Interference effects in $(\pi,\pi N)$ reactions

Y. Ohkubo and L. C. Liu
Isotope and Nuclear Chemistry Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545
(Received 8 February 1984)

Contributions of quasifree and nonquasifree reaction processes to $(\pi,\pi N)$ reactions have been investigated across the (3,3) resonance region. We demonstrate that the quantum mechanical interference between these two types of reaction processes significantly affects the calculated π^-- and π^+-induced cross sections, as well as their ratio. In the case of incident π^-, the contributions from the interference term even exceed those from the pure nonquasifree term. This investigation of the $(\pi,\pi N)$ reaction thus indicates that the interpretations given to the parameter introduced into various published semiclassical models, in which interference effects were ignored, are incorrect.

I. INTRODUCTION

Pion-induced single-nucleon removal (SNR), the $(\pi,\pi N)$ reaction, represents a valuable tool to probe nuclear structure and to investigate pion-nucleus dynamics. In particular, since both decay products of the (3,3) resonance, the pion and the nucleon, can be simultaneously measured, the $(\pi,\pi N)$ reaction can be used to study more directly the propagation of the $\Delta(1232)$ in a nuclear medium. To take full advantage of these features of the $(\pi,\pi N)$ reaction, it is essential to have a complete understanding of the reaction mechanisms.

Experimental investigations of $(\pi,\pi N)$ reactions can be divided into two complementary groups. In one group of experiments the outgoing pion and nucleon are detected by coincidence techniques. These experiments have usually been carried out under specifically selected geometries optimal for studying the quasifree aspects of the reaction.[1,2] In the other group of experiments, the outgoing pion and nucleon are not measured. Instead, the reaction was studied via the identification of the residual nucleus, either by using activation techniques,[3,4] or by detecting prompt gamma rays emitted by the residual nucleus.[5] In these latter approaches, all aspects of the SNR, quasifree as well as nonquasifree, are reflected in the measured integrated $(\pi,\pi N)$ cross sections. We show in this work that contributions to $(\pi,\pi N)$ activation cross sections arising from both quasifree and nonquasifree processes are important and that a careful quantum mechanical treatment of these processes is crucial to the correct interpretation of the data.

The first accurate experimental measurements that reveal the important nonquasifree characteristics of $(\pi,\pi N)$ reactions were due to Dropesky *et al.* and Batist *et al.*[3] These authors noted that throughout the entire resonance region, the activation cross section ratio R of $^{12}C(\pi^-,\pi N)^{11}C$ to $^{12}C(\pi^+,\pi N)^{11}C$ differ considerably from theoretical predictions based only on the quasifree knockout mechanism. In a pure quasifree model the theoretical ratio can be schematically expressed as $R_{th}=N/D$ with

$$N=\sigma[^{12}C(\pi^-,\pi^-n)^{11}C]$$

and

$$D=\sigma[^{12}C(\pi^+,\pi^+n)^{11}C]+\sigma[^{12}C(\pi^+,\pi^0p)^{11}C] .$$

Here, σ stands for the cross sections. At the pion energy of 180 MeV, one obtains $R_{th}\simeq 2.9$, when the plane-wave impulse approximation (PWIA) is used, and $R_{th}\simeq 2.5$, when the distorted-wave impulse approximation (DWIA) is used. The experimental value is 1.59 ± 0.07 (Ref. 3). Deviations from theoretical predictions based on the quasifree model have also been noted, but less dramatic, in coincidence experiments.[2]

In 1969, Hewson proposed a mechanism for nonquasifree knockout processes[6] by considering $R'=N'/D'$ with

$$N'=|A[^{12}C(\pi^-,\pi^-n)^{11}C]+B[^{12}C(\pi^-,\pi^-p\to n)^{11}C]|^2 \tag{1.1}$$

and

$$D'=|A[^{12}C(\pi^+,\pi^+n)^{11}C]+B[^{12}C(\pi^+,\pi^+p\to n)^{11}C]|^2 + |A[^{12}C(\pi^+,\pi^0p)^{11}C]|^2 . \tag{1.2}$$

Here, $A[^{12}C(\pi,\pi N)^{11}C]$ denotes quasifree knockout amplitudes and $B[^{12}C(\pi,\pi p\to n)^{11}C]$ denotes amplitudes for the sequential processes $\pi+{}^{12}C\to\pi+p+{}^{11}B \to\pi+n+{}^{11}C$. In sequential processes, quasifree knockout is followed by final-state charge exchange (CX) between the outgoing nucleon and the residual nucleus. Hewson estimated the contributions of the final-state nucleon charge exchange (NCX) processes using plane waves for pions and only the P_{33} wave in the pion-nucleon scattering amplitude. By postulating various *ad hoc* nucleon-nucleus charge-exchange potentials, he found that the inclusion of NCX can give a value for R' between 1.50 and 1.97. Owing to the omission of distortions of pion wave functions, Hewson was, however, unable to predict the absolute magnitude of the cross sections for $^{12}C(\pi^+,\pi N)^{11}C$ and $^{12}C(\pi^-,\pi N)^{11}C$. Furthermore, as a result of using only the πN P_{33} wave in his calculations Hewson could not predict the correct energy dependence of R. Other researchers[7,8] have since simplified Hewson's NCX model to obtain the energy dependence of the cross sections and of the cross-section ratio. In Ref. 7, for example, the ratio is defined as $R''=N''/D''$ with

$$N''=(1-P_{CX})\sigma(\pi^-n\to\pi^-n)+P_{CX}\sigma(\pi^-p\to\pi^-p) \tag{1.3}$$

and

$$D''=(1-P_{CX})\sigma(\pi^+n\to\pi^+n)+P_{CX}\sigma(\pi^+p\to\pi^+p) +(1-P_{CX})\sigma(\pi^+n\to\pi^0p)\ . \tag{1.4}$$

Here, the quantity P_{CX} is assumed to represent the probability of having NCX in the final state. The σ's are free πN scattering cross sections. The P_{CX} was estimated from a rough calculation with one parameter, which was fit to the data at one energy. A formalism similar to Eqs. (1.3) and (1.4) has been employed in Ref. 9 to estimate pion charge exchange contributions to (π,πN) reactions.

We note the crucial difference between Eqs. (1.1) and (1.2) and Eqs. (1.3) and (1.4). In the former equations, the amplitudes are added. In Eqs. (1.3) and (1.4), the cross sections are added. Consequently, the interference between the amplitudes A and B is left out in the calculation of R''. Clearly, the use of Eqs. (1.3) and (1.4) to calculate ^{12}C(π^+,πN)^{11}C and ^{12}C(π^-,πN)^{11}C cross sections and their ratio is incorrect. This is because when more than one amplitude contributes to the same final state, according to the basic principle of quantum mechanics, it is the amplitudes, not the cross sections, that should be added. Using Eqs. (1.3) and (1.4) might be acceptable only if the interference between A and B is unimportant. However, as we shall show, this is not the case with (π,πN) reactions on ^{12}C.

In Sec. II, we present the formalism for the ^{12}C($\pi^\pm,\pi$N)^{11}C reactions. The formalism can be readily generalized to deal with (π,πN) reactions on nuclei other than ^{12}C. In addition to direct knockout and NCX, we have also included in our calculations *pion* charge exchange (πCX) in the final states, and have fully taken into account interferences among all those processes which contribute coherently to the reaction. We have also examined πCX in the initial state. In the case of ^{12}C, it corresponds to charge exchange leading to nonanalog nuclear states. We have found that contributions by initial-state πCX are negligible in the present case, and thus we will leave them out in the present work. In calculating the distortions and final-state charge exchange processes, we have used the published pion-nucleus and nucleon-nucleus optical potentials. These basic interactions are not treated as adjustable quantities. Results and discussion are given in Sec. III. In brief, we find that: (a) both NCX and its interference with the quasifree process contribute significantly to (π,πN) cross sections; and (b) the inclusion of the interference (i.e., the coherent aspect of the reaction) is essential for reaching a true understanding of experimental data. Finally, the formalism developed in this work represents an extension of existing quantum-mechanical direct reaction theories used for (p,2p) and (π,πN) reactions.[10]

II. FORMALISM

With the normalization $\langle\vec{p}\,'\,|\,\vec{p}\rangle=\delta(\vec{p}\,'-\vec{p})$, the total cross section of the (π,πN) reaction in the c.m. frame of the pion-nucleus system is given by

$$\sigma=(2\pi)^4\frac{E_\pi(k)E_A(k_A)}{kW}\mathcal{N}\int d\vec{K}\,d\vec{Q}\,d\vec{P}\,\delta[W-E_\pi(K)-E_N(Q)-E_R(P)]\delta(\vec{K}+\vec{Q}+\vec{P}) \times\sum_{\{\alpha\}J_0'\beta_0'}\left|\sum_i\langle\vec{K}t';\vec{Q}\mu'\tau';\vec{P}J_0'M_s'\,\tfrac{1}{2}M_T'\beta_0'\,|\,A_i\,|\,\vec{k}t;\vec{k}_A0\rangle\right|^2 . \tag{2.1}$$

Here, $\vec{k}$, $\vec{k}_A(=-\vec{k})$, $\vec{K}$, $\vec{Q}$, and $\vec{P}$ denote, respectively, the momentum of the incoming pion, the target nucleus, the outgoing pion, the nucleon, and the residual nucleus. The total energy of the system is denoted by W with

$$W=E_\pi(k)+E_A(k) =(k^2+m_\pi^2)^{1/2}+(k^2+m_A^2)^{1/2}\ .$$

In Eq. (2.1), $\{\alpha\}=(t',\tau',M_T')\otimes(\mu',M_s')$ with the first and second sets of the quantum numbers denoting, respectively, the isospin and spin projections of the particles. The quantum numbers J_0' and β_0' refer to the spins and the energy eigenvalues of different final states of the residual nucleus. Furthermore, $\mathcal{N}$ is an antisymmetrization coefficient arising from the indistinguishability of the A nucleons. It reflects the fact that there are $\mathcal{N}$ equivalent ways of grouping A nucleons in a final state with one nucleon in the continuum and $(A-1)$ nucleons in a bound state. For the study of the ^{12}C($\pi^\pm,\pi$N)^{11}C reactions that leave ^{11}C in its particle-stable states ($E^*<12$ MeV),[3] we only need to consider $j=\frac{3}{2}$ nucleons. Consequently, $\mathcal{N}=2(2j+1)=8$.

A. Reaction model

For the purpose of examining various direct SNR reaction processes which originate on a 1p-shell nucleon, we can express the nuclear part of the final state in Eq. (2.1) according to

$$\langle\vec{Q}\mu'\tau';\vec{P}J_0'M_s'\,\tfrac{1}{2}M_T'\beta_0'\,|\equiv\langle\vec{Q}\mu'\tau'\,|\,\langle\vec{P}J_0'M_s'\,\tfrac{1}{2}M_T'\beta_0'[(j=\tfrac{1}{2})^4J_c=T_c=0]\rangle\ . \tag{2.2}$$

Here, J_c and T_c stand, respectively, for the spin and the isospin of the 1s-shell core. We decompose the initial ^{12}C state into

$$|{}^{12}\mathrm{C}(J_A=T_A=0)\rangle=\sum_{\beta_0 J_0 \tau M_T M_s} c(\beta_0 J_0) C_{\tau}^{1/2}{}_{M_T}^{1/2}{}_{0}^{0} C_{-M_s}^{J_0}{}_{M_s}^{J_0}{}_{0}^{0} \,|J_0 -M_s;\tfrac{1}{2}\tau\rangle\, |J_0 M_s \tfrac{1}{2} M_T \beta_0[(j=\tfrac{1}{2})^4 J_c=T_c=0]\rangle \;, \tag{2.3}$$

where

$$c(\beta_0 J_0)=\langle J_A T_A\{ \,|J_0 T_0(=\tfrac{1}{2})\beta_0, j=T_0\rangle$$

is the coefficient of fractional parentage.[11]

We emphasize that several amplitudes can contribute to a given $(\pi,\pi\mathrm{N})$ reaction. In this work, we take into account the following seven amplitudes.

(a) Quasifree (QF) amplitudes: $A_1[\pi^- \, {}^{12}\mathrm{C}\rightarrow(\pi^-\mathrm{n})^{11}\mathrm{C}]$; $A_2[\pi^+ \, {}^{12}\mathrm{C}\rightarrow(\pi^+\mathrm{n})^{11}\mathrm{C}]$; and $A_3[\pi^+ \, {}^{12}\mathrm{C}\rightarrow(\pi^0\mathrm{p})^{11}\mathrm{C}]$.

(b) Final-state nucleon charge exchange (NCX) amplitudes: $A_4[\pi^- \, {}^{12}\mathrm{C}\rightarrow\pi^-\mathrm{p}^{11}\mathrm{B}\rightarrow(\pi^-\mathrm{n})^{11}\mathrm{C}]$ and $A_5[\pi^+ \, {}^{12}\mathrm{C} \rightarrow\pi^+\mathrm{p}^{11}B\rightarrow(\pi^+\mathrm{n})^{11}\mathrm{C}]$.

(c) Final-state pion charge exchange (πCX) amplitudes: $A_6[\pi^- \, {}^{12}\mathrm{C}\rightarrow\mathrm{n}\pi^0 \, {}^{11}\mathrm{B}\rightarrow(\mathrm{n}\pi^-)^{11}\mathrm{C}]$ and $A_7[\pi^+ \, {}^{12}\mathrm{C} \rightarrow\mathrm{p}\pi^+ \, {}^{11}\mathrm{B} \rightarrow(\mathrm{p}\pi^0)^{11}\mathrm{C}]$. The reaction processes associated with the QF, NCX, and πCX amplitudes are illustrated in Figs. 1(a)—(c), where the pions, nucleons, and nuclei are represented by dashed, thin, and multiple lines, respectively. The Ω is the wave operator which distorts the pion or nucleon wave functions. The cross sections of π^-- and π^+-induced SNR and their ratio are then given by

$$\sigma^- = \sigma[{}^{12}\mathrm{C}(\pi^-,\pi\mathrm{N})^{11}\mathrm{C}] \propto |A_1+A_4+A_6|^2 \;, \tag{2.4}$$

$$\sigma^+ = \sigma[{}^{12}\mathrm{C}(\pi^+,\pi\mathrm{N})^{11}\mathrm{C}] \propto |A_2+A_5|^2 + |A_3+A_7|^2 \;. \tag{2.5}$$

We define the cross-section ratio as

$$R=\sigma^-/\sigma^+ \;. \tag{2.6}$$

In momentum space, the quasifree amplitude associated with Fig. 1(a) is given by

$$\begin{aligned} A_{\mathrm{QF}} = \sum_{\tau\mu M_s} \int & d\vec{\mathrm{k}}' d\vec{\mathrm{k}}'' d\vec{\mathrm{q}}' \langle \vec{\mathrm{k}}'' t'; \vec{\mathrm{q}}' \mu' \tau' \,|\, t_{\pi\mathrm{N}}(\sqrt{s}\,) \,|\, \vec{\mathrm{k}}' t; \vec{\mathrm{q}} \mu\tau\rangle \\ & \times \Psi^{(-)*}_{(\pi)t'M_T'; \vec{\kappa}'}(\vec{\kappa}'') \Phi^{(-)*}_{(N)\tau' M_T'; \vec{\mathrm{Q}}+\vec{\mathrm{k}}''/A}(\vec{\mathrm{q}}'+\vec{\mathrm{k}}''/A) \Psi^{(+)}_{(\pi)t0; \vec{\mathrm{k}}}(\vec{\mathrm{k}}') \\ & \times \frac{c(\beta_0' J_0')}{\sqrt{2(2J_0'+1)}} (-1)^{J_0'+M_s+(1/2)-\tau} C_{\mu}^{1/2}{}_{-\mu-M_s}^{1/2}{}_{-M_s}^{J_0'} \delta_{M_s M_s'} \phi_{nlm}(\vec{\mathrm{q}}+\vec{\mathrm{k}}'/A) \;, \end{aligned} \tag{2.7}$$

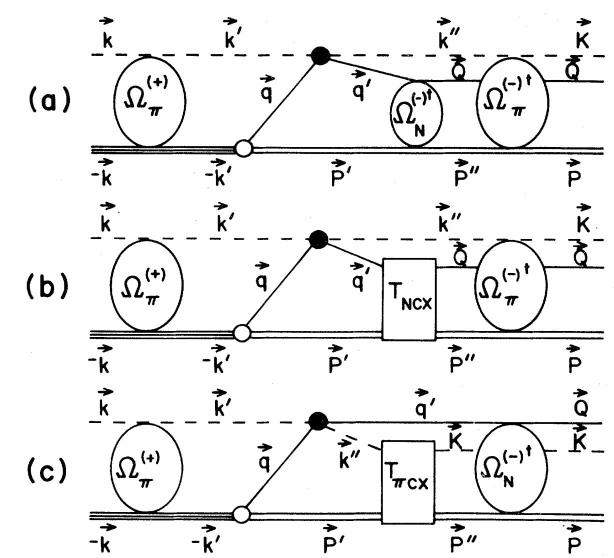

FIG. 1. Diagrams for $(\pi,\pi\mathrm{N})$ reactions: (a) quasifree knockout; (b) direct knockout followed by final-state nucleon charge exchange; and (c) quasifree knockout followed by final-state pion charge exchange. The filled and open circles denote, respectively, the pion-nucleon scattering amplitude and the one-nucleon form factor. The Ω are the wave operators. The pions, the nucleons, and the nuclei are represented by the dashed, solid, and multiple lines, respectively.

where $\vec{\mathrm{q}}=\vec{\mathrm{k}}''-\vec{\mathrm{k}}'+\vec{\mathrm{q}}'$ and $\sqrt{s}$ is the invariant mass of the πN system. The $\vec{\kappa}'$ and $\vec{\kappa}''$ are the relative momenta between the pion and the residual nucleus. Similarly, the momentum variables of $\Phi^{(-)}_{(\mathrm{N})}$ and $\Psi^{(+)}_{(\pi)}$ refer to nucleon-residual nucleus and pion-target nucleus relative momenta, respectively. Furthermore, ϕ is the Fourier transform of the bound nucleon wave function. We use the harmonic oscillator wave function[12]

$$\phi_{nlm}(\vec{\mathrm{r}})=R_{nl}(r)\times Y_{lm}(\hat{r})$$

with $n=l=1$, $m=-M_s-\mu$, and an oscillator parameter $a=1.64$ fm for ${}^{12}\mathrm{C}$ (Ref. 13). The Ψ and Φ represent, respectively, the distorted wave functions of the pion and the nucleon. In obtaining Eq. (2.7), from Fig. 1(a), we have used the following identities which relate the matrix elements of the wave operators to the distorted wave functions:

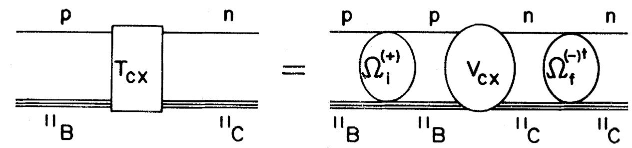

FIG. 2. The diagram for Eq. (2.16), which replaces the final-state charge-exchange parts of Figs. 1(b) and (c) in our distorted-wave calculations.

$$\Psi^{(-)*}_{(\pi)t'M'_T;\vec{\kappa}'}(\vec{\kappa}'')=\langle\vec{K}t';\vec{P}M'_T\,|\,\Omega_\pi^{(-)^\dagger}\,|\,\vec{k}''t';\vec{P}''M'_T\rangle\ ; \tag{2.8}$$

$$\Phi^{(-)*}_{(N)\tau'M'_T;\vec{Q}+\vec{k}''/A}(\vec{q}\,'+\vec{k}''/A)=\langle\vec{Q}\tau';\vec{P}''M'_T\,|\,\Omega_N^{(-)^\dagger}\,|\,\vec{q}\,'\tau';\vec{P}'M'_T\rangle\ ; \tag{2.9}$$

and

$$\Psi^{(+)}_{(\pi)t0;\vec{k}}(\vec{k}\,')=\langle\vec{k}\,'t;-\vec{k}\,'0\,|\,\Omega_\pi^{(+)}\,|\,\vec{k}t;-\vec{k}0\rangle\ . \tag{2.10}$$

Since $m_\pi \ll m_{A-1}$, we have $\vec{\kappa}\,'\simeq\vec{K}$ and $\vec{\kappa}''\simeq\vec{k}''$ in Eqs. (2.7) and (2.8). To make the discussion transparent, we shall use the eikonal distorted wave functions. Consequently, we write

$$\Psi^{(-)*}_{(\pi)t'M'_T;\vec{K}}(\vec{k}'')=(2\pi)^{-3/2}\int d\vec{r}\exp[-i(\vec{K}-\vec{k}'')\cdot\vec{r}]\exp\left[-i(m_\pi/K)\int_0^\infty\langle U_\pi(\vec{r}+\hat{K}s'_\pi)\rangle_{t'M'_T}ds'_\pi\right]\ , \tag{2.11}$$

$$\Phi^{(-)*}_{(N)\tau'M'_T;\vec{Q}+\vec{k}''/A}(\vec{q}\,'+\vec{k}''/A)=(2\pi)^{-3/2}\int d\vec{r}\exp[-i(\vec{Q}-\vec{q}\,')\cdot\vec{r}]\exp\left[-i(\eta m_N/Q)\int_0^\infty\langle U_N(\vec{r}+\hat{Q}s'_N)\rangle_{\tau'M'_T}ds'_N\right]\ , \tag{2.12}$$

and

$$\Psi^{(+)}_{(\pi)t0;\vec{k}}(\vec{k}\,')=(2\pi)^{-3/2}\int d\vec{r}\exp\left[-i(\vec{k}\,'-\vec{k})\cdot\vec{r}-i(m_\pi/K)\int_{-\infty}^{z}\langle V_\pi(\vec{b},\hat{k}z')\rangle_{t0}dz'\right]\ , \tag{2.13}$$

where $\eta=(A-1)/A$. (Refer to the Appendix for the geometry used for the eikonal distortion.) In Eqs. (2.11)–(2.13), the $\langle V_\pi\rangle$, $\langle U_\pi\rangle$, and $\langle U_N\rangle$ stand, respectively, for the optical potentials for π-^{12}C, π-^{11}C, and N-^{11}C systems in the elastic channels specified by the isospin projections t, t', τ', and M'_T. We assume that the distorted waves peak around their respective asymptotic momenta so that we can take the pion-nucleon scattering amplitude $t_{\pi N}$ out of the integration at the asymptotic momentum of each particle. To improve this factorization approximation, we use a medium-modified πN amplitude $t'_{\pi N}$ (see Sec. II B). We obtain, after straightforward but tedious algebra,

$$A_{QF}=\sum_{\mu\tau}\langle\vec{K}t';\vec{Q}\mu'\tau'\,|\,t'_{\pi N}(w')\,|\,\vec{k}t;(\vec{K}+\vec{Q}-\vec{k})\mu\tau\rangle c(\beta'_0J'_0)[2(2J'_0+1)]^{-1/2}$$
$$\times(-1)^{J'_0+M'_s+(1/2)-\tau}C^{1/2\ \ 1\ \ \ \ \ \ \ \ J'_0}_{\mu\ \ -\mu-M'_s\ -M'_s}G_{QF}(\vec{K},\vec{Q},\vec{k};t'tM'_T\tau';M'_s\mu)\ . \tag{2.14}$$

Here, the distorted-wave nuclear form factor is given by

$$G_{QF}(\vec{K},\vec{Q},\vec{k};t'M'_T\tau';M'_s\mu)=(2\pi)^{-3/2}\theta\int d\vec{r}\exp[-i(\vec{K}+\vec{Q}-\eta\vec{k})\cdot\vec{r}]\exp\left[-i(m_\pi/K)\int_0^\infty\langle U_\pi(\vec{r}+\hat{K}s'_\pi)\rangle_{t'M'_T}ds'_\pi\right]$$
$$\times\exp\left[-i(\eta m_N/Q)\int_0^\infty\langle U_N(\vec{r}+\hat{Q}s'_N)\rangle_{\tau'M'_T}ds'_N\right]$$
$$\times\exp\left[-i(m_\pi/k)\int_{-\infty}^{z}\langle V_\pi(\vec{b},\hat{k}z')\rangle_{t0}dz'\right]\phi_{1,-\mu-M'_s}(\vec{r})\ . \tag{2.15}$$

In Eq. (2.14), the variable w' indicates the total c.m. energy (or the invariant mass) of the πN system at which the $t'_{\pi N}$ will be evaluated. The determination of w' will be discussed in Sec. II B. The quantity θ is defined by $\theta=\theta(E'_{rel}-E_{rel}-E_s)$. Here, $E_{rel}=(-\vec{P}-\eta\vec{k}\,')^2/2\eta m_N$ and $E'_{rel}=(-\vec{P}-\eta\vec{K})^2/2\eta m_N$ represent, respectively, the nucleon kinetic energies in the nucleus before and after the πN collision. The step function θ has been introduced to ensure that SNR will occur only when the energy transfer to the nucleon is greater than the separation energy E_s of the nucleon ($E_s=18$ MeV for the neutron and 16 MeV for the proton in ^{12}C). The use of this phenomenological θ has been shown necessary to suppress spurious SNR events in the kinematical region characterized by small pion scattering angles.[14]

It is useful to express the charge-exchange T matrices T_{NCX} and $T_{\pi CX}$ in Figs. 1(b) and (c) in terms of the matrix elements of corresponding charge-exchange potentials between distorted-wave states. More specifically, we have[15]

$$\langle\phi_f\,|\,T_{CX}\,|\,\phi_i\rangle=\langle\Phi_f^{(-)}\,|\,V_{CX}\,|\,\Phi_i^{(+)}\rangle\ , \tag{2.16}$$

where ϕ and $\Phi^{(\pm)}$ denote, respectively, plane-wave and distorted-wave states (see Fig. 2). With the aid of Eq. (2.16), we obtain from Figs. 1(b) and 2 the following expression for the NCX amplitude:

$$A_{NCX}=\sum_{\beta_0J_0M_T\tau\tau''\mu\mu''M_s}\int d\vec{k}\,'d\vec{k}''d\vec{q}\,'\Psi^{-*}_{(\pi)t'M'_T;\vec{K}}(\vec{k}'')\Psi^{(+)}_{(\pi)t0;\vec{k}}(\vec{k}\,')$$
$$\times\langle\Phi^{(-)}_{(N)\tau'\mu';(1/2)M'_TJ'_0M'_s\beta'_0;\vec{Q}+\vec{k}''/A}\,|\,V_{NCX}\,|\,\Phi^{(+)}_{(N)\tau''\mu'';(1/2)M_TJ_0M_s\beta_0;\vec{q}\,'+\vec{k}''/A}\rangle$$
$$\times(Q^2/2m_N-q'^2/2m_N+i\epsilon)^{-1}\langle\vec{k}''t';\vec{q}\,'\tau''\mu''\,|\,t_{\pi N}(\sqrt{s})\,|\,\vec{k}\,'t;\vec{q}\tau\mu\rangle$$
$$\times\delta_{-\tau,M_T}(-1)^{J_0+M_s+(1/2)+M_T}c(\beta_0J_0)[2(2J_0+1)]^{-1/2}\,C^{1/2\ \ 1\ \ \ \ \ \ \ J_0}_{\mu\ \ -\mu-M_s\ -M_s}\phi_{nlm}\left[\vec{q}\,'+\frac{\vec{k}\,'}{A}\right]\ , \tag{2.17}$$

where $\vec{q}=\vec{k}''+\vec{q}'-\vec{k}'$. We note the presence of a nucleon propagator in Eq. (2.17). The eikonal representation of this propagator can be written as

$$(Q^2/2m_N-q'^2/2m_N+i\epsilon)^{-1}=\int d\vec{\Delta}e^{-i\vec{q}'\cdot\vec{\Delta}}G_{\vec{q}'}(\vec{Q};\vec{r}',\vec{r})$$
$$=\int d\vec{\Delta}e^{-i\vec{q}'\cdot\vec{\Delta}}(-i)\int_0^\infty dt\, e^{itQ^2/m_N}\delta(\vec{\Delta}-t\vec{Q}/m_N)\ , \tag{2.18}$$

where $\vec{\Delta}=\vec{r}'-\vec{r}$. In obtaining Eq. (2.18), we have used for the Green's function $G_{\vec{q}'}(\vec{Q};\vec{r}',\vec{r})$ the parametric form of Sugar and Blankenbecler.[16] Using Eq. (2.18) and the same approximations leading to Eq. (2.14), we obtain

$$A_{NCX}=\sum_{J_0\beta_0\tau\mu\mu''M_s}\langle\vec{K}t';\vec{Q}\mu''\tau''\,|\,t'_{\pi N}(w')\,|\,\vec{k}t;(\vec{K}+\vec{Q}-\vec{k})\mu\tau\rangle c(\beta_0J_0)[2(2J_0+1)]^{-1/2}$$
$$\times(-1)^{J_0+M_s+(1/2)-\tau}C^{1/2\ \ 1\ \ \ \ \ \ \ J_0}_{\mu\ \ -\mu-M_s\ -M_s}G_{NCX}(\vec{K},\vec{Q},\vec{k};\beta_0J_0t'tM'_T\tau'\tau;M'_sM_s\mu''\mu'\mu) \tag{2.19}$$

with

$$G_{NCX}(\vec{K},\vec{Q},\vec{k};\beta_0J_0t'tM'_T\tau'\tau;M'_sM_s\mu''\mu'\mu)$$
$$=(2\pi)^{-3/2}\theta\int d\vec{r}\exp[-i(\vec{K}-\eta\vec{k})\cdot\vec{r}]\exp\left[-i\frac{m_\pi}{K}\int_0^\infty\langle U_\pi(\vec{r}+\hat{K}s'_\pi)\rangle_{t'M'_T}ds'_\pi\right]$$
$$\times\int_0^\infty ds''_N\exp\left[-i\frac{\eta m_N}{Q}\int_{s''_N-s_N}^\infty\langle U_N(\vec{r}+\hat{Q}s'_N)\rangle_{\tau'M'_T}ds'_N\right]$$
$$\times\langle\tau'\mu'\,|\,\langle\tfrac{1}{2}M'_TJ_0M'_s\beta'_0\,|\,e^{-i\vec{Q}\cdot\vec{r}}V_{NCX}(\vec{r}+\hat{Q}s''_N)\,|\,\tfrac{1}{2}-\tau J_0M_s\beta_0\rangle\,|\,\tau''\mu''\rangle(-im_N/Q)$$
$$\times\exp\left[-i\frac{\eta m_N}{Q}\int_0^{s''_N-s_N}\langle U_N(\vec{r}+\hat{Q}s'_N)\rangle_{\tau'',-\tau}ds'_N\right]\exp\left[-i\frac{m_\pi}{k}\int_{-\infty}^{z}\langle V_\pi(\vec{b},kz')\rangle_{t0}dz'\right]$$
$$\times\phi_{1,-\mu-M_s}(\vec{r})\ . \tag{2.20}$$

Here, $\vec{r}''=\vec{b}''+\vec{z}''$ and $\tau''=t+\tau-t'=\tau'+M'_T+\tau$. The physical picture associated with Eq. (2.20) is as follows: The incoming pion knocks out a proton at position $\vec{r}=(\vec{b},\vec{z})$; the proton propagates from $\vec{r}$ to $\vec{r}''$ where the NCX reaction $p+{}^{11}B\to n+{}^{11}C$ takes place. The momentum-space πCX amplitude [cf. Figs. 1(c) and 2] has a structure similar to Eq. (2.17) and is given by the following:

$$A_{\pi CX}=\sum_{\beta_0J_0\tau t''\mu M_s}\langle\vec{K}t'';\vec{Q}\tau'\mu'\,|\,t'_{\pi N}(w')\,|\,\vec{k}t;(\vec{K}+\vec{Q}-\vec{k})\tau\mu\rangle c(\beta_0J_0)[2(2J_0+1)]^{-1/2}$$
$$\times(-1)^{J_0+M_s+(1/2)-\tau}C^{1/2\ \ 1\ \ \ \ \ \ \ J_0}_{\mu\ \ -\mu-M_s\ -M_s}G_{\pi CX}(\vec{K},\vec{Q},\vec{k};\beta_0J_0t''t'tM'_T\tau'\tau;M'_sM_s\mu'\mu)\ , \tag{2.21}$$

with

$$G_{\pi CX}(\vec{K},\vec{Q},\vec{k};\beta_0J_0t''t'tM'_T\tau'\tau;M'_sM_s\mu'\mu)$$
$$=(2\pi)^{-3/2}\theta\int d\vec{r}\exp[-i(\vec{Q}-\eta\vec{k})\cdot\vec{r}]$$
$$\times\exp\left[-i\frac{\eta m_N}{Q}\int_0^\infty\langle U_N(\vec{r}+\hat{Q}s'_N)\rangle_{\tau'M'_T}ds'_N\right]\int_0^\infty ds''_\pi\exp\left[-i\frac{m_\pi}{K}\int_{s''_\pi-s_\pi}^\infty\langle U_\pi(\vec{r}+\hat{K}s'_\pi)\rangle_{t'M'_T}ds'_\pi\right]$$
$$\times\langle t'\,|\,\langle\tfrac{1}{2}M'_TJ'_0M'_s\beta'_0\,|\,e^{-i\vec{K}\cdot\vec{r}}V_{\pi CX}(\vec{r}+\hat{K}s''_\pi)\,|\,\tfrac{1}{2}-\tau J_0M_s\beta_0\rangle\,|\,t''\rangle(-im_\pi/K)$$
$$\times\exp\left[-i\frac{m_\pi}{K}\int_0^{s''_\pi-s_\pi}\langle U_\pi(\vec{r}+\hat{K}s'_\pi)\rangle_{t'',-\tau}ds'_\pi\right]\exp\left[-i\frac{m_\pi}{k}\int_{-\infty}^{z}\langle V_\pi(\vec{b},kz')\rangle_{t0}dz'\right]\phi_{1,-\mu-M_s}(\vec{r})\ , \tag{2.22}$$

where $t''=t+\tau-\tau'=t'+M'_T+\tau$. It is worth noting that in the plane-wave limit, G_{QF} reduces to $\phi_{nlm}(\vec{Q}_B)$ with $\vec{Q}_B=\vec{K}+\vec{Q}-\eta\vec{k}$ equal to the initial momentum of the nucleon in the target nucleus. However, G_{NCX} and $G_{\pi CX}$ do not reduce to the Fourier transform of the bound-state nucleon wave function even in the plane-wave limit. These form factors are absent in standard reaction theories for SNR and are the result of the explicit introduction of charge-exchange scatterings in the final state.

B. Theoretical input

The πN amplitude is parametrized according to[17]

$$\langle \vec{K}t';\vec{Q}\mu'\tau' \,|\, t_{\pi N}(\sqrt{s}) \,|\, \vec{k}t;\vec{p}\mu\tau\rangle = \left[\frac{E_\pi(k_c')E_N(k_c')E_\pi(k_c)E_N(k_c)}{E_\pi(K)E_N(Q)E_\pi(k)E_N(p)k_c'k_c}\right]^{1/2} \langle t';\mu'\tau' \,|\, \sum_{i=0}^{3} O_i f_i(\sqrt{s};\vec{k}_c',\vec{k}_c) \,|\, t;\mu\tau\rangle$$
$$\equiv \langle t';\mu'\tau' \,|\, \sum_{i=0}^{3} O_i F_i(\sqrt{s};\vec{k}_c',\vec{k}_c) \,|\, t;\mu\tau\rangle \,, \qquad (2.23)$$

where $O_0=1$, $O_1=-i\vec{\sigma}\cdot\hat{k}_c\times\hat{k}_c'$, $O_2=\vec{t}\cdot\vec{\tau}$, and $O_3=O_1O_2$ are operators in the spin-isospin space with $\vec{t}$ and $\vec{\tau}$ denoting the isospins of the pion and the nucleon. The F_i $(i=0,1,2,3)$ depend on the πN phase shifts and off-shell form factors. They are functions of three independent variables which have been chosen as the invariant mass $\sqrt{s}$, the initial c.m. momentum $\vec{k}_c$, and the final c.m. momentum $\vec{k}_c'$ of the πN system. (Refer to Sec. IV of Ref. 18 for details.)

The pion-nucleus optical potentials used for calculating the distortions of pion wave functions by ^{12}C and ^{11}C (or ^{11}B) are defined by

$$V_\pi(\vec{r})=-\frac{2\pi}{\mu_\pi}[Z\rho_p(\vec{r})+N\rho_n(\vec{r})]F_0'(0) \qquad (2.24)$$

and

$$U_\pi(\vec{r})=-\frac{2\pi}{\mu_\pi}\{[Z\rho_p(\vec{r})+N\rho_n(\vec{r})]F_0'(0)$$
$$\pm[Z\rho_p(r)-N\rho_n(\vec{r})]F_2'(0)\} \,, \qquad (2.25)$$

with μ_π being the reduced pion mass. The ρ_p and ρ_n are normalized proton and neutron densities. In Eq. (2.25), the plus sign is for the π^+ interaction and the minus sign for the π^- interaction. The F_0' and F_2' are effective forward πN scattering amplitudes in the sense that $V_\pi(\vec{r})$ [and, similarly, $U_\pi(\vec{r})$] are effective local potentials. To see this, we consider the Fourier transform

$$\langle \vec{k}' \,|\, V_\pi \,|\, \vec{k}\rangle=(2\pi)^{-3}\int e^{i\vec{k}'\cdot\vec{r}'}V_\pi(\vec{r}',\vec{r})$$
$$\times e^{-i\vec{k}\cdot\vec{r}}d\vec{r}\,d\vec{r}' \,, \qquad (2.26)$$

where $V_\pi(\vec{r}',\vec{r})=\delta(\vec{r}'-\vec{r})V_\pi(\vec{r})$ for a local potental. Since $\langle \vec{k}' \,|\, V_\pi \,|\, \vec{k}\rangle$ can be nonlocal in momentum space, it follows that $V_\pi(\vec{r})$ has the meaning of an effective local potential in coordinate space. Combining Eqs. (2.24) and (2.26), we obtain for $\vec{k}'=\vec{k}$ (the forward scattering):

$$\frac{-2\pi}{\mu_\pi}AF_0'(0)=(2\pi)^3\langle \vec{k} \,|\, V_\pi \,|\, \vec{k}\rangle \,. \qquad (2.27)$$

Equation (2.27) defines the effective forward amplitude $F_0'(0)$ in terms of the forward-direction momentum-space π-^{12}C optical potential. This latter quantity can be exactly evaluated according to[17]

$$(2\pi)^3\langle \vec{k}' \,|\, V_\pi \,|\, \vec{k}\rangle=-\frac{2\pi}{\mu_{\pi N}}\int d\vec{Q}_R F_0(\tilde{\omega};\vec{k}_c';\vec{k}_c)$$
$$\times(Z\tilde{\rho}_p+N\tilde{\rho}_n) \qquad (2.28)$$

for $\vec{k}'=\vec{k}$ (which corresponds to $\vec{k}_c'=\vec{k}_c$). In Eq. (2.28), the $\mu_{\pi N}$, $\tilde{\omega}$, $\vec{k}_c$, and $\vec{k}_c'$ are, respectively, the reduced pion mass, the invariant mass (or the total c.m. energy) of the πN system, the initial c.m. πN relative momentum, and the final c.m. πN relative momentum. The $\tilde{\rho}_p$ and $\tilde{\rho}_n$ are the momentum-space proton and neutron density distributions. The quantities $\tilde{\omega}$, $\vec{k}_c$, $\vec{k}_c'$, $\tilde{\rho}_p$, and $\tilde{\rho}_n$ are functions of the momentum $\vec{Q}_R$, which in turn is related to the Fermi motion of the struck nucleon.[18] Using an algebra similar to the one leading to Eq. (2.27), we obtain from Eq. (2.25) the relation

$$-\frac{2\pi}{\mu_{\pi N}}(\pm)(Z-N)F_2'(0)=(2\pi)^3\langle \vec{k};tM_T \,|\, (V_\pi-U_\pi) \,|\, \vec{k};tM_T\rangle$$
$$=-\frac{2\pi}{\mu_{\pi N}}\int d\vec{Q}_R\langle \vec{k};tM_T \,|\, \sum_i \vec{t}\cdot\vec{\tau}_{(i)}F_2(\tilde{\omega};\vec{k}_c;\vec{k}_c) \,|\, \vec{k};tM_T\rangle$$
$$=-\frac{2\pi}{\mu_{\pi N}}\int d\vec{Q}_R(\pm)F_2(\tilde{\omega};\vec{k}_c;\vec{k}_c)(Z\tilde{\rho}_p-N\tilde{\rho}_n) \,. \qquad (2.29)$$

From the second line of Eq. (2.29), we obtain the effective coordinate-space potential operator $V_{\pi CX}$ of Eq. (2.22):

$$V_{\pi CX}(\vec{r}'')=-\frac{2\pi}{\mu_{\pi N}}F_2'(0)\,\vec{t}\cdot\sum_i \vec{\tau}_{(i)}\rho_{(i)}(\vec{r}'') \,. \qquad (2.30)$$

Here, $\vec{t}$ is the isospin operator for the pion and $\vec{\tau}_{(i)}$ and $\rho_{(i)}$ are, respectively, the (Pauli) isospin operator and the density distribution associated with the nucelon i.

An inspection of the systematics of $\langle \vec{k} \,|\, V_\pi \,|\, \vec{k}\rangle$ of several nuclei has shown that it has a higher resonance energy and a broader resonance width than those of the free (3,3) resonance.[18] It has also been noted[18] that this modification is a consequence of a detailed treatment of nuclear binding, off-shell effects, and the integration over the Fermi motion of the struck nucleon in Eq. (2.28). A similar feature has been noted for $\langle \vec{k};tM_T \mid (V_\pi - U_\pi) \mid \vec{k};tM_T \rangle$ defined in Eq. (2.29). We have found that it is possible to numerically reproduce the $F'_0(0)$ and $F'_2(0)$ of Eqs. (2.28) and (2.29) by, respectively, evaluating $F_0(0)$ and $F_2(0)$ in the fixed scatterer approximation (FSA) defined in Ref. 14, but with a modification of the P_{33} interaction. This latter observation forms the basis of the approach that we have used to simplify the numerical computations of the amplitude $t'_{\pi\mathrm{N}}$ appearing in Eqs. (2.14), (2.19), and (2.21). In the following, we describe in detail this approach.

We parametrized the P_{33} phase shifts according to[19]

$$\tan\delta_{33}/q^3 = b + cq^2 + dq^4 + \frac{0.99\Gamma_0\omega_0}{q_0^3(\omega_0^2-\omega^2)} , \tag{2.31}$$

where b, c, and d are constants given in Ref. 19, and

$$\omega_0 = (q_0^2 + m_\pi^2)^{1/2} + (q_0^2 + m_\mathrm{N}^2)^{1/2} .$$

We kept b, c, and d unchanged and varied Γ_0 and ω_0 such that with the modified δ_{33} and unmodified other πN partial-wave phase shifts, the evaluation of the product $-AF'_0(2\pi)^{-2}\mu_\pi^{-1}$ in the FSA reproduces the exactly calculated $\langle \vec{k} \mid V_\pi \mid \vec{k} \rangle$ of the π-^{12}C system.[18] A solution was obtained with $\Gamma_0 = 180$ MeV and $\omega_0 = 1280$ MeV denoted henceforth Γ'_0 and ω'_0. It remains to specify the energy variable w' which enters into the calculation of the F'_i, $i=0,1,2,3$, and hence the calculation of $t'_{\pi\mathrm{N}}$. Before introducing the factorization approximation, the energy variable $\sqrt{s}$ of $t_{\pi\mathrm{N}}$ in Eqs. (2.7) and (2.17) is completely determined, and can be evaluated from the four-momenta of the off-mass-shell pion and nucleon. After the factorization, the energy variable of $t'_{\pi\mathrm{N}}$, denoted w', is no longer defined by first principles. A reasonable choice is

$$w' = \tilde{w} - \langle \mathrm{Re} V_\pi \rangle_\mathrm{av} . \tag{2.32}$$

Here, $\tilde{w}$ is the energy that enters into the $t_{\pi\mathrm{N}}$ for the exact calculation of the pion-nucleus optical potential [Eq. (2.28)], and $\langle \mathrm{Re} V_\pi \rangle_\mathrm{av}$ represents the average pion potential energy in the nucleus. It is useful to express the $\tilde{w}$ in terms of the incoming pion momentum $\vec{k}$ and the momentum of the recoil nucleus $\vec{P}$. The result is[20]

$$\tilde{w} \simeq E_\pi(k) + \frac{k^2}{2m_A} + m_\mathrm{N} - E_s - \frac{P^2}{2m_{A-1}} - \frac{P^2}{2(m_\pi + m_\mathrm{N})} , \tag{2.33}$$

where E_s is the separation energy of the nucleon. Equation (2.33) represents the relevant πN collision energy in the absence of pion self-energy. This is correct since in the evaluation of the optical potential, the pions are by definition in the free state. (Medium effects on the pion will be generated self-consistently when the pion-nucleus scattering amplitude is calculated with the Lippmann-Schwinger equation which iterates the optical potential.) However, Eq. (2.7) indicates that in the $(\pi,\pi\mathrm{N})$ reaction, the πN amplitude $t_{\pi\mathrm{N}}$ appears between the distorted pion wave functions Ψ and Φ. Consequently, the relevant local pion energy will be changed from the free pion energy by an amount equal to $-\mathrm{Re} V_\pi$. This justifies Eq. (2.32). We have approximated $\langle \mathrm{Re} V_\pi \rangle_\mathrm{av}$ by calculating the $\mathrm{Re} V_\pi$ [Eq. (2.24)] at the pion asymptotic energy $E_\pi(k)$ and at the half-density radius.

Equation (2.33) can be rewritten in the form

$$\tilde{w} \simeq E_\pi(k) + \frac{k^2}{2m_A} + m_\mathrm{N} - \delta\omega_0 \tag{2.34}$$

with

$$\delta\omega_0 = E_s + \frac{P^2}{2m_{A-1}} + \frac{P^2}{2(m_\pi + m_\mathrm{N})} \tag{2.35}$$

representing the decrease of the energy available to the πN collision. The three terms in Eq. (2.35) represent, respectively, the binding correction, the recoil correction, and the c.m. motion correction due to the propagation of the πN system. Using the most probable value of P in the $(\pi,\pi\mathrm{N})$ reaction (which is largely controlled by the argument of the bound-state nucleon wave function $\vec{P} + \eta\vec{k}$), we have found that $\delta\omega_0$ is about 45 MeV, in agreement with the previous finding, $\omega'_0 - \omega_0 = 1280 - 1232 = 48$ MeV. Indeed, microscopically, it is precisely the same three terms of Eq. (2.35) that have caused, via Eq. (2.28), the apparent shift of the resonance position. We recall that the shift $\delta\omega_0$ has already been taken into account in generating the modified P_{33} phase shift. Consequently, to calculate $t'_{\pi\mathrm{N}}$ based on the use of ω'_0 and Γ'_0, the relevant energy variable is

$$w' = E_\pi(k) + k^2/2m_A + m_\mathrm{N} - \langle \mathrm{Re} V_{\pi,k} \rangle_\mathrm{av} .$$

For the nucleon-nucleus interaction, we have used the optical potential of Watson, Singh, and Segel.[21] The spin-nonflip part of the potential is given by

$$U_{\mathrm{N,opt}} = -V_R f(r,r_R,a_R) - iW_V f(r,r_I,a_I) + 4ia_I W_S \frac{d}{dr} f(r,r_I,a_I) + V_\mathrm{Coul}(r,r_c) . \tag{2.36}$$

Here, f is the usual Woods-Saxon form factor

$$f(r,r_0,a_0) = \{1 + \exp[(r - r_0 A^{1/3})/a_0]\}^{-1} . \tag{2.37}$$

The Coulomb potential is that of a uniformly charged sphere, namely,

$$V_\mathrm{Coul} = (zZe^2/2R_c)(3 - r^2/R_c^2), \text{ for } r \le R_c = r_c A^{1/3} ; \quad = zZe^2/r, \text{ for } r \ge R_c , \tag{2.38}$$

z and Z being the charges of the incident nucleon and the target. From a systematic fit to nucleon scattering on $1p$-shell nuclei at energies between 10 and 50 MeV, they determined that

$$V_R = 60 + 0.4Z/A^{1/3} \pm 27(N-Z)/A - 0.3E_\mathrm{c.m.} , \tag{2.39}$$

$$W_S = W_S(E_\mathrm{c.m.}) \pm 10(N-Z)/A , \tag{2.40}$$

$$W_V = W_V(E_\mathrm{c.m.}) , \tag{2.41}$$

$r_R = r_I = r_c = 1.15 - 0.001E_\mathrm{c.m.}$, $a_R = 0.57$, and $a_I = 0.5$. We refer to Ref. 21 for the functions $W_S(E_\mathrm{c.m.})$ and $W_V(E_\mathrm{c.m.})$. In Eqs. (2.39)–(2.41), $E_\mathrm{c.m.}$ is the nucleon energy in the c.m. frame of the nucleon-nucleus system. Furthermore, the energies are in MeV units and the lengths in fm. The plus sign is for incident protons and the minus sign for incident neutrons. From the $[(N-Z)/A]$ part of the V_R and W_S, we obtain

$$V_{\mathrm{NCX}}(\vec{r}) = \mp \left[\frac{27}{A} f(r, r_R, a_R) - \frac{10}{A} 4 i a_S \frac{d}{dr} f(r, r_I, a_I) \right] \left[\vec{\tau} \cdot \sum_i \vec{\tau}_{(i)} \right] , \tag{2.42}$$

where $\vec{\tau}$ and $\vec{\tau}_{(i)}$ stand for the (Pauli) isospin operators of the propagating nucleon and the ith nucleon of the residual nucleus.

III. RESULTS AND DISCUSSION

We compare our calculated cross sections with the experimental data of Dropesky *et al.*[3] for ^{12}C(π^-,πN)^{11}C and ^{12}C(π^+,πN)^{11}C reactions in Fig. 3. As we can see, our calculations are able to reproduce, reasonably well, the energy dependence and the absolute magnitude of the cross sections.

In Fig. 4, we show the cross-section ratios for the ^{12}C($\pi^\pm$,πN)^{11}C reactions. The dotted curve is the ratio of free πN cross sections

$$\sigma(\pi^- n \to \pi^- n)/[\sigma(\pi^+ n \to \pi^+ n) + \sigma(\pi^+ n \to \pi^0 p)] .$$

The dashed curve corresponds to the ratio obtained with conventional quasifree models which do not have final state charge exchanges. Results due to our full theory are shown as the solid curve. As we can see the theory is able to account for the energy dependence of the ratio, although the calculated ratios are slightly higher than the experimental data (the shaded area). We have found that the πCX has very small effects on the calculated cross sections. This agrees with an earlier observation by Silbar.[9] The contributions from πCX and its interference with the quasifree process are at most 1 mb at all the energies studied. Consequently, the difference between the solid and dashed curves is mainly caused by NCX and its interference with the quasifree direct knockout process. It is noteworthy that the cross-section ratios given by our detailed quasifree calculations differ already from the free πN scattering cross-section ratios, even when NCX is absent (compare dashed and dotted curves). We have noted that this modification of free ratios is mainly caused by the difference of n-^{11}C and p-^{11}C optical potentials used for the distortion calculations.

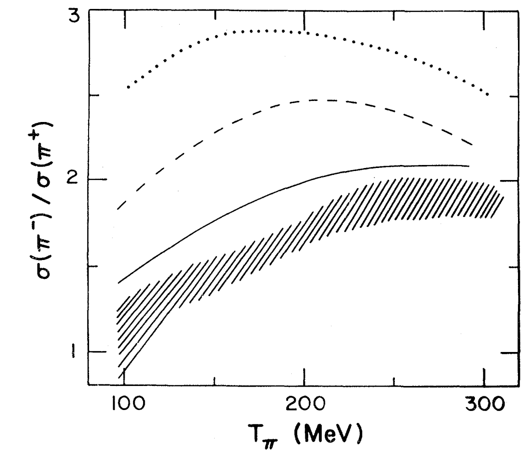

FIG. 4. Cross-section ratios for the ^{12}C($\pi^\pm$,πN)^{11}C reaction. The dotted curve represents the ratio of free π-N cross sections,

$$\sigma(\pi^- n \to \pi^- n)/[\sigma(\pi^+ n \to \pi^+ n) + \sigma(\pi^+ n \to \pi^0 p)] .$$

The dashed curve is due to the calculations based on pure quasifree scattering. The solid curve and the shaded area have the same meanings as for Fig. 3.

To exhibit the relative importance of the interference effects between NCX and QF processes, we show in Fig. 5 the percent contributions to the calculated $\sigma(\pi^\pm)$. The

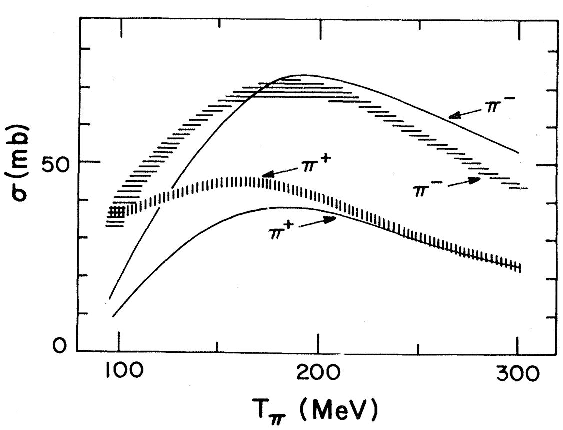

FIG. 3. Cross sections for the ^{12}C($\pi^\pm$,πN)^{11}C reaction. Shaded areas are the experimental data of Dropesky *et al.* (Ref. 3). The solid curves are calculated results based on the full theory which includes quasifree, NCX, πCX, and the interference between these processes.

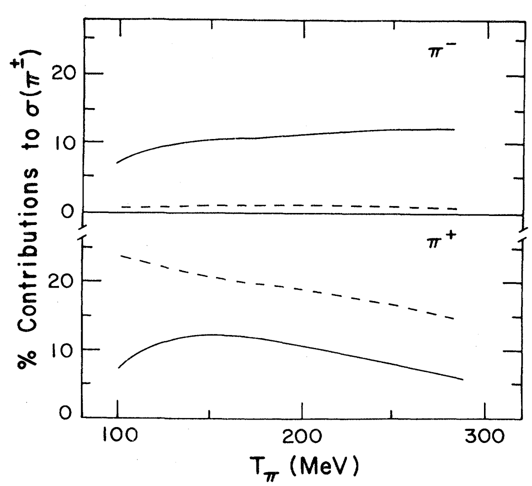

FIG. 5. Percent of NCX and interference contributions to $\sigma(\pi^\pm)$. Solid curves: interference; dashed curves: NCX.

solid curves represent contributions to σ^- and σ^+ from the interference terms $2\,\mathrm{Re}(A_1A_4^*)$ and $2\,\mathrm{Re}(A_2A_5^*)$, respectively. On the other hand, the dashed curves represent contributions from pure NCX: $|A_4|^2$ in the case of σ^- and $|A_5|^2$ in the case of σ^+. It is interesting to note that in the NCX theories of Refs. 7 and 8, the interference terms are not present. In those theoretical treatments, the competition between quasifree and nonquasifree processes were taken into account via the parameter P_{CX} which was given the meaning of the percent contribution from pure NCX. It is clear from this work that the P_{CX}, introduced in Refs. 7 and 8, simulates the summed effects of interferences and NCX, but not the pure NCX. Indeed, the interpretation of P_{CX} in those analyses could be very misleading. As we can see from Fig. 5, in the case of incident π^-, the contributions from the interference term greatly exceed those from the pure NCX. If the methods of Refs. 7 and 8 were used, these interference effects would be incorrectly identified as a part of pure NCX effects.

For ^{12}C, we have found that the interference is constructive at all energies. Since the potential for NCX is proportional to A^{-1}, we can expect that the importance of NCX and its interference with the quasifree knockout process will be smaller in nuclei heavier than ^{12}C. This agrees with the observations by Kaufman *et al.* and Ohkubo *et al.*[4] who have found that there is no evidence for NCX contributions in ^{197}Au(π,πN) and ^{127}I(π,πN) reactions, respectively. Although in the case of ^{12}C, the inclusion of interference happened to have little effect on the calculated cross-section ratios, we have no reason to assume this insensitivity to be true in general.

The calculations presented in this work can be improved in many respects. For example, one might forgo the use of the factorization approximation and the use of eikonal distortions. This would require the performance of a nine-dimensional integration for each of the (π,πN) amplitudes in Eqs. (2.7) and (2.17). Also, it demands a tedious partial-wave decomposition of each of the distorted waves.[22] On the other hand, such calculations would allow a treatment of the medium effect from first principles. They also would enable the use of momentum-space pion-nucleus and momentum-space nucleon-nucleus optical potentials which can be more directly related to the elementary πN and NN scattering amplitudes. However, we believe that the basic features brought out by the present calculations, namely, the importance of the interference between different processes of the (π,πN) reaction, will not be changed.

As noted at the end of Sec. II A, the presence of G_{NCX} and $G_{\pi\mathrm{CX}}$ is a novel feature and is a direct result of treating explicitly the coupling between elastic scattering (the usual distortion) and charge exchange scattering in the final state. Since the distorted-wave approximation is valid only when the coupling between different reaction channels is weak, the need for introducing explicitly the NCX and/or πCX processes is a strong indication of the limitation of conventional distorted wave (DW) theories for SNR. In this respect, the formalism developed in this work represents a natural extension of simpler direct-reaction theories for (e,e′p), (p,2p), and (π,πN) reactions.[10]

ACKNOWLEDGMENTS

We would like to acknowledge useful discussions with R. R. Silbar. This work was performed under the auspices of the Division of Nuclear Physics, U. S. Department of Energy.

APPENDIX

We show in this section the method used in calculating the eikonal distortion factors contained in the nuclear form factors G_{QF} [Eq. (2.15)], G_{NCX} [Eq. (2.20)], and $G_{\pi\mathrm{CX}}$ [Eq. (2.22)]. We will first give the derivation and calculation of the quantity $\int_0^\infty \langle U_\pi(\vec{r}+\hat{K}s'_\pi)\rangle ds'_\pi$ corresponding to the distortion of the outgoing pion having the momentum $\vec{K}$. Here, s_π is the distance traveled by the pion in the direction of $\vec{K}$. The $\langle U_\pi\rangle$ is the pion-nucleus optical potential which depends on the nuclear density $\rho(|\vec{r}_\pi|)$, with $r_\pi=(L_\pi^2+z_\pi'^2)^{1/2}$ being the distance between the moving pion and the nuclear center. (Discussion of the distortions of the incoming pion and outgoing nucleon is formally identical.) Then, we will examine the analytical structure and the physical content of the G's, taking G_{QF} as a specific example.

The geometry associated with the distortion of the outgoing pion is illustrated in Fig. 6, where $\vec{r}=\vec{b}+\vec{z}$ is the position of the πN collision site, and $\vec{b}$ the impact parameter of the incoming pion. The direction of the outgoing pion momentum $\hat{K}$ is defined by the polar and azimuthal angles of the momentum $\vec{K}$, denoted θ_K and ψ_K. From Fig. 6, we have

$$L_\pi=[b^2+a^2+2ba\cos(\pi-\phi-\psi_K)]^{1/2}\,, \tag{A1}$$

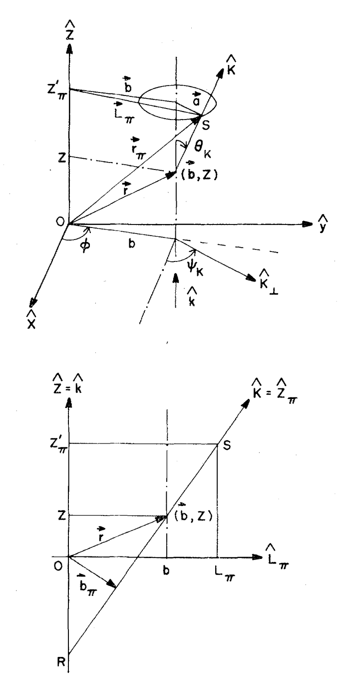

FIG. 6. The geometry associated with the outgoing pion of momentum $\vec{K}$.

with

$$a=(z'_\pi - z)\tan\theta_K \ . \qquad \text{(A2)}$$

Equations (A1) and (A2) imply that $L_\pi = L_\pi(\vec{r},\theta_k,\psi_k,z'_\pi)$, which depends on six variables. To simplify the calculations, we eliminated the ψ_K dependence by averaging L_π with respect to ψ_K. The average distance $\bar{L}_\pi$ is then given by

$$\bar{L}_\pi = \frac{2}{\pi}(a+b)E\left[\frac{2\sqrt{ab}}{a+b},\frac{\pi}{2}\right] . \qquad \text{(A3)}$$

Here, E is the second-kind elliptical integral. As a result of the axial symmetry of the whole system, $\bar{L}_\pi$ is also independent of ϕ. The distortion is then calculated as follows:

$$\exp\left[-i\frac{m_\pi}{K}\int_0^\infty \langle U_\pi(\vec{r}+\hat{K}s'_\pi)\rangle ds'_\pi\right] \simeq \exp\left[-i\frac{m_\pi}{K}\int_0^\infty \langle U_\pi(\bar{L}_\pi, z+s'_\pi\cos\theta_K)\rangle ds'_\pi\right] \equiv D_{(\pi)}^{(-)}(K,\theta_K,b,z) \qquad \text{(A4)}$$

which possesses axial symmetry. We recall that $\bar{L}_\pi$ is not a constant, and it varies as the pion moves along its trajectory. However, we have found that it is a good numerical approximation to set $\bar{L}_\pi = b$, for $b>a$, and $\bar{L}_\pi = a$, for $b<a$. The distortion factors of the outgoing nucleon and the incoming pion can be written in a form similar to Eq. (A4):

$$\exp\left[-i\frac{\eta m_N}{Q}\int_0^\infty \langle U_N(\vec{r}+\hat{Q}s'_N)\rangle ds'_N\right] \simeq \exp\left[-i\frac{\eta m_N}{Q}\int_0^\infty \langle U_N(\bar{L}_N, z+s'_N\cos\theta_Q)\rangle ds'_N\right] \equiv D_{(N)}^{(-)}(Q,\theta_Q,b,z) \ ; \qquad \text{(A5)}$$

$$\exp\left[-i(m_\pi/k)\int_{-\infty}^{z}\langle V_\pi(\vec{b},\hat{k}z')\rangle dz'\right] \equiv D_{(\pi)}^{(+)}(k,b,z) \ . \qquad \text{(A6)}$$

The geometry used to obtain Eq. (A3) becomes ambiguous only when θ_K or $\theta_Q = 90°$ exactly. However, we have found that the D's defined by Eqs. (A4) and (A5) are smooth functions of θ (even when θ approaches 90°) and that Gaussian quadratures can be used for the numerical integration to avoid the extreme situation corresponding to $\theta=90°$. The nuclear form factor G_{QF} becomes

$$G_{QF} = (2\pi)^{-3/2}\theta\int b\,db\,d\phi\,dz \exp[iz(P\cos\theta_P + \eta k) + iPb\sin\theta_P\cos(\psi_P - \phi)] D_{(\pi)}^{(-)}(K,\theta_K,b,z) D_{(N)}^{(-)}(Q,\theta_Q,b,z) D_{(\pi)}^{(+)}(k,b,z)$$
$$\times R_{nl}[(b^2+z^2)^{1/2}]\left[\frac{2l+1}{4\pi}\right]^{1/2}\left[\frac{(l-m)!}{(l+m)!}\right]^{1/2} P_l^m[z/(b^2+z^2)^{1/2}]\exp(im\phi) \ . \qquad \text{(A7)}$$

If we let $\lambda = \psi - \psi_P$, and integrate with respect to λ, we obtain

$$G_{QF} = (2\pi)^{-1/2}\theta\int b\,db\,dz \exp[iz(P\cos\theta_P + \eta k)] J_m(Pb\sin\theta_P) D_{(\pi)}^{(-)}(K,\theta_K,b,z) D_{(N)}^{(-)}(Q,\theta_Q,b,z) D_{(\pi)}^{(+)}(k,b,z)$$
$$\times R_{nl}[(b^2+z^2)^{1/2}]\left[\frac{2l+1}{4\pi}\right]^{1/2}\left[\frac{(l-m)!}{(l+m)!}\right]^{1/2} P_l^m[z/(b^2+z^2)^{1/2}]\exp(im\pi/2 + im\psi_P) \ , \qquad \text{(A8)}$$

where J_m is the Bessel function of integer order m. Equation (A8) requires only a two-dimensional integration. The presence of $\exp(im\psi_P)$ in Eq. (A8) will further simplify the calculation of σ with Eq. (2.1), since $\int d\psi_p \exp(im\psi_P)\exp(-im'\psi_P) = (2\pi)\delta_{mm'}$. Thus, Bessel functions with different m orders contribute incoherently to σ. However, since A_{NCX} and $A_{\pi CX}$ also contain J_m, Bessel functions of the same order but associated with different reaction processes contribute coherently to σ.

[1]H. O. Ziock *et al.*, Phys. Rev. C **24**, 2674 (1981); R. J. Ellis *et al.*, Phys. Lett. **88B**, 253 (1979).
[2]E. Piasetzky *et al.*, Phys. Lett. **114B**, 414 (1982).
[3]B. J. Dropesky, G. W. Butler, C. J. Orth, R. A. Williams, G. Friedlander, M. A. Yates, and S. B. Kaufman, Phys. Rev. Lett. **34**, 821 (1975); Phys. Rev. C **20**, 1844 (1979); L. H. Batist, V. D. Vitman, V. P. Koptev, M. M. Makarov, A. A. Naberezhov, V. V. Nelyubin, G. Z. Obrant, V. V. Sarantsev, and G. V. Scherbakov, Nucl. Phys. **A254**, 480 (1975).
[4]S. B. Kaufman, E. P. Steinberg, and G. W. Butler, Phys. Rev. C **20**, 262 (1979); Y. Ohkubo, N. T. Profile, and C. J. Orth, *ibid.* **26**, 198 (1982).
[5]B. J. Lieb, H. S. Plendl, C. E. Stronach, H. O. Funsten, and V. G. Lind, Phys. Rev. C **26**, 2084 (1982).
[6]P. W. Hewson, Nucl. Phys. **A133**, 659 (1969); Hewson's calculation addressed the experimental data obtained by Chivers *et al.*, published in Nucl. Phys. **A126**, 129 (1969). However, it turned out that their data were incorrect. The reliable data were obtained by the authors of Ref. 3.
[7]M. M. Sternheim and R. R. Silbar, Phys. Rev. Lett. **34**, 824 (1975).
[8]Y. Ohkubo and N. T. Porile, Phys. Rev. C **25**, 2638 (1982). The charge-exchange parameter P_{CX} introduced in this reference can be related to the one used in Ref. 7 and in Eqs. (1.3) and (1.4) by a simple renormalization; J. Hufner, H. J. Pirner, and M. Thies, Phys. Lett. **59B**, 215 (1975).
[9]R. R. Silbar, Phys. Rev. C **12**, 341 (1975).
[10]T. Berggren, Annu. Rev. Nucl. Sci. **16**, 153 (1966); G. Jacob and A. J. Maris, Rev. Mod. Phys. **45**, 6 (1973); B. K. Jain and S. C. Phatak, Nucl. Phys. **A302**, 401 (1978); D. F. Jackson, A. A. Ioannides, and A. W. Thomas, *ibid.* **A322**, 493 (1979); E. Levin and J. M. Eisenberg, *ibid.* **A355**, 277 (1982); N. S. Chant, L. Rees, and P. G. Roos, Phys. Rev. Lett. **48**, 1784 (1982).

[11]S. Cohen and D. Kurath, Nucl. Phys. **A101**, 1 (1967).

[12]H. Überall, *Electron Scattering from Complex Nuclei* (Academic, New York, 1971), Vol. 1.

[13]L.-C. Liu, Phys. Rev. C **17**, 1787 (1978).

[14]L.-C. Liu, Nucl. Phys. **A223**, 523 (1975). The use of θ represents an approximation to restore the lack of orthogonality between the bound-state nuclear wave function and the continuum wave function of the ejected nucleon. Formal discussion of the orthogonality scattering can be found in C. M. Shakin and R. M. Thaler, Phys. Rev. C **7**, 494 (1973); J. P. Dedonder, Nuovo Cimento **76A**, 478 (1983).

[15]L. S. Rodberg and R. M. Thaler, *Introduction to the Quantum Theory of Scattering* (Academic, New York, 1967), Ch. 12-1; A. Picklesimer, P. C. Tandy, and R. M. Thaler, Phys. Rev. C **25**, 1233 (1982).

[16]R. L. Sugar and R. Blankenbecler, Phys. Rev. **183**, 1388 (1969).

[17]L. Celenza, L.-C. Liu, and C. M. Shakin, Phys. Rev. C **11**, 1593 (1975); L.-C. Liu and C. M. Shakin, in *Progress in Particle and Nuclear Physics,* edited by D. Wilkinson (Pergamon, New York, 1980), Vol. 5, Ch. 5, p. 211.

[18]R. S. Bhalerao, L.-C. Liu, and C. M. Shakin, Phys. Rev. C **21**, 2103 (1980). The $\langle \vec{k} \mid V_\pi \mid \vec{k} \rangle$ for ^{12}C can be obtained by multiplying the curves for ^{16}O in this reference by the scaling factor $\frac{12}{16}=\frac{3}{4}$.

[19]G. Rowe, M. Salomon, and R. Landau, Phys. Rev. C **18**, 584 (1978).

[20]L.-C. Liu and C. M. Shakin, Phys. Rev. C **16**, 1963 (1977).

[21]B. A. Watson, P. P. Singh, and R. E. Segel, Phys. Rev. **182**, 977 (1969).

[22]E. Levin, Ph.D. thesis, Tel Aviv University, Israel (unpublished).