Unified analysis of pion single- and double-charge-exchange scattering in the resonance region

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(Received 2 July 1984)

We present the results of a phenomenological analysis of all existing data on pion single- and double-charge-exchange scattering to isobaric analog states at $T_\pi \simeq 164$ MeV. We use a theory in which both reactions are described by the same optical potential, U. The form of U is theoretically motivated and separates explicitly the effects of nuclear structure and reaction dynamics. The latter is characterized phenomenologically by two complex numbers, one for the isovector and one for the isotensor term in U. Elastic scattering from selected $N=Z$ nuclei is independently fitted to determine the isoscalar terms in U. Realistic Skyrme III densities are used to describe the nuclear structure. We find one set of parameters which describes the scattering throughout the periodic table. The striking N, Z, and A dependence predicted by the lowest-order U and observed in the data is preserved.

I. INTRODUCTION

With the new spectrometer facilities at the meson factories, it is now possible to make high-quality measurements of reactions, such as single charge exchange[1–4] (SCX) and double charge exchange[5–12] (DCX) to isobaric analog states, which are promising sources of information on nuclear structure and Δ-nucleus dynamics. In the last few years, there has been a concerted effort to make survey studies of these reactions. Reasonably complete sets of data exist for $T_\pi \simeq 164$ and 292 MeV. One of the striking features of the results is the scaling of the cross section with N, Z, and A: the 0° SCX follows an $(N-Z)A^{-4/3}$ behavior and the 5° DCX an approximate $(N-Z)(N-Z-1)A^{-10/3}$. These trends follow from the lowest-order optical model and the diffractive pictures[13] in which the pion scatters sequentially from the nucleons of the target nucleus through the free pion-nucleon scattering amplitude. This simplicity, however, is lost when one examines the magnitude of the SCX cross sections and the shape of the DCX angular distributions. The former is two to four times greater at resonance than the results of the lowest-order optical potential model. In the case of $^{18}O(\pi^+,\pi^-)^{18}Ne$, the first minimum of the angular distribution is at an angle around 10° smaller than the predictions of the lowest-order model.

Theoretical arguments show that the amplitude must contain other terms in addition to the sequential scattering arising from the lowest-order optical potential $U^{(1)}$. Thus, one expects to augment $U^{(1)}$ by terms which describe Δ-nucleus interactions and other dynamical effects. Large contributions of this type have been found in phenomenological studies of elastic pion scattering.[14,15] This information may be systematically incorporated into the theory in part through addition of higher-order terms in U. Similar considerations may be applied to charge exchange leading to isobaric analog states.[16–18] One challenge presented to theory by the recent charge-exchange data is to show how the simple scaling systematics characteristic of the cross sections predicted by $U^{(1)}$ are compatible with the more complicated picture in which significant higher-order terms contribute. The data also afford us an opportunity to obtain a quantitative characterization of the Δ-nucleus interaction complementary to that obtained in other experiments[19] and in a form which would permit direct comparison to theoretical models.

Recent theoretical work[18] indicates that it is possible to include appreciable second-order terms in the optical potential without destroying the quantitative scaling behavior. This conclusion encouraged us to attempt a global fit to all of the existing data for SCX and DCX to analog states at a single value of pion incident energy, viz., 164 MeV. Section II of this paper reviews the isobaric invariant optical model adopted as the basis of our phenomenology, Sec. III discusses the detailed results of our parameter search, and Sec. IV summarizes our main findings and discusses their implications.

II. THEORY

Charge-exchange scattering to isobaric analog states and elastic scattering are connected at a fundamental level by isospin symmetry, which holds to a rather high level of accuracy for the strong interaction. This leads to a very general form for the optical potential

$$U=U_0+U_1\vec{\Phi}\cdot\vec{\mathrm{T}}+U_2(\vec{\Phi}\cdot\vec{\mathrm{T}})^2 , \tag{1}$$

where $\vec{\Phi}$ is the pion and $\vec{\mathrm{T}}$ the nuclear isospin operator, and where U_0, U_1, and U_2 are referred to as the isoscalar, isovector, and isotensor components of the optical potential. The specific value of U_i may be determined by solving the appropriate many-body theory.

The theory of this paper is based on a model in which the pion field is coupled to static nucleon sources.[20] One is then able to express U diagrammatically for spherical $J=0$ nuclei as a function of the underlying absorption and emission vertices and the nuclear densities, over which the scattering amplitude must eventually be averaged. One is led to an optical potential for the Klein-Gordon equation. It has a form similar to that applied successfully by Stricker, McManus, and Carr[21] to low-energy elastic scattering and pionic atoms, but it is generalized[17] to include the operator dependence of Eq. (1). The optical potential is naturally expressed as a series expansion in the nuclear density matrices. We assume that the only terms which need to be explicitly considered at resonance are those up to quadratic in density because of the surface (low density) dominance of the scattering. The result[18] is

$$\begin{aligned}U=&\vec{\nabla}\cdot[\xi(r)+\Delta\xi(r)]\vec{\nabla}-k^2[\bar{\xi}(r)+\Delta\bar{\xi}(r)]\\&-\tfrac{1}{2}(p_1-1)\nabla^2\xi(r)-\tfrac{1}{2}(p_2-1)\nabla^2\Delta\xi(r) ,\end{aligned} \tag{2}$$

where $\bar{\xi}$ and ξ represent the S- and P-wave terms of the lowest-order optical potential, and where $\Delta\bar{\xi}$ and $\Delta\xi$ represent the corresponding pieces of the second-order potential. For both S and P waves, these have the form

$$\xi=\xi_0+\vec{\Phi}\cdot\vec{\mathrm{T}}\xi_1 \tag{3}$$

and

$$\Delta\xi=\Delta\xi_0+\Delta\xi_1(\vec{\Phi}\cdot\vec{\mathrm{T}})+\Delta\xi_2(\vec{\Phi}\cdot\vec{\mathrm{T}})^2 . \tag{4}$$

The quantities p_n $(n=1,2)$ arise as a correction to the fixed-nucleon assumption and are defined as

$$p_n=\frac{1+\epsilon/n}{1+\epsilon/A} , \tag{5}$$

where $\epsilon=\omega/M$, $\omega=(k^2+m_\pi^2)^{1/2}$ is the incident pion total energy, and M is the nucleon mass.

The lowest-order quantities depend upon the nuclear density and isospin in the following manner:

$$\xi_0=\lambda_0^{(1)}\rho(r) , \tag{6}$$

$$\xi_1=\frac{\lambda_1^{(1)}}{2T}\Delta\rho(r) , \tag{7}$$

where ρ is the total nuclear density and $\Delta\rho$ the excess neutron density. These expressions hold for both S-wave $(\bar{\xi})$ and P-wave (ξ) terms. The parameters $\lambda^{(1)}$ are related to free pion-nucleon scattering through the phase shifts $\delta(\kappa)$ in the usual way,

$$\begin{aligned}\bar{\lambda}_0^{(1)}&=\frac{4\pi}{3\kappa k^2}p_1(2\alpha_3+\alpha_1) ,\\ \bar{\lambda}_1^{(1)}&=\frac{8\pi}{3\kappa k^2}p_1(\alpha_3-\alpha_1)\end{aligned} \tag{8}$$

and

$$\begin{aligned}\lambda_0^{(1)}&=\frac{4\pi}{3\kappa^3 p_1}(4\alpha_{33}+2\alpha_{31}+2\alpha_{13}+\alpha_{11}) ,\\ \lambda_1^{(1)}&=\frac{8\pi}{3\kappa^3 p_1}(2\alpha_{33}+\alpha_{31}-2\alpha_{13}-\alpha_{11}) ,\end{aligned} \tag{9}$$

where for S waves (α_{2I}) and P waves $(\alpha_{2J,2I})$

$$\alpha(\kappa)=\exp[i\delta(\kappa)]\sin\delta(\kappa)$$

and κ is the wave number in the pion-nucleon center-of-momentum frame. In applying this theory in the resonance region, it may be necessary to shift the incident pion energy to account for the Δ_{33} propagation.[22,24] We have allowed for such a shift by modifying the (3,3) channel so that α_{33} is replaced by $\Omega\alpha_{33}$, where

$$\begin{aligned}\Omega&=\frac{\omega-\omega_R+i\Gamma}{\omega-(\omega_R+\Delta E_1)+i(\Gamma+\Delta E_2)} ,\\ \omega_R&=1232\ \mathrm{MeV}-(\kappa^2+M^2)^{1/2} ,\\ \Gamma&=0.08\times(\tfrac{4}{3})\kappa^3\omega_R/(\tilde{\omega}m_\pi^2) ,\end{aligned} \tag{10}$$

and

$$\tilde{\omega}=(\kappa^2+m_\pi^2)^{1/2} .$$

Here, we have incorporated a real (ΔE_1) and imaginary (ΔE_2) energy shift to allow for the possibility of shifting the position and changing the width from the free Δ_{33} resonance values.

As we indicated, the nuclear density and isospin dependence of the second-order terms can be obtained by making a low-density expansion. One is led to a form for the second-order optical potential $U^{(2)}$ which separates explicitly the nuclear-structure input from the reaction dynamics. The general result for the form of $\Delta\xi$ in P waves is

$$\Delta\xi_0=\lambda_0^{(2)}\frac{\rho^2(r)}{\rho_0}-\frac{1}{2T-1}\lambda_2^{(2)}\frac{\Delta\rho^2(r)}{\rho_0} , \tag{11}$$

$$\Delta\xi_1=\frac{\lambda_1^{(2)}}{2T}\frac{\rho(r)\Delta\rho(r)}{\rho_0}+\frac{\lambda_2^{(2)}}{2T(2T-1)}\frac{\Delta\rho^2(r)}{\rho_0} , \tag{12}$$

$$\Delta\xi_2=\frac{\lambda_2^{(2)}}{T(2T-1)}\frac{\Delta\rho^2(r)}{\rho_0}+\frac{\lambda_4^{(2)}}{T^2}\frac{\Delta\rho^2(r)}{\rho_0} . \tag{13}$$

A similar result holds for S waves; however, in the region of 164 MeV we assume $\Delta\bar{\xi}=0$ and we only consider the effects of $\Delta\xi$. The reaction dynamics are contained in the complex numbers $\lambda_0^{(2)}$, $\lambda_1^{(2)}$, $\lambda_2^{(2)}$, and $\lambda_4^{(2)}$, and the nuclear structure is contained in the explicit factors of density. In Ref. 17, it is shown how to calculate the four complex

numbers in terms of a fixed-source field theory. A few of the λ parameters in Eqs. (11)—(13) have been calculated from the theory, and it is verified that they are virtually independent of N and Z and weakly dependent on A and position r. For our application to data in Sec. III, we assume these to be universal constants which characterize the scattering throughout the periodic table. The quantity $\lambda_4^{(2)}$ is calculated from the theory and not treated as an adjustable parameter; as discussed in Refs. 17 and 18, $\lambda_4^{(2)}$ is the correction which permits the inclusion of sequential scattering through nonanalog intermediate states and is needed to avoid double counting of the sequential scattering to the double analog through the single analog state. To describe the nuclear structure, we use the Hartree-Fock densities of the Skyrme III interaction,[24] which consists of parameters fitted to properties of spherical nuclei for $A \geq 16$.

III. RESULTS

Elastic scattering is needed in order to determine the distortions of the pion wave before and after charge exchange has taken place. We, therefore, made a χ^2 fit of the energy shift in the lowest-order optical potential and of the coefficient $\lambda_0^{(2)}$ of Eq. (11) to $\pi^{\pm}$ scattering from selected $N=Z$ nuclei, i.e., ^{16}O (Ref. 25), ^{28}Si (Ref. 26), and ^{40}Ca (Ref. 25). The comparison to data is shown in Fig. 1; the data for ^{12}C are from Refs. 27 and 28; the curves are predictions. We find

$$\Delta E = 35.0 + 0.3i \text{ MeV} , \tag{14}$$

$$\lambda_0^{(2)} = 0.75 + 3.67i \text{ fm}^3 , \tag{15}$$

with $\chi^2 = 20$, where χ^2 is the chi square per degree of freedom. For reference we also give the single-scattering coefficients

$$\lambda_0^{(1)} = 4.63 + 5.33i \text{ fm}^3$$

and

$$\lambda_1^{(1)} = 5.17 + 5.31i \text{ fm}^3 .$$

The sign of ΔE is such that the resonance is raised by 35 MeV and only very slightly broadened. One also finds an increase in the resonance energy and in its width in the phenomenological isobar-hole model (Refs. 15 and 23). The shift of 35 MeV is not inconsistent with the rough estimates of Ref. 23. The broadening is interpreted as an effect of true pion absorption and of multiple inelastic and quasielastic excitation. The resonance is expected to be narrowed by the Pauli effect. The amount of broadening in the phenomenological isobar-hole model is about 40 MeV at the center of the nucleus and falls off proportional to the density. If we assume that the important density region near resonance is about 10% of the central density, we would expect 4 MeV (compared to the half-width of 55 MeV). The effect of Pauli correlations on the second-order U was considered in Ref. 17. Calculating the Pauli effect at $T_\pi = 164$ MeV according to the theory developed there, we find

$$\lambda_0^{(2)} = 7.1 + 0.63i \text{ fm}^3 . \tag{16}$$

It is tempting to associate the broadening of the width in Eq. (10) and the narrowing through $\lambda_0^{(2)}$ in Eq. (15) with the mechanisms discussed in Refs. 15 and 23 and those of Ref. 17, respectively. One cannot draw very strong conclusions, however, because of the strong correlation between ΔE and $\lambda^{(2)}$ in our fitting procedure. It would be

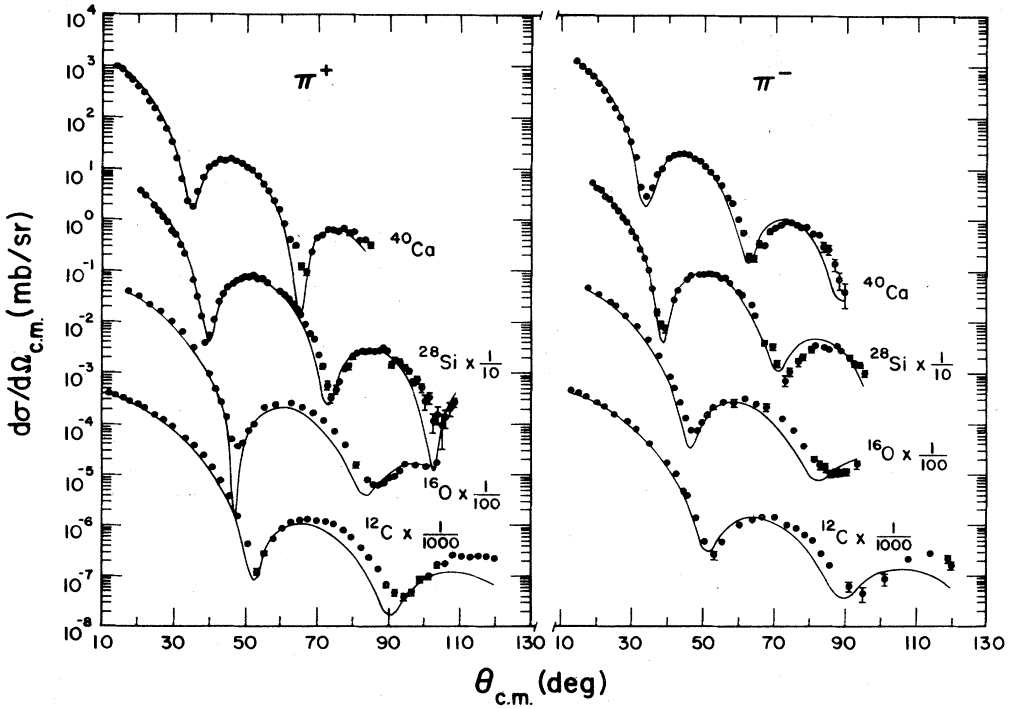

FIG. 1. Comparison of theoretical π^+ and π^- elastic scattering cross sections to data at $T_\pi = 164$ MeV for ^{16}O, ^{28}Si, ^{40}Ca, and ^{12}C.

useful to have available a theoretically derived U in which the energy shift is predicted as a consequence of the theory. Progress along this line is being made in Ref. 29. When this goal is accomplished, we would have greater confidence in our ability to interpret the phenomenological isoscalar parameters.

To determine $\lambda_1^{(2)}$ and $\lambda_2^{(2)}$, we simultaneously fit the 0° SCX and 5° DCX cross sections. The value of $\lambda_4^{(2)}$ is calculated theoretically, $\lambda_4^{(2)}=2.89-1.13i$ fm^3. The parameter $\lambda_2^{(2)}$ affects SCX very little. However, when $\lambda_1^{(2)}$ is large enough to change SCX, it also has a large effect on DCX. Our results are

$$\lambda_1^{(2)}=7.71+15.5i \text{ fm}^3 , \tag{17}$$

$$\lambda_2^{(2)}=1.66+10.8i \text{ fm}^3 , \tag{18}$$

with a reduced $\chi^2=3.3$.

The comparison to SCX data (Ref. 3) is shown in Fig. 2. We see that the magnitude and trend of the data are well reproduced for the $J=0$ nuclei to which the theory is applicable. The largest discrepancies exist for ^{7}Li and ^{13}C, for which the phenomenological result is low by a factor of 1.5–2.0. It would be interesting to see whether adding the spin-dependent piece of the density matrix in the derivation of $U^{(2)}$ (Ref. 17) could improve the description in these nuclei. In any case, single spin-flip contributions are known to enhance the total SCX cross section in the case of ^{7}Li (Ref. 30) and are needed for a complete description.

The results for DCX are shown in Fig. 3. The 5° points included in the fit are those for ^{14}C (Ref. 10), ^{18}O (Ref. 8), ^{26}Mg (Ref. 10), ^{42}Ca (Ref. 12), ^{48}Ca (Ref. 12), and ^{56}Fe (Ref. 9). The values for ^{42}Ca and ^{48}Ca were interpolated from the results given in Ref. 12. The theoretical point for ^{90}Zr is a prediction. The trend of the data is very well reproduced. As stressed in Ref. 18, the ratio of cross sections for ^{42}Ca and ^{48}Ca differs appreciably from the ratio of the number of pairs of valence neutrons because of the influence of nonanalog intermediate states; the physics appears to be properly taken into account in the theory by the interplay between the parameters $\lambda_2^{(2)}$ and $\lambda_4^{(2)}$.

The large isovector coefficient $\lambda_1^{(2)}$ in Eq. (17) arises because the lowest-order optical potential gives SCX cross sections that are much too small. However, in contrast to

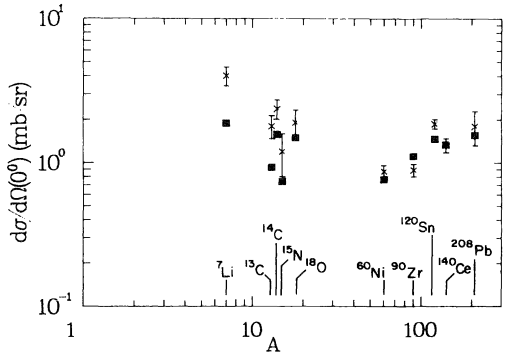

FIG. 2. Comparison of theoretical single charge exchange $d\sigma/d\Omega(0°)$ to data at $T_\pi=165$ MeV. The × represent data and the ■ represent theoretical result.

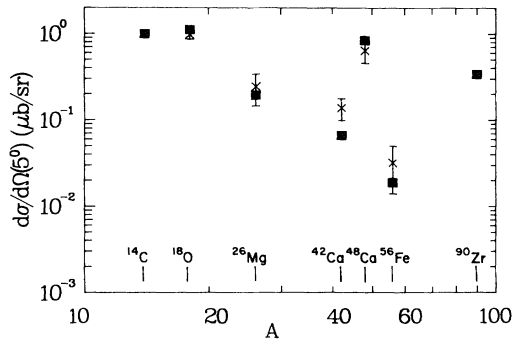

FIG. 3. Comparison of theoretical double charge exchange $d\sigma/d\Omega(5°)$ to data at $T_\pi=164$ MeV. The × represent data and the ■ represent theoretical result.

SCX, the magnitude of the DCX cross sections is reasonably well reproduced by the lowest-order theory. Because of the large adjustment in the isovector potential required to reproduce the SCX cross sections, the magnitude of DCX is no longer correct without the large compensating adjustment in the isotensor coefficient $\lambda_2^{(2)}$ in Eq. (18).

Angular distributions for SCX and DCX were not included in our fitting procedure. Thus, they play an important role in corroborating the validity of our phenomenological analysis. The angular distributions for DCX are shown in Fig. 4. The ^{18}O data are from Ref. 8 and the ^{14}C and ^{26}Mg data from Ref. 9. The dashed curves are the results without any second-order optical potential. The predicted minima are located much too far out in angle, which has been a difficulty with all theories up to now. With the same second-order parameters that give the A dependence of the zero-degree cross sections, we also find the minima in the proper location (solid curves). Small adjustments in the parameters could im-

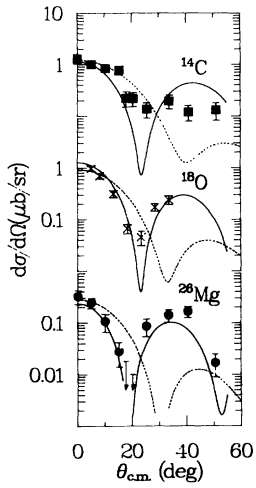

FIG. 4. Comparison of theoretical double-charge-exchange angular distributions to data at $T_\pi=164$ MeV. The dashed curves are the lowest order result ($\lambda_i^{(2)}=0$ and $\Delta E=0$). The solid curves show the final result and are based on a fit to only the forward-angle data shown in Figs. 2 and 3 and the elastic angular distributions of ^{16}O, ^{28}Si, and ^{40}Ca.

prove the description of the angular distributions, but uncertainties arising from other sources discussed later in this section do not warrant such fine tuning at this time.

The measured angular distributions for SCX on ^{13}C and ^{15}N at 164 MeV are compared with our predictions in Fig. 5. As stated, we used the Skyrme III densities[24] in determining the parameters of U. The corresponding angular distribution is the solid curve in Fig. 5. For comparison, we show the results of using the Negele-Vautherin density matrix expansion (DME) densities[31] (dashed curve) with the same optical potential parameters as already determined. The latter give rise to cross sections 15% greater at small angles because in the surface the DME neutron distributions are larger than the Skyrme neutron distributions.

In Fig. 6 we show elastic scattering[32] from ^{14}C evaluated with and without the isospin dependent terms in $U^{(2)}$. Because the ground state of ^{14}C has $T=1$, these contribute to elastic scattering, in contrast to the case of $T=0$ nuclei, used for determining the isoscalar piece of $U^{(2)}$. Clearly, the second-order isospin-dependent terms make an important contribution for both π^+ and π^- scattering, bringing the theory closer to experiment. We do not understand the source of the residual discrepancies which appear at larger angles. It is worth pointing out, however, that a forward scattering approximation was used to obtain the general form of the second-order potential we have adopted.[17]

How unique is the parameter set we have determined? The answer to this question is required if we are to use the parameters in Eqs. (11)—(13) as the basis of evaluating

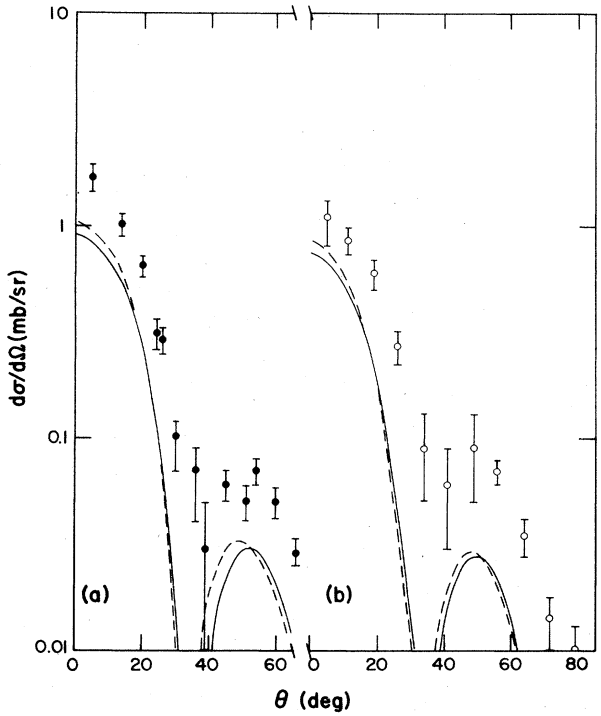

FIG. 5. Comparison of theoretical single-charge-exchange angular distributions to data at $T_\pi=165$ MeV: (a) ^{13}C; (b) ^{15}N. The solid curve is with Skyrme III densities and the dashed curve with the Negele-Vantherin DME densities.

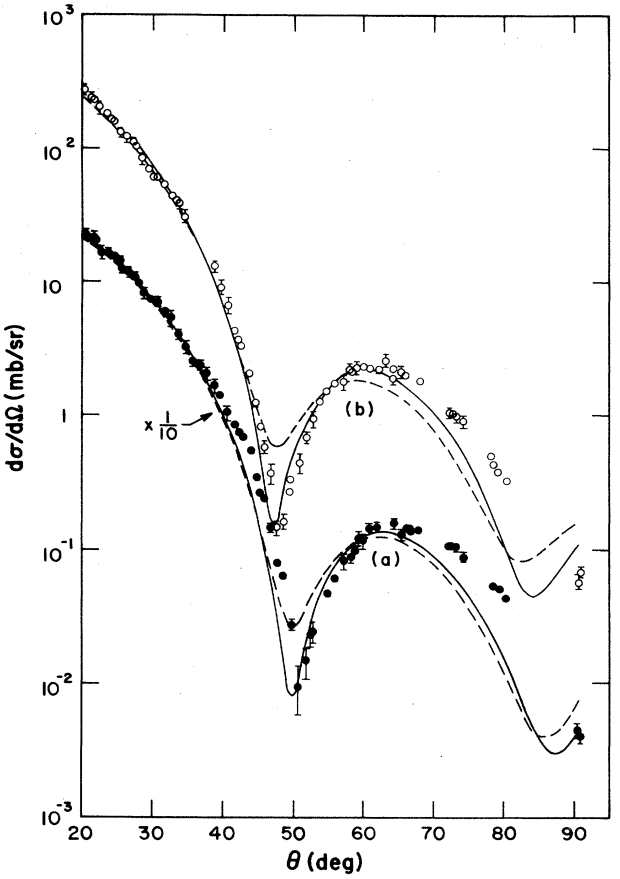

FIG. 6. Comparison of theoretical π^+ (a) and π^- (b) elastic scattering cross section from ^{14}C to data at $T_\pi=164$ MeV. The solid and dashed curves are, respectively, results with and without the isospin-dependent terms in $U^{(2)}$.

theoretical calculations. We have already stated that a strong correlation exists between the energy shift and the isoscalar ρ^2 parameter. This correlation leads to some ambiguity in the extracted values of the parameters because the magnitudes of the SCX and DCX cross sections depend on the *ratio* of isovector to isoscalar potential.[18] We have tried to estimate the resulting uncertainty in $\lambda_1^{(2)}$ and $\lambda_2^{(2)}$ by arbitrarily setting the imaginary part of $\lambda_0^{(2)}$ to -1. There is some deterioration in the fit to elastic scattering, which could be at least partially repaired by a compensatory shift in ΔE. We then refitted charge exchange and found

$$\lambda_0^{(2)}=4.07-1.0i \text{ fm}^3 , \tag{19}$$

$$\lambda_1^{(2)}=12.5+11.8i \text{ fm}^3 , \tag{20}$$

$$\lambda_2^{(2)}=2.81+10.8i \text{ fm}^3 , \tag{21}$$

with a reduced $\chi^2=4$. Compared to Eqs. (17) and (18), these are modest changes in $\lambda_1^{(2)}$ and $\lambda_2^{(2)}$.

A second source of ambiguity which should be addressed quantitatively in a theory is the possibility of a different energy shift in the isoscalar and isovector term in the lowest-order optical potential. One might expect such a difference given that the charge exchange occurs

on the valence neutrons, which are less bound than the average nucleon. Consequently, we tried a third fit in which the energy shift was turned off in the isovector term of the lowest-order U. We recalculated $\lambda_4^{(2)}$ to be

$$\lambda_4^{(2)}(\Delta E=0)=5.3+2.6i \text{ fm}^3 \tag{22}$$

and found

$$\lambda_0^{(2)}=-0.95-1.0i \text{ fm}^3 , \tag{23}$$

$$\lambda_1^{(2)}=2.3+9.8i \text{ fm}^3 , \tag{24}$$

$$\lambda_2^{(2)}=-5.5+8.3i \text{ fm}^3 , \tag{25}$$

with a reduced $\chi^2=2.4$. Putting these results together, we find

$$\lambda_1^{(2)}=(7.4\pm4.5)+(12.4\pm2.7)i \text{ fm}^3 , \tag{26}$$

$$\lambda_2^{(2)}=(-0.5\pm4)+(10.0\pm1.3)i \text{ fm}^3 , \tag{27}$$

There are additional uncertainties in the $\lambda_1^{(2)}$ arising from the limitations of our knowledge of nuclear structure. In most of the nuclei involved, the nuclear structure has been expected to be rather well described by Hartree-Fock theory. However, different models lead to some differences in the results. Calculations in heavy[33] and light[34] nuclei show that uncertainties of 15—30% in SCX cross sections arise from different choices of the effective strong interaction used for the nuclear structure. Additional corrections would arise in heavy nuclei from a more complete treatment of the Coulomb interaction. These Coulomb corrections are very awkward to include and are subject to large uncertainties at the present time. In Ref. 33, they were estimated to give rise to as much as a 30% increase in the cross sections. We conclude that there may be appreciable theoretical uncertainties in $\lambda^{(2)}$ arising from this source. Effects of core polarization can apparently have a large effect on double charge exchange. Liu has estimated an increase of about a factor of 2 in ^{18}O (Ref. 35) and ^{42}Ca (Ref. 11) targets. We need an enhancement of about this size in ^{42}Ca, but such a large effect in ^{18}O would be difficult to accommodate. Clearly, a systematic study of these and other nuclear correlation effects would be useful.

IV. SUMMARY AND DISCUSSION

We have made a phenomenological optical model analysis of single- and double-charge-exchange scattering to isobaric analog states for incident pions of 164 MeV. The form of the optical potential U was deduced from a microscopic theory in a previous study.[17] According to this theory, the nuclear structure and pion-nucleus reaction dynamics separate in a simple, characteristic fashion in the second-order terms, $U^{(2)}$. The factorization of nuclear structure and reaction dynamics in the different pieces of $U^{(2)}$ make it possible to use the theory to isolate the dynamical content of the data. The strong sensitivity of cross sections to nuclear structure requires, however, that the best possible nuclear densities be used to accomplish this goal.

The charge-exchange reaction dynamics are characterized in $U^{(2)}$ by an isoscalar and isovector coefficient, $\lambda_1^{(2)}$ and $\lambda_2^{(2)}$, respectively. We used Skyrme III Hartree-Fock densities and fitted data throughout the periodic table to determine $\lambda_1^{(2)}$ and $\lambda_2^{(2)}$, whose values are given in Eqs. (26) and (27). Elastic-scattering data were used independently to determine the isoscalar terms in $U^{(2)}$ and the Δ_{33} resonance energy shift, given in Eqs. (15) and (14), respectively.

In a nontrivial and perhaps unexpected fashion, the phenomenological description that we have obtained resolves the outstanding puzzles of (1) understanding the magnitude of the SCX cross sections, and (2) the shape of the DCX angular distributions. The magnitude of SCX is improved simply by an enhancement of the isovector terms in $U^{(2)}$. The shape and magnitude of the DCX angular distribution arises as a cooperation between the enhanced isovector scattering and the introduction of the isotensor scattering, which describes the simultaneous interaction of the pion with the two nucleons. An interference between the sequential single-charge-exchange scattering from two nucleons and the isotensor term moves the DCX minimum to smaller angles. The enhanced isovector term in $U^{(2)}$ is essential to achieve the correct magnitude in DCX. Isotopic spin invariance guarantees that the physics in these channels is combined in the proper way. The requirements of reproducing both the SCX and DCX cross sections provides a strong constraint on the phase and magnitude of the parameters $\lambda_1^{(2)}$ and $\lambda_2^{(2)}$, and serves to emphasize the importance of making a unified analysis to understand either of these reactions.

Our success in parametrizing the striking regularities apparent in the charge-exchange data lends empirical support to the theoretical form of $U^{(2)}$ and its characteristic separation of nuclear structure and reaction dynamics. Our result is significant because it suggests that the pion optical potential scales in a smooth and predictable fashion throughout the periodic table. It also demonstrates that it is possible to characterize the pion-nucleus dynamics of single and double charge exchange at a given energy in terms of a few parameters, which are related to theory in a well-defined manner.

In order to compare our results to theory, it is necessary to calculate and sum the different dynamical effects $\Delta\lambda_i^{(2)}(j)$ which contribute

$$\lambda_i^{(2)}=\sum_j \Delta\lambda_i^{(2)}(j) .$$

Examples include long- and short-range correlations,[17] π-π interactions,[36] direct Δ-nucleus interactions,[37] and true absorption and multiple reflections,[38] to name a few. It is also possible that there are processes which cannot be expressed directly in meson theory and which require the theory of quarks and gluons. We look forward to a time when the isospin dependence of the theory is evaluated with sufficient completeness that a meaningful comparison to the phenomenological parameters is possible.

We have attempted to fit the data at $T_\pi=164$ MeV only. To study the energy dependence of $U^{(2)}$, it is necessary to make a similar analysis of data at other energies. From our 164-MeV results we expect that a similar optical-model analysis will provide a compact and physi-

cally meaningful representation of data at other energies as well. Of course, elastic, single, and double-charge-exchange measurements are all needed for the analysis because they provide independent sources of information. It should also be pointed out that although our fits are generally quite good, there is some fluctuation of the theoretical results about the data. One consequence is that parameters are not reliably determined unless data are available over a large region of the periodic table. A reasonably complete set of data is now becoming available at $T_\pi = 292$ MeV, but we would like to see corresponding data sets at several other energies over the region of the (3,3) resonance.

We express our appreciation to H. T. Fortune for his comments and careful reading of the manuscript.

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