch curves, providing three different values of ω for the same q. This special behavior comes out as an interference of the p-h and Δ-h contribution to the pion self-energy. At smaller momenta there are also two short branches with values of ω close to zero. With respect to the π^-, the corresponding branch is shown as a pole of the π^+ propagator, which appears at negative values of ω. We know that we should change ω to $-\omega$ to get the π^- branch. We can also appreciate the appearance of three branches providing three values of ω for the same q over a large range of momenta. At $q=0$, $\omega>m_\pi$, which shows the repulsive character of the s-wave pion self-energy for negative pions of positive energy. Also shown in the figure is the value of $\mu_n-\mu_p$ with an opposite sign. This allows for a comparison with the π^- branch, drawn there with an opposite sign too.

As noted in Ref. 1 the scheme developed there does not provide a reliable determination of $\mu_n-\mu_p$, since the aim is only to calculate the quasiparticle properties which involve derivatives of the self-energy and not absolute magnitudes of the self-energy. We have thus used here the results of Sjöberg[13] obtained by using the Brueckner-Bethe expansion, which give an appropriate symmetry energy and are also in agreement with more recent results with the hypernetted chain method[14] for the binding energy of neutron matter as a function of the density. For densities above $\rho=2\rho_0$ a smooth extrapolation of the results of Sjöberg had to be done.

We can now discuss the conditions for the threshold of pion condensation by looking at the figure. The threshold for a π^- condensate would come when the π^- branch is tangent to the line $-(\mu_n-\mu_p)$. As seen in the figure the curve is still far away from that situation. A π^+ condensate would require that the negative branch of the π^+ touch the line $-(\mu_n-\mu_p)$, which as we see from the figure is also quite far away. Finally, the possibility of a $\pi^+\pi^-$ couple would require that the negative branch of the π^+ and the π^- branch join at some value of the momentum, which also does not show up in the figure. We might think that by increasing the density the chances for the condensate would increase as is the case for symmetric nuclear matter, but this is not the case here since an increase in ρ implies more repulsion for the π^-, from the s wave, which overcomes the increased attraction from the p wave. As a consequence we find the smallest difference between ω_{π^-} and $\mu_n-\mu_p$ at $\rho\approx 2.5\rho_0$, and this difference is still as large as 70 MeV.

In Fig. 3 we show the spectrum for a π^0, which shows similar features to the π^+, except in the origin, where $\omega\approx m_\pi$ because the s-wave pion self-energy is negligible for the π^0. No extra branches appear with $\omega\approx 0$ in the range of densities studied, thus also excluding the possibility of a π^0 condensate.

We have changed the value of C_ρ to implement changes in g'. In Fig. 4, $C_\rho=2.3$ ($g'\approx 0.7$) and there is an increased repulsion in the pion self-energy. The chances for the pion condensate are equally dim in this case. One could think that by decreasing C_ρ, and consequently g', thus gaining attraction in the pion self-energy, one would increase the chances for the condensate. In Fig. 5 we have plotted the pion spectrum for $C_\rho=1.1$ ($g'\approx 0.4$) and, although the distance to the line $-(\mu_n-\mu_p)$ is reduced, the difference is still significant. An inspection of the figures shows that, unlike the case of symmetric nuclear matter, the parameter g' now does not play such an important role in generating a pion condensate. Indeed, at small values of q, where the curve of the π^- spectrum is closer to the difference of the chemical potentials, the p-wave attraction (that is interfered with by g') cannot compete with the s-wave repulsion, and at large values of q, where the p wave is important, the distance to $-(\mu_n-\mu_p)$ is too large.

The values of g' from empirical determinations in magnetic nuclear excited states[15] or the axial current renormalization in finite nuclei[16,17] suggest values of $g'\approx 0.6$–0.7 and hence, according to our calculations, would rule out pion condensates in neutron stars. One should point out that the values of g' determined there

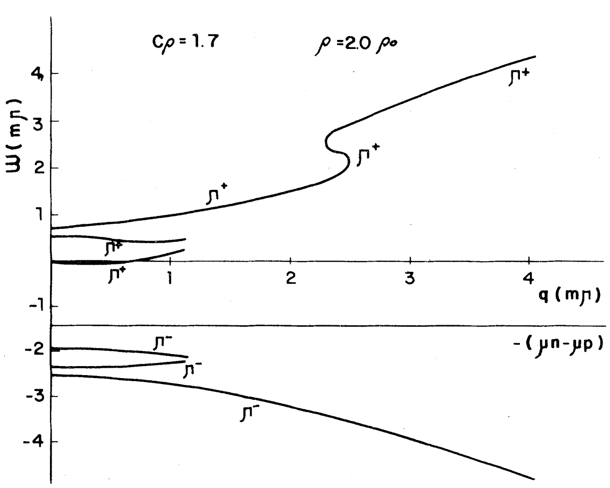

FIG. 2. The spectrum for positive and negative pions at $\rho=2\rho_0$ and $C_\rho=1.7$. The straight line shows the value of the difference between proton and neutron chemical potentials.

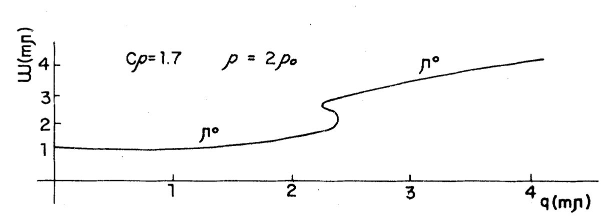

FIG. 3. The spectrum of neutral pions at $\rho=2\rho_0$ and $C_\rho=1.7$.

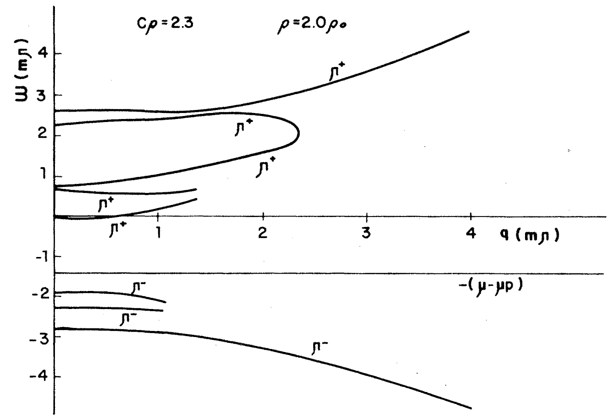

FIG. 4. Same as in Fig. 2 but with $C_\rho=2.3$.

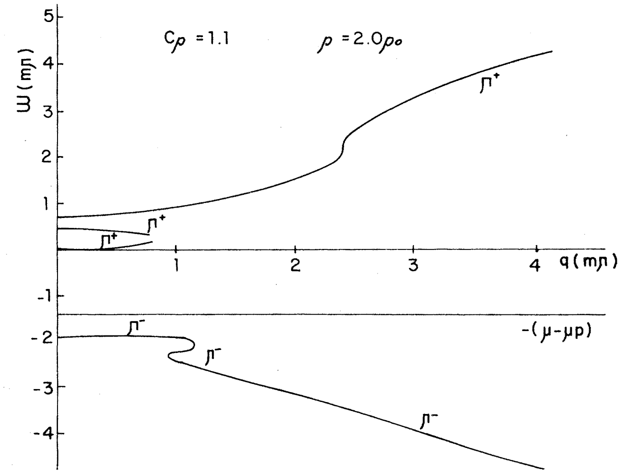

FIG. 5. The spectrum of charged pions at $\rho=2\rho_0$ and $C_\rho=1.1$.

refer to one kinematical region, the region of small momenta, which is quite different to the one where the pion condensate might appear. Indeed, the pion condensates could appear at a momentum of around 2—3 times the pion mass, thus the equivalent value of g' could be smaller in this domain. Though the argument is correct, one should bear in mind that $g'(\omega,\vec{q})$ represents the short range part of the interaction and thus the dependence on $\vec{q}$ must be quite smooth. It is difficult on these grounds to argue that it could go from $g'=0.7$ to 0.35 at $q\approx 2m_\pi$, which is much smaller than the mass of the heavy mesons, assumed in this model to be the source of the short range part. The model of Ref. 11 generates, in fact, a slowly decreasing function on $\vec{q}$, $g'(\omega,\vec{q})$, which certainly does not give that jump. Furthermore, we should stress that in the calculations used here we keep the ω and $\vec{q}$ dependence of $g'(\omega,\vec{q})$, and when we quote the values of g' they are the values calculated at $\omega=\vec{q}=0$, so they should be directly compared to those determined empirically. Other determinations of g' by using the G matrix calculated in nuclear matter also provide a smooth dependence on $\vec{q}$.[18]

We would also like to make some comments with respect to earlier results from Migdal.[2] In the π^0 spectrum, shown in Fig. 3, there is also a small soundlike branch at q^0, $\vec{q}\approx 0$,[2] which is omitted in the figure, but nothing that would lead to a π^0 condensate. On the other hand, the Δ branch, which appears separated from the optical branch in Ref. 2, interferes in our case with the optical branch to give the results of Figs. 2 and 3. We can see that these results are quite sensitive to the input since in Fig. 4, where C_ρ has been increased, the Δ branch and the optical branch appear decoupled.

Although our function $g'(\omega,\vec{q})$ has a smooth dependence on ω and $\vec{q}$, we have kept it independent of the density. Calculations of g' by means of a G matrix that incorporates the induced interaction in the exchange p-h channel lead to a value of g' which increases slightly with the density.[19,20] This would further prevent pion condensation in the neutron medium.

IV. CONCLUSIONS

We have studied the spectrum of charged and neutral pions in a neutron medium by using a many-body scheme that renormalizes simultaneously the pions and the nucleons. The pions are shown to develop several branches, depending on the density and the value of the meson-baryon coupling constants. Apart from the usual "optical" branch, the pions develop other "acoustical" branches, by means of which the phenomenon of pion condensation might appear.

We have studied the pion spectrum for a wide range of values of the spin-spin correlation parameter g' from 0.4 up to 0.7 in the lower side of the empirical estimates of this quantity. In this range the pion spectrum appeared far away from the conditions required for a pion condensate of the π^-, π^+, π^0, or $\pi^+\pi^-$ type. Unlike the situation in symmetric nuclear matter, the increase of the density did not help the condensation because the repulsion from the s-wave πN interaction for a π^- grows faster with the density than the attraction from the p wave. The results of this paper would thus rule out the possibility of any condensates in pure neutron matter and, by a smooth extrapolation, in a neutron star.

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