Comparison of approximate chiral-dynamical $\pi N \to \pi\pi N$ models used in $A(\pi,2\pi)$ calculations

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The relative importance of the diagrams contributing to the $\pi N \to \pi\pi N$ reaction in the framework of the Weinberg effective Lagrangian theory is investigated. Total production cross sections, pion-pion angular correlations, and the energy spectrum of the outgoing nucleon given by various approximate schemes proposed in the literature are found to be very different from those predicted by the Weinberg theory. We indicate the inadequacies of these approximate schemes and discuss their implications to the predicted $(\pi,2\pi)$ total cross sections on complex nuclei. The relevance of our findings to microscopic models for pion-nucleus double charge exchange reactions is also discussed.

I. INTRODUCTION

The pion-induced single pion production on a free nucleon has been studied for a long time. Theoretical interest in this reaction near threshold stems to a large extent from its ability to discriminate among various chiral-dynamics models. Specifically, the chiral-symmetry-breaking parameter ξ appearing in the model Lagrangian can be determined by comparing near-threshold experimental cross sections for the reaction $\pi N \to \pi\pi N$ with those obtained in the soft-pion theory.[1] One of the most recent attempts in this direction has been described in Ref. 2. This reaction also serves as a probe of low energy $\pi\pi$ scattering which otherwise is difficult to study experimentally.[1,3]

The $(\pi,2\pi)$ reaction on a complex nucleus is also of great interest, since it may provide a new avenue for studying the pion field inside a nucleus. Eisenberg[4,5] has proposed this reaction as a means to selectively excite pionlike levels ($J^\pi = 0^-, 1^+, 2^-, \ldots;\ T=1$), to study the spin-isospin strength distribution in closed-shell nuclei, and to look for possible indications of pion condensation precursor phenomena. Since the basic $\pi N \to \pi\pi N$ cross section becomes comparable to the πN elastic scattering cross section at pion kinetic energies $T_\pi \sim 600$ MeV and exceeds the elastic scattering cross section at $T_\pi > 1$ GeV, we believe that the $(\pi,2\pi)$ reaction in complex nuclei represents one of the most powerful tools to study nuclei at pion energies above the much studied (3,3) resonance region.

In recent years, several authors have predicted total cross sections for the $A(\pi,2\pi)$ reaction as a function of the target nucleus mass and/or incident pion energy.[5-10] All of these authors use as a basic input the $\pi N \to \pi\pi N$ amplitude derived from Weinberg's effective Lagrangian[11,1] in some approximation. It is the primary purpose of this paper to point out the limitations of these approximations and to discuss what effects these approximations may have on the predicted behavior of the *nuclear* cross sections. We also show the effects of using the various approximate schemes on theoretical angular correlations of outgoing pions and on the energy spectrum of the outgoing nucleon in the $\pi N \to \pi\pi N$ reaction. These two quantities were not calculated in Refs. 5—10. As we shall see, approximate results for them also differ considerably from those obtained with the exact theory. We believe the results of this investigation are of vital importance to the theoretical interpretation of the $(\pi,2\pi)$ reaction in complex nuclei. Many such experimental studies are currently being planned at the Clinton P. Anderson Meson Physics Facility (LAMPF).

In Sec. II, we present the necessary theoretical framework and describe serious limitations of the various approximate procedures used to calculate the $\pi N \to \pi\pi N$ amplitude. These procedures have been used in the recent literature to predict $(\pi,2\pi)$ cross sections on complex nuclei. In Sec. III, we give our numerical results based on the full Weinberg effective Lagrangian theory. We discuss in particular the implications of our results to the published theoretical $A(\pi,2\pi)$ cross sections. We also discuss the relevance of our findings to other pion-nucleus reactions, taking the double charge exchange reaction as an example. The conclusions are presented in Sec. IV.

II. THEORY

We consider the reaction

$$\pi^-(Q)+p(p_i)\to\pi^+(q_1)+\pi^-(q_2)+n(p_f)\ , \tag{1}$$

since it is the most important channel of the $\pi N \to \pi\pi N$ reaction at $T_\pi < 1$ GeV. In Eq. (1) the four-momenta of the respective particles are given in parentheses. The cross section is given by[12]

$$\sigma=\int \delta^{(4)}(p_i+Q-p_f-q_1-q_2)\frac{|T|^2}{(2\pi)^5 v} \times\frac{d^3p_f d^3q_1 d^3q_2}{[(E_i/m)(E_f/m)2\omega_Q 2\omega_1 2\omega_2]}\ , \tag{2}$$

where v is the relative velocity between the initial π^- and p; m is the nucleon mass; E_i and E_f are the energies of the initial and final nucleons, respectively; and ω_Q, ω_1, and ω_2 are the energies of the pions. The invariant amplitude, T, given by Weinberg's effective Lagrangian in the lowest order perturbation theory, has the form

$$T=T^{(1)}+T^{(2)}+T^{(3)}+T^{(K)}\ , \tag{3}$$

where $T^{(1)}$, $T^{(2)}+T^{(K)}$, and $T^{(3)}$ represent, respectively, the contributions of the one-point, two-point, and three-point diagrams in Fig. 1. The term $T^{(K)}$ arises due to the anomalous magnetic moment term in the Lagrangian. We have[13]

$$T^{(1)}=i\frac{g_\pi}{2m}\frac{\sqrt{2}}{4f_\pi^2}\bar{u}(p_f)\left[4m+2q_2+8m\frac{2Q\cdot q_1-m_\pi^2\xi}{(p_f-p_i)^2-m_\pi^2}\right]\gamma_5 u(p_i)\ , \tag{4}$$

$$T^{(2)}=-i\frac{g_\pi}{2m}\frac{\sqrt{2}}{4f_\pi^2}\bar{u}(p_f)\left[4m+6q_2-\frac{2mq_1(q_2+Q)}{2p_f\cdot q_1+m_\pi^2}-\frac{2mQ(q_2-q_1)}{2p_f\cdot Q-m_\pi^2}+\frac{2m(q_2+Q)q_1}{2p_i\cdot q_1-m_\pi^2}+\frac{2m(q_2-q_1)Q}{2p_i\cdot Q+m_\pi^2}\right]\gamma_5 u(p_i)\ , \tag{5}$$

$$\begin{aligned}T^{(3)}=-i\left[\frac{g_\pi}{2m}\right]^3 2\sqrt{2}\bar{u}(p_f)\Bigg[&2q_2+\frac{2mq_2Q}{2p_i\cdot Q+m_\pi^2}+\frac{2mq_2q_1}{2p_i\cdot q_1-m_\pi^2}-\frac{2mQq_2}{2p_f\cdot Q-m_\pi^2}-\frac{2mq_1q_2}{2p_f\cdot q_1+m_\pi^2}\\&-\frac{4m^2Qq_2q_1}{(2p_f\cdot Q-m_\pi^2)(2p_i\cdot q_1-m_\pi^2)}-\frac{4m^2q_1q_2Q}{(2p_i\cdot Q+m_\pi^2)(2p_f\cdot q_1+m_\pi^2)}\Bigg]\gamma_5 u(p_i)\ ,\end{aligned} \tag{6}$$

and

$$\begin{aligned}T^{(K)}=-i\frac{g_\pi}{2m}\frac{K_v}{4f_\pi^2}2\sqrt{2}\bar{u}(p_f)\Bigg[&-q_2\cdot Qq_1\left[\frac{1}{2p_f\cdot q_1+m_\pi^2}+\frac{1}{2p_i\cdot q_1-m_\pi^2}\right]-q_1\cdot q_2Q\left[\frac{1}{2p_i\cdot Q+m_\pi^2}+\frac{1}{2p_f\cdot Q-m_\pi^2}\right]\\&+\frac{q_1q_2Q}{2p_f\cdot q_1+m_\pi^2}+\frac{Qq_2q_1}{2p_f\cdot Q-m_\pi^2}+\frac{q_2Qq_1}{2p_i\cdot q_1-m_\pi^2}+\frac{q_2q_1Q}{2p_i\cdot Q+m_\pi^2}\Bigg]\gamma_5 u(p_i)\ .\end{aligned} \tag{7}$$

Here g_π is the πNN effective (strong) coupling constant, f_π is the pion decay constant, m_π is the pion mass, and $K_v=1.85$.

Various approximations to the above amplitudes have been proposed in the literature. We discuss them in the following paragraphs.

Olsson and Turner[1] have neglected the contributions of $T^{(3)}$ and $T^{(K)}$ (see Ref. 14). They have further calculated $T^{(1)}$ and $T^{(2)}$ in the threshold approximation, which, in the c.m. frame, is defined by $Q=(\omega(\vec{Q}_{\rm thr}),\vec{Q}_{\rm thr})$, $p_i=(E_i(\vec{p}_{i,\rm thr}),\vec{p}_{i,\rm thr})$, $q_1=q_2=(m_\pi,\vec{0})$, and $p_f=(m,\vec{0})$. Using $|\vec{Q}_{\rm thr}|=|\vec{p}_{i,\rm thr}|=214.8$ MeV/c, one has $\omega(\vec{Q}_{\rm thr})=256.2$ MeV and $E_i(\vec{p}_{i,\rm thr})=962.6$ MeV. With these two approximations (denoted OT)

$$T^{(1)}=i\frac{g_\pi}{2m}\frac{\sqrt{2}}{4f_\pi^2}(-2m)\left[-2-\frac{m_\pi}{m}+\frac{2\omega_Q-m_\pi\xi}{\omega_Q-m_\pi}\right]\times\frac{\chi_f^\dagger(-\vec{\sigma}\cdot\vec{Q})\chi_i}{\sqrt{2m(E_i+m)}}\ , \tag{8}$$

$$T^{(2)}=-i\frac{g_\pi}{2m}\frac{\sqrt{2}}{4f_\pi^2}(-2m)\left[-\frac{3m_\pi}{m}+\frac{2m_\pi}{2m+m_\pi}-\frac{2m_\pi}{2E_i-m_\pi}\right]\times\frac{\chi_f^\dagger(-\vec{\sigma}\cdot\vec{Q})\chi_i}{\sqrt{2m(E_i+m)}}\ , \tag{9}$$

and therefore

$$T=i\frac{g_\pi}{2m}\frac{\sqrt{2}}{4f_\pi^2}(-2m)\left[\frac{2\omega_Q-m_\pi\xi}{\omega_Q-m_\pi}+\frac{2m_\pi}{m}-2-\frac{2m_\pi}{2m+m_\pi}+\frac{2m_\pi}{2E_i-m_\pi}\right]\frac{\chi_f^\dagger(-\vec{\sigma}\cdot\vec{Q})\chi_i}{\sqrt{2m(E_i+m)}}\ , \tag{10}$$

where χ are the two-component Pauli spinors. In Ref. 1, total cross sections for the reaction $\pi^-p\to\pi^+\pi^-n$ have been calculated for $T_\pi\le 300$ MeV with the use of Eqs. (2) and (10). We have further noted that in Ref. 1 the

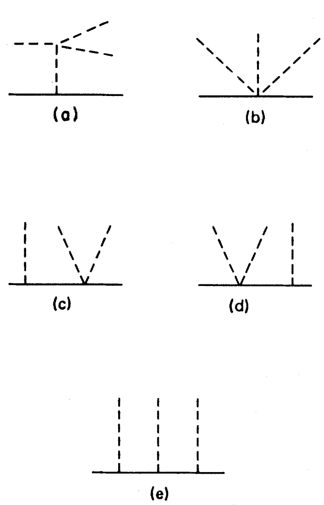

FIG. 1. (a) and (b) one-point, (c) and (d) two-point, and (e) three-point tree diagrams for the reaction πN$\to\pi\pi$N. Diagram (a) corresponds to the pion-pole term and diagram (b) to the contact term.

kinematical factors in the square brackets in Eq. (2) have also been replaced by their threshold values, i.e., $E_f = m$, $\omega_1 = \omega_2 = m_\pi$, $E_i = 962.6$ MeV, and $\omega_Q = 256.2$ MeV.

Rockmore in Ref. 9 has neglected $T^{(2)}$, $T^{(3)}$, and $T^{(K)}$. He used Eq. (8) to calculate $T^{(1)}$ and thus the total cross sections for the reaction $\pi^- p \to \pi^+ \pi^- n$ for $T_\pi \leq 290$ MeV. As in Ref. 1, the quantities inside the square brackets in Eq. (2) were also approximated by their threshold values. (Calculations based on this approximate scheme will be labeled *R*1.) Based on this approximate $\pi N \to \pi\pi N$ amplitude, he has predicted cross sections for the (π^-, π^+, π^-) reaction on ^{4}He, ^{12}C, and ^{16}O.

Eisenberg,[5] Cohen and Eisenberg,[6,8] and Cohen[7] have employed the approximate *center-of-mass* amplitude, Eq. (10), in the *laboratory* frame. Noting that for $E_i = m$, $\omega_Q = 311.95$ MeV, and $\xi = 0$ the quantity inside the square brackets in Eq. (10) is $\simeq 2 \times 0.93$, they wrote

$$T^{(1)} + T^{(2)} = -i \frac{g_\pi}{2m} \frac{\sqrt{2}}{4f_\pi^2} 2(0.93) \chi_f^\dagger (-\vec{\sigma} \cdot \vec{Q}) \chi_i \ , \quad (11)$$

and used Eq. (11) to calculate various $A(\pi, 2\pi)$ cross sections at T_π below ~ 410 MeV. (We shall denote this approximation as EC.)

Recently, Rockmore[10] calculated $(\pi^+, 2\pi^+)$ total cross sections on complex nuclei for $T_\pi \leq 280$ MeV, using yet another approximation for the $\pi N \to \pi\pi N$ amplitude. The basic reaction in this case is

$$\pi^+(Q) + p(p_i) \to \pi^+(q_1) + \pi^+(q_2) + n(p_f) \ , \quad (12)$$

for which[13]

$$T^{(1)} = -i \frac{g_\pi}{2m} \frac{\sqrt{2}}{4f_\pi^2} \bar{u}(p_f) \times \left[2(q_1 + q_2) + 8m \frac{2q_i \cdot q_2 + \xi m_\pi^2}{(Q - q_1 - q_2)^2 - m_\pi^2} \right] \times \gamma_5 u(p_i) \ . \quad (13)$$

He neglected the contributions of $T^{(2)}$, $T^{(3)}$, and $T^{(K)}$, as well as of the first term on the right-hand side of Eq. (13). The second term which corresponds to Fig. 1(a) was then evaluated in the threshold approximation. (We shall denote this approximation as *R*2.) In the c.m. frame he thus has

$$T^{(1)} = -i \frac{g_\pi}{2m} \frac{\sqrt{2}}{4f_\pi^2} (-2m) \frac{m_\pi (2 + \xi)}{\omega_Q - m_\pi} \frac{\chi_f^\dagger (-\vec{\sigma} \cdot \vec{Q}) \chi_i}{\sqrt{2m(E_i + m)}} \ . \quad (14)$$

If the approximation *R*2 is made for the reaction $\pi^- p \to \pi^+ \pi^- n$, Eq. (4) would become

$$T^{(1)} = i \frac{g_\pi}{2m} \frac{\sqrt{2}}{4f_\pi^2} (-2m) \frac{2\omega_Q - m_\pi \xi}{\omega_Q - m_\pi} \frac{\chi_f^\dagger (-\vec{\sigma} \cdot \vec{Q}) \chi_i}{\sqrt{2m(E_i + m)}} \ . \quad (15)$$

In spite of the diversity of these approximations, we note from the structure of Eq. (2) that as soon as the threshold approximation is invoked, the amplitude T becomes independent of the energy and momentum of the particles in the final state. Consequently, *the results of these different approximate theories differ from each other and from the phase space only by constant multiplicative factors.* That is to say, they all have the same energy and angular dependence as that predicted by the phase space.

In the following section, we examine the quality of the above approximate schemes by comparing the $\pi N \to \pi\pi N$ cross sections obtained by using these schemes with those given by the exact calculation of Eq. (3).

III. NUMERICAL RESULTS AND DISCUSSION

We first study the relative importance of the diagrams in Fig. 1 by calculating without any approximation their contributions to the $\pi^- p \to \pi^+ \pi^- n$ total cross sections (Fig. 2). The result based upon a calculation that includes all the diagrams in Fig. 1 is labeled *exact*. The remaining curves in Fig. 2 are obtained by including only some of the diagrams in the calculation (see Fig. 2 caption). The Monte Carlo technique has been employed to perform the multidimensional integration in Eq. (2). An inspection of Fig. 2 indicates that even at T_π as low as 190 MeV, the diagrams (b), (c), and (d) in Fig. 1 are far from negligible with respect to the pion-pole diagram [Fig. 1(a)]. On the

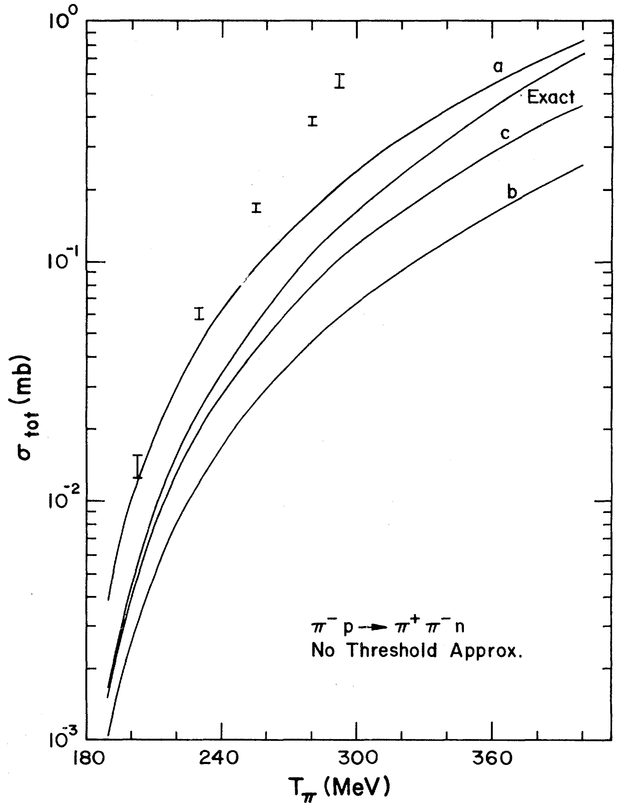

FIG. 2. Total cross section for the reaction $\pi^- p \to \pi^+ \pi^- n$ as a function of the incident pion kinetic energy in the laboratory frame. The curve based on a calculation that includes all the diagrams in Fig. 1 is labeled *exact*. The remaining curves are based on calculations that include only some of the diagrams: (a) diagram 1(a) only, (b) diagrams 1(a) and (b), and (c) diagrams 1(a)–(d). Experimental data are from Refs. 17 and 18.

other hand, the diagram (e) in Fig. 1 is relatively unimportant below $T_\pi \simeq 240$ MeV. The term $T^{(K)}$ is negligible over the entire energy region considered. As in Ref. 13, we have used $g_\pi = 13.5$ and $f_\pi = 87$ MeV in all the calculations presented in this paper. The results are not insensitive to the choice of these parameters. For example, if we use $g_\pi = 13.4$ and $f_\pi = 82$ MeV as in Ref. 10, the cross sections are increased by about 20%.

The $\pi^- p \to \pi^+ \pi^- n$ total cross sections based on the four approximate schemes described in Sec. II are compared with those based on the full theory in Figs. 3 and 4. In view of the diversity of approximations employed in Refs. 5—10, we find it convenient to consider the following two cases separately:

(a) Threshold approximation No. 1: T as well as E_f, ω_1, and ω_2 in the square brackets in Eq. (2) are calculated at threshold.

(b) Threshold approximation No. 2: T and *all* the five quantities in the square brackets in Eq. (2) are calculated at threshold.

The corresponding total cross sections are presented in Figs. 3 and 4, respectively.

It is evident from Fig. 3 that even in the near-threshold region, there are sizable differences among the curves

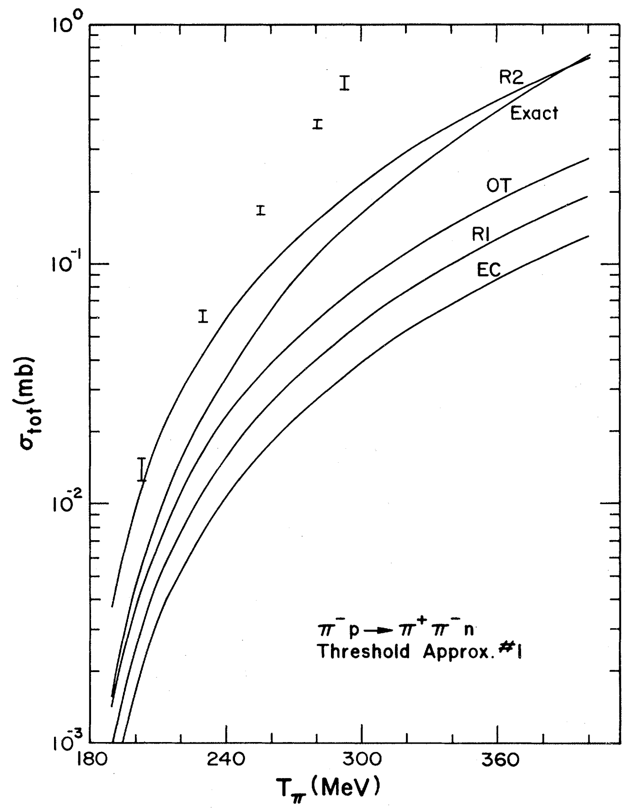

FIG. 3. Total cross section for the reaction $\pi^- p \to \pi^+ \pi^- n$ as a function of the incident pion kinetic energy in the laboratory frame. Curve labeled *exact* is the same as in Fig. 2. The remaining curves represent the four approximate schemes described in Sec. II: OT (Ref. 1), $R1$ (Ref. 9), $R2$ [Eq. (15)], and EC (Refs. 5—8). Threshold approximation No. 1 defined in Sec. III was used. Experimental data are from Refs. 17 and 18.

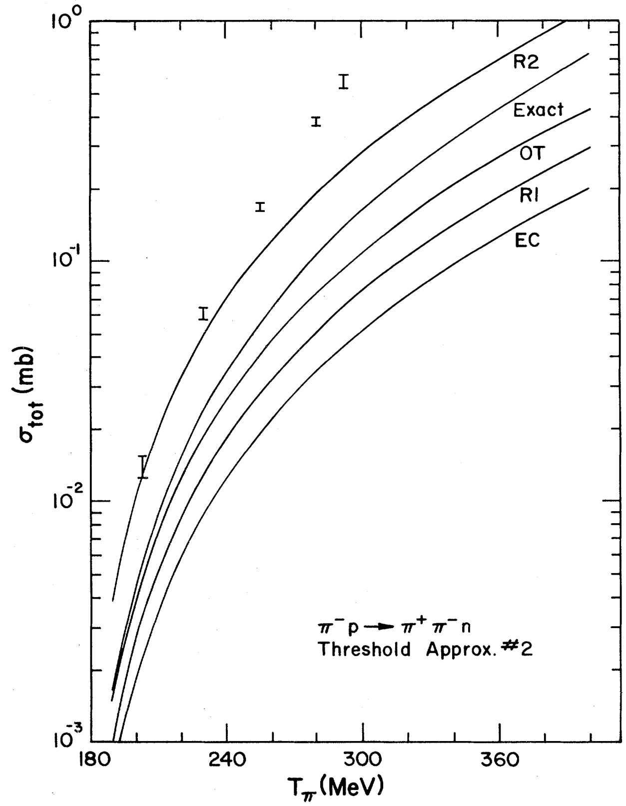

FIG. 4. Same as Fig. 3 for the threshold approximation No. 2.

given by the exact and approximate theories. With the exception of $R2$, cross sections calculated with approximate amplitudes are much lower than those obtained with the full theory. Because of the fact that the theoretical cross sections labeled *exact*, in turn, are much smaller than the experimental data, *the approximate amplitudes for* $\pi^- p \to \pi^+ \pi^- n$ *when used in the calculation of* (π^-, $\pi^+\pi^-$) *cross sections on nuclei would tend to underestimate the nuclear cross sections, in some cases, by even an order of magnitude.* Results in Refs. 5—9 should be viewed in light of these remarks.

It has been suggested in Ref. 5 that the existence of pion condensation precursor phenomena would enhance the $A(\pi, 2\pi)$ cross sections considerably above what would be observed otherwise, and such an enhancement may provide partial evidence in support of the existence of such phenomena. However, it is evident from the inspection of curves EC and *exact* in Fig. 3 that a similar large enhancement can already be achieved by simply making an exact calculation of the $\pi N \to \pi\pi N$ amplitude. Interpretation of the future $A(\pi, 2\pi)$ data[15,16] will have to be made bearing this in mind.

In Fig. 4, the curves $R2$, OT, $R1$, and EC are somewhat higher than they are in Fig. 3. This increment is the result of using an additional approximation, namely, $\omega_Q = \omega(\vec{Q}_{thr})$ and $E_i = E_i(\vec{p}_{i,thr})$, as in case (b). Apparently, this is the procedure adopted in Refs. 1 and 9. For the energy range being investigated, this approximation does not seem to be appropriate.

A closer examination of the approximation $R2$ is in order. One should note that this approximation was originally proposed for the calculation of the total cross sections of the $(\pi^+,2\pi^+)$ reactions on complex nuclei.[10] In these reactions, if the impulse approximation holds, the basic interaction will be $\pi^+p\to\pi^+\pi^+n$. Indeed, as can be seen from Fig. 5, $R2$ is in somewhat better agreement with the data for this reaction than was the corresponding curve in Fig. 3 with the data for the $\pi^-p\to\pi^+\pi^-n$ reaction. However, we believe that this apparent success of $R2$ should not be considered as an indication of its general validity. Our reservations are based on the following observations: (a) $R2$ has been formulated within the framework of the Weinberg theory, yet it takes into account only one single term, the pion-pole term. We have noted that there are large contributions of other diagrams to all branches of the $\pi N\to\pi\pi N$ reaction. A simple neglect of the other diagrams is thus hard to justify. (b) We have calculated total cross sections for the reactions $\pi^-p\to\pi^0\pi^-p$ and $\pi^+p\to\pi^0\pi^+p$, using the same set of assumptions as in Ref. 10. These cross sections, together with those based on the exact theory, are compared with experimental data[21,22] in Fig. 6. We see that the curve $R2$ fails to describe the data by about an order of magnitude. Similar deviations were also found in the case of the $\pi^-p\to\pi^0\pi^0n$ reaction (curves not shown). Our point of view is that all different charge channels of the $\pi N\to\pi\pi N$ process should be described by one single theory, and a good theory should be able to describe the stronger channels. As we know, in the energy region under consideration $\pi^+p\to\pi^+\pi^+n$ is the weakest of the five single-pion production channels; at $T_\pi\sim 250$ MeV, it is about an order of magnitude weaker than the $\pi^-p\to\pi^+\pi^-n$ reaction. A reasonable modeling of only the weakest channel, as is the case with the approximation $R2$, is thus of limited applicability.

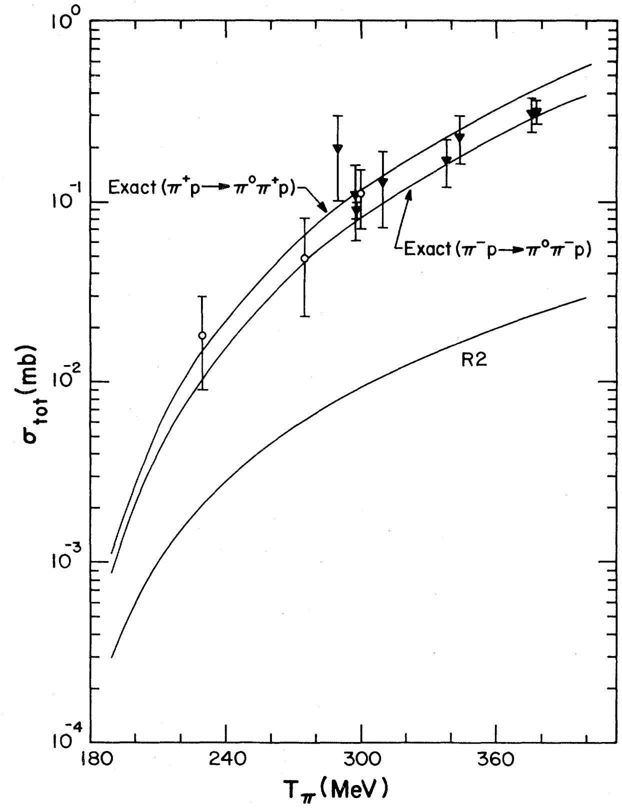

FIG. 6. Same as Fig. 3 for the reactions $\pi^+p\to\pi^0\pi^+p$ and $\pi^-p\to\pi^0\pi^-p$. The curve $R2$ is the same for the two reactions. Experimental data for $\pi^+p\to\pi^0\pi^+p$ (denoted ○) are from Ref. 21 and those for $\pi^-p\to\pi^0\pi^-p$ (denoted ▼) are from Ref. 22.

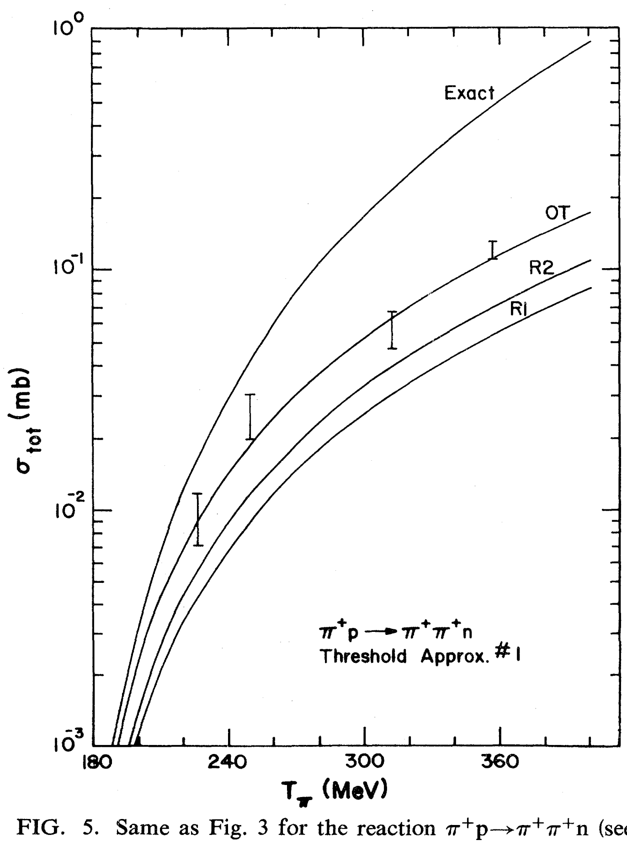

FIG. 5. Same as Fig. 3 for the reaction $\pi^+p\to\pi^+\pi^+n$ (see Ref. 19). Experimental data are from Ref. 20.

So far, we have concentrated on the dependence of the total production cross section on theoretical approximation schemes used. It is equally interesting to differentiate their predictions for other observables. To our knowledge, this apsect has not been examined in the literature. In Figs. 7 and 8, we present for the $\pi^-p\to\pi^+\pi^-n$ reaction the energy spectrum of the outgoing neutron and the angular correlations of the outgoing pions. Curves labeled *exact* represent results obtained in the exact theory. Curves *a* and *b* correspond to results obtained with the approximate scheme of Eisenberg and Cohen[5–8] in the threshold approximation No. 1 and No. 2, respectively. It is noteworthy that in Figs. 7 and 8 the curves labeled *exact* and EC are significantly different in shape and magnitude.

Our remark in Sec. II concerning the essential phase-space nature of the calculations of Refs. 5–10 is borne out by the inspection of curves $R2$, OT, $R1$, and EC in Figs. 3 and 4. As we have anticipated, these four curves differ from each other only by constant multiplicative factors. The same remark applies to Figs. 7 and 8, in which we have presented results based on only one approximate scheme, namely, that of Eisenberg and Cohen.[5–8] Results

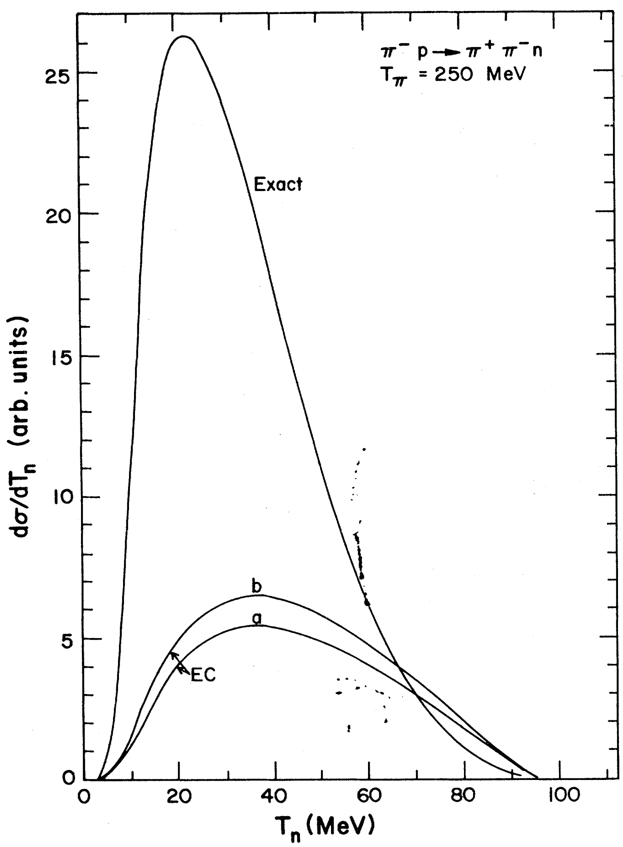

FIG. 7. Energy spectrum of the outgoing neutron in the reaction $\pi^- p \rightarrow \pi^+ \pi^- n$ in the laboratory frame. The curve labeled *exact* represents an exact calculation which includes all the terms in Eq. (3). Curves *a* and *b* are based on the Eisenberg-Cohen scheme with threshold approximations No. 1 and No. 2, respectively.

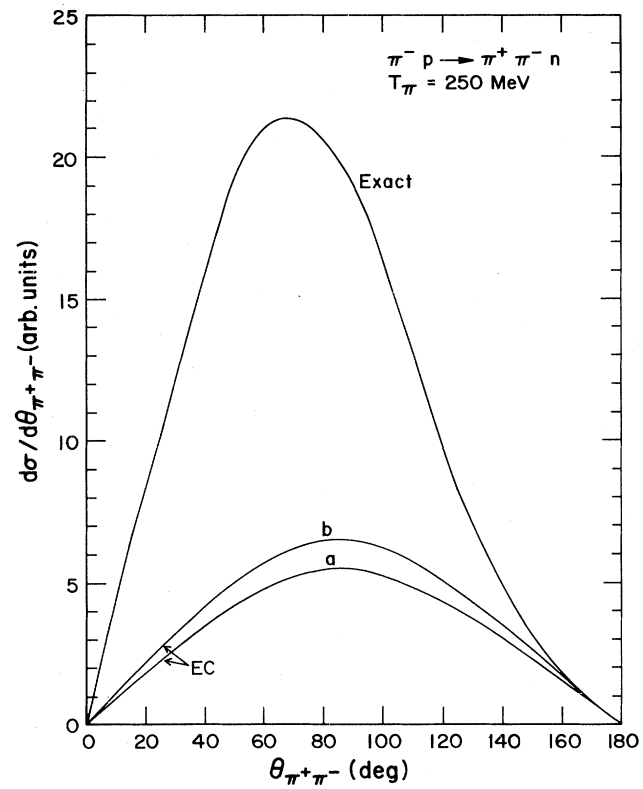

FIG. 8. Angular correlations of the outgoing pions in the reaction $\pi^- p \rightarrow \pi^+ \pi^- n$ in the laboratory frame. Meanings of the curves are the same as in Fig. 7.

based on other approximate schemes (*R*1, *R*2, and OT) and on the phase-space calculations will be identical to curves *a* and *b* in shape, but will differ in magnitude. In this respect, we can regard all the approximate theories as containing only minimal information on the dynamics.

In this paper, we paid particular attention to the various approximate schemes used in the recent literature to calculate the $\pi N \rightarrow \pi\pi N$ amplitude. We did not concern ourselves with other approximations that the authors of Refs. 5—10 have made in the calculation of $(\pi,2\pi)$ cross sections on *nuclei*, for example, the neglect of off-shell effects of the basic production amplitude. In fact, the expressions for $T^{(i)}$ in Sec. II are valid only if all the external particles in Fig. 1 are on mass shell, which is not the case if the reaction takes place in a nuclear medium. Moreover, unlike the fully on-shell case, the off-shell amplitude can be singular.

Pion-nucleus double charge exchange reaction

In recent years, there has been considerable interest in calculating meson exchange current (MEC) contributions to the pion-nucleus double charge exchange reactions (DCX).[23,24] Germond and Wilkin calculated one such diagram in which the incoming pion scatters off a virtual pion in a nucleus.[23] This diagram was found to give a large contribution to the total DCX cross section on ^{4}He for $T_\pi \leq 475$ MeV. More recently, Oset *et al.*[24] calculated the excitation function for $^{18}O(\pi^+,\pi^-)^{18}Ne$ in the (3,3) resonance region by adding the contribution of the $\pi\pi$ scattering diagram to that due to conventional sequential single charge exchanges. The MEC effect was found to increase the cross section by at least 50% at $T_\pi = 250$ MeV. Here, our findings of the large cancellations between the pion-pole diagram and other "tree" diagrams should serve as a caution in calculating pion DCX by adding only the $\pi\pi$ scattering diagram. More specifically, since the contact term [Fig. 1(b)] is mainly responsible for the noted cancellation in low-energy $\pi N \rightarrow \pi\pi N$ reactions, one expects that the important MEC effects on DCX cross sections, found in Ref. 24, could be largely canceled by the DCX process via the contact interaction:

$$(\pi^+ n)n \xrightarrow{\text{contact}} (\pi^- p \pi^+)n \rightarrow \pi^- pp \ .$$

A similar cancellation was noted by Robilotta and Wilkin[25] in connection with the calculation of the pion-deuteron scattering length. In Ref. 24, this was also discussed, but no numerical evidence was presented.

Recently, Johnson *et al.*[26] also calculated $^{18}O(\pi^+,\pi^-)^{18}Ne$ cross sections by considering the conventional sequential mechanism together with diagrams which involve $\pi\Delta\Delta$ and $\rho\Delta\Delta$ vertices; but no $\pi\pi$ scattering diagram was considered. The main source of uncertainties embedded in the calculations that include $\pi\Delta\Delta$ and $\rho\Delta\Delta$ vertices (as well as those proposed in Refs. 23 and 24) is the poor knowledge of the coupling constants and interaction ranges associated with these vertices. Since these vertices also appear in a full theory for the $\pi N \rightarrow \pi\pi N$ reaction, the study of pion-induced pion production should yield useful information and provide constraints on these coupling constants and range parameters. Clearly, the same information can be used for the modeling of many other nuclear reactions than DCX, in which pionic degrees of freedom and/or virtual pion production may be important. In this regard, we believe that the development of an adequate microscopic theory for pion-induced pion production is called for.

IV. CONCLUSIONS

We summarize our findings as follows. (1) Contributions made by the contact term and the two-point diagrams to the total cross section for the $\pi^- p \rightarrow \pi^+\pi^- n$ reaction are far from negligible when compared to the contribution of the pion-pole term. (2) With the exception of the curve labeled $R2$ (Fig. 3), cross sections for $\pi^- p \rightarrow \pi^+\pi^- n$ given by the approximate theories are much smaller than those given by the full theory. Disagreement between the former results and the experimental data is even more marked. Although $R2$ happens to be closer to the experimental data than are the other curves, we have given in Sec. III our reservations about this approximate scheme. (3) Approximate schemes used in the literature are shown to contain only minimal dynamical information in that their predictions differ from the phase space prediction by at most a multiplicative constant. (4) Predicted cross sections for $A(\pi,2\pi)$ reactions suffer from these drawbacks. In particular, if the existence of pion condensation precursor phenomena is to be tested in $A(\pi,2\pi)$ reactions, then the calculation of the basic $\pi N \rightarrow \pi\pi N$ amplitude will have to be done more accurately than has been reported. Otherwise, the importance of precursor phenomena, if they exist at all, would be overestimated to compensate for the low theoretical cross sections arising from the use of the approximate theories.[5–9] (5) Angular correlations of the outgoing pions and the energy spectrum of the outgoing neutron calculated in the exact theory and with the approximate schemes of Refs. 5–10 differ significantly in magnitude and shape, indicating the additional inadequacies of the latter theories. In particular, analysis of the measured $\pi\pi$ angular correlation in $A(\pi,2\pi)$ reactions with an approximate theory may lead to erroneous conclusions about pion propagation in a nucleus. (6) We have stressed the interrelationship between the $\pi N \rightarrow \pi\pi N$ reaction and many other pion-nucleus reactions, taking the double charge exchange as a specific example. We have expressed caution against making calculations of these reactions with an inadequate set of diagrams. We believe a thorough study of $\pi N \rightarrow \pi\pi N$ reactions will provide useful guidance in improving such calculations.

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