$C2$ contributions to back-angle inelastic electron scattering from ^{166}Er and ^{181}Ta

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We present results of distorted-wave Born approximation calculations of longitudinal contributions to 180° inelastic electron scattering, in the limit of zero electron mass and pure Coulomb distortion. A rough estimate of said contributions is given for the transitions $0^+ \rightarrow 2^+$ in ^{166}Er, and $\frac{7}{2}^+ \rightarrow \frac{9}{2}^+$ and $\frac{7}{2}^+ \rightarrow \frac{11}{2}^+$ in ^{181}Ta. Results are compared to plane-wave Born approximation calculations of transverse form factors.

In a previous paper,[1] one of us pointed out that, in distorted wave Born approximation (DWBA), the differential cross section at 180° for inelastic electron scattering from nuclei contains charge multipole contributions that are different from zero even for ultrarelativistic electron ($m_{e/\epsilon_e} \rightarrow 0$). In this Brief Report we give a rough estimate of these contributions in the limit of zero electron mass. As is well known,[2] in this limit the differential cross section at 180° in plane wave Born approximation (PWBA) is proportional to the transverse form factor,

$$\frac{d\sigma(\epsilon_i,\theta=\pi)}{d\omega} = \frac{Z^2\alpha^2|F_T(q)|^2}{4\epsilon_i^2(1+2\epsilon_i/M_{\text{targ}})} , \tag{1}$$

with $\epsilon_i = K_i$, $\epsilon_f = K_f$, the initial and final electron energies and momenta, $q = K_i + K_f$, and F_T is the transverse form factor involving only transverse multipoles of the nuclear transition vector current. The PWBA result expressed in Eq. (1) has motivated[3] many experiments of electron scattering at 180° to get information on current distributions in nuclei, since electron scattering at lower angles is, in general, dominated by longitudinal form factors. This is particularly so for transitions within ground-state rotational bands in axially symmetric deformed nuclei. In particular, for $0^+ \rightarrow \lambda^+$ (λ = even ≥ 2) transitions within the ground-state band of even-even rotational nuclei, the cross section at back-angle measures, according to Eq. (1), the transverse $E\lambda$ multipole of the collective rotational current in the band,[4] and thus, experiments at 180° offer a way to obtain direct and unique information on the nature of the nuclear collective rotational motion. However, as said before, if we take into account distortion effects, the longitudinal $C\lambda$ multipole also contributes. It is then very important, for the correct interpretation of experimental data, to know the order of magnitude of the longitudinal contribution and, in particular, to know whether the latter is comparable or not to the transverse form factor in Eq. (1). If the $C\lambda$ contributions in DWBA turn out to be much smaller than F_T, we may expect Eq. (1) to hold, but otherwise a complete DWBA analysis of the experimental data at 180° will be required.

In DWBA, the $C\lambda$ contribution to the differential cross section at 180° is given by[1]

$$\left.\frac{d\sigma_{C\lambda}(\epsilon_i,\theta=\pi)}{d\Omega}\right|^{\text{DWBA}} = \frac{\alpha^2 64\pi\epsilon_F^2}{(1+2\epsilon_i/M_{\text{targ}})} \frac{|\mathscr{M}^C_{\lambda 1}|^2}{(2I_i+1)} , \tag{2}$$

where $\mathscr{M}^C_{\lambda 1}$ ($\equiv \mathscr{M}^C_{\lambda\,\mu=1}$) is the individual reduced amplitude corresponding to the total contribution of the $C\lambda$ multipole (ρ_λ) of the nuclear transition charge. The notation and conventions are as in Ref. 1. To compare with the transverse form factor in Eq. (1), we computed the $C2$ "form factor"

$$|F_{C2}|^2 = \frac{4\epsilon_i^2(1+2\epsilon_i/M_{\text{targ}})}{Z^2\alpha^2} \left.\frac{d\sigma_{C2}(\epsilon_i,\theta=\pi)}{d\Omega}\right|^{\text{DWBA}}$$
$$= \frac{256\pi\epsilon_i^2\epsilon_f^2}{Z^2(2I_i+1)} |\mathscr{M}^C_{21}|^2 . \tag{3}$$

For the transition $0^+ \rightarrow 2^+$ in ^{166}Er, this is the whole longitudinal contribution; for the transitions $\frac{7}{2}^+ \rightarrow \frac{9}{2}^+$ and $\frac{7}{2}^+ \rightarrow \frac{11}{2}^+$ in ^{181}Ta, higher longitudinal multipoles ($C4, C6, C8$) also contribute, but since these peak at higher q values and the transition energies involved are small, we may forget them in a first estimate.[1] Note that $\mathscr{M}^C_{\lambda 1}$ [see Eqs. (2) and (4) in Ref. 1] changes sign under the interchange of ϵ_i and ϵ_f, then for small $\omega (= \epsilon_i - \epsilon_f)$ we may expand $\mathscr{M}^C_{\lambda 1}$ in powers of $\omega/2K$ [with $K = (\epsilon_i + \epsilon_f)/2$] and get to second order $\mathscr{M}^C_{\lambda 1} \propto \omega/q$. We have taken this expansion for the evaluation of $\mathscr{M}^C_{21}$.

Following Ref. 1, we write the $C2$ amplitude as

$$\mathscr{M}^C_{21} = \int r^2 dr \int r'^2 dr' \rho_2(r') \frac{r_<^2}{r_>^3} \frac{3}{8\sqrt{6}} \times \sum_{l\geq 1} (-1)^l [G_l(x,x') - G_l(x',x)] , \tag{4}$$

with $x = K_i r$, $x' = K_f r$, and

$$G_l(x,x') = \frac{l(l+1)}{(l^2-\frac{1}{4})(l+\frac{3}{2})} [(l+2)(l-\tfrac{1}{2}) e^{i(\delta_l+\delta_{l+2})} f_l(x) f_{l+2}(x')$$
$$- (l+\tfrac{1}{2}) e^{i(\delta_l+\delta_{l+1})} f_l(x) g_{l+1}(x') - (l-1)(l+\tfrac{3}{2}) e^{i(\delta_{l-1}+\delta_{l+1})} g_{l-1}(x) g_{l+1}(x')$$
$$- (l+\tfrac{1}{2}) e^{i(\delta_l+\delta_{l+1})} f_{l+1}(x) g_l(x')] , \tag{5}$$

where we have rearranged the sum over partial waves l so that in the plane wave limit each term is identically zero. The notation and conventions for phase shifts (δ_l) and radial solutions (f_l, g_l) of the Dirac equation for the electron in the electrostatic field $V(r)$ are as in Ref. 5. To evaluate Eqs. (4) and (5), we used the well-known expressions[5,6] of phase

shifts and radial functions corresponding to a pure Coulomb potential [$V(r)=-\alpha Z/r$], in the limit of zero electron mass. Consistently with the point charge approximation [$V(r)=-\alpha Z/r$] we took $r_<$ and $r_>$ in Eq. (4) to be the nuclear and electronic coordinates, respectively, and evaluated the electronic integrals

$$I_l=\int_0^\infty \frac{dr}{r}[G_l(x,x')-G_l(x',x)] \ . \tag{6}$$

These are somewhat different from the radial integrals of Dirac-Coulomb functions usually discussed in the literature,[6] for if we neglect the energy difference I_l is automatically zero.

The easiest way to evaluate these integrals is to make an expansion in $\alpha Z/l$. Using the recurrence relations for the confluent hypergeometric functions, one can show, after a straightforward but tedious algebra, that the integrand in Eq. (6) can be written to lowest order in $\alpha Z/l$ as

$$\begin{aligned}
&G_l(x,x')-G_l(x',x)\\
&\cong e^{-(z+z')/2}(-iy)(-zz')^l e^{\pi y}\left[\frac{\Gamma(l+1-iy)}{\Gamma(2l+3)}\right]^2\frac{(2l+1)(l+1)^2}{(l^2-\frac{1}{4})(l+\frac{3}{2})}\\
&\times\left[(2l-1)\frac{z'}{z}\left\{{}_1F_1(l,2l;z)\left[{}_1F_1(l+2,2l+4;z')\left[2l+1+\frac{2l(l+2)}{z'}+\frac{2l(l+1)}{z}\right]+{}_1F_1(l+3,2l+6,z')\frac{lz'}{2(2l+5)}\right]\right.\right.\\
&\left.+{}_1F_1(l+1,2l+2;z)\,{}_1F_1(l+2,2l+4;z')\frac{z(l+1)}{2(2l+1)}\right\}+{}_1F_1(l+1,2l+2;z)\,{}_1F_1(l,2l;z')\frac{2(l+1)(2l+1)}{z'}\\
&\left.-{}_1F_1(l+2,2l+4;z)\,{}_1F_1(l+1,2l+2;z')\frac{lz}{2(2l+3)}-(z\rightleftharpoons z')\right] \ ,
\end{aligned} \tag{7}$$

with $z=2ix$, $z'=2ix'$, $y=\alpha Z$. Then, the integrals involved in Eq. (6) reduce to integrals of products of spherical Bessel functions and powers having simple analytical solutions (for more details see Ref. 7), and after the expansion in ω/K we get

$$\begin{aligned}
I_l&\cong\frac{\omega}{K}a_l\equiv\frac{\omega}{K}\frac{2\pi}{5}y\frac{\exp[\pi y-2iy\psi(l+1)]}{(4l^2-1)(2l+3)}\\
&\times\left[i\frac{(2l+1)^2(2l-1)(2l+3)}{3\pi l(l+1)}+\frac{l^2+l+1}{2l+1}\right.\\
&\left.-l(l+1)\left[\frac{1}{2l-1}+\frac{1}{2l+3}\right]\right] \ ,
\end{aligned} \tag{8}$$

with $\psi(n)=\Gamma'(n)/\Gamma(n)$. Then $\mathscr{H}_{21}^C$ in Eq. (4) takes the form

$$\mathscr{H}_{21}^C\cong\frac{\omega}{K}I(\alpha Z)\int r^4\rho_2(r)\,dr \ , \tag{9}$$

with

$$I(\alpha Z)=\frac{3}{8\sqrt{6}}\sum_{l\geq 1}(-1)^l a_l \ . \tag{10}$$

To sum the infinite series in Eq. (10) we used the reduction method based on the recurrence relations of the Legendre polynomials [note that $(-1)^l=P_l(\cos\theta)$ for $\theta=\pi$] described in Sec. II, Pt. 4 of Ref. 8. Since a_l goes like $1/l$, the series converges quite rapidly after just one reduction.

Results for $|I(\alpha Z)|$ are given in Table I for different Z values. For the cases of interest here ($\alpha Z=0.5$) the expansion in $\alpha Z/l$ is not well justified for partial waves $l<10$. Therefore, for $\alpha Z\gtrsim 0.5$, we used an alternative method to compute the integrals I_l (see results within parenthesis in Table I). This method is based on the work of Gargaro and Onley.[9] We first made a second order expansion in ω/K of the integrand using the relation[10]

$${}_1F_1(a,b;\lambda z)=\lambda^{-a}\sum_{n=0}^{\infty}\frac{\Gamma(a+n)}{n!\Gamma(a)}(1-\lambda^{-1})^n \times {}_1F_1(a+n,b;z) \ ,$$

with $\lambda=1\pm\omega/2K$, $z=2iKr$. Then the integrals involved in Eq. (6) are of the form

$$I=\int_0^\infty dr r^{\nu-1}e^{-2iKr}\,{}_1F_1(a,b;2iKr)\,{}_1F_1(\bar{a},\bar{b};2iKr) \ , \tag{11}$$

with K real, $R_e(b)>R_e(a)>0$,

$$R_e(\bar{b}-\bar{a}+a-\nu)>0, \quad \nu-b+1\neq 0,-1,-2,\ldots , \tag{12}$$

whose solution is given in Eq. (14) of Ref. 9 (see also Ref. 7). In our case, that solution has to be handled with care. For some of the integrals (11) involved in Eq. (6), the conditions (12) are only satisfied to order $(\alpha Z/l)^2$, leading to great numerical instabilities when that solution is used for large l values. The values of $|I(\alpha Z)|$ given within parenthesis in Table I were obtained using this last method for $l<10$ and the result of Eq. (8) for $l\geq 10$. We should mention, however, that since in the approach taken here we have neglected the effect of the finite nuclear size which is

TABLE I. Calculated $|I(\alpha Z)|$ vs αZ values.

αZ	0.04	0.1	0.2	0.3	0.4	0.5	0.6
$\lvert I(\alpha Z)\rvert$	1.18×10^{-3}	3.61×10^{-3}	1.00×10^{-2}	2.09×10^{-2}	3.89×10^{-2}	6.80×10^{-2} (2.65×10^{-1})	1.15×10^{-1} (3.07×10^{-1})

more important for the smaller l values, we cannot expect to get a very realistic answer with either of the two methods for $l \lesssim 10$. Complete DWBA calculations taking into account the nuclear extension and the electron mass will be presented in a forthcoming publication.[11]

In the approach of point charge nucleus, taken here, the nuclear structure only enters the calculation [see Eq. (9)] via the (q independent) quadrupole moment. To compare with the transverse form factor, we simulate the appropriate q dependence (finite nuclear size) of F_{C2}, defined in Eq. (3), replacing in Eq. (9) $\int r^4 \rho_2(r) dr$ by $(15/q^2) \times \int j_2(qr) \rho_2(r) r^2 dr$, having the same behavior as the nuclear radius goes to zero. Finally, for application to the nuclear transitions within ground state rotational bands under consideration, we write[4]

$$\rho_2(r) = \sqrt{2I_i+1}\langle I_i K 20 | I_f K \rangle \rho_2^{\text{int}}(r) \ , \tag{13}$$

where $\rho_2^{\text{int}}(r)$ is the intrinsic quadrupole charge distribution in the expansion

$$\rho^{\text{int}}(\vec{r}) = \sum_{\lambda=\text{even}} \rho_\lambda^{\text{int}}(r) Y_{\lambda 0}(\Omega) \ . \tag{14}$$

Approaching the intrinsic charge distribution by

$$\rho^{\text{int}}(\vec{r}) = \rho\left[r\left[1 - \sum_\lambda \alpha_\lambda P_\lambda(\cos\theta)\right]\right] \ ,$$

with $\rho(r)$ a two-parameter Fermi function $\rho(r) = \rho_F(1+e^{(r-c)/a})^{-1}$, we get to first order in the deformation parameters α_λ

$$\rho_2^{\text{int}}(r) = -\left(\frac{4\pi}{5}\right)^{1/2} \alpha_2 r \frac{\partial \rho^{(r)}}{\partial r} \ . \tag{15}$$

For ^{166}Er, we took $c = 6.16$ fm, $t (= 4a \ln 3) = 2.37$ fm, $\alpha_2 = 0.2$ adjusted to the experimental values of the intrinsic quadrupole moment $Q_0 = 756$ fm^2, and rms radius,[12] $\langle r^2 \rangle^{1/2}$

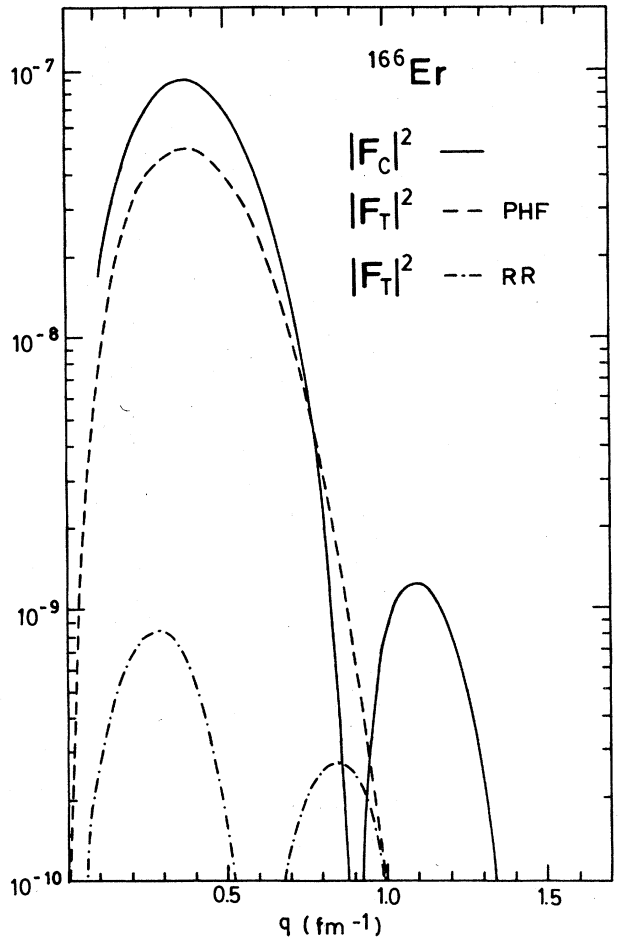

FIG. 1. Calculated longitudinal form factor $|F_C|^2 (= |F_{C2}|^2)$ described in the text, compared with PWBA calculations of transverse form factors in the rigid rotor (RR) and projected Hartree-Fock (PHF) approaches, for the transition $0^+ \rightarrow 2^+$ in the ground-state band of ^{166}Er.

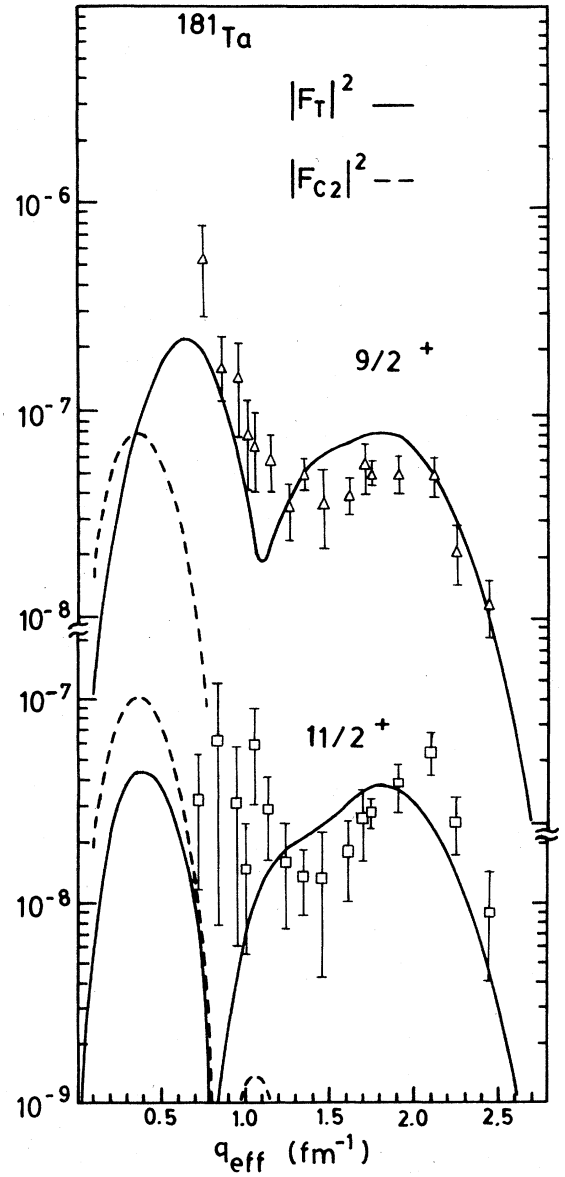

FIG. 2. Same as Fig. 1 for the transitions $\frac{7}{2}^+ \rightarrow \frac{9}{2}^+$ and $\frac{7}{2}^+ \rightarrow \frac{11}{2}^+$ in the ground-state band of ^{181}Ta. Also shown are the experimental data plotted vs q_{eff} (Ref. 17).

$= 5.24$ fm. For ^{181}Ta, we took $c = 6.33$ fm, $t = 2.5$ fm, $\alpha_2 = 0.168$, adjusted to the experimental values (Refs. 13 and 14), $Q_0 = 738$ fm^2, $\langle r^2 \rangle^{1/2} = 5.48$ fm.

The results obtained with these approximations for $|F_{C2}|^2$ are shown in Figs. 1 and 2 for the transition $0^+ \rightarrow 2^+$ in ^{166}Er and for the transitions $\frac{7}{2}^+ \rightarrow \frac{9}{2}^+, \frac{11}{2}^+$ in ^{181}Ta, respectively. To be conservative, we took the smaller value of $|I(\alpha Z)|$ in Table I corresponding to $\alpha Z = 0.5$. In Fig. 1 we show, for comparison, the theoretical results[15] of PWBA calculations of the transverse form factor, using the rigid rotor (RR) model and the projected Hartree-Fock (PHF) approach to describe the nuclear collective current. It can be seen that the first peak of $|F_{C2}|^2$ and $|F_T|^2$ PHF are located at the same q value and of the same order of magnitude. Similarly, in Fig. 2 we show the results[16] of PWBA calculations of the transverse form factors in the PHF approach, also shown in this figure are the experimental data[17] plotted versus q_{eff}. Again, for the transition $\frac{7}{2}^+ \rightarrow \frac{11}{2}^+$, the first peak of the transverse form factor (which is due to the $E2$ multipole) is roughly located at the same q value and of the same order of magnitude as $|F_{C2}|^2$. For the transition $\frac{7}{2}^+ \rightarrow \frac{9}{2}^+$, the first peak of the transverse form factor is dominated by the $M1$ multipole (see Ref. 16) and it is larger than the $|F_{C2}|^2$ peak. It should also be mentioned that, for the small transition energies involved in the cases considered here, taking into account the electron mass, as well as the finite nuclear size, in the DWBA calculations enhances the $C2$ contribution. This is especially so for the transition $\frac{7}{2}^+ \rightarrow \frac{9}{2}^+$ in ^{181}Ta (Ref. 11), and presumably, for the transition $0^+ \rightarrow 2^+$ in ^{166}Er, where $\omega \ll m_e$. However, as rough as the evaluation of $|F_{C2}|^2$ presented here is, the results clearly indicate that, at least for inelastic scattering on heavy nuclei, Eq. (1) is a very poor approximation and complete DWBA calculations have to be made to compare with experimental data at 180°, taking into account all permitted charge and current multipoles. This, we feel, is rather unfortunate for it makes the extraction of information on transverse nuclear currents much harder, especially so since longitudinal and transverse electric multipoles interfere in DWBA. A review of some of the approaches and methods currently used in DWBA codes for the extraction of transition densities from experimental data can be found in Ref. 18.

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