Effective decoupling from the particle-rotor model

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The results of a schematic one-BCS-quasiparticle—plus—rotor band mixing calculation, carried out for the $i_{13/2}$ neutron orbital at all deformations and all locations of the Fermi energy, have been used to bring out many new features, which are also supported by the experimental data. It is thus observed that decoupled bands of a generalized nature exist almost everywhere. For the Fermi energy lying in the lower half of the j shell, it is found that a particle with an effective angular momentum $j' < j$ is decoupled from the core and gives rise to decoupled bands having band head spin $I = j'$. The value of j' decreases as the Fermi level rises. However, the nature of the results changes dramatically if the Fermi energy is raised beyond the $K = \frac{5}{2}$ level. This produces new types of decoupled bands, wherein a particle with an effective aligned angular momentum j' seems to be decoupled from a core having rotational angular momentum $R = 2$, 4, or 6. The systematics from the experimental data is in complete agreement with the results from the calculations. The $^{173-179}$W isotopes, however, completely disobey the general trends and may be explained by assuming a very small deformation only.

I. INTRODUCTION

For the past three decades, the particle-plus-rotor model first proposed by Bohr and Mottelson[1] has continued to provide the framework for the analysis of the rotational band structure of odd mass nuclei. In spite of its approximate nature, it continues to be one of the most simple and elegant models in nuclear structure, still revealing a great deal of physics. While most of the data on rotational bands in odd-mass nuclei may be interpreted in the strong coupling limit, the strongly perturbed bands (SPB's) are known to defy this interpretation. Most common examples are the $i_{13/2}$ neutron orbital based bands in the rare earth region. The large Coriolis force leads to considerable band mixing, and a new type of coupling, called the rotation aligned coupling, is known to arise.[2,3] This coupling is seen to be a limiting situation of the particle-plus-rotor model when the last particle occupies a large-j and low-K single particle level at small deformations.[3,4] These conditions are, however, very stringent and, in moving from one nucleus to a neighboring nucleus, quick deviations may occur. This is best reflected, for example, in a group of Er nuclei having $A = 157$ to 167. It is thus found that ^{157}Er is closest to the decoupling limit, and as one moves towards ^{167}Er large deviations set in.[4] The art of analyzing these bands has considerably advanced in recent years, and the decrease in alignment has been quantitatively discussed in the framework of the cranking model,[5] and to some extent, also the particle-plus-rotor model.[6]

In this paper we use the results of one-BCS-quasiparticle—plus—rotor band mixing calculations for the $i_{13/2}$ neutron orbital to discuss some obscure features of these calculations. These calculations have been carried out at all deformations and Fermi energies, and some Coriolis attenuation has also been included at lower lying Fermi energies. While some of the results, particularly those for the lower half of the $i_{13/2}$ shell, seem to supplement the finding of the more sophisticated calculations,[5] we also obtain many new and interesting results. In particular, we find that the decoupling picture of Stephens *et al.* remains valid, in a greatly generalized fashion, at all deformations and all locations of the Fermi energy. When the Fermi energy lies in the lower half of the j shell ($K = \frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$ single particle levels), we obtain decoupled bands[7] which have their band heads $I = \frac{9}{2}$ or $\frac{5}{2}$ instead of the usual $I = \frac{13}{2}$. While this result is not surprising in view of the decreasing alignment as the Fermi energy rises,[5] we find a dramatic change in the nature of the results near the middle of the shell (the $K = \frac{7}{2}$ level). For the Fermi energy lying near the $K = \frac{7}{2}$ level or above, we again get well-defined decoupled bands but of a different nature. It now appears as if the last particle has an effectively aligned spin j' which is coupled to a core having angular momentum $R' = 2$, 4, or 6, depending on the location of the Fermi energy. The higher the Fermi energy in the upper-half j shell, the greater the core rotational angular momentum. The band head spin is thus $I = R' + j'$, where $R' \neq 0$. We find ample evidence for the existence of such bands from the existing experimental data. That the strongly coupled bands may also be interpreted as decoupled bands which are seemingly based on an excited core was recently pointed out by one of us[8] and is a novel feature. We also show in this paper that SPB's of the lighter W nuclei fall completely out of these systematics and may be explained by assuming a very small deformation.

In Sec. II we present very briefly, for the sake of completeness, the schematic particle-plus-rotor model we used. In Sec. III, we present the results of our calculations. In Sec. IV, we compare the general features and systematics as emerging from the calculations with the experimental data. The distinct behavior of W nuclei is also pointed out. We summarize our results in Sec. V.

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II. THE PARTICLE-PLUS-ROTOR MODEL

The Hamiltonian for an axially symmetric core plus particle is usually written as

$$H = H_{\rm qp} + \frac{\hbar^2}{2\mathscr{I}}(R_x^2 + R_y^2)\,, \tag{1}$$

where the terms have their usual meaning. Since the total angular momentum $\vec{I} = \vec{R} + \vec{j}$, where $\vec{j}$ is the particle angular momentum and $\vec{R}$ is the rotational angular momentum of the core, we can rewrite the Hamiltonian as

$$H = H_{\rm qp} + \frac{\hbar^2}{2\mathscr{I}}[I(I+1) - \langle K^2 \rangle] + \frac{\hbar^2}{2\mathscr{I}}[\langle j^2 \rangle - \langle K^2 \rangle] + H_c\,, \tag{2}$$

where $I_z = K = \Omega = j_z$ and

$$H_c = -\kappa \frac{\hbar^2}{2\mathscr{I}}(I_+ j_- + I_- j_+)\,. \tag{3}$$

Here H_c is the usual Coriolis term, and its off-diagonal matrix elements may be multiplied by an attenuation factor κ. We have assumed j to be a good quantum number and have mixed all the seven bands based on the $i_{13/2}$ neutron orbitals. The Nilsson model single particle energies[9] corresponding to the mass number $A = 163$ were used to obtain the quasiparticle energies with a pairing gap $\Delta = 1.0$ MeV. A moment of inertia parameter value of $\hbar^2/2\mathscr{I} = 14$ keV was used.

III. THE EFFECTIVELY "DECOUPLED" BANDS

The results of the calculation at a small deformation $\delta = 0.19$ and no Coriolis attenuation are shown in Fig. 1, where we have plotted the yrast level energies of the favored band for different locations of the Fermi energy in the $i_{13/2}$ shell. Here we have chosen the $I = \frac{13}{2}$ level as the reference level, and instead of plotting the level energies as such, we have plotted the ratio

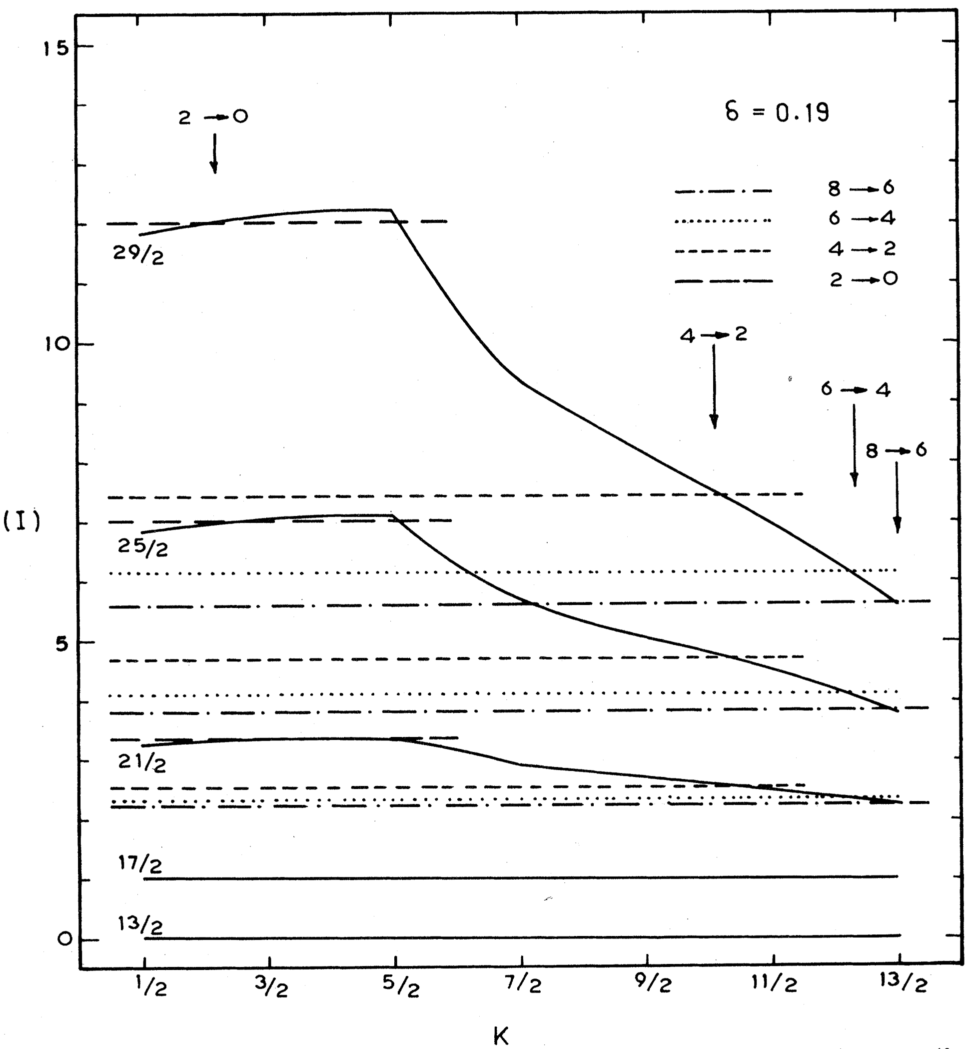

FIG. 1. The ratio $\Gamma(I) = (E_I \to E_{13/2})/\Delta E(\frac{17}{2} \to \frac{13}{2})$ plotted for the Fermi energy lying near the $K = \frac{1}{2}$ to $K = \frac{13}{2}$ levels. The different sets of lines, as shown in the figure, are obtained by assuming the band head $I = \frac{13}{2}$ to correspond to $R = 0$, 2, 4, and 6 and a rotational spectrum for the even-even core.

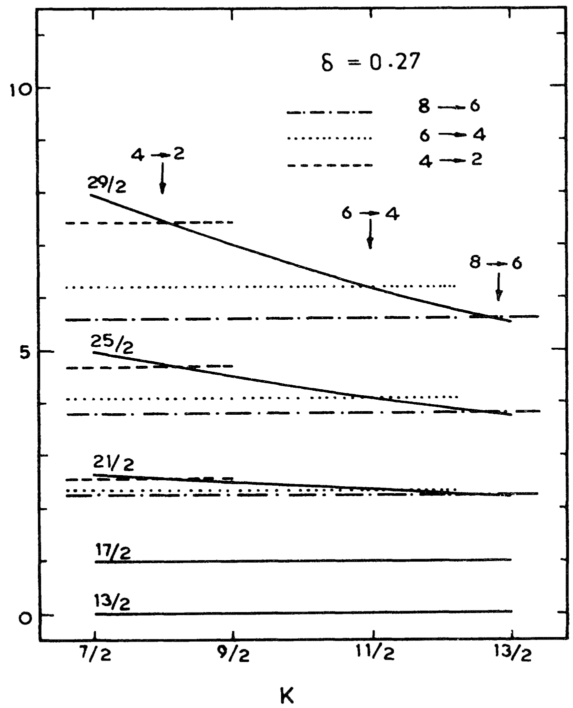

FIG. 2. Same as Fig. 1 but for a deformation $\delta=0.27$.

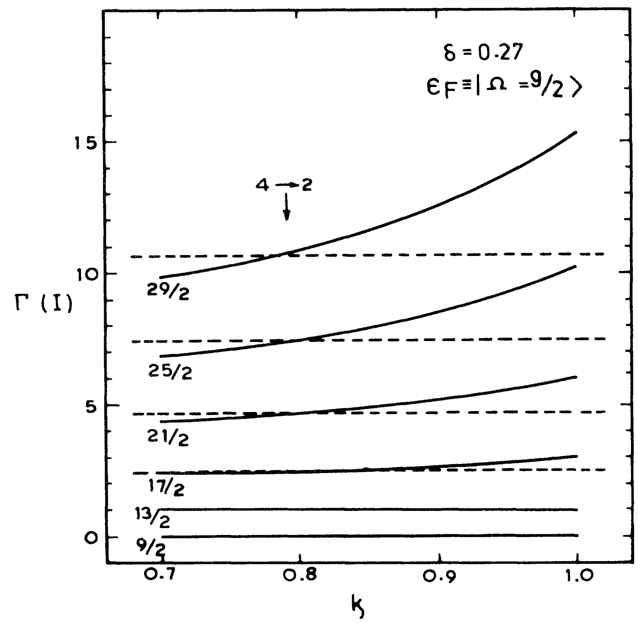

FIG. 3. The ratio

$$\Gamma(I)=(E_I \rightarrow E_{9/2})/\Delta E(\tfrac{13}{2} \rightarrow \tfrac{9}{2})$$

as a function of the Coriolis attenuation parameter κ. The Fermi energy lies near the $K=\frac{5}{2}$ level.

$$\Gamma(I)=(E_I \rightarrow E_{13/2})/\Delta E(\tfrac{17}{2} \rightarrow \tfrac{13}{2}) .$$

This ratio can be compared directly with the ratio (E_I / E_{2^+}) for the core rotational band levels which have a fixed value of 1,3.3,7,12, . . . , etc. for $I=2,4,6,8, \ldots$, respectively, and are shown in Fig. 1 by dashed lines. Wherever the ratios $\Gamma(I)$ cross these lines simultaneously, we get the usual decoupled band. This crossing is seen to occur somewhere between the $K=\frac{1}{2}$ and the $K=\frac{3}{2}$ Fermi energy. This is a well-known result. The interesting

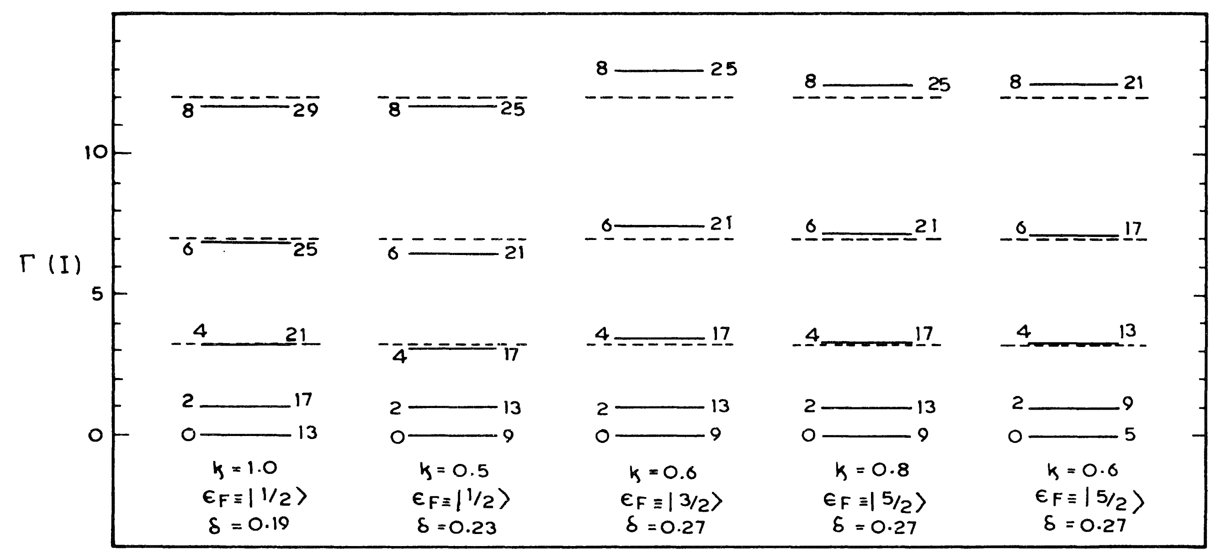

FIG. 4. Typical changes in the band structure as one moves from small deformation and low lying Fermi energy towards large deformation and higher lying Fermi energy in the lower half of the $i_{13/2}$ orbital. The ratio $\Gamma(I)$ is plotted instead of the level energies and compared with the rigid rotor values (dashed lines) to show the emergence of effectively decoupled bands.

point to note is that the ratio $\Gamma(I)$ remains almost constant in going from the $K=\frac{1}{2}$ to $\frac{5}{2}$ Fermi energy. The insensitivity of the band mixing calculations towards the location of the Fermi energy near the low-K single particle levels is an interesting feature and was recently pointed out by us.[10] After $K=\frac{5}{2}$, the values of $\Gamma(I)$ show a dramatic fall and there seems to be a drastic change in the nature of the results.

We have also plotted, in Fig. 1, rigid rotor ratios defined differently and shown by different sets of lines. For example, we assume that the lowest level (here $I=\frac{13}{2}$) corresponds to a core having angular momentum $R=2$, 4, or 6; we then get three sets of values for each of these possibilities and they are plotted in Fig. 1 as different sets of lines. If $\Gamma(I)$ crosses any one set of lines simultaneously, it implies the existence of a decoupled band which is based on a core having angular momentum $R=2$, 4, or 6. It is interesting to note that all the crossings occur in the upper-half portion of the j shell, i.e., higher lying Fermi energies. Thus we get the decoupled band based on the $R=2$ core at a Fermi energy lying between the $K=\frac{9}{2}$ and $\frac{11}{2}$. For the Fermi energy lying between the $K=\frac{11}{2}$ and $\frac{13}{2}$ levels, we get a decoupled band based on the $R=4$ and also the $R=6$ core. In general, we call these bands the "effectively" decoupled bands since a particle with an effective angular momentum j' seems to be aligning itself with a core having angular momentum R not necessarily zero.

If the deformation is increased to say $\delta=0.27$, similar features persist, except for the shifting of the Fermi energy where these bands may be found (Fig. 2). We can see the existence of a decoupled band based on an $R=2$ core

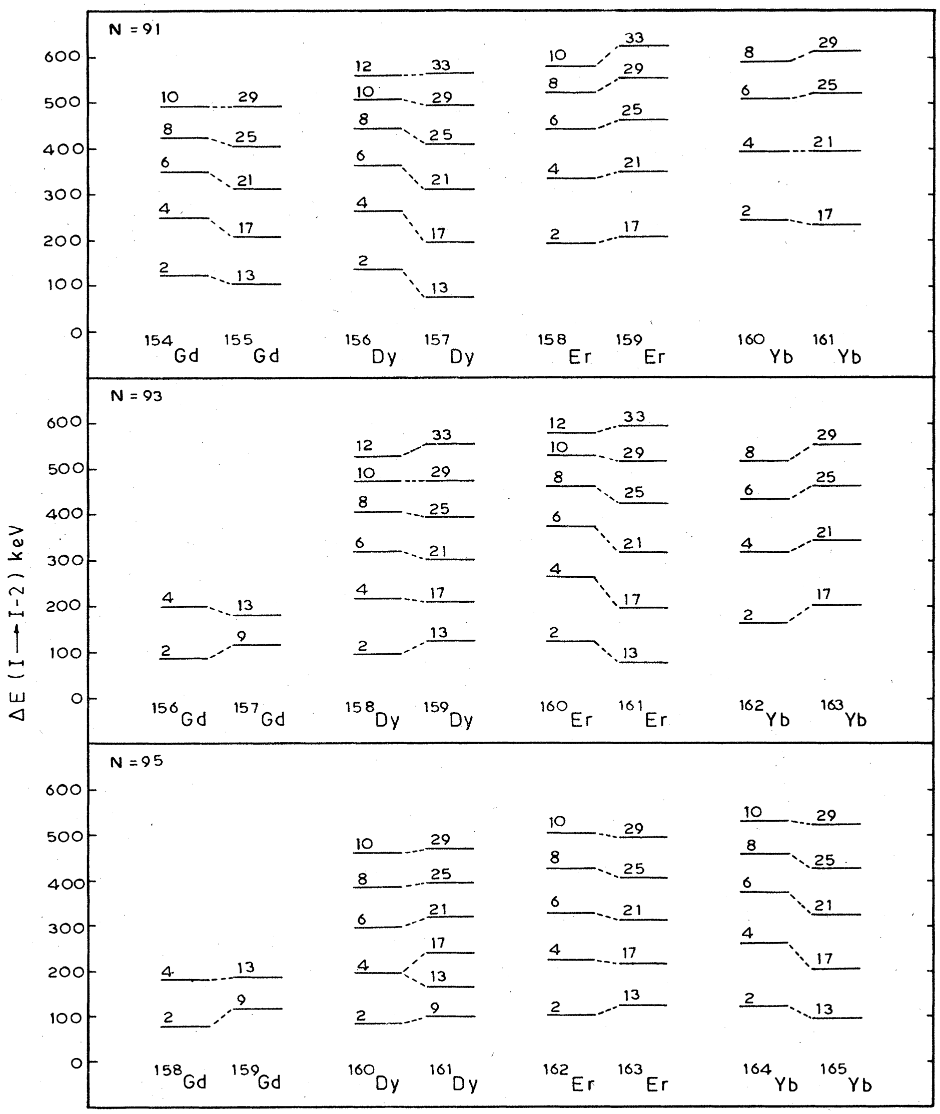

FIG. 5. Comparison of the experimental transition energies of the favored bands belonging to 12 odd-A nuclei with those of the respective even-even core band. The odd nuclear energy levels are labeled by $2I$. The data have been taken from Refs. 11 and 12 and a private compilation.

for the Fermi energy in between the $K=\frac{7}{2}$ and $\frac{9}{2}$. Those based on an $R=4$ and 6 core are seen to occur near the $K=\frac{11}{2}$ and $\frac{13}{2}$ Fermi energies, respectively. The situation when the Fermi energy lies in the lower-half portion of the j shell has not been shown here, but the usual decoupled band obtained at $\delta=0.19$ is not seen, as expected. However, if we bring in some attenuation ($\kappa=0.6$ to 0.8), we do get effectively decoupled bands which have their lowest state spin $I=\frac{9}{2}$ or $\frac{5}{2}$. In other words, the band head angular momentum is $I=R+j'$, where $R=0$ and $j'<j$. This merely implies a decrease in the aligned angular momentum, which is now well established.[6]

If we assume that some Coriolis attenuation (say 10—20%) exists even at higher lying Fermi energies, we may get a particular type of effectively decoupled band at a Fermi energy of our choice. This is shown in Fig. 3 for the Fermi energy lying near the $K=\frac{9}{2}$ level. For no attenuation, we do not get any decoupled band. However, an attenuation $\kappa\approx 0.8$ gives an effectively decoupled band based on the $R=2$ core. From Fig. 2, this band was seen to occur in between the $K=\frac{7}{2}$ and $\frac{9}{2}$ Fermi levels. Thus slight adjustments are possible with small variations in the strength of the Coriolis term within acceptable limits. However, the Coriolis attenuations have almost no effect for Fermi energy lying near the $K=\frac{11}{2}$ or $\frac{13}{2}$ level.

IV. SYSTEMATICS FROM THE THEORY AND THE EXPERIMENTAL DATA

The general features and systematics that emerge from these numerical calculations are shown in Figs. 4—8. In Fig. 4, we show the pattern of bands expected when the Fermi energy lies near the $K=\frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$ levels, i.e., the lower half of the $i_{13/2}$ shell. It is known that a Coriolis attenuation up to 50% is needed in this region and therefore has been taken into account in obtaining these results. The lightest nuclei may lie at the threshold of the deformation region ($\delta=0.19$ to 0.23) and the Fermi energy anywhere near the $K=\frac{1}{2}$ level; we get the usual decoupled band even without any attenuation. As the neutron number increases, the deformation increases, and also the Fermi energy rises. As shown in Fig. 4, for attenuations in the range of $\kappa=0.5$—0.8, we get decoupled bands firstly with the $I=\frac{9}{2}$ band head and then with the $I=\frac{5}{2}$ band head. The aligned angular momentum thus goes on decreasing from $j=\frac{13}{2}$ to $\frac{5}{2}$ in going from $K=\frac{1}{2}$ to $\frac{5}{2}$ Fermi energy, a result also obtained from cranking model calculations.[6] If we thus consider a sequence of isotopes, the effectively aligned angular momentum should decrease, but in a sequence of isotones, the deformation may decrease, and therefore we expect effectively decoupled bands with increasing aligned angular momentum.

In Fig. 5, we show the experimental data for nuclei having $N=91$—95, where the transition energies of SPB's have been compared with the transition energies of the ground rotational bands of the respective cores. It is clear that in going from left to right (a sequence of isotones), the nature of the decoupled band changes from $I=\frac{5}{2}$ and/or $\frac{9}{2}$ to $I=\frac{13}{2}$. In moving from top to bottom (a sequence of isotopes), however, the reverse happens. These types of bands are also obtained for $N=97$ nuclei, whose data are contained in Fig. 7.

As pointed out earlier, the nature of the results and hence the pattern of bands changes dramatically after the Fermi energy is raised beyond the $K=\frac{5}{2}$ level. In Fig. 6 we show the typical band structures expected when the Fermi energy lies in the upper-half portion of the $i_{13/2}$ shell. For the Fermi energy lying near the $K=\frac{7}{2}$ or $\frac{9}{2}$ level, we get effectively decoupled bands having a band head $I=\frac{9}{2}$ and which are based on a core having angular momentum $R=2$. On raising the Fermi energy further, we may expect a decrease in deformation, and therefore at $\delta=0.19$ say, and $\epsilon_F=|\frac{9}{2}\rangle$ we get an effectively decoupled band having $I=\frac{13}{2}$ and $R=2$. Near $\epsilon_F=|\frac{11}{2}\rangle$, we get an effectively decoupled band with $I=\frac{13}{2}$ and $R=4$, and for $\epsilon_F=|\frac{13}{2}\rangle$ the band head spin is $I=\frac{13}{2}$ while $R=6$. In the last two cases, i.e., for $\epsilon_F=|\frac{11}{2}\rangle$ or

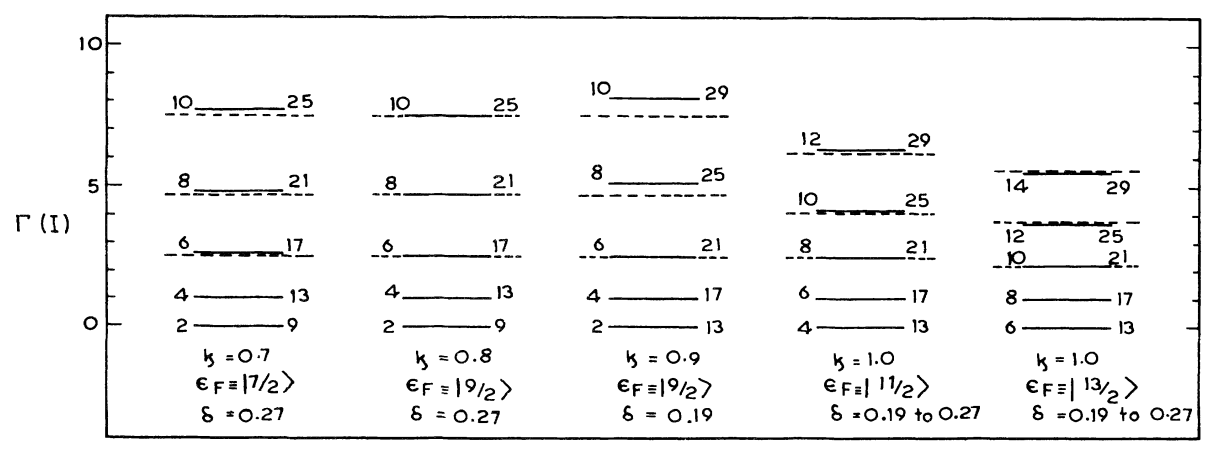

FIG. 6. Typical band structures expected when the Fermi energy lies in the upper-half portion of the $i_{13/2}$ orbital. The dashed lines correspond to the rigid rotor values obtained by assuming the band head to correspond to $R=2$, 4, or 6 as indicated on the left of each band.

$\epsilon_F = |\frac{13}{2}\rangle$, no Coriolis attenuation is needed nor does it have any effect on the results. Similarly the deformation also does not affect these results. However, for the Fermi energy lying near the $K=\frac{7}{2}$ and $\frac{9}{2}$ levels, some attenuation (≈ 0.8) is needed to obtain the effectively decoupled bands.

That a dramatic change in the coupling does occur is indeed confirmed by the experimental data shown in Fig. 7. In this figure, we compare the transition energies of nuclei having $N \geq 97$ with the rotational transition energies of the respective even-even core bands. It can be seen that the $N=97$ isotones exhibit effectively decoupled bands characteristically similar to those of $N=91-95$ nuclei. However, in going from $N=97$ to 99, the nature of bands has completely changed and now we get effectively decoupled bands based on an $R=2$ core. It is clear from Fig. 7 that all the SPB's belonging to $N=99-103$ nuclei have the lowest transition $(\frac{13}{2} \rightarrow \frac{9}{2})$, which matches well with the $(4 \rightarrow 2)$ transition of the core band and so on. In $N=105$ and 107 nuclei, while Hf isotopes still show the same type of behavior, the Os isotopes now have the lowest transition $(\frac{17}{2} \rightarrow \frac{13}{2})$, which matches with the $(4 \rightarrow 2)$ core band transition. This change reflects a decreased deformation for these Os isotopes. The $N=109$ Os isotope exhibits a lowest transition $(\frac{17}{2} \rightarrow \frac{13}{2})$ having a correspondence with the $(6 \rightarrow 4)$ transition of the core band, indicating that the Fermi energy lies near the $K=\frac{11}{2}$ level.

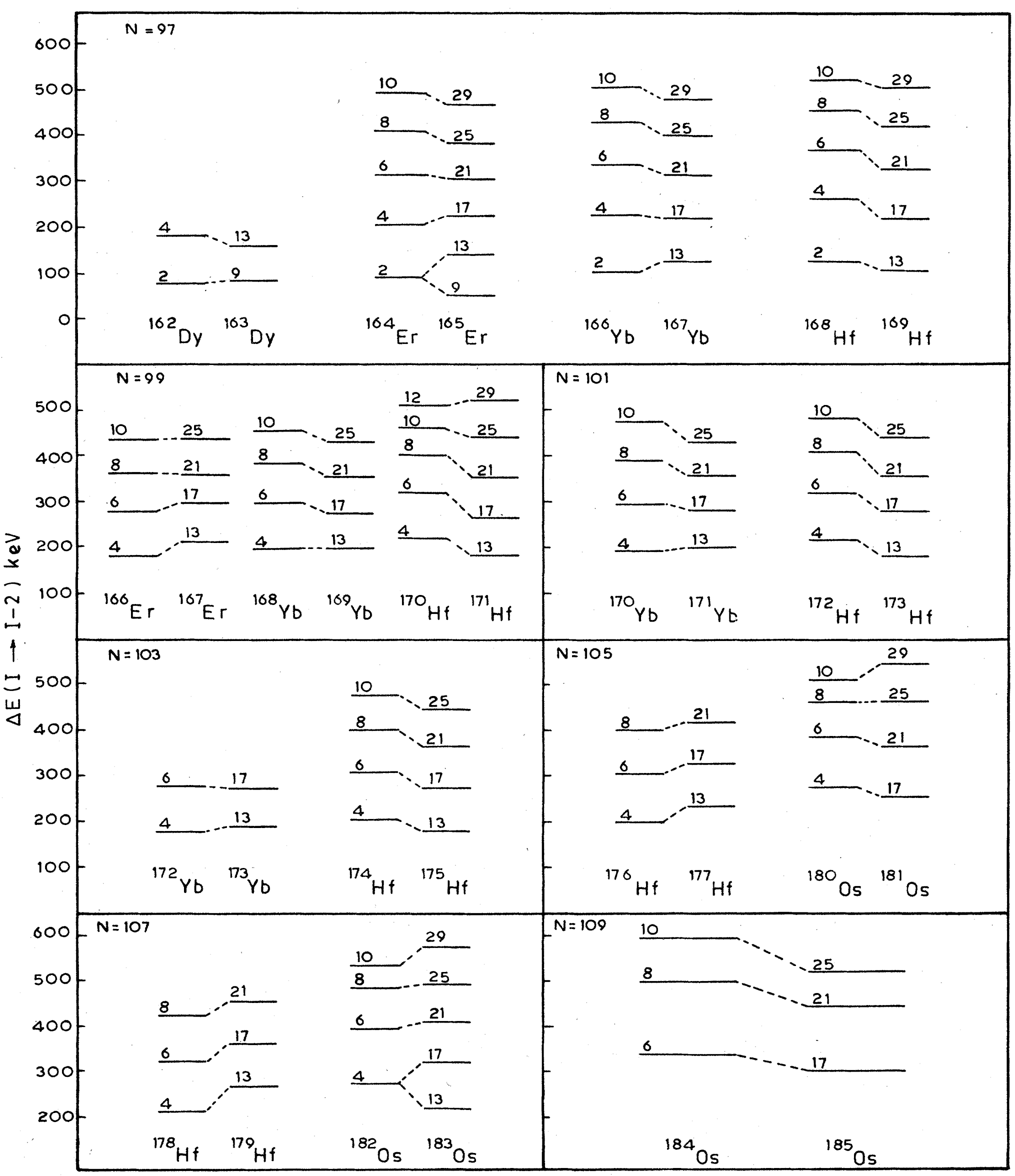

FIG. 7. Comparison of the experimental transition energies of the favored bands belonging to 16 nuclei with those of the respective core band transition energies. The $N=97$ isotones behave similarly to those shown in Fig. 5, but there is a sudden change at $N=99$ as explained in the text.

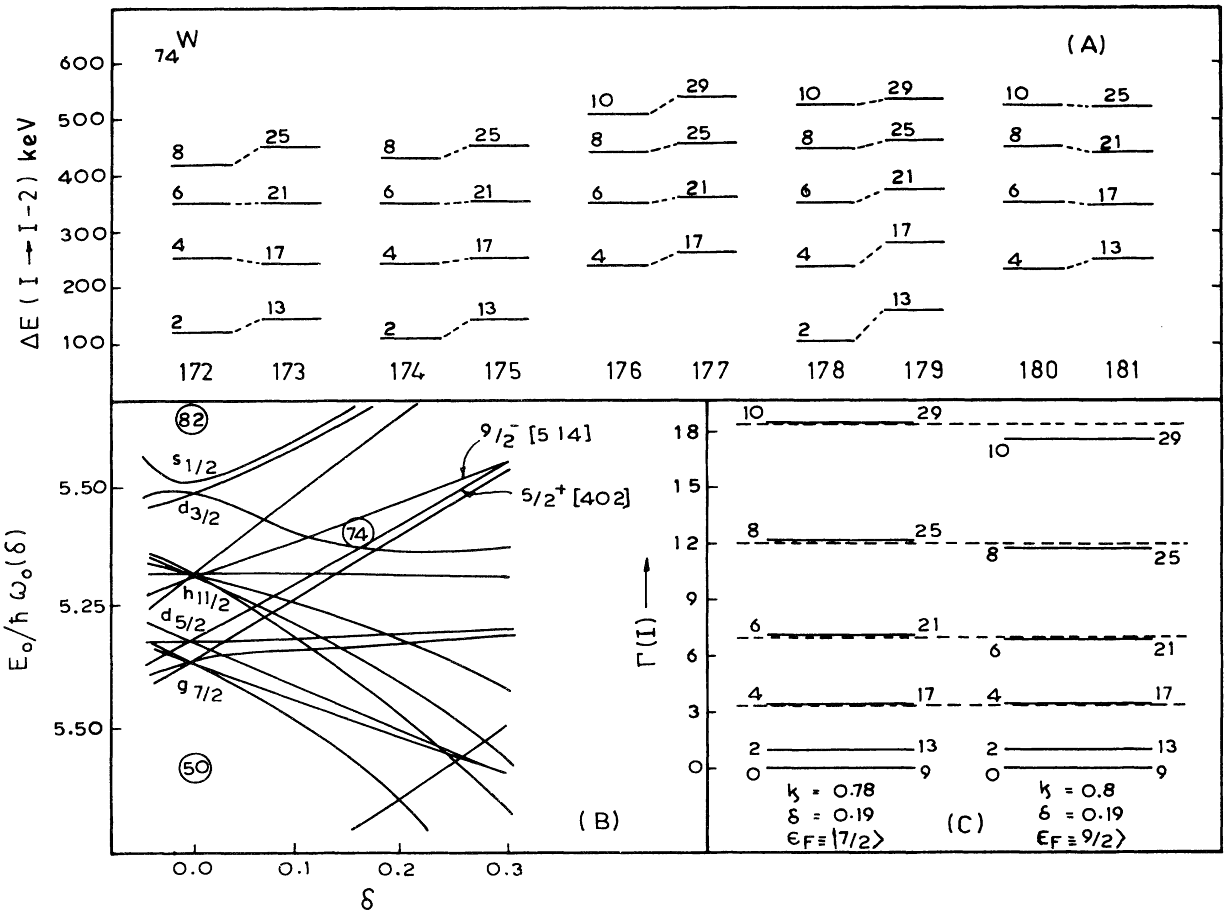

FIG. 8. (a) The same as Fig. 7 but for W isotopes only. (b) The Nilsson level diagram relevant to the protons in W nuclides. (c) The typical band structure expected if the W nuclides are assumed to possess a very small deformation.

The band structure exhibited by certain W isotopes, however, falls completely out of these general trends. This is particularly surprising in view of the fact that the W and Os isotopes are supposed to have similar deformation systematics. In Fig. 8(a) we show the transition energies of all the W isotopes. It is clear from the figure that the ^{173,175,179}W isotopes have $(\frac{13}{2} \rightarrow \frac{9}{2})$ as the lowest transition, which can be matched with the $(2 \rightarrow 0)$ core transition. This kind of spectrum is expected, as observed earlier, in the lower half of the $i_{13/2}$ shell. The $N=107$ isotope, ^{181}W, however falls in line with the other $N=107$ isotones. We may point out here that the $\frac{9}{2}$ level has not been observed in the ^{177}W isotope, and from the systematics it is clear that the $(\frac{13}{2} \rightarrow \frac{9}{2})$ transition should exist having a value of the order of 150 keV. This rather anomalous behavior of the $^{173-179}$W isotopes may be understood if we look at the Nilsson diagram for the protons $50 \leq Z \leq 82$, [Fig. 8(b)]. The proton Fermi energy of W is expected to occupy the $\frac{5}{2}^+$ [402] level, which is strongly sloped towards small deformation. This, coupled with the fact that there is a substantial gap at $Z=74$ for small deformation, indicates that the W isotopes may well have a deformation considerably smaller than the other isotones. In Fig. 8(c) we show the band structure expected for such a small deformation and the Fermi energy lying near the $K=\frac{7}{2}$ or $\frac{9}{2}$ level. It is clear that a pattern similar to that observed experimentally emerges.

V. RESULTS AND DISCUSSION

We have analyzed the results of a schematic one-BCS-quasiparticle—plus—rotor band mixing calculation in the context of the SPB's. It is known that the SPB's are interpreted in terms of the decoupled bands, but only when the deformation is small and the Fermi energy lies near the low-K single particle levels. We show that, taking the Coriolis attenuation to be part and parcel of these calculations, at least for lower Fermi energies, the decoupling picture remains valid, although in a more generalized form, at all deformations and all locations of the Fermi energy. When the Fermi energy lies in the lower half of the $i_{13/2}$ shell, we obtain decoupled bands having an effective aligned angular momentum $\frac{9}{2}$ or even $\frac{5}{2}$, instead of the usual $\frac{13}{2}$. This merely indicates the decrease in alignment as the Fermi energy rises. Furthermore, we find that while the results remain almost insensitive towards the location of the Fermi energy in the lower half of the $i_{13/2}$ shell, there is a dramatic change in the nature of the results when the Fermi energy crosses the $K=\frac{5}{2}$ level. We still get well-defined decoupled bands, but of a different nature, when the Fermi energy is located near the $K=\frac{7}{2}$ to $\frac{13}{2}$ levels. We find that now the band head spin I is obtained by coupling an effective particle angular momentum j' to a rotational core having angular momentum $R=2$, 4, or 6 and not the usual $R=0$. That such a change does take place is clearly supported by the experimental data.

The W isotopes having $A=173-179$, however, display an anomalous behavior in the sense that they fall completely out of the general systematics. The reason for this seems to be a very small deformation for these isotopes as compared to other isotones.

It is thus clear that in going from lower Fermi energies to higher Fermi energies, the decoupling type of picture is maintained in a more general form. Apparently, a new type of coupling, that of a particle aligned to an excited core, seems to emerge. This may have far reaching consequences on determining the amount of pairing correlation in odd-mass nuclei. It is now well accepted that the slow rise of the moment of inertia at lower spins in the even-even nuclei is mainly due to a slow but gradual decrease in the pairing correlations. Since the SPB's closely follow the ground state band of the core nucleus at lower spins, it is possible to obtain information on the pairing correlations by direct comparison. Thus a decoupled band based on an $R=4$ core should have a lesser amount of pairing correlation than the one based on an $R=2$ or 0 core. Since the core angular momentum seems to be increasing with a rise in the Fermi energy towards large-K levels, the amount of pairing correlation at the band head should be decreasing at higher lying Fermi energies in the $i_{13/2}$ shell.

In conclusion, we find that an interpretation in terms of effectively decoupled bands can explain the data on SPB's starting from the beginning of the j shell to the top of the j shell. The bands are seen to evolve in a continuous manner as the Fermi energy rises from low-K to large K levels. This kind of interpretation may turn out to be helpful in determining the pairing correlations in odd-A bands.

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