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Nonmesonic decay of heavy Λ hypernuclei

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We have calculated the ratio of the rate for nonmesonic decay of the Λ in nuclear matter to that of the free Λ decay rate using a pion and rho meson exchange model. Including tensor force effects and a final-state correlation function generated from the Reid-soft-core potential, we estimate $\Gamma_{\rm nm}/\Gamma_{\rm free}$ to be of order 1. A discrepancy with respect to the prior estimate of Adams is resolved.

I. INTRODUCTION

The free Λ particle decays principally into a pion and a nucleon, the nucleon having a momentum of 101 MeV/c. Even if one neglects the Λ binding energy, a Λ at rest in nuclear matter cannot decay into a nucleon of this momentum because the nucleon would lie below the Fermi surface ($k_F=268$ MeV/c). Early in the history of hypernuclear physics it was recognized that this Pauli blocking effect would severely inhibit such a decay of heavy Λ hypernuclei, and that the major decay mode would be via the weak nonleptonic reaction process ΛN→NN.[1] In fact, this nonmesonic decay process is already quite important for $\mathscr{A}=5$.[2]

We know of two early attempts to estimate the nonmesonic decay rate $\Gamma_{\rm nm}$ of a heavy Λ hypernucleus. Dalitz[3] obtained for the ratio of $\Gamma_{\rm nm}$ to the free decay rate Γ_Λ

$$\frac{\Gamma_{\rm nm}}{\Gamma_\Lambda}=2\ , \tag{1}$$

while Adams[4] included correlation effects and obtained as his final result

$$\frac{\Gamma_{\rm nm}}{\Gamma_\Lambda}=0.06\ . \tag{2}$$

Cheng, Heddle, and Kisslinger[5] recently reported a quark model calculation giving $\Gamma_{\rm nm}/\Gamma_\Lambda=0.33$. The only measurement of the lifetime of a heavy hypernucleus of which we are aware is that of Nield *et al.*,[6] who obtained in the $\mathscr{A}=16$ system

$$\tau({}^{16}z_\Lambda)=(0.86^{+0.33}_{-0.26})\times10^{-10}\ {\rm sec}$$

from 22 events, which yields

$$\frac{\Gamma_{\rm nm}}{\Gamma_\Lambda}=3\pm1\ . \tag{3}$$

The discrepancy between Adam's result and the experimental value on the one hand and Dalitz's estimate on the other, and the fact that further experimental studies are proposed[7] demands that the calculation of the decay rate be reinvestigated. We report here such a reinvestigation.

We adopt a pion and rho meson exchange model, depicted in Fig. 1, for the weak nonleptonic ΛN→NN interaction. This is totally analogous to the models of the weak nucleon-nucleon interaction which have been used to study parity mixtures in nuclei.[8] The π exchange mechanism was considered very early by Karplus and Ruderman[9] and is the better defined, because the weak Λ→Nπ vertex can be determined from experiment. The ρ exchange process must, in the factorization approximation, approach the contact interaction considered by Block and Dalitz[10] as m_ρ becomes infinite. The weak ΛNρ coupling is not directly accessible experimentally (perhaps a study of the decay modes of heavy hypernuclei will enable us to determine this coupling), and we have taken the simplest factorization approximation as our guide, knowing that it could be in error by a factor of 3 or so if our experience with weak parity violation in nuclei is any indication.[8,11]

In a preliminary account of this work[12] we reported re-

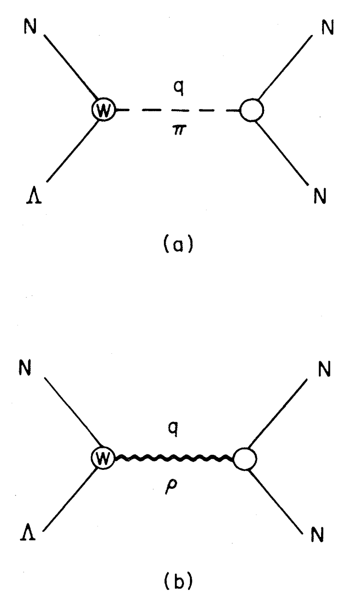

FIG. 1. (a) The pion exchange contribution to ΛN→NN. (b) The ρ meson exchange contribution to ΛN→NN.

sults for $\Gamma_{nm}/\Gamma_\Lambda$ which were obtained using simplified correlation functions and omitting tensor effects. We found ρ exchange terms contributed significantly to Γ_{nm}, reporting as the final result of that calculation

$$\frac{\Gamma_{nm}}{\Gamma_\Lambda}=2.9 \text{ or } 0.1 \; . \tag{4}$$

The value of 2.9 resulted when the π and ρ exchange terms were added and the 0.1 value when they were subtracted, because the relative sign is not known *a priori*. In this paper we include tensor force effects and use correlation functions generated by the Reid-soft-core (RSC) potential. The net effect is that we estimate the ratio $\Gamma_{nm}/\Gamma_\Lambda$ to be of order 1, independent of whether the π and ρ terms are added or subtracted.

In our earlier report we quoted for the Karplus-Ruderman π exchange term alone a value of $\Gamma_{nm}/\Gamma_\Lambda=4.1$ when no correlations were included, passing over without comment the discrepancy between this value and Adams's value of $\Gamma_{nm}/\Gamma_\Lambda=0.51$ for what should have been the same calculation. We have since discovered the cause of this discrepancy: Adams employed a value of the $\Lambda N\pi$-coupling strength which was too small to reproduce the experimental value of the free lifetime Γ_Λ. Of course, this would have no effect upon the ratio $\Gamma_{nm}/\Gamma_\Lambda$ if the same coupling constant were used to calculate both Γ_{nm} and Γ_Λ, as it is in our calculations. However, Adams used his coupling constant to calculate only Γ_{nm}; he used the experimental value of Γ_Λ to form the ratio. All of his results are therefore too small, and should be multiplied by 6.81. When this is done, Adams's result becomes 3.5 in the uncorrelated case, and the discrepancy is almost removed.

In this paper we describe the $\Lambda N\rightarrow NN$ amplitudes that we have constructed in Sec. II. In Sec. III we describe how these amplitudes are modified in nuclear matter, how they may be related to the decay rate for heavy hypernuclei, and the results from our detailed calculations. In Sec. IV we outline a simple, back-of-the-envelope calculation which proves to be a useful way of estimating Γ_{nm}. Finally, in Sec. V we summarize our conclusions.

II. THE ΛN→NN AMPLITUDE

A. The pion exchange contribution

The pion exchange contribution to $\Lambda N\rightarrow NN$ is illustrated in Fig. 1(a). The $\Lambda N\pi$ vertex is fixed by the experimental decay rate and asymmetry parameters. It may be represented by the effective Hamiltonian[12]

$$\mathscr{H}=G_F\mu^2 A\bar{\psi}_N(1+\lambda\gamma_5)\vec{\tau}\Psi_\Lambda\cdot\vec{\phi} \; , \tag{5}$$

where μ is the pion mass and $G_F=1.02\times10^{-5}m_p^{-2}$ is the Fermi constant. The $\Psi_\Lambda=\binom{0}{\psi_\Lambda}$ is the spurion containing the Λ field (which is introduced to incorporate the $\Delta I=\frac{1}{2}$ rule), and A and λ are dimensionless, empirical constants for which we take the values[13]

$$|A|=1.05 \; , \tag{6a}$$

$$\lambda=-6.87 \; . \tag{6b}$$

We note (1) that the phase of A is not determined experimentally, and (2) that a factorization model for the vertex gives the wrong magnitude for A and the wrong sign for λ, although the magnitude of λ is predicted quite well.[13]

Taking the nonrelativistic limit of Eq. (5), the effective Hamiltonian with the usual πNN vertex gives for the amplitude of Fig. 1(a),

$$M_\pi(\vec{q})=G_F\mu^2\frac{Af}{\mu}\left[1+\frac{\lambda}{2m}\vec{\sigma}_1\cdot\vec{q}\right]\frac{\Phi_\pi(\vec{q}^{\,2})}{\vec{q}^{\,2}+\mu^2}i\vec{\sigma}_2\cdot\vec{q}\,\vec{\tau}_1\cdot\vec{\tau}_2 \; , \tag{7}$$

where $\Phi_\pi(\vec{q}^{\,2})$ is the product of the form factors at the weak and strong vertices.

Because we want to include the effects of correlations in determining the amplitude in nuclear matter it is convenient to Fourier transform Eq. (7) to obtain a transition potential $V_\pi(\vec{r})$ in configuration space

$$\begin{aligned} V_\pi(\vec{r}) &= \int\frac{d^3q}{(2\pi)^3}e^{i\vec{q}\cdot\vec{r}}M_\pi(\vec{q}) \\ &= V_c^\pi+V_{pv}^\pi+V_t^\pi \; , \end{aligned} \tag{8}$$

which we have separated into central, parity violating, and tensor parts. The central term has the structure

$$V_c^\pi=\tfrac{1}{3}V_s^\pi(r)\vec{\sigma}_1\cdot\vec{\sigma}_2\vec{\tau}_1\cdot\vec{\tau}_2 \tag{9a}$$

with

$$V_s^\pi(r)=G_F\mu^2\frac{Af}{\mu}\frac{\lambda}{2m}\frac{1}{2\pi^2}W_{0,2}(r,\mu;\Phi_\pi) \; , \tag{9b}$$

while the parity violating term is of the form

$$V_{pv}^\pi=V_p^\pi(r)\vec{\sigma}_1\cdot\hat{r}\vec{\tau}_1\cdot\vec{\tau}_2 \tag{10a}$$

with

$$V_p^\pi(r)=iG_F\mu^2\frac{Af}{\mu}\frac{1}{2\pi^2}W_{1,1}(r,\mu;\Phi_\pi) \; , \tag{10b}$$

and the tensor term is

$$V_t^\pi=V_d^\pi(r)[(\vec{\sigma}_1\cdot\hat{r})(\vec{\sigma}_2\cdot\hat{r})-\tfrac{1}{3}\vec{\sigma}_1\cdot\vec{\sigma}_2]\vec{\tau}_1\cdot\vec{\tau}_2 \tag{11a}$$

with

$$V_d^\pi(r)=G_F\mu^2\frac{Af}{\mu}\frac{\lambda}{2m}\frac{1}{2\pi^2}W_{2,2}(r,\mu;\Phi_\pi) \; . \tag{11b}$$

In Eqs. (9)—(11), the operators subscripted 1 act on the Λ particle, and we have adopted the notation

$$W_{l,m}(r,\mu;\Phi)=\int_0^\infty k^{2+m}dk\frac{j_l(kr)}{k^2+\mu^2}\Phi(r) \; . \tag{12}$$

Note that except in the case of $W_{1,1}$, the integrals required in Eqs. (9)—(11) diverge when no form factor is introduced (i.e., $\Phi_\pi=1$). The divergence corresponds to the well-known delta function in the one pion exchange potential.

However, the strong πNN vertex must be momentum dependent. It is required by the Goldberger-Treiman discrepancy,[14] it is obtained in dispersion relation calculations of the vertex function,[15] and it is obtained in the quark model.[16] There is no reason to suspect that a fundamentally different form factor is required at the weak vertex—indeed for the parity conserving coupling in the pole model, the weak and strong form factors are the same. Following conventional procedure, we combine the strong and weak vertex form factors, and any propagator corrections, into a single form factor $\Phi_\pi(\vec{k}^2)$ which we parametrize with a monopole form

$$\Phi_\pi(\vec{k}^2)=\frac{\Lambda_\pi^2-\mu^2}{\Lambda_\pi^2+\vec{k}^2}\ . \tag{13}$$

Reasonable choices for Λ_π^2 are the following: $\Lambda_\pi^2=10\mu^2$ and $\Lambda_\pi^2=20\mu^2$. The first comes from a semipole (or square root) approximation to the cloudy bag model πNN form factor,[16] and the second is based upon the same approximation to the dispersion relation πNN form factor.[15] The semipole approximation to the πNN form factor, with the assumption that the weak and strong form factors are identical, leads to the monopole form for $\Phi_\pi(\vec{k}^2)$ given in Eq. (13).

Using Eq. (13) we find[17]

$$W_{0,2}=\Lambda_\pi^3 k_0(\Lambda_\pi r)-\mu^3 k_0(\mu r)\ , \tag{14a}$$

$$W_{1,1}=\mu^2 k_1(\mu r)-\Lambda_\pi^2 k_1(\Lambda_\pi r)\ , \tag{14b}$$

$$W_{2,2}=\Lambda_\pi^3 k_2(\Lambda_\pi r)-\mu^3 k_2(\mu r)\ , \tag{14c}$$

where $k_l(x)$ is the spherical Bessel function of the third kind:[18]

$$k_l(x)=\sqrt{(\pi/2x)}K_{l+1/2}(x)\ . \tag{15}$$

In particular, for $l=0$, 1, and 2 one has

$$k_0(x)=\frac{\pi}{2}\frac{e^{-x}}{x}\ , \tag{16a}$$

$$k_1(x)=\frac{\pi}{2}\frac{e^{-x}}{x}\left[1+\frac{1}{x}\right]\ , \tag{16b}$$

$$k_2(x)=\frac{\pi}{2}\frac{e^{-x}}{x}\left[1+\frac{3}{x}+\frac{3}{x^2}\right]\ . \tag{16c}$$

In the limit $\Lambda_\pi\to\infty$, one has $\Phi_\pi\to 1$ and therefore $\Lambda_\pi^3 k_0(\Lambda_\pi r)\sim r^{-2}\delta(r)$, such that one recovers the δ function term in the potential. The correct identification of the singularity at $r=0$ in the case of Eq. (14c) when we let Λ_π become infinite is more subtle, but the singularity in the difference $\Lambda_\pi^3 k_2(\Lambda_\pi r)-\mu^3 k_2(\mu r)$ is a delta function. In any practical case, one is interested in the matrix elements of these potentials taken between wave functions which vanish at the origin, and the delta function singularity is of no consequence. However, in the uncorrelated plane wave calculation different results may be obtained depending upon how the singularity is treated. Thus, one must exercise care in comparing the results of different authors in that case. Having defined the pion exchange transition potential we now turn to the rho exchange contribution.

B. The rho exchange contribution

The ρ meson exchange contribution to the ΛN$\to$NN amplitude is depicted in Fig. 1(b). This time, however, the weak ΛNρ vertex cannot be determined from experiment. It must be estimated theoretically. These theoretical estimates will be discussed below, when we assign numerical values to the constants α, β, and ϵ in the effective Hamiltonian

$$\mathscr{H}_{\rm eff}=\frac{G_F}{\sqrt{2}}m_\rho^2\bar{\psi}_{\rm N}\left[\alpha\gamma^\mu+\beta i\frac{\sigma^{\mu\nu}k_\nu}{2m}+\epsilon\gamma^\mu\gamma_5\right]\frac{\vec{\tau}}{2}\Psi_\Lambda\cdot\vec{\rho}_\mu\ . \tag{17}$$

In writing the effective Hamiltonian we have used $\partial^\mu\rho_\mu=0$ to eliminate possible terms in k^μ and $k^\mu\gamma_5$, assumed the absence of second class currents to eliminate the $\sigma^{\mu\nu}k_\nu\gamma_5$ term, and assumed that the $\Delta T=\frac{1}{2}$ rule remains valid.

In this manner we construct the ρ exchange contributions to the ΛN$\to$NN transition potential (in momentum space):

$$V_\rho=-\frac{Gm_\rho^2}{4\sqrt{2}}g_\rho\frac{\vec{\tau}_1\cdot\vec{\tau}_2}{\vec{q}^{\,2}+m_\rho^2}\left[F_1\alpha-\frac{(\alpha+\beta)(F_1+F_2)}{(2m)^2}(\vec{\sigma}_1\times\vec{q})\cdot(\vec{\sigma}_2\times\vec{q})+i\epsilon\frac{(F_1+F_2)}{2m}\vec{\sigma}_1\times\vec{\sigma}_2\cdot\vec{q}\right]\ . \tag{18}$$

Here we have used as the strong vertex

$$\Gamma_{\mu,i}^{(s)}=g_\rho\left[F_1\gamma_\mu+\frac{F_2}{2m}\sigma_{\mu\nu}k^\nu\right]\frac{\tau_i}{2}\ , \tag{19}$$

where $\vec{k}$ is directed towards the vertex. We have employed the conventions of Bjorken and Drell regarding metric, γ matrices, etc.

Using the well-known identity

$$(\vec{\sigma}_1\times\vec{q})\cdot(\vec{\sigma}_2\times\vec{q})=(\vec{\sigma}_1\cdot\vec{\sigma}_2)\vec{q}^{\,2}-(\vec{\sigma}_1\cdot\vec{q})(\vec{\sigma}_2\cdot\vec{q})\ , \tag{20}$$

the parity conserving part of Eq. (18) may be transformed to configuration space following the procedures of Sec. II A. The parity violating term also has the structure $f(q)\vec{\mathscr{A}}\cdot\vec{q}$, which was the momentum space structure of the π exchange piece; although the spin vector $\vec{\mathscr{A}}$ is different in the two cases, the previous transformation to configuration space applies to this case also.

The resulting transition potential in configuration space is of the same structure as the π exchange potential of Eq. (8)

$$V_\rho(\vec{r})=V_c^\rho+V_{\rm pv}^\rho+V_t^\rho\ . \tag{21}$$

Here we have

$$V_c^\rho=[-V_s^\rho(r)+\tfrac{2}{3}V_{s,\sigma}^\rho(r)\vec{\sigma}_1\cdot\vec{\sigma}_2]\vec{\tau}_1\cdot\vec{\tau}_2\ , \tag{22a}$$

$$V_{\rm pv}^\rho=-V_p^\rho(r)\vec{\sigma}\times\vec{\sigma}_2\cdot\hat{r}\ , \tag{22b}$$

and

$$V_t^\rho = -V_d^\rho(r) T_2(\hat{r}\hat{r}) \cdot T_2(\vec{\sigma}_1 \vec{\sigma}_2) \ . \tag{22c}$$

The radial shapes are given by

$$V_s^\rho(r) = \frac{Gm_\rho^2}{4\sqrt{2}} g_\rho \frac{1}{2\pi^2} W_{0,0}(r, m_\rho; F_1 \alpha_1) \ , \tag{23a}$$

$$V_{s,\sigma}^\rho(r) = \frac{Gm_\rho^2}{4\sqrt{2}} g_\rho \frac{1}{2\pi^2} \frac{1}{(2m)^2} \times W_{0,2}[r, m_\rho; (F_1+F_2)(\alpha+\beta)] \ , \tag{23b}$$

$$V_p^\rho(r) = \frac{iGm_\rho^2}{4\sqrt{2}} g_\rho \frac{1}{2\pi^2} \frac{1}{2m} W_{1,1}[r, m_\rho; (F_1+F_2)\epsilon] \ , \tag{23c}$$

$$V_t^\rho = \frac{Gm_\rho^2}{4\sqrt{2}} g_\rho \frac{1}{2\pi^2} \frac{1}{(2m)^2} \times W_{2,2}[r, m_\rho; (F_1+F_2)(\alpha+\beta)] \ , \tag{23d}$$

where $W_{l,m}(r,\mu;\Phi)$ was defined in Eq. (12). To complete the definition of the ρ exchange potential we must specify the functions α, β, and ϵ which characterize the weak vertex and must specify the form factors F_1 and F_2 used at the strong vertex.

The weak vertex $(\Lambda \to N\rho)_w$ should be calculated using the weak, $\Delta S = 1$ effective Hamiltonian at the quark level,[19] taking matrix elements of this Hamiltonian between bag model or oscillator states. For the case $\Lambda \to N\pi$, in which one can use current algebra to reduce the matrix elements to $\langle B' \mid H_w \mid B \rangle$, this procedure does not give entirely satisfactory results for the hyperon decays.[20] Alternatively, one could use $SU(6)_w$ to relate $(\Lambda \to N\rho)_w$ to the weak vertex $(N \to N\rho)_w$, reviving the suggestion of McKellar and Pick.[21] However, while a consensus seems to be emerging on the magnitude of the parity violating part of $(N \to N\rho)_w$,[22] the parity conserving part is totally unknown. In any event, a knowledge of $(N \to N\rho)_w$ does not uniquely determine the reduced matrix elements in $SU(6)_w$. For these reasons we have not attempted to make a definitive calculation of $(\Lambda \to N\rho)_w$, preferring to use simple models for the amplitudes. In the present experimental circumstances these models should be an adequate guide to the order of magnitude of the ρ meson effect.

The first and simplest model is the factorization model, which has a long history in weak interaction physics. Because the factorization model result is known[23] to be about a factor of $\sin\theta_c \cos\theta_c$ too small for pion decay of the Λ, and because the model gives the wrong relative sign between the parity conserving and parity violating terms, we omit the $\sin\theta_c \cos\theta_c$ factor which naturally appears and treat the relative signs of the pc and pv terms as arbitrary. In this approximation, illustrated in Fig. 2, one has

$$\alpha = \sqrt{(3/2)} \frac{F_1(q^2)}{g_\rho} \frac{m_\rho^2}{m_\rho^2 - q^2} \ , \tag{24}$$

$$\beta = \frac{3}{2} \frac{3}{5} \frac{F_2(q^2)}{g_\rho} \frac{m_\rho^2}{m_\rho^2 - q^2} \ , \tag{25}$$

$$\epsilon = 0.71\sqrt{(3/2)} \frac{F_A(q^2)}{g_\rho} \frac{m_\rho^2}{m_\rho^2 - q^2} \ , \tag{26}$$

where F_1 and F_2 were defined above and F_A is an axial form factor. To obtain these results we have used SU(3) symmetry for the weak currents with D/F ratios of 0, $\frac{3}{2}$, and 2 for the vector, tensor, and axial vector form factors, respectively. The additional factor of $m_\rho^2/(m_\rho^2 - q^2)$ arises from the vector meson dominance representation of the vector form factors.

The second model we consider applies only to the parity conserving weak amplitudes $(\Lambda \to N\pi)_w$—which we shall see below give the dominant contribution to the Λ lifetime. This model is based on the observation that, in the current algebra approach to the pionic decays of the hyperons,[24] the parity conserving decay of the Λ is dominated by the pole graphs of Fig. 3. This pole model determines the coefficient $A\lambda$ in Eq. (5) as

$$\sqrt{2}A\lambda = g_{\pi NN} \left[\frac{(1-\alpha)\sqrt{(2/3)}(D-F)}{m_\Sigma - m_N} + \frac{\sqrt{(3/2)}(F+\frac{1}{3}D)}{m_\Lambda - m_N} \right] , \tag{27}$$

where $(1-\alpha)/\alpha = 1.7$ is the D/F ratio of the strong baryon-baryon-meson vertex where the overall strength is given by the πNN coupling constant, $g_{\pi NN}$. Here, D and F are the D and F reduced matrix elements of the $\Delta S = 1$ weak Hamiltonian (assumed to be the 6 component of an octet), defined by

$$\langle B_i \mid H_w \mid B_j \rangle = G_f \mu_\pi^2 (F f_{i6j} + D d_{i6j}) \ . \tag{28}$$

A fit to the p-wave hyperon decay amplitudes shown in Table I gives

$$g_{\pi NN} \frac{D-F}{m_\Sigma - m_N} = 65.3 \ , \tag{29}$$

$$g_{\pi NN} \frac{(F+\frac{1}{3}D)}{m_\Lambda - m_N} = 35.1 \ . \tag{30}$$

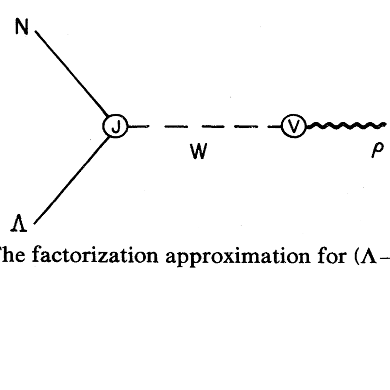

FIG. 2. The factorization approximation for $(\Lambda \to Nm)_w$.

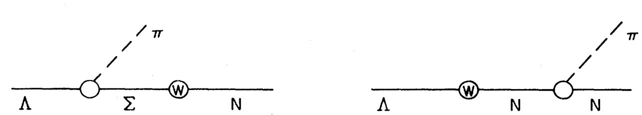

FIG. 3. The baryon pole graphs for the parity conserving part of $(\Lambda \to N\pi)_w$.

TABLE I. Pole fit to $B \to B'\pi$ parity conserving amplitudes. The fit has been chosen to favor the Λ and Σ amplitudes. The data are from Ref. 25. The fitted values are $F = 4.81\mu$, $D = -4.12\,\mu$, and $D/F = -0.88$.

Process	Amplitude/$(G_F\mu^2)$ Observed	Fitted
$\Xi^- \to \Lambda\pi^-$	6.73 ± 0.41	13.7
$\Lambda \to p\pi^-$	10.17 ± 0.24	9.4
$\Sigma^+ \to n\pi^+$	19.05 ± 0.16	19.0
$\Sigma^+ \to p\pi^0$	12.04 ± 0.59	12.0
$\Sigma^- \to n\pi^-$	-0.65 ± 0.08	2.0

We can then approximate the pc part of $(\Lambda \to N\rho)_w$ with pole diagrams similar to those of Fig. 3 but with a ρ meson replacing the π meson. The result may be expressed as

$$\alpha = F_1 \frac{\mu^2}{m_\rho^2} \left[\frac{(1-\alpha_1)\sqrt{(2/3)}(D-F)}{m_\Sigma - m_N} + \frac{\sqrt{(3/2)}(F + \frac{1}{3}F)}{m_\Lambda - m_N} \right] \tag{31}$$

and

$$\beta = F_2 \frac{\mu^2}{m_\rho^2} \left[\frac{(1-\alpha_2)\sqrt{(2/3)}(D-D)}{m_\Sigma - m_N} + \frac{\sqrt{(3/2)}(F + \frac{1}{3}D)}{m_\Lambda - m_N} \right], \tag{32}$$

with the same weak Hamiltonian parameters D and F as in the π case, but with $(1-\alpha_1)/\alpha_1$ and $(1-\alpha_2)/\alpha_2$ being the vector current D/F ratios for the γ_μ and $\sigma_{\mu\nu}k^\nu$ coupling, namely 0 and $\frac{3}{2}$. Numerically, one has

$$\alpha = (0.088)\sqrt{(3/2)}F_1 \tag{33}$$

and

$$\beta = -(0.053)\tfrac{3}{5}\sqrt{(3/2)}F_2 \ . \tag{34}$$

The effect of using the pole model has been to substantially reduce the value of the weak form factors α and β with respect to the factorization model values, and even to change the sign of one of the form factors. The q^2 dependence of the form factors is also altered. We note that the tensor potential of Eq. (23d) is proportional to $(\alpha+\beta)$. When evaluated at $q^2=0$ [where $F_1(0)=1$, $F_2(0)=3.7$], one finds from Eqs. (33) and (34)

$$(\alpha+\beta)_{q^2=0} = -0.036 \ ; \tag{35}$$

but one finds

$$(\alpha+\beta)_{q^2=0} = 0.79 \tag{36}$$

from Eqs. (24) and (25) with $g_\rho^2/4\pi = 2$. These results differ by a factor of -20, which is a very significant difference, both in magnitude and sign.

We can only conclude from these model studies that the $(\Lambda \to N\rho)_w$ vertex is very model dependent. The critical question is whether the dimensionless parameter measuring the strength of the $B \to B'M$ weak interaction type is $G_F m_M^2$—as occurs naturally in the factorization models—or is $G_F\mu^2$ independent of M—as will occur in pole models fitted to the $B \to B'\pi$ data. Perhaps experiments such as the observation of the nonmesonic decay rate of heavy Λ hypernuclei will help decide this question.

Finally, it is necessary to specify the form factors F_1 and F_2. These have recently been discussed extensively in the context of three-body forces.[26] It has been suggested that $F_1/F_2 \neq$ constant. It was also pointed out that one can use form factors to extrapolate between the Höhler and Pietarinen value of $\kappa = F_2/F_1 = 6.6$ (the value at $q^2 = m_\rho^2$) and the vector dominance model value of $\kappa = 3.7$ (the value at the photon mass shell $q^2=0$). To simplify the structure of the resulting potential we have fixed the ratio at $q^2 = m_\rho^2$,

$$F_2/F_1 = 6.6 \ , \tag{37}$$

but we utilize

$$F_1(q^2)/F_2(q^2) = \frac{\Lambda_\rho^2 - m_\rho^2}{\Lambda_\rho^2 - q^2} \tag{38}$$

with a value of

$$\Lambda_\rho^2 = 2.27 m_\rho^2 \ . \tag{39}$$

This rather low value of Λ_ρ reflects the known rapid variation of F_2. Because it happens that the tensor potential produces the principal effect, we have chosen to select our form factor to match the variation of the dominant $\sigma_{\mu\nu}k^\nu$ contribution to that part of the potential.

Now that the form factors have been specified, we may obtain the radial forms of the potential in Eq. (23). For the case in which the factorization approximation is employed, these are

$$V_s^\rho(r) = \frac{Gm_\rho^2}{4\sqrt{2}} \frac{1}{2\pi^2} \sqrt{(3/2)} m_\rho^2 \left\{ \frac{r}{2}[k_0(m_\rho r) + k_0(\Lambda_\rho r)] + \frac{2}{\Lambda_\rho^2 - m_\rho^2}[\Lambda_\rho k_0(\Lambda_\rho r) - m_\rho k_0(m_\rho r)] \right\}, \tag{40a}$$

$$V_{s,\sigma}^\rho(r) = \frac{Gm_\rho^2}{4\sqrt{2}} \frac{1}{2\pi^2} \sqrt{(3/2)} \frac{(1+\kappa)(1+\frac{3}{5}\kappa)}{(2m)^2} m_\rho^2 \left\{ \frac{r}{2}[\Lambda_\rho^2 k_0(\Lambda_\rho r) + m_\rho^2 k_0(m_\rho r)] + \frac{\Lambda_\rho^2 + m_\rho^2}{\Lambda_\rho^2 - m_\rho^2}[\Lambda_\rho k_0(\Lambda_\rho r) - m_\rho k_0(m_\rho r)] \right\}, \tag{40b}$$

$$V_p^\rho(r)=i\frac{Gm_\rho^2}{4\sqrt{2}}\frac{1}{2\pi^2}\frac{(1+\kappa)(0.71g_A)}{2m}\sqrt{(3/2)}m_\rho^2\left\{\frac{r}{2}[m_\rho k_0(m_\rho r)+\Lambda_\rho k_0(\Lambda_\rho r)]+\frac{2}{\Lambda_\rho^2-m_\rho^2}[\Lambda_\rho^2 k_1(\Lambda_\rho r)-m_\rho^2 k_1(m_\rho r)]\right\}, \tag{40c}$$

$$V_d^\rho(r)=\frac{Gm_\rho^2}{4\sqrt{2}}\frac{1}{2\pi^2}\frac{(1+\kappa)(1+\frac{3}{5}\kappa)}{(2m)^2}\sqrt{(3/2)}m_\rho^2\left\{\frac{r}{2}[m_\rho^2 k_1(m_\rho r)+\Lambda_\rho^2 k_1(\Lambda_\rho r)]+\frac{2}{\Lambda_\rho^2-m_\rho^2}[\Lambda_\rho^3 k_2(\Lambda_\rho r)-m_\rho^3 k_2(m_\rho r)]\right\}. \tag{40d}$$

In the case of the pole model, different radial forms are obtained because of the different form factor structure of the model. As an example we quote the tensor potential from Eqs. (33) and (34),

$$V_d^\rho(r)=\frac{G\mu^2}{4\sqrt{2}}\frac{1}{2\pi^2}\frac{(1+\kappa)(1-0.36\kappa)}{(2m)^2}\times(2.70)\sqrt{(3/2)}(\Lambda_\rho^2-m_\rho^2)\left\{\frac{1}{\Lambda_\rho^2-m_\rho^2}[m_\rho^3 k_2(m_\rho r)-\Lambda_\rho^3 k_2(\Lambda_\rho r)]-\tfrac{1}{2}\Lambda_\rho^2 r k_1(\Lambda_\rho r)\right\}. \tag{41}$$

Note the appearance of $G\mu^2$ in Eq. (41) compared to Gm_ρ^2 in Eq. (40), a reflection of the difference in the models of the weak vertex. The strength is further reduced by the $(1-0.36\kappa)$ factor, resulting in a ρ exchange potential in the pole model which is signficantly weaker than that in the factorization approximation.

The complete transition potential is obtained by adding the ρ exchange potential obtained in this subsection to the π exchange potential derived in Sec. II B. It may then be used to compute the transition rate.

III. THE ΛN→NN TRANSITION RATE FOR A Λ PARTICLE IN NUCLEAR MATTER

The rate due to the ΛN→NN transition for a Λ particle in nuclear matter is obtained by calculating the matrix element of the transition potential $V_\pi+V_\rho$ between the initial ΛN state and the final NN state, antisymmetrizing with respect to the final nucleons, squaring, dividing by the initial flux, integrating over the possible initial (nuclear matter) nucleon states, and summing over the final state phase space. It is useful to make a number of simplifying approximations in this calculation. The validity of some of these approximations has been tested by Adams,[4] and the corrections are quite small. In particular we will assume that:

(i) The initial ΛN state is an s-wave state, with zero relative momentum between the Λ and the N.

(ii) The relative momentum of the final N-N pair ranges from 407 to 429 MeV/c. We have computed the matrix element at a fixed relative momentum which we took to be $Q=420$ MeV/c. We have verified that the matrix elements vary by only some 10% over the range quoted.

(iii) We ignore the spin dependence of the initial- and final-state wave functions.

This has little effect on the final rate. Because of the $\kappa=6.6$ factor, the $V_{s,\sigma}^\rho$ part of V_ρ is larger by a factor of 6.3 (for the longest $e^{-m_\rho r}$ term) than the central V_s^ρ term; we therefore neglect the latter contribution from the outset.

In our initial survey to establish the dominant effect, we make two additional approximations, which we later relax:

(a) We neglect the tensor correlations in the initial and final states induced by the strong nucleon-nucleon interaction.

(b) We multiply the potential by an empirical correlation function, $1-e^{-r^2/R^2}$ with $R\cong0.75$ fm and calculate matrix elements using plane wave states.

The rate for the pion induced transition is given by

$$\Gamma_{\rm nm}^{(\pi)}=\frac{mQ\rho}{\pi^3}(G_F f A)^2\left[\frac{3}{2}\left[\frac{\lambda\mu}{2m}\right]^2|F_{00}|^2+\tfrac{9}{2}|F_{10}|^2+6\left[\frac{\lambda\mu}{2m}\right]^2|F_{20}|^2\right], \tag{42}$$

where

$$F_{l0}=\mu^{1+m_l}\int_0^\infty r^2dr\,\psi_l(Q,r)\phi_0(\bar{k}r)W_{l,m_l}(r,\mu;\Phi_\pi) \tag{43}$$

describes transitions from an initial s state to a final state with angular momentum l, and

$$m_l=\tfrac{1}{2}[3+(-1)^l]=\begin{cases}2, & l \text{ even}\\ 1, & l \text{ odd}\,.\end{cases} \tag{44}$$

This leads to

$$\frac{\Gamma_{\rm nm}^{(\pi)}}{\Gamma_{\rm free}}=1.009(0.388|F_{00}|^2+4.5|F_{10}|^2+1.55|F_{20}|^2) \tag{45}$$

which yields the results in Table II, where we illustrate the effects of including a form factor and of including correlations with different correlation lengths R.

First we note that the formula (42) is consistent with that given by Adams, and that our uncorrelated value for $\Gamma_{\rm nm}^{(\pi)}/\Gamma_{\rm free}$ is in reasonable agreement with Adams's value, after the latter has been corrected as was discussed in the Introduction. Next we note that including correlations decreases $\Gamma_{\rm nm}^{(\pi)}$ and increasing the correlation length decreases $\Gamma_{\rm nm}^{(\pi)}$ still more, as one would expect intuitively.

TABLE II. $(\Gamma_{\rm nm}^{(\pi)}/\Gamma_{\rm free})$ from π exchange.

	$s\to s$	$s\to p$	$s\to d$	Total
No form factor, No correlations	0.01	1.00	3.12	4.13
No form factor, $R=0.7$ fm	0.003	0.54	1.95	2.49
No form factor, $R=0.75$ fm	0.001	0.44	1.87	2.31
No form factor, $R=1.0$ fm	2×10^{-4}	0.25	1.31	1.56
Form factor, $\Lambda_\pi^2=20\mu^2$, $R=0.75$ fm	0.031	0.005	1.03	1.06

Our results at this point are consistent with Adams's results without tensor correlations. The $s\to d$ transition dominates the amplitude, but the $s\to p$ parity violating transition gives a significant contribution, until the form factor effects which reduce the $s\to p$ transition contribution drastically are included.

We remark in passing that, even with $R=1.41$ fm, we find $\Gamma_{\rm nm}^{(\pi)}/\Gamma_{\rm free}=0.75$. Cheng, Heddle, and Kisslinger[5] quote $\tau_{\rm nm}^{(\pi)}=0.16\times10^{-6}$ sec, which when corrected for their use of Adams's parameters becomes equivalent to $\Gamma_{\rm nm}^{(\pi)}/\Gamma_{\rm free}=1.11$, for the π contribution cut off at 0.8 fm. This seems to correlate reasonably with our Gaussian cutoff calculations.

Including a form factor reduces the total rate by a factor of about $\frac{1}{2}$, but reduces the $s\to p$ contribution to the total rate substantially, so that all but 3% of the nonmesonic decay process in the nucleons proceeds through the $s\to d$ transition induced by the tensor component of the one pion exchange $\Lambda N\to NN$ transition potential.

Next we must investigate the consequence of adding the ρ exchange parts of the transition potential. The central and tensor parts of the transition potential are readily included by straightforward modifications of Eq. (42), which we will specify below. The parity violating part of the ρ exchange potential has a new spin structure, and the modification of the $s\to p$ transition rate is not so obvious. The result is that the $s\to p$ transition rate becomes

$$(\Gamma_{\rm nm}^{\pi+\rho})_{s\to p}=\frac{mQ\rho}{\pi^3}\left[\tfrac{9}{2}(G_FfA)^2|F_{10}^\pi|^2+\tfrac{4}{3}\left[\frac{G_Fg_\rho}{4\sqrt{2}}\frac{m_\rho}{2m}\right]^2|F_{10}^\rho|^2+\tfrac{2}{3}(G_FfA)\left[\frac{G_Fg_\rho}{4\sqrt{2}}\frac{m_\rho}{2m}\right]\mathrm{Re}(F_{10}^\pi)^*(F_{10}^\rho)\right], \tag{46}$$

where

$$F_{10}^\rho=m_\rho^2\int_0^\infty r^2dr\,\psi_l(Q,r)\phi_0(\bar{k}r)\times W_{1,1}[r,m_\rho;(F_1+F_2)\epsilon]\,. \tag{47}$$

Without form factors, this alters the $s\to p$ decay rate induced by π exchange by 30% for the factorization parameters. It is increased or decreased depending on the unknown relative sign of the π and ρ exchange potentials. This partial rate is also greatly reduced when the form factors are included.

Similar results hold for the $s\to s$ transitions, induced by the central potential, but there remain to be considered the $s\to d$ transitions induced by the tensor potential. For these transitions we may write

$$(\Gamma_{\rm nm}^{\pi+\rho})_{s\to d}=\frac{6mQ\rho G_F^2}{\pi^3}\left|\frac{Af\lambda\mu}{2m}F_{20}^\pi-\frac{m_\rho^2}{(2m)^2}\frac{g_\rho}{\sqrt{2}}F_{20}^\rho\right|^2, \tag{48}$$

where

$$F_{20}^\rho=m_\rho^3\int_0^\infty r^2dr\,\psi_2(Q,r)\phi_0(\bar{k},r)\times W_{2,2}[r,m_\rho;(F_1+F_2)(\alpha+\beta)]\,. \tag{49}$$

Numerical results from this expression are presented in Table III for the factorization form of the potential. The ρ contribution is $\frac{1}{4}$ to $\frac{2}{3}$ of the π contribution in the rate, but because the contributions add in the amplitude the effect of including both terms can be quite dramatic. Since we cannot trust the factorization approximation to give the relative signs of the amplitudes correctly[3] we quote answers for both constructive and destructive interference of the amplitudes. Had we used the pole model, the contribution of the ρ exchange transition potential would have been reduced by a factor of order 10^{-3} and would be quite negligible.

It is clear that our results with form factors included are consistent with the datum.[6] Adams[4] reported that including tensor-force, final-state interactions significantly reduced the $s\to d$ transition rate. For this reason we relax

TABLE III. ($\Gamma_{nm}/\Gamma_{free}$) including π and ρ exchange ($s \to d$ transitions only). The ρ potential is the factorization type.

	π alone	ρ alone	$\pi+\rho$	$\pi-\rho$
No form factor No correlations $\kappa_\rho=3.7$	3.12	0.49	6.08	1.13
No form factor $R=0.75$ fm $\kappa_\rho=6.6$	1.86	0.26	3.52	0.72
No form factor $R=0.75$ fm $\kappa_\rho=6.6$	1.86	1.13	6.13	0.06
Form factor $R=0.75$ fm $\kappa_\rho=6.6$ $\Lambda_\pi^2=20\mu^2,\Lambda_\rho^2=2.27m_\rho^2$	1.03	0.49	2.91	0.10

assumptions (a) and (b) above and recalculate the dominant parity conserving transition rate.

The tensor force couples the s and d partial waves, both in nuclear matter and in the final state. Because we have assumed that the relative momentum of the initial Λ-nucleon pair is very small, we can neglect the d-wave component in the initial state. In this case, Eq. (48) becomes

$$(\Gamma_{nm})_{s+d}=96mQ\rho\pi \sum_{i=\alpha,\gamma} |\mathscr{F}_{(i,0)}^{(0,s)}+2\sqrt{2}\mathscr{F}_{(i,0)}^{(0,d)}|^2 , \quad (50)$$

where the potential is written in the form

$$V=[\tfrac{1}{3}V_{s,\sigma}(r)\vec{\sigma}_1\cdot\vec{\sigma}_2+V_d(r)T_2(\vec{r}\,\vec{r})\cdot T_2(\vec{\sigma}_1\vec{\sigma}_2)](\vec{\tau}_1\cdot\vec{\tau}_2) \quad (51)$$

and

$$\mathscr{F}_{(i,l')}^{(0,l)}=\int_0^\infty r^2dr\, u_{(l',i)}^{j(-)*}(Q,r)V_l(r)u_{01;01}^1(\bar{k},r) . \quad (52)$$

Here, the $u_{(l_f,i)}^{j(-)}$ are the scattering-state, Blatt-Biedenharn eigenphase solutions, and $u_{01;01}^1(\bar{k},r)$ is the s-state component of the initial $^3S_1+{}^3D_1$ wave function. The $^1S_0 \to {}^1S_0$ transition is omitted in Eq. (50); but as we have seen that all $s \to s$ transitions are negligible, this is an acceptable approximation.

Using the factorization approximation for the NΛρ vertex, the Reid soft core potential to generate the scattering states, and a nuclear matter correlation function due to Negele,[27] we obtain the results given in Table IV. Comparing these results to those in Table III we see that the tensor force in the final state, and a more realistic correlation function have not altered the results very much, in contrast to the results obtained by Adams. Once again we note that the pole model for the ΛNρ vertex would have given a negligible contribution.

IV. A SIMPLE ESTIMATE OF ΛN→NN

It is useful, especially when the literature contains widely different estimates of the value of Γ_{nm}, to be able to generate a simple estimate to establish the order of magnitude of the effect. The transition appears to be dominated by the parity conserving π exchange transition potential and it can therefore be represented, using pole dominance of the ΛNπ vertex, as in Fig. 4. If we consider just the first diagram and compensate for this by renormalizing the matrix element of H_w to give the total $\Lambda \to N\pi$ p-wave amplitude, we can then estimate

$$\sigma_{\Lambda N\to NN} \sim \left[\frac{K}{m_\Lambda - m_N}\right]^2 \sigma_{NN\to NN} , \quad (53)$$

where K is the renormalized matrix element of H_w. We then take

$$\Gamma_{\Lambda N\to NN} \sim \bar{v}\rho\sigma_{\Lambda N\to NN}$$
$$\sim \bar{v}\rho\left[\frac{K}{m_\Lambda - m_N}\right]^2 \sigma_{NN\to NN} . \quad (54)$$

Using $\bar{v}=3k_F/4m$, $\rho=0.17$ fm^{-3}, and $K/(m_\Lambda - m_N) =1.1\times10^{-7}$, we find

$$\frac{\Gamma_{nm}}{\Gamma_{free}} \sim 0.3\left[\frac{\sigma_{NN}}{100\text{ mb}}\right] , \quad (55)$$

where 100 mb is a reasonable estimate of the appropriate NN cross section. Because this simple estimate is of the

TABLE IV. Parity conserving transitions contributing to the ΛN→NN rate in nuclear matter. The values quoted are for $\Gamma_{nm}/\Gamma_\Lambda$ and include correlations and tensor force effects.

	$\Gamma_{nm}/\Gamma_\Lambda$
π only, no form factor	2.03
π only, form factor $\Lambda_\pi^2=10\mu^2$	0.54
π only, form factor $\Lambda_\pi^2=20\mu^2$	0.97
ρ only, form factor $\Lambda_\rho^2=2.27m_\rho^2$	0.52
$\pi+\rho$, form factor $\Lambda_\pi^2=20\mu^2$, $\Lambda_\rho^2=2.27m_\rho^2$	2.33
$\pi-\rho$, form factor $\Lambda_\pi^2=20\mu^2$, $\Lambda_\rho^2=2.27m_\rho^2$	0.71

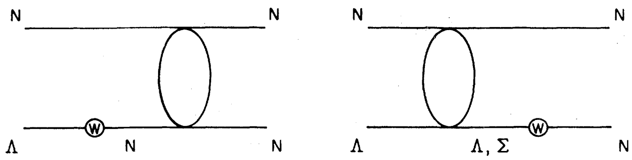

FIG. 4. Pole representation of the transition potential.

same order as our detailed results, we feel confident that we have not omitted any significant effects.

V. CONCLUSIONS

Because of the sensitivity of the results to the unknown $\Lambda N\rho$ vertex, we cannot give a precise prediction of Γ_{nm}. It is clear from Tables III and IV that the estimates can vary by factors of 2 or 3.

However, although the results are also sensitive to the choice of form factors, as are many results in nuclear physics today, they are not particularly sensitive to the nuclear wave functions. This holds open the possibility of using experimental results on Γ_{nm} to fix the $\Lambda N\rho$ coupling. This will not be easy, but it would be informative because obtaining information on these couplings from other experiments is difficult.

Lacking this information, perhaps the best way to quote our final result is as

$$\log_{10}\frac{\Gamma_{nm}}{\Gamma_\Lambda}=0\pm0.3 \ . \qquad (56)$$

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