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Mean-field theory of prompt, high-energy nucleon emission

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A time-dependent mean-field theory is developed for fast nucleon emission in nuclear heavy-ion reactions. The essence of the model is to treat the relative motion of the ions classically while treating the internal excitations quantum mechanically. General properties of the model are calculated by assuming a phenomenological external field. The model is applied to study fast neutron emission from the reaction $^{16}O+^{93}Nb$.

I. INTRODUCTION

One of the more interesting facets of nuclear heavy-ion reactions, at energies above the Coulomb barrier, is the character of light particle emission such as nucleons, d, t, ^{3}He, and α particles. A large portion of these particles can be interpreted as statistical evaporation from the compound system or emission from excited fragments in deep-inelastic collisions. However, there is also a component of the light particle spectra which is believed to be nonstatistical in nature. Several models have been proposed to understand the origin of these nonstatistical, fast, light particles in the hope that the source of these particles would enhance our understanding of the evolution of the ion-ion reactions. A variety of models based on different underlying physical assumptions are apparently successful in describing the data and are briefly discussed in the following.

The rotating hot spot model[1-4] is based on the assumption that a large frictional force converts the energy of the relative motion into a locally excited region. This local region is assumed to develop early in the reaction. The energy distribution in this local region is assumed to correspond to an equilibrium Maxwell Boltzmann distribution with a specific temperature. Typically, a temperature of 6—7 MeV is assumed, while the evaporation spectra yield temperatures of 1—2 MeV. A classical, equilibrium, statistical treatment of the emission process is used to deduce the energy spectrum. One is inclined to question this model in view of the fact that it assumes a time scale of approximately 10^{-22} sec for equilibration. Considering the relatively long mean-free path[5] for nucleon-nucleon collisions at these energies, the assumption of equilibration is difficult to understand.

Reactions induced by light particles have also been interpreted in terms of precompound models[6] in which the projectile is assumed to excite a limited number of particle-hole states ("excitons") with the subsequent emission of light particles via two-body collisions. These processes are calculated from the classical Boltzmann master equation. This approach has been generalized[7-9] to include ion-induced reactions. The results which are in fair agreement with the data, however, are sensitive to the number and the character of the excitons employed in the calculation. An interesting feature of this model is that the relaxation time which emerges from the solution of the master equation with a finite number of excitons is about 10^{-21} sec.

It has also been proposed that the emission of fast, light particles may result directly from hard, two-nucleon collisions. This direct knockout model has been applied at low energies[8] and high energies.[10,11] The low-energy model assumes a zero-range force and quasifree scattering of a projectile nucleon with a target nucleon. The agreement with data depends on the choice of the force and particularly the choice of the form factors for finding nucleons in their respective nuclei.

Unlike these two models, the Fermi jets[12] or prompt emission of fast particles (PEP's) model[13-15] assumes that the particles are emitted by coupling the internal Fermi motion of the nucleons to the relative motion of the ions. This model assumes that during the early stages of the reaction, a large mean-free path allows nucleons from the projectile (or target) to pass through the target (or projectile). The coupling to the relative motion gives the nucleons sufficient energy to overcome the potential energy and be emitted. The emission time is simply the transit time for a fast nucleon to traverse a nucleus, about 10^{-22} sec. The transit of the projectile (target) nucleon across the target (projectile) is treated classically, and the emitted particle is assumed to travel freely with modifications only for the refraction at the edge of the nucleus.

The energy of the emitted particle relative to the nucleus from which it is emitted is large on the scale of the thermal energies found in these reactions, but it is not large on the scale of typical nuclear kinetic energies, binding energies, or potential energies. This sheds doubt on the validity of the classical treatment of the motion of the emitted particles. This concern is alleviated by using a time-dependent Hartree-Fock (TDHF) approach[16,17] to the problem and fast nucleons are seen to be emitted in TDHF calculations. The technical difficulty of extracting cross sections from TDHF calculations prevents one from using this approach as a systematic data analysis tool. Even if this technical difficulty could be surmounted, there would still remain a need for a simpler, quantum-mechanical model since it is difficult to extract from TDHF calculations the physical mechanism or combination of mechanisms which produce the emission.

This collection of models clearly assumes different underlying physics. The ability of the system to equilibrate is viewed differently in each model. The hot spot model assumes that local equilibrium is reached in a very short time scale. The exciton model solves numerically a master equation to follow the approach to equilibrium, while the PEP's and knockout models assume an underlying direct process as the cause for the emission. The role that the two-body collisions play is also different in each model. In the hot spot model, two-body collisions are implicitly assumed to dominate the interaction, as they must produce a short mean-free path and a short relaxation time. In the exciton model, two-body collisions produce both the equilibration of energy and the emission of fast particles. The knockout model assumes a single two-body collision as the underlying mechanism, while the PEP's model incorporates the two-body interaction only through its role as the source of a mean field. All the models, except TDHF, assume to some degree a classical treatment of the emitted particle.

A systematic study of the promptly emitted, fast particles thus holds great promise at discerning between these models. A study of these particles could thus serve to illucidate the dynamics of the early phases of the heavy-ion reactions and help us to understand the evolution and energy loss mechanisms during this stage. A systematic theoretical study would first require a large body of systematic experimental data. Experimentally, the emission of light, fast particles has been seen in coincidence with deep-inelastic,[21-25] fusion,[7,26-30] and fusion-fission[8,24,31] fragments. Present data, however, are insufficient to tell us quantitatively how the spectrum of these particles depends on the mass and energy of the ions. In addition, one would like to know how this dependence is modified when the particles are detected in coincidence with other fragments.

In this work we develop a model for the emission of fast, light particles which is based on the following observations seen in TDHF calculations. In TDHF calculations we see a very short time scale (on the order of 10^{-22} sec) for particle emission to occur. Since TDHF calculations are done by initially setting up a many-body wave function, which is the product of target and projectile wave functions, and then evolving this wave function in time to complete a full nuclear reaction (nuclear reaction times of 10^{-21} sec or greater), the short time scale implies that these particles are emitted before the entrance channel values for heavy-ion masses and energies are changed by dynamical rearrangements and exchange processes. Therefore, if we consider particle emission from a reaction of the type,

$$A+B\rightarrow N+X\ ,$$

where N denotes a single emitted nucleon and X stands for everything else, then during this short time scale for particle emission, one can approximate A and B to retain their identity while being perturbed by the fields generated by each other. However, since the wavelength of the particles is not small compared to the nuclear sizes, we treat the motion of the emitted particles quantum mechanically. This distinguishes our approach from the other models (except the TDHF calculations). The relative motion of the centers of the two ions is characterized by a short wavelength, and we thus take a classical limit to describe this motion. This decoupling of the relative motion and intrinsic degrees of freedom has also been studied in atomic physics.[18,19] In our present work we treat the interactions of particles in one nucleus with those in the other as a time-dependent mean field. Within this context, nucleons from one of the ions are excited into the continuum via the mean field produced by the other. For mass asymmetric collisions, nucleons are preferentially emitted from the light fragment. This behavior is observed in TDHF work,[16,17] as well as in the Fermi jet[12] and PEP's (Refs. 13 and 15) models which employ a mean field. On elementary terms, the interaction potential which excites nucleons out of the light fragment is proportional to the mass of the heavy fragment, while the interaction potential which excites nucleons out of the heavy fragment is proportional to the mass of the light fragment. These assumptions lead to a model that is sufficiently tractable so as to allow for a systematic survey of a large body of nucleon emission data.

In the next section we derive the model starting with a many-body Hamiltonian describing the dynamics of the two ions. We then take the classical limit for the relative motion of the ions and obtain equations which govern the intrinsic dynamics of the ions. In Sec. III we calculate the exclusive and inclusive one- and two-particle cross sections for fast nucleon emission. In Sec. IV we derive approximations to the model which enable us to investigate its general features. In particular, we study neutron emission from the asymmetric $^{16}O+^{93}Nb$ reaction at $E_{\rm lab}=204$ MeV in both fusion and deep-inelastic reactions. We also investigate the dependence of our results on the laboratory energy. Results are then compared to experimental observations. Some of these calculations have been published previously.[20] Section V looks at two-body cross sections and two-particle correlations with and without final-state interactions.

II. EQUATIONS OF MOTION

In this section we develop a model for the emission of nucleons during the collision of two ions. The reaction we will consider will be the one discussed in the previous section. The $^{16}O+^{93}Nb$ collision was previously studied in TDHF calculations where prompt fast neutrons were emitted from the light heavy-ion fragment. Here, for simplicity, we will consider emission from fragment A while we freeze the intrinsic degrees of freedom of B. The extension of the model to include incoherent emission of nucleons from both the light and heavy ions is, in principle, possible but not considered. We study the particle emission from fragment A which is perturbed by an external time-dependent field. This field is produced by a structureless particle of mass B located at a distance $\vec{R}$ from the center of A. The equations, as presented here, are Galilean invariant; thus, we work in a coordinate system which is fixed at the center of fragment A. With these assumptions, the Hamiltonian which governs our system is

$$H(\{\vec{r}_i\},\vec{R})=H_0(\{\vec{r}_i\})+T_{\vec{R}}+U(\vec{R},\{\vec{r}_i\}) , \tag{2.1}$$

where H_0 is the intrinsic Hartree-Fock Hamiltonian for A, depending on the intrinsic coordinates of the nucleus $\{\vec{r}_i\}$. The spin and isospin quantum numbers have been left out for notational simplicity. $T_{\vec{R}}$ is the relative kinetic energy of A and B, and U is the external potential which induces the coupling between the internal degrees of freedom of A and the relative motion of B. Explicitly,

$$\begin{aligned} H_0 &= \sum_{i=1}^{A} t_i+\tfrac{1}{2}\sum_{i\neq j=1}^{A} v_{ij} , \\ T_{\vec{R}} &= -\frac{\hbar^2}{2M}\vec{\nabla}^2_{\vec{R}} , \\ U(\vec{R},\{\vec{r}_i\}) &= \sum_{i=1}^{A} u(\vec{r}_i-\vec{R}) , \end{aligned} \tag{2.2}$$

where U is a one-body potential which specifies the interaction of the particles in A with particle B. The one-body nature of U is an assumption of our model.

Let Ψ denote the state of the system at some time t. Then the equations governing the evolution of the system can be obtained through the variational principle as

$$\delta S=\delta\int dt\langle\Psi(t)\,|\,(H-i\hbar\partial_t)\,|\,\Psi(t)\rangle=0 , \tag{2.3}$$

where δ denotes the variation with respect to Ψ^*. We seek an approximate solution for Ψ in the form of a product of two terms,

$$\Psi(\{\vec{r}_i\},\vec{R},t)=G(\vec{R},t)\Phi(\{\vec{r}_i\},t) , \tag{2.4}$$

where Φ is a Slater determinant of single-particle wave functions ϕ_λ,

$$\Phi(\{\vec{r}_i\},t)=\frac{1}{\sqrt{A!}}\det||\phi_\lambda(\vec{r}_i,t)|| ,$$

which describes the motion of the particles in A. The function G describes the motion of B. With this form of the wave function, the variational principle (2.3) becomes

$$\frac{\delta S}{\delta G^*}=0,\quad \frac{\delta S}{\delta\phi^*_\lambda}=0 . \tag{2.5}$$

Using Eqs. (2.1) and (2.4) in (2.3), Eq. (2.5) yields dynamical equations for ϕ_λ,

$$[h(\vec{r},t)+E(\vec{r},t)-i\hbar\partial_t]\phi_\lambda(\vec{r},t)=0 \tag{2.6}$$

and for G,

$$\left[-\frac{\hbar^2}{2M}\vec{\nabla}^2_{\vec{R}}+V(\vec{R},t)-i\hbar\partial_t\right]G(\vec{R},t)=0 , \tag{2.7}$$

with

$$\int d^3R\,|\,G(\vec{R},t)\,|^2=1 ,$$
$$\int d^3r\,|\,\phi_\lambda(\vec{r},t)\,|^2=1 .$$

In these equations E and V are time-dependent potentials which are given by

$$E(\vec{r},t)=\int d^3R\,|\,G(\vec{R},t)\,|^2u(\vec{r}-\vec{R})$$

and

$$V(\vec{R},t)=\int d^3r\,\rho(\vec{r},t)u(\vec{r}-\vec{R}) , \tag{2.8}$$

where ρ is the single-particle density of nucleus A,

$$\rho(\vec{r},t)=\sum_{\lambda=1}^{A}|\,\phi_\lambda(\vec{r},t)\,|^2 ,$$

and h is the single-particle Hartree-Fock Hamiltonian.[38] These Eqs. (2.6)—(2.8) describe the motion of our system.

In solving Eq. (2.7) for the motion of fragment B, we will utilize a wave packet formalism to generate a classical approximation. The previous TDHF work treats the motion of the heavy ions classically using the sharp cutoff approximation.[16] First, we construct a general wave packet G and derive dynamical conditions under which G will propagate without appreciable dispersion or spreading. A wave packet satisfying these conditions we call a "classical wave packet." We begin by considering the case in which the two ions are initially far apart so that we have $V\to 0$ and $G\to G_0$ as $t\to-\infty$. We solve this case in a form that can be easily generalized to include the time-dependent potential $V(\vec{R},t)$. Ignoring for the moment long-range Coulomb forces, Eq. (2.7) becomes

$$\left[-\frac{\hbar^2}{2M}\vec{\nabla}^2_{\vec{R}}-i\hbar\partial_t\right]G_0(\vec{R},t)=0 . \tag{2.9}$$

Equation (2.9) may be solved as follows. At time $t=0$, G_0 is chosen to be a wave packet centered about $\vec{R}=0$

$$G_0(\vec{R},0)=e^{i/\hbar\Gamma_0(\vec{R},0)}g(\vec{R}) ,$$

where g is a function which is sharply peaked about $\vec{R}=0$. For simplicity, we choose

$$g(\vec{x})=(\sqrt{\pi}\sigma)^{-3/2}e^{-x^2/2\sigma^2} ,$$

with $\sigma^2=1.31B^{2/3}$ determined from the size of nucleus B. This choice of σ will minimize the dispersion in G_0. At an arbitrary time G_0 is given by

$$G_0(\vec{R},t)=e^{i/\hbar\Gamma_0(\vec{R},t)}g(\vec{R}-\vec{a}_0) ,$$

with Γ_0 given by

$$\Gamma_0(\vec{R},t)=M\dot{\vec{a}}_0\cdot\vec{R}-\int_{-\infty}^{t}dt'\tfrac{1}{2}M\dot{\vec{a}}_0^{\,2} ,$$

and $g(\vec{x})$ given by

$$-\frac{\hbar^2}{2M}\vec{\nabla}^2_{\vec{x}}g(\vec{x})=\tfrac{1}{2}M(\dot{\vec{a}}_0)^2 g(\vec{x}) .$$

This last result follows if we neglect terms which cause the spreading of the wave packet. In these equations, $\vec{a}_0$ and $\dot{\vec{a}}_0$ are the mean position and mean velocity of the wave packet

$$\vec{a}_0=\int d^3R\,\vec{R}\,|G_0(\vec{R},t)|^2,$$

$$\dot{\vec{a}}_0=-\frac{i\hbar}{M}\int d^3R\,G_0^*(\vec{R},t)\vec{\nabla}_{\vec{R}}G_0(\vec{R},t) .$$

The condition that the wave packet behave completely classically over the time scale of the collision requires

$$\frac{t_{\text{collision}}}{\tau_{\text{classical}}}\ll 1, \quad \tau_{\text{classical}}=\left[\frac{\hbar}{M\sigma^2}\right]^{-1} .$$

Since at $t=0$ the wave packet is localized in a region which is characterized by the size of the actual nucleus, we have $\tau_{\text{classical}}=19.6B^{5/3}$ fm/c. The heavy-ion collision times which we will consider are less than 10^3 fm/c. Using this value for $t_{\text{collision}}$ we show in Fig. 1 the dependence of the ratio $t_{\text{collision}}/\tau_{\text{classical}}$ on the heavy-ion mass. Note that collision times for ions with masses less than about 40 are about a factor of 4 shorter than that assumed in Fig. 1. Thus, we see that the noninteracting heavy ions can reasonably be treated as localized, nonspreading wave packets.

Following the above argument, we write the solution for the interacting system as

$$G(\vec{R},t)=e^{i/\hbar\Gamma(\vec{R},t)}g(\vec{R}-\vec{a})$$

with the initial condition

$$\Gamma(\vec{R},t)\underset{t\to-\infty}{\longrightarrow}\Gamma_0(\vec{R},t)$$

and

$$\vec{a}(t)\underset{t\to-\infty}{\longrightarrow}\vec{a}_0(t) .$$

In terms of Γ and g, Eq. (2.7) becomes

$$\left[\frac{(\vec{\nabla}_{\vec{R}}\Gamma)^2}{2M}+V(\vec{R},t)+\partial_t\Gamma+i\hbar\left[\dot{\vec{a}}-\frac{\vec{\nabla}_{\vec{R}}\Gamma}{M}\right]\cdot\vec{\nabla}_{\vec{R}}\right]g(\vec{R}-\vec{a})=0 . \quad (2.10)$$

In obtaining Eq. (2.10), we have used the relation

$$i\partial_t g[\vec{R}-\vec{a}(t)]=-\dot{\vec{a}}\vec{\nabla}_{\vec{R}}g[\vec{R}-\vec{a}(t)] ,$$

and have ignored terms which correspond to the zero-point energy of the packet, namely

$$\xi_0=\int d^3R\,g[\vec{R}-\vec{a}(t)]\left[-\frac{\hbar^2}{2M}\vec{\nabla}^2_{\vec{R}}\right]g[\vec{R}-\vec{a}(t)]$$

$$=\frac{3}{4}\frac{\hbar^2}{\sigma^2}=\frac{3}{4}\frac{\hbar}{\tau_{\text{classical}}} .$$

In Fig. 2 we plot ξ_0 as a function of nuclear mass M. We see that ξ_0 is generally small and can be neglected. In Eq. (2.10) $\vec{\nabla}\Gamma$ plays the role of the classical momentum of the heavy-ion B. The phase factor Γ will be of the form

$$\Gamma(\vec{R},t)=M\dot{\vec{a}}\cdot\vec{R}-\int_{-\infty}^{t}dt'\mathscr{I}(t')$$

with

$$\lim_{t\to-\infty}\mathscr{I}(t)\to\tfrac{1}{2}M(\dot{\vec{a}}_0)^2 .$$

The function $\mathscr{I}(t)$ can be found by taking the expectation value of Eq. (2.10). Requiring that terms in lowest order of an expansion of $\vec{R}$ about $\vec{a}$ satisfy Eq. (2.10) gives $\mathscr{I}(t)$ in terms of $\vec{a}(t)$

$$\mathscr{I}(t)=\tfrac{1}{2}M\dot{\vec{a}}(t)^2+M\ddot{\vec{a}}(t)\cdot\vec{a}(t)+V[\vec{a}(t),t] , \quad (2.11)$$

where we have used an expansion in $(\vec{r}-\vec{a})$ for the expectation of V,

$$\int d^3R\,|g[\vec{R}-\vec{a}(t)]|^2V(\vec{R},t)=V[\vec{a}(t),t]+\tfrac{3}{4}\sigma^2|\vec{\nabla}_a V[\vec{a}(t),t]|^2+\cdots .$$

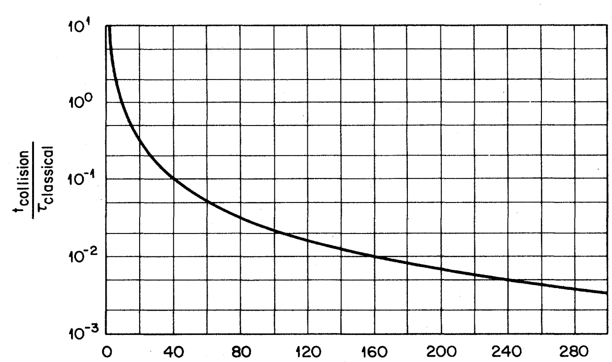

FIG. 1. The ratio $t_{\text{collision}}/\tau_{\text{classical}}$ as a function of nuclear mass number B, $\tau_{\text{collision}}$ is assumed to be 10^3 fm/c.

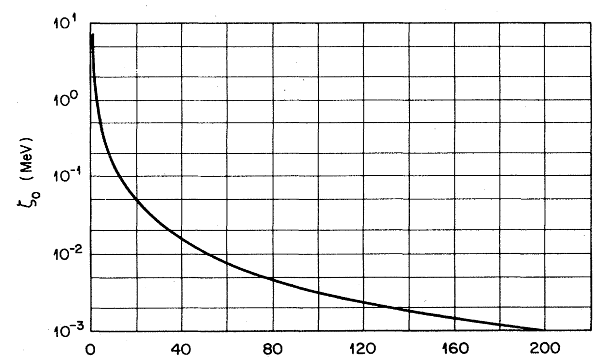

FIG. 2. Zero-point energy ξ_0 (MeV) of the wave packet as a function of nuclear mass number B.

Keeping only the first term in the series requires

$$V[\vec{a}(t),t] \gg \frac{3}{4}\sigma^2 \,|\, \vec{\nabla}_a V[\vec{a}(t),t] \,|^2 \,.$$

Equation (2.11) has a simple interpretation in terms of classical mechanics. The first term is the kinetic energy of the mass as it moves in the presence of a time-dependent potential $V(\vec{a},t)$. The second term is the work done by the mass M as it moves along the trajectory $\vec{a}(t)$. Since the potential is time dependent, the energy $\epsilon(t)$ defined by

$$\epsilon(t)=\frac{1}{2}M[\dot{\vec{a}}(t)]^2+V[\vec{a}(t),t] \tag{2.12}$$

is not a constant of motion.

If we require the terms linear in the expansion of $\vec{R}$ about $\vec{a}$ to vanish when taking the expectation value of Eq. (2.10), we find

$$M\ddot{\vec{a}}(t)+\vec{\nabla}_{\vec{a}}V[\vec{a}(t),t]=0 \,, \tag{2.13}$$

and the rate of change of the energy $\epsilon(t)$ becomes

$$\frac{d\epsilon(t)}{dt}=\{M\ddot{\vec{a}}(t)+\vec{\nabla}_{\vec{a}}V[\vec{a}(t),t]\}\dot{\vec{a}}+\frac{\partial V[\vec{a}(t),t]}{\partial t} \,,$$
$$=\frac{\partial V[\vec{a}(t),t]}{\partial t} \,.$$

These are precisely Newton's equations for the motion of a particle in a time-dependent potential. We are thus in a region where the center of the wave packet follows a classical trajectory, and we can neglect the spreading of the packet as it traverses the nucleus.

The reduction of the field Eqs. (2.6) and (2.8) follows in a similar manner

$$E(\vec{r},t)=\int d\vec{R}\,|\,G(\vec{R},t)\,|^2 u(\vec{r}-\vec{R})$$
$$=u[\vec{r}-\vec{a}(t)]+\frac{3}{4}\sigma^2\,|\,\nabla_{\vec{a}}u[\vec{r}-\vec{a}(t)]\,|^2+\cdots$$
$$\simeq u[\vec{r}-\vec{a}(t)] \,.$$

With this approximation, the equation for ϕ_λ, the basic equation of our model, becomes

$$\{h_0(\vec{r})+u[\vec{r}-\vec{a}(t)]\}\phi_\lambda(\vec{r},t)=i\hbar\partial_t\phi_\lambda(\vec{r},t) \,. \tag{2.14}$$

In the absence of long-range Coulomb forces, u is a finite-range interaction. Because nucleons in nucleus A are localized near the origin, we have

$$E(0,t)=u[\vec{a}(t)]$$

and E goes to zero as $|\,t\,|\to\infty$. Thus the initial solution for ϕ_λ satisfies the equation,

$$[h_0(\vec{r})-i\hbar\partial_t]\phi_\lambda(\vec{r},t)=0 \,,$$

where h_0 is the static Hartree-Fock Hamiltonian for the nucleons in A. These solutions are

$$\phi_\lambda(\vec{r},t)=e^{-i\epsilon_\lambda t/\hbar}\chi_\lambda(\vec{r}) \,, \tag{2.15}$$

with

$$h_0\chi_\lambda=\epsilon_\lambda\chi_\lambda \,.$$

We also have that $h(t)$ approaches h_0 for both initial and final asymptotic times.

III. INVARIANT EXCLUSIVE AND INCLUSIVE CROSS SECTIONS

The initial and final channel Hamiltonian consists in part of H_0, the many-body Hamiltonian of A as given in Eq. (2.2), which can be written in the second quantized representation as

$$H_0=E_0+\sum_\lambda \epsilon_\lambda :b_\lambda^\dagger b_\lambda:$$
$$+\sum_{\lambda,\mu,\nu,\sigma}\langle\lambda\mu\,|\,v\,|\,\mu\sigma\rangle :b_\lambda^\dagger b_\mu^\dagger b_\sigma b_\nu: \,, \tag{3.1}$$

where ":" denotes the normal ordering with respect to the reference state $|X_0\rangle$, $b_\lambda^\dagger(b_\lambda)$ are the fermion creation (annihilation) operators in the Hartree-Fock basis of Eq. (2.15) which satisfy the usual anticommutation relations

$$\{b_\lambda^\dagger,b_\mu\}=\delta_{\lambda\mu} \,.$$

In this representation the many-body state $|X_0\rangle$, defined by

$$|X_0\rangle=\prod_{\lambda=1}^{A} b_\lambda^\dagger\,|0\rangle \,, \tag{3.2}$$

minimizes the energy E_0,

$$E_0=\langle X_0\,|\,H_0\,|\,X_0\rangle \,. \tag{3.3}$$

If the Hamiltonian in this representation is truncated to contain only one-body terms,

$$H_0=E_0+\sum_\lambda \epsilon_\lambda :b_\lambda^\dagger b_\lambda: \,, \tag{3.4}$$

then H_0 has a complete set of eigenstates, each of which is uniquely specified by a set of occupation numbers N,

$$N=\{\lambda_1,\lambda_2,\ldots,\lambda_A\}$$

with

$$H_0\,|\,X_N\rangle=E_N\,|\,X_N\rangle \,,$$
$$|\,X_N\rangle=\prod_{i=1}^{A} b_{\lambda_i}^\dagger\,|0\rangle \,. \tag{3.5}$$

Thus a complete set of orthonormal states which span the Hilbert space of H_0 are single Slater determinants, having a set of single-particle quantum numbers, denoted by N, in which each single-particle state may assume any of the allowed values. In this representation the projection operator onto the state $|X_N\rangle$ is

$$\hat{P}_N=|\,X_N\rangle\langle X_N\,| \,,$$

which is normalized so that

$$\sum_{N=0}^{\infty}\frac{1}{A!}\hat{P}_N=1 \,. \tag{3.6}$$

In Eq. (3.6) the summation is overcomplete and exactly cancels the factor $A!$. We chose to work in this basis of many-body Slater determinants.

We next must derive expressions for the differential cross sections that would result from the solution of our model. The general principles of elastic and inelastic scattering in a wave packet formalism have been given by Low.[39] Our formulation is similar to that of Low's with modifications so that we may treat the relative motion of the ions classically. We are considering the collision of two heavy ions A and B in which nucleus B is assumed to be a structureless probe, and A can undergo inelastic excitations

$$A+B\rightarrow A'+B\ . \tag{3.7}$$

Processes corresponding to more complex reaction channels are beyond the framework of this model. The excited states of A in Eq. (3.7) are either bound or continuum particle-hole excitations produced by the time-dependent field of Eq. (2.2). The geometry of the scattering event is shown in Fig. 3.

The cross sections can be written in terms of an initial current density $J^0(\vec{R}_i,t)$ of ions B incident on the nucleus A in its ground state, and in terms of the outgoing currents of ions B which leave the nucleus A in a state S, $\vec{J}^{(S)}(\vec{R}_f,t)$. Following Low, the number of incident and scattered particles, N_i and N_f, respectively, which pass through a surface element $d\vec{S}_i$ and $d\vec{S}_f$ are

$$\begin{aligned} N_i &= \int_{-\infty}^{+\infty} dt\, d\vec{S}_i\cdot\vec{J}^{\,0}(\vec{R}_i,t)\ , \\ N_f &= \int_{-\infty}^{+\infty} dt\, d\vec{S}_f\cdot\vec{J}^{\,S}(\vec{R}_f,t)\ . \end{aligned} \tag{3.8}$$

The currents in Eq. (3.8) are obtained from the state $\Psi(t)$ and the exclusive current operator,

$$\hat{J}^{(N)}(\vec{R})=\frac{\hbar}{M}\mathrm{Im}\{\hat{P}_N\vec{\nabla}_{\vec{R}}\}\ , \tag{3.9}$$

by

$$\begin{aligned} \vec{J}^{(N)}(\vec{R},t) &= \langle\Psi(t)\,|\,\hat{J}^{(N)}(\vec{R})\,|\,\Psi(t)\rangle \\ &= |\langle\Phi(t)\,|\,X_N\rangle|^2\frac{\hbar}{M}\mathrm{Im}\{G^*(\vec{R},t)\vec{\nabla}_{\vec{R}}G(\vec{R},t)\}\ , \end{aligned} \tag{3.10}$$

which follows from Eq. (2.4). Note that in Fig. 3 the surfaces $d\vec{S}_i$ and $d\vec{S}_f$ are specified by coordinates $\vec{R}_i$ and $\vec{R}_f$, respectively. Using the form of G obtained in Sec. II, Eq. (3.8) becomes

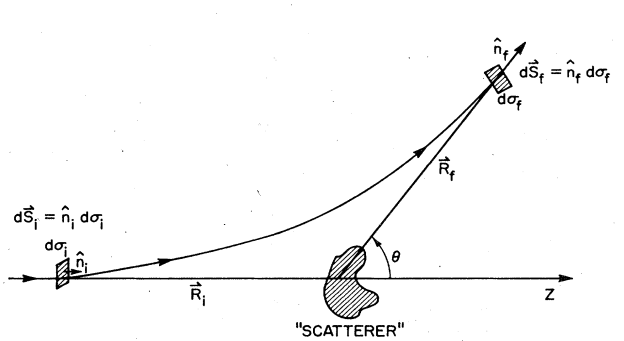

FIG. 3. A classical trajectory characterizing a scattering event in our reference frame.

$$N_i=\Delta\sigma_i\int_{-\infty}^{+\infty}dt\,|\langle\Phi(t)\,|\,X_0\rangle|^2\dot{\vec{a}}_0(t)\,|\,g[\vec{R}_i-\vec{a}_0(t)]\,|^2\ , \tag{3.11}$$

where, as in Fig. 3, we have chosen the incident beam direction along the z axis, and

$$\lim_{t\rightarrow-\infty}\vec{a}_0(t)\rightarrow\vec{b}+\vec{V}_0t\ .$$

The initial trajectory is specified in terms of the impact parameter $\vec{b}$ and an initial velocity $\vec{V}_0$. Since $g(\vec{R}_i-\vec{a}_0)$ is nonzero only when $\vec{a}_0\approx\vec{R}_i$, we have $\dot{\vec{a}}_0=\hat{e}_z\dot{a}_0$ and $|\langle\Phi(t)\,|\,X_0\rangle|^2=1$, so that

$$N_i\simeq\Delta\sigma_i\int_{-\infty}^{+\infty}d\rho\,|\,g(\rho)\,|^2\ . \tag{3.12}$$

Similarly, the number of scattered particles is

$$N_f=\Delta\sigma_f\int_{-\infty}^{+\infty}dt\,|\langle\Phi(t)\,|\,X_S\rangle|^2\dot{\vec{a}}(t)\hat{n}_f\,|\,g[\vec{R}_f-\vec{a}(t)]\,|^2\ . \tag{3.13}$$

This may be simplified as follows. Particle-hole excitations in A occur only when B is in the scattering region. Since $\vec{R}_f$ is arbitrarily far from this region and the function g is appreciable only when $\vec{a}(t)\simeq\vec{R}_f$, so that we have $\vec{a}(t)\simeq\hat{n}_f\dot{a}(t)$, Eq. (3.13) becomes

$$N_f=\Delta\sigma_f\,|\langle\Phi(+\infty)\,|\,X_S\rangle|^2\int_{-\infty}^{+\infty}d\rho\,|\,g(\rho)\,|^2\ . \tag{3.14}$$

The inelastic cross section to the many-body state S is defined by

$$\frac{d\sigma_S}{d\Omega}=\frac{\Delta A}{\Delta\Omega}\frac{N_f}{N_i}=\frac{\Delta A}{\Delta\Omega}\,|\langle\Phi(\infty)\,|\,X_S\rangle|^2\ ,$$

where ΔA is a small area of the incident beam. Since the wave packet propagates at all times along a classical trajectory without dispersion, we have $\Delta\sigma_i=\Delta\sigma_f$. In this limit, there are no scattered outgoing quantum mechanical waves. The final state moves as a classical particle. In the axially symmetric geometry of Fig. 3, ΔA is just $2\pi b\,db$, and the solid angle is given by $d\Omega=2\pi\sin\theta\,d\theta$. The exclusive differential cross section becomes

$$\frac{d\sigma_S}{d\Omega}=\left[\frac{d\sigma}{d\Omega}\right]|_{\mathrm{cl}}\langle\Phi(+\infty)\,|\,X_S\rangle|^2\ ,$$

with (3.15)

$$\left[\frac{d\sigma}{d\Omega}\right]_{\mathrm{cl}}=\frac{b\,db}{\sin\theta\,d\theta}\ .$$

Equation (3.15) has been obtained by other authors.[19] However, our derivation is the only treatment in terms of a classical wave packet limit and provides insight into the limits of its applicability.

The matrix element $\langle\Phi(\infty)\,|\,X_S\rangle$ is easily found in the second quantized representation. The wave function $|\Phi(t)\rangle$ is a Slater determinant of single-particle states which are time dependent

$$|\Phi(t)\rangle=\prod_{\lambda=1}^{A}a_\lambda^\dagger(t)\,|0\rangle\ , \tag{3.16}$$

where the operators $a_\lambda^\dagger(t)$ $[a_\lambda(t)]$ define a complete set of single-particle states ϕ_λ which are the solutions of Eq. (2.14). These states form a complete and orthonormal basis, and we can expand the operators $b_s^\dagger$ in terms of them,

$$b_s^\dagger = \sum_\lambda C_{s\lambda}(t) a_\lambda^\dagger(t) \; . \tag{3.17}$$

The state $|X_S\rangle$ is given by Eq. (3.5),

$$|X_S\rangle = \sum_{s=1}^{A} b_s^\dagger |0\rangle \; ,$$

so that the overlap matrix element becomes a determinant of the coefficients $C_{s\lambda}(\infty)$,

$$|\langle \Phi(+\infty)|X_S\rangle|^2 = |\det||C_{s_i\lambda_j}(\infty)|||^2 \; . \tag{3.18}$$

The exclusive inelastic differential and total cross sections are then given by

$$\left[\frac{d\sigma_S}{d\Omega}\right] = \left[\frac{d\sigma}{d\Omega}\right]_{\rm cl} |\det||C_{s_i\lambda_j}(\infty)|||^2 \tag{3.19}$$

and

$$\sigma_S = 2\pi \int_0^\infty db\, b\, |\det||C_{s_i\lambda_j}(b,t=\infty)|||^2 \; . \tag{3.20}$$

The inclusive, one- and two-particle cross sections are deduced by summing over the allowed states of the $A-1$ and $A-2$ nucleons, respectively. We find

$$\frac{d\sigma(\alpha)}{d\Omega} = \frac{1}{A!}\sum_S \left[\sum_{i=1}^{A} \delta_{s_i\alpha}\right] \frac{d\sigma_S}{d\Omega} \tag{3.21}$$

and

$$\frac{\delta\sigma(\alpha,\beta)}{d\Omega} = \frac{1}{A!}\sum_S \left[\sum_{i,j=1}^{A} \delta_{s_i\alpha}\delta_{s_j\beta} - \sum_{i=1}^{A} \delta_{\alpha s_i}\delta_{\beta s_i}\right] \frac{d\sigma_S}{d\Omega} \; . \tag{3.22}$$

The indices α and β denote single-particle states in the final channel Hamiltonian. From these, we obtain the one-particle total inclusive cross section

$$\sigma^{(1)} = \sum_\alpha \int d\Omega \frac{d\sigma(\alpha)}{d\Omega} = 2\pi \int_0^\infty b\, db \sum_\alpha \sum_{\lambda=1}^{A} |C_{\alpha\lambda}(b,t=\infty)|^2 \tag{3.23}$$

and the two-particle total inclusive cross section

$$\sigma^{(2)} = \sum_{\alpha,\beta} \int d\Omega \frac{d\sigma(\alpha,\beta)}{d\Omega} = 2\pi \int_0^\infty db\, b \sum_{\alpha,\beta} \left[\sum_{\lambda,\mu=1} |C_{\lambda\alpha}(b,t=\infty)|^2 |C_{\mu\beta}(b,t=\infty)|^2 - \left|\sum_{\lambda=1}^{A} C^*_{\alpha\lambda}(b,t=\infty) C_{\beta\lambda}(b,t=\infty)\right|^2\right] \; . \tag{3.24}$$

In Eqs. (3.23) and (3.24) $\sigma^{(1)}$ and $\sigma^{(2)}$ are independent scalar invariants which characterize a particular inclusive reaction. From these cross sections and the usual definition of one- and two-particle invariant cross sections,

$$\sigma^{(1)} = \int d^3q \frac{d^3\sigma^{(1)}}{dq^3} \; , \qquad \sigma^{(2)} = \int d^3k\, d^3q \frac{d^6\sigma^{(2)}}{d^3k\, d^3q} \; , \tag{3.25}$$

we obtain

$$\frac{d^3\sigma^{(1)}}{dk^3} = \frac{1}{(2\pi)^2} \sum_{\lambda=1}^{A} \int_0^{b_{\max}} b\, db\, |C_{\vec{k}\lambda}(\infty)|^2 \tag{3.26}$$

and

$$\frac{d^6\sigma^{(2)}}{d^3k\, d^3q} = \frac{1}{(2\pi)^5} \int_0^{b_{\max}} db\, b \left[\sum_{\lambda,\mu=1}^{A} |C_{\lambda\vec{k}}|^2 |C_{\mu\vec{q}}|^2 - \left|\sum_{\lambda=1}^{A} C^*_{\lambda\vec{k}} C_{\lambda\vec{q}}\right|^2\right] \; . \tag{3.27}$$

These equations, (3.26) and (3.27), are valid in a reference frame in which nucleus A is at rest at the origin. However, the invariance of the total cross section allows one to easily obtain the corresponding differential cross sections in either the laboratory or the center-of-mass frames. Also, in deriving Eqs. (3.25) and (3.27), we have assumed that the continuum states in our model saturate the particle-state summation in Eqs. (3.23) and (3.24), $\beta \to \vec{k}$, and

$$\sum_\beta \to \int \frac{d^3k}{(2\pi)^3} \; .$$

IV. NUMERICAL STUDIES

The general features of the time-dependent equations discussed in the previous sections are easily computed using a rank-one separable potential for the external field u

$$\langle \vec{r}' - \vec{a}(t)|u|\vec{r} - \vec{a}(t)\rangle = \Lambda_0 Q(|\vec{r}' - \vec{a}(t)|) Q(|\vec{r} - \vec{a}(t)|) \; . \tag{4.1}$$

The theory of replacing a local potential by a separable one is reasonably well understood.[32,33] We will impose the symmetry condition that the external field is rotationally invariant at all times. Thus the rotational symmetries of the initial nuclear state are unchanged by the time evolution. The momentum representation of u truncated to maintain this rotational invariance can be written as

$$\langle \vec{q} \,|\, u(t) \,|\, \vec{q}\,' \rangle = 4\pi\Lambda_0 \sum_{lm} F_l[q,a(t)]F_l[q',a(t)] \times Y^*_{lm}(\hat{q}\,')Y_{lm}(\hat{q}) \tag{4.2}$$

with

$$F_l[qa(t)] = j_l[qa(t)]Q(q) \,.$$

Solutions to Eq. (2.15) are obtained by expanding $\phi_\lambda(t)$ in the basis of the static Hartree-Fock Hamiltonian h_0 of Eq. (2.15)

$$\phi_\lambda(\vec{r},t) = \sum_\alpha C_{\alpha\lambda}(t)\chi_\alpha(\vec{r})e^{-i\epsilon_\alpha t/\hbar} + \int \frac{d^3k}{(2\pi)^3} C_{\vec{k}\lambda}(t)\chi^+_k(\vec{r})e^{-i\epsilon_k t/\hbar} \tag{4.3}$$

with the initial condition that

$$\lim_{t\to-\infty} C_{\alpha\lambda}(t) = \delta_{\alpha\lambda} \,.$$

The states χ_α in Eq. (4.3) are the bound state eigenfunctions of h_0 obtained by solving the self-consistent Hartree-Fock equations.[35] Whereas the states $\chi^{+}_{\vec{k}}$ are the continuum state solutions with outgoing boundary conditions having an energy $\epsilon_{\vec{k}}$. The scattering states $\chi^{+}_{\vec{k}}$ can be generated using the separability of the Hartree-Fock single-particle potential[34] v,

$$h_0 = t + v \,,$$
$$\langle \vec{r} \,|\, v \,|\, \vec{r}\,' \rangle = \sum_{\alpha,\beta=1}^{A} \chi_\alpha(\vec{r}\,')v_{\alpha\beta}\chi^*_\beta(\vec{r}) \,, \tag{4.4}$$

where the matrix element is given as

$$v_{\alpha\beta} = \langle \chi_\alpha \,|\, v \,|\, \chi_\beta \rangle \,.$$

From the Lippman-Schwinger equation, we obtain in momentum space,

$$\chi^{+}_{\vec{k}}(\vec{q}) = (2\pi)^3\delta(\vec{q}-\vec{k}) + \frac{2m}{\hbar^2} \sum_{\alpha,\beta=1}^{A} \frac{\chi_\alpha(\vec{q})D_{\alpha\beta}(\vec{k})\chi^*_\beta(\vec{k})}{k^2-q^2+i\eta} \,, \tag{4.5}$$

where $D_{\alpha\beta}$ is an element of the matrix D defined by

$$D = (1 - v\cdot Y)^{-1}v \,.$$

The elements of the matrix Y are given by

$$Y_{\alpha\beta}(\vec{k}) = \frac{2m}{\hbar^2} \int \frac{d^3q}{(2\pi)^3} \frac{\chi^*_\alpha(\vec{q})\chi_\beta(\vec{q})}{k^2-q^2+i\eta} \,.$$

With the expansion of Eq. (4.1), the time evolution Eq. (2.14) becomes a set of coupled linear differential equations for the expansion coefficients C

$$i\hbar\dot{C}_{\alpha\lambda}(t) = \sum_{\alpha'} A_{\alpha\alpha'}(t)C_{\alpha'\lambda}(t)e^{-i(\epsilon_{\alpha'}-\epsilon_\alpha)t/\hbar} + \int \frac{d^3k}{(2\pi)^3} A_{\alpha\vec{k}}(t)C_{\vec{k}\lambda}(t)e^{-i(\epsilon_{\vec{k}}-\epsilon_\alpha)t/\hbar} \,,$$
$$i\hbar\dot{C}_{\vec{k}\lambda}(t) = \sum_{\alpha'} A_{\vec{k}\alpha'}(t)C_{\alpha'\lambda}(t)e^{-i(\epsilon_{\alpha'}-\epsilon_\alpha)t/\hbar} + \int \frac{d^3k}{(2\pi)^3} A_{\alpha\vec{k}}(t)C_{\vec{k}\lambda}(t)e^{-i(\epsilon_{\vec{k}}-\epsilon_\alpha)t/\hbar} \,, \tag{4.6}$$

with

$$A_{\alpha\alpha'}(t) = \int d^3r\, \chi^*_\alpha(\vec{r})u[\vec{r}-a(t)]\chi_{\alpha'}(\vec{r})$$

and with a similar definition for the matrix elements involving the scattering states.

We choose $Q(q)$ to be of the Yukawa form

$$Q(q) = (q^2+\kappa^2)^{-1} \,. \tag{4.7}$$

With this choice, the matrix element $A_{\alpha\vec{k}}(t)$ becomes

$$A^l_{\alpha\vec{k}}(t) = 4\pi\Lambda_0 \left\{ \int_0^\infty q^2dq\, \chi^*_{\alpha l}(q)F_l[qa(t)] \right\} \times \left\{ \int_0^\infty q^2dq\, \chi^{+}_{\vec{k}l}(q)F_l[qa(t)] \right\} , \tag{4.8}$$

where $\chi_{\alpha l}(q)$ $[\chi^{+}_{\vec{k}l}(q)]$ is the radial part of the wave function $\chi_\alpha(\vec{q})$ $[\chi^{+}_{\vec{k}}(\vec{q})]$. Similarly, if we substitute

$$C_{\vec{k}\lambda}(t) = C^l_{k\lambda}(t)Y^*_{lm}(\hat{k}) \tag{4.9}$$

into Eq. (4.6), the resulting equations are uncoupled in the variable $\hat{k}$ and reduce the three-dimensional integrals in Eq. (4.6) to one-dimensional integrations

$$i\hbar\dot{C}^l_{\alpha\lambda}(t) = \sum_{\alpha'} A^l_{\alpha\alpha'}C^l_{\alpha'\lambda}(t)e^{-i(\epsilon_{\alpha'}-\epsilon_\alpha)t/\hbar} + \int_0^\infty \frac{k^2dk}{(2\pi)^3} A^l_{\alpha k}C^l_{k\lambda}(t)e^{-i(\epsilon_k-\epsilon_\alpha)t/\hbar} \,,$$
$$i\hbar\dot{C}^l_{k\lambda}(t) = \sum_{\alpha'} A^l_{k\alpha'}(t)C^l_{\alpha'\lambda}(t)e^{-i(\epsilon_\alpha-\epsilon_k)t/\hbar} + \int_0^\infty \frac{dk'k'^2}{(2\pi)^3} A^l_{kk'}(t)C^l_{k'\lambda}(t)e^{-i(\epsilon_{k'}-\epsilon_k)t/\hbar} \,. \tag{4.10}$$

The numerical advantages of our choice for the form of u are several. First, the rotational approximation renders l a good quantum number, so that Eq. (4.10) does not couple different values of l. Second, the calculation of matrix elements in Eq. (4.8) requires one-dimensional integrations only. Third, the matrix A formed from elements of Eq. (4.8) can be written as a product of two vectors which allows the integration of the time over a step Δt to be performed exactly as shown in the Appendix.

Computations of Eq. (4.10) have been carried out for the prompt neutron emission observed in coincidence with the deep-inelastic branch of the $^{16}O+^{93}Nb$ reaction at

$E_{\text{lab}}=204$ MeV.[25] We calculated neutron emission only from the ^{16}O fragment, and we solved Eq. (4.10) in its rest frame. As previously stated, we ignore the emission from the Nb-like fragment. For our model of ^{16}O we have used the Hartree-Fock wave functions of Ref. 35. The radial scale κ of the external field was adjusted so that the interaction was confined within the strong absorption radius of the $^{16}\text{O}+^{93}\text{Nb}$ system for which $\kappa=0.5$ fm^{-1}. The classical trajectory equation, Eq. (2.13), was solved using a modified Coulomb potential of the form

$$V(R)=\begin{cases} \dfrac{Z_A Z_B e^2}{2R_C}\left[3-\left[\dfrac{R}{R_C}\right]^2\right] & R<R_C \\ \dfrac{Z_A Z_B e^2}{R} \quad R>R_C\,, \end{cases} \tag{4.11}$$

with

$$R_C=1.30(A^{1/3}+B^{1/3})\ \text{fm}\,,$$

where $A=16$, $Z_A=8$, $B=93$, and $Z_B=41$. For the deep-inelastic branch of the reaction, we determined the impact parameter range from the fusion and total neutron cross sections[36] using the sharp cutoff model. We obtained

$$b_{\min}\simeq 6.0\ \text{fm}\,,$$

$$b_{\max}\simeq 10.0\ \text{fm}\,.$$

We adjusted the field strength Λ_0 to obtain a neutron multiplicity of 0.134 in agreement with the corresponding experimental value.[25] Equation (4.10) was solved on a discrete momentum time lattice

$$t_i=i\Delta t \quad i=1,\ldots,N_T\,,$$

$$k_j=j\Delta k \quad j=1,\ldots,N_k$$

with

$$\Delta t=0.4\ \text{fm}/c,\quad N_T=1{,}000$$

and

$$\Delta k=0.022\ \text{fm}^{-1},\ N_k=100\,.$$

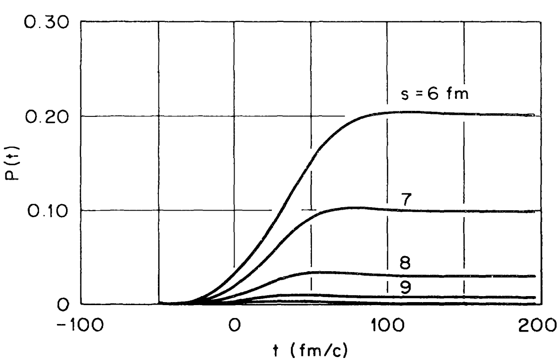

FIG. 4. The continuum occupation probability for the deep-inelastic reaction as a function of time (fm/c). The variable s labeling the curves is the heavy-ion impact parameter.

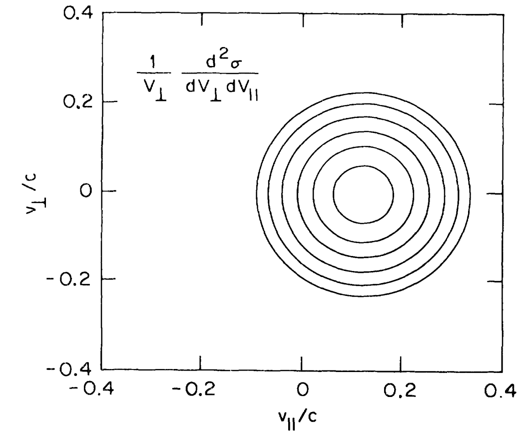

FIG. 5. Contours of the invariant cross section for the deep-inelastic reaction as a function of velocities parallel ($V_{\parallel}/c$) and perpendicular ($V_{\perp}/c$) to the initial beam direction in units of fm$^2c^3$. The interior contour has a value 1.01×10^5 fm$^2c^3$; each subsequent contour decreases in magnitude by a factor of 5.

The maximum continuum energy was thus 100 MeV. The s-wave contribution from ^{16}O to the following results is less than 1%, in agreement with the previous time-dependent Hartree-Fock calculation.[16]

In Fig. 4 we show the time dependence of the total emission probability

$$p(t)=\frac{1}{A}\sum_{\lambda=1}^{A}\int\frac{d^3k}{(2\pi)^3}\,|C_{\vec{k}\lambda}(t)|^2 \tag{4.12}$$

for various impact parameters as a function of time. Us-

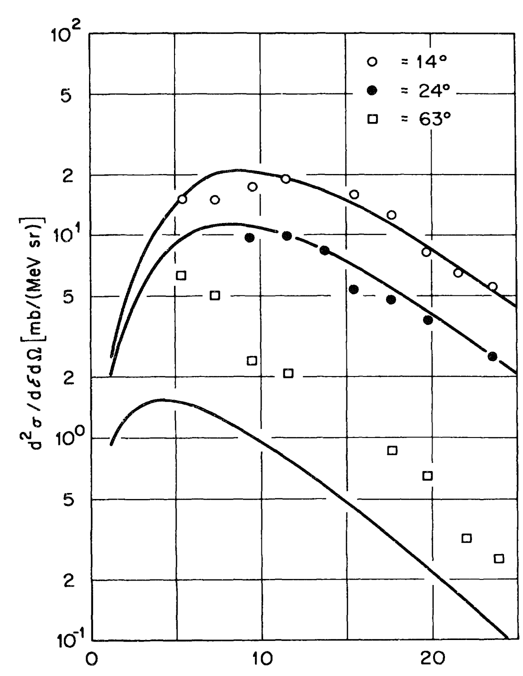

FIG. 6. The double-differential cross sections $d^2\sigma/d\epsilon\,d\Omega$ (mb/MeV sr) for the deep-inelastic reaction versus the experimental results of laboratory angles 14°, 24°, and 63°. The data are from Ref. 25.

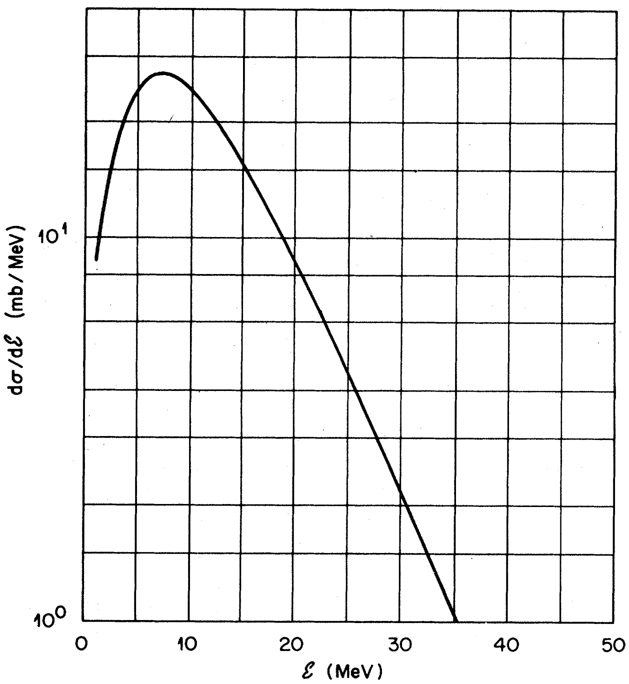

FIG. 7. The energy spectrum $d\sigma/d\epsilon$ (mb/MeV) for the deep-inelastic reaction obtained by angle integration of Fig. 5 as a function of energy (MeV).

ing this definition of $p(t)$, we calculate the mean velocity of the frame from which the particles are emitted

$$\overline{V}=\left[2\pi\int_{b_{\min}}^{b_{\max}} db\, bV_l(b)\right]\left[2\pi\int_{b_{\min}}^{b_{\max}} db\, b\right]^{-1} \qquad (4.13)$$

with

$$V_l=\int_{-\infty}^{+\infty} dt\, p(t)V_T(t)\Big/\int_{-\infty}^{+\infty} dt\, p(t)\;,$$

where V_T is the relative velocity of the heavy ions on the trajectory for a given impact parameter. We found $\overline{V}/c=0.124$. In Fig. 5 we show the contours of the invariant singles cross section as a function of the longitudinal and transverse laboratory velocities. From this figure,

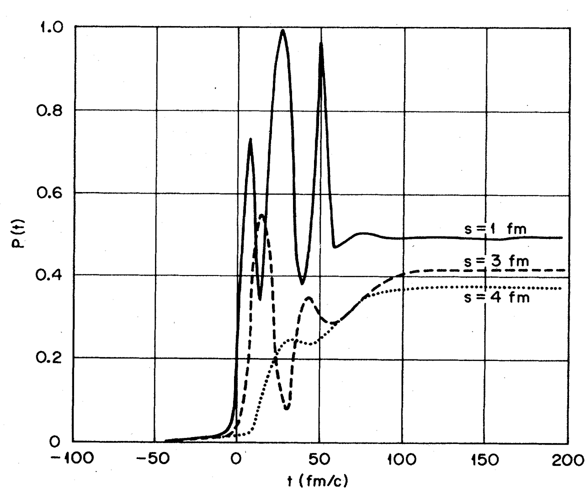

FIG. 8. Same as Fig. 4 except for the fusion reaction.

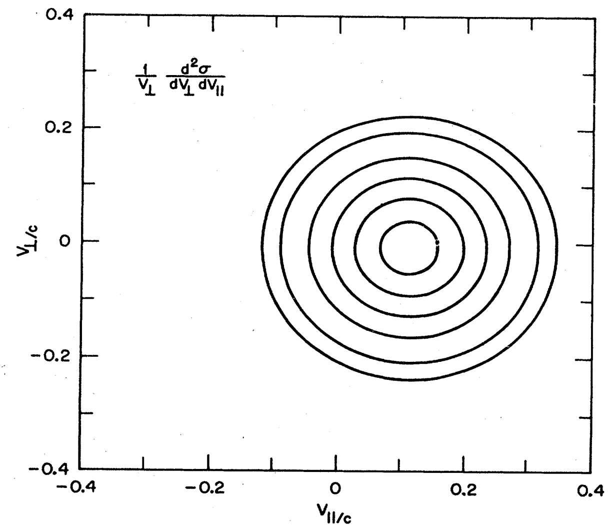

FIG. 9. Same as Fig. 5 except for the fusion reaction. The interior contour has a value 3.1×10^5 fm$^2c^3$, and each subsequent contour decreases in magnitude by a factor of 5.

we note the following: the contours are centered about the mean velocity $\overline{V}$; they are isotropic and have a characteristic exponential dependence on the total velocity $V=\sqrt{V_\perp^2+V_\parallel^2}$.

The energy-angle differential cross section $d^2\sigma/d\epsilon\, d\Omega$

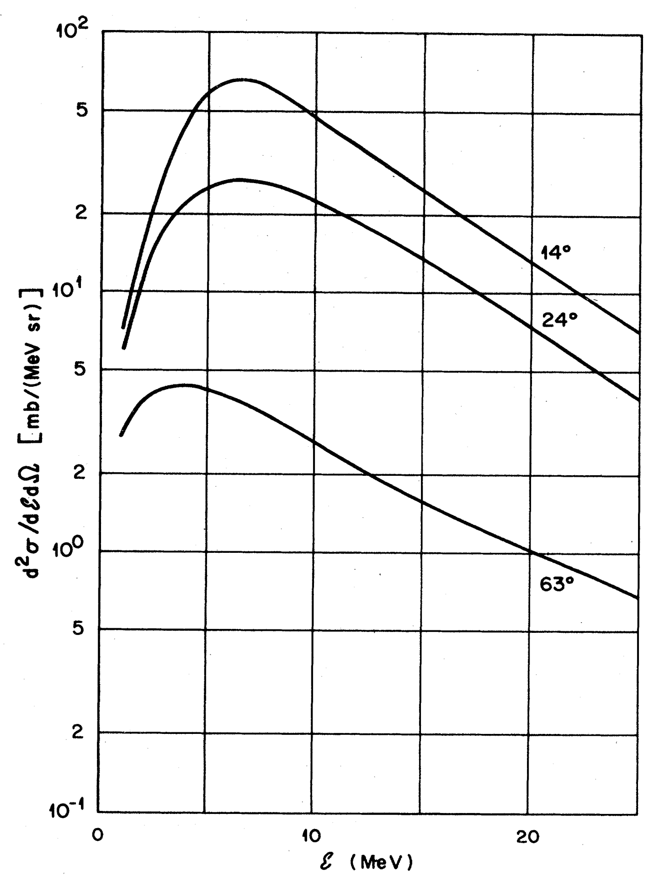

FIG. 10. Same as Fig. 6 except for the fusion reaction.

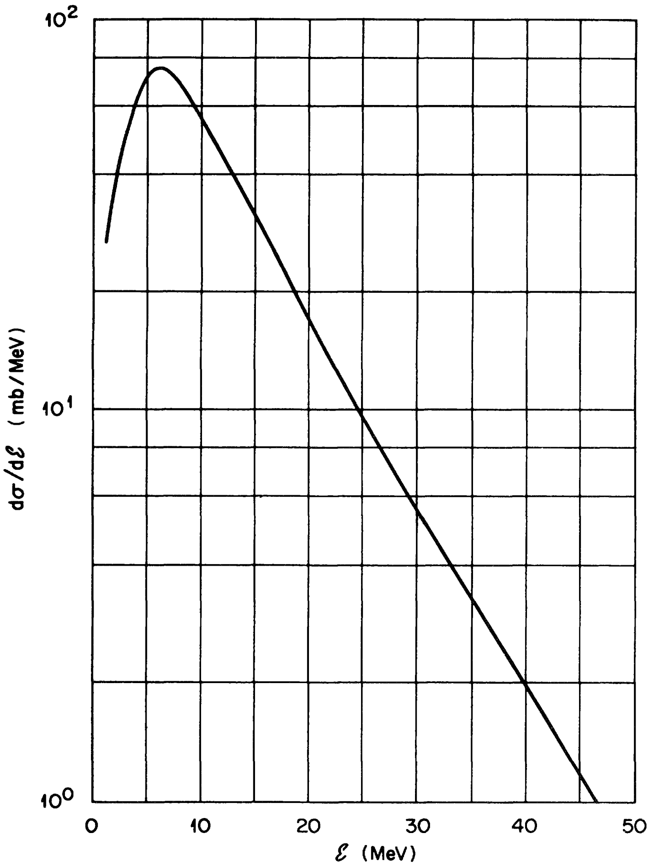

FIG. 11. Same as Fig. 7 except for the fusion reaction.

is shown in Fig. 6 as a function of energy for angles 14°, 24°, and 63°. The experimental results[25] are also shown, and we see fairly good agreement with the data with the exception of the backward angle of 63°. This disagreement may be due to the neglect of emission from the ^{93}Nb target in these calculations. In Fig. 7 we show the angle integrated energy spectrum $d\sigma/d\epsilon$. In order to extract a temperature parameter, we have fitted the calculated energy spectrum to the functional form of

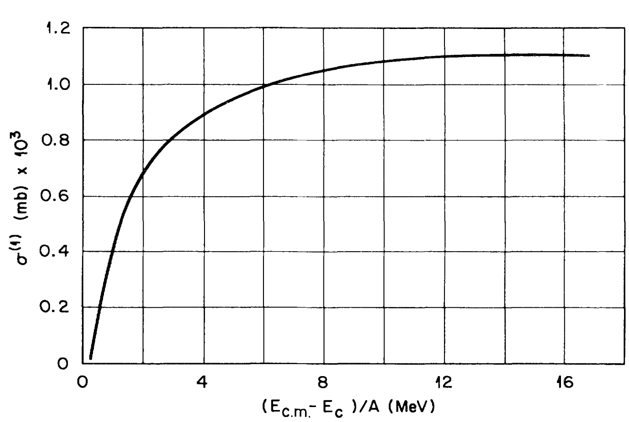

FIG. 12. Neutron emission cross section (mb) for the total reaction (fusion plus deep inelastic) as a function of the available energy per particle above the Coulomb barrier (MeV).

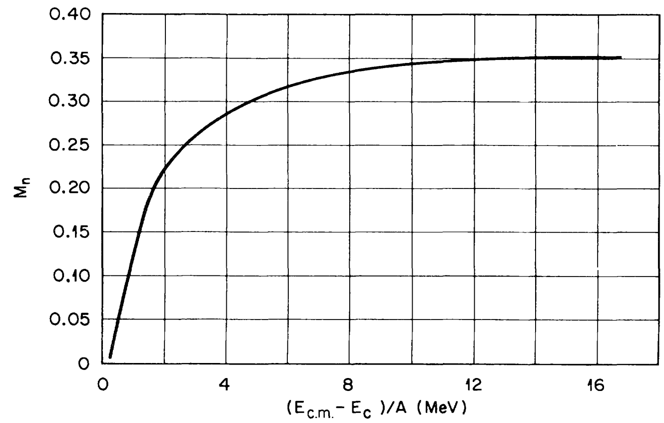

FIG. 13. Multiplicities for the total reaction (fusion plus deep inelastic) as a function of the available energy per particle above the Coulomb barrier (MeV).

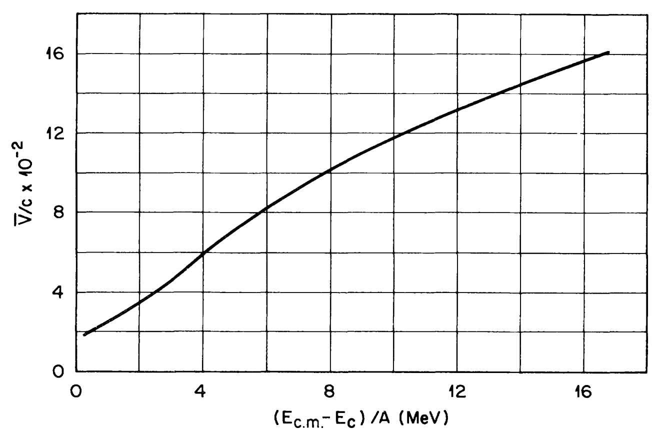

FIG. 14. The velocity, $\overline{V}/c$ of the frame from which the particles are emitted for the total reaction (fusion plus deep inelastic) as a function of available energy per particle above the Coulomb barrier (MeV).

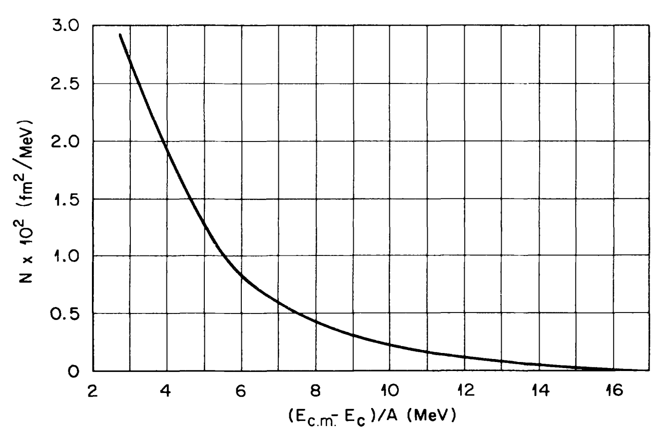

FIG. 15. The normalization factor N (fm^2/MeV) of Eq. (4.14) as a function of available energy per particle above the Coulomb barrier (MeV).

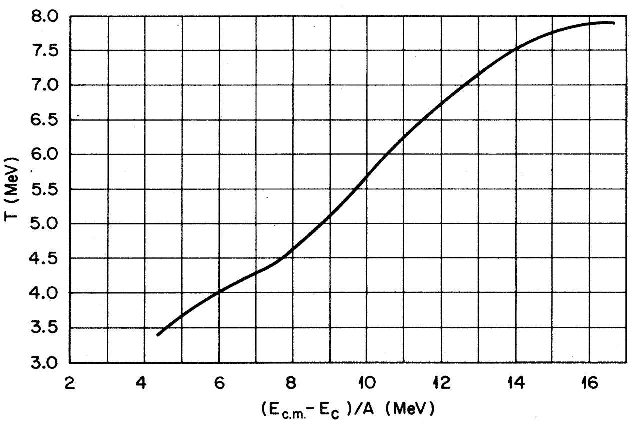

FIG. 16. The temperature parameter (MeV) of Eq. (4.14) as a function of available energy per particle above the Coulomb barrier (MeV).

$$\frac{d\sigma}{d\epsilon} \cong N \left[\frac{\epsilon}{T} \right]^{\alpha} e^{-\epsilon/T} , \tag{4.14}$$

and found $N=69.3$ fm^2 MeV^{-1}, $T=5.2$ MeV, and $\alpha=1.32$.

Although there are no data for neutron emission in a fusion reaction for this system, we present results for this reaction in Figs. 8—11. In Fig. 8 we plot the total emission probability $p(t)$ as a function of time. For this case, we see interesting oscillations in the emission probabilities which are absent in the deep-inelastic reaction. This dramatically demonstrates why simple approximations to the time evolution such as the Born approximation would be inadequate. Figures 9—11 again show the observed characteristics of the emission spectrum. The invariant cross section contours of Fig. 9 are now centered about a velocity $\overline{V}/c=0.114$. We obtain a multiplicity of 0.334 corresponding to a total cross section of 1048 mb. As before, a functional fit to the energy spectrum, $d\sigma/d\epsilon$, of Fig. 11 yields $N=191.1$ fm^2 MeV^{-1}, $T=4.8$ MeV, and $\alpha=1.202$. Without changing the values of κ and Λ_0, we have also investigated the beam energy dependence of the particle emission from the combined fusion and deep-inelastic reactions. Figures 12 and 13 show the dependence of the total cross section and the multiplicity on the available energy per particle above the Coulomb barrier. As a function of this energy, we also show the source velocity $\overline{V}/c$ in Fig. 14, and the energy spectrum parameters of Eq. (4.14), N,T, and α in Figs. 15—17.

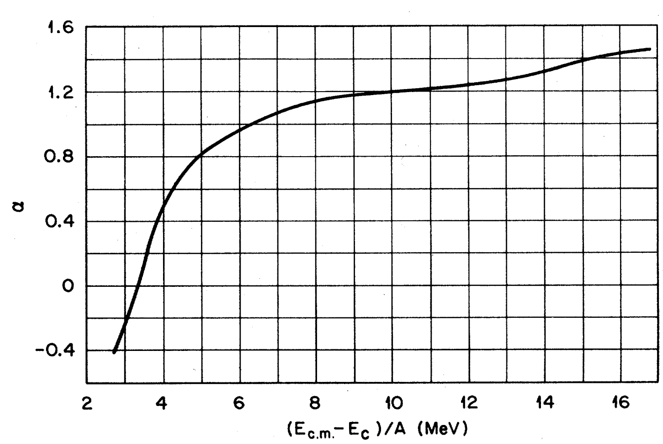

FIG. 17. The power, α, of Eq. (4.14) as a function of available energy per particle above the Coulomb barrier (MeV).

V. PAIR CORRELATIONS IN INCLUSIVE EMISSION

The approximations and assumptions which have been discussed here, at each stage, satisfied Galilean invariance. Thus the inclusive two-particle cross sections derived [Eq. (3.27)] are invariant with respect to Galilean transformations. Since we have derived a one-body theory for the emission process, it is important to understand the two-particle correlations which are included in Eq. (3.27). First, the wave function of the nucleus is at all times a Slater determinant. The antisymmetry of the wave function leads to the second term on the right-hand side of Eq. (3.27). The second type of correlations are the dynamical correlations which enter into our equations through the time-dependent mean field. As in time-dependent Hartree-Fock theory, the strong short-range correlations serve to renormalize the interaction and produce, in lowest order, a time-dependent, one-body potential. In our work, we ignore the contribution to the mean field from dynamical particle exchange and polarization and determine it solely from entrance channel nuclear configurations.

There is another source of correlation between the detected particles that is not yet in our model and that does not principally depend on the dynamics of the emission process. If two nucleons are emitted in nearly the same direction, then they interact substantially before they reach the detector. Because of the strong nucleon-nucleon interaction at low energies, we would expect this interaction to alter the two-particle cross sections for small relative momenta.

We can approximately account for this effect by including a residual nucleon-nucleon potential in our Hamiltonian. If we were to add this interaction at the onset of the time evolution, we would add a substantial numerical complexity to our model. Among the effects would be the emission of two nucleons from correlations which are present in the initial nuclei, or from correlations which are dynamically induced during the reaction. Here, instead, we will consider only those final-state interactions which take place after the completion of the emission process. This we can accomplish if we replace the current operator by a new current operator in which the two-particle state is replaced by a correlated two-particle state. The current operator then becomes

$$\hat{J}_{\vec{k}\vec{q}}=\left[\frac{\hbar}{2Mi}\vec{\nabla}_{\vec{R}}+\text{H.c.}\right]\int\frac{d^3k'd^3q'd^3k''d^3q''}{(2\pi)^{12}}\psi^{*(-)}_{\vec{k}\vec{q}}(\vec{k}',\vec{q}')a^\dagger_{\vec{k}'}a^\dagger_{\vec{q}'}\psi^{(-)}_{\vec{k}\vec{q}}(\vec{k}'',\vec{q}'')a_{\vec{q}''}a_{\vec{k}''}\,. \tag{5.1}$$

The two-body scattering state can be written in relative and center-of-mass coordinates

$$\vec{P}=\vec{k}+\vec{q}\quad \vec{p}=\frac{\vec{k}-\vec{q}}{2}\,,$$

$$\vec{P}'=\vec{k}'+\vec{q}'\quad \vec{p}'=\frac{\vec{k}'-\vec{q}'}{2}\,,$$

as

$$\psi^{(-)}_{\vec{k}\vec{q}}(\vec{k}',\vec{q}')=(2\pi)^3\delta(\vec{P}'-\vec{P})\bar{\psi}^{(-)}_{\vec{p}}(\vec{p}')\,,$$

and Eq. (5.1) becomes

$$\hat{J}_{\vec{k}\vec{k}}=\left[\frac{\hbar}{2Mi}\vec{\nabla}_{\vec{R}}+\text{H.c.}\right]\int\frac{d^3P\,d^3p\,d^3Q\,d^3q}{(2\pi)^6}\delta(\vec{K}-\vec{P})\bar{\psi}^{*(-)}_{\vec{k}}(\vec{p})a^\dagger_{\vec{P}+\vec{p}/2}a^\dagger_{\vec{P}-\vec{p}/2}\otimes\delta(\vec{K}-\vec{Q})\bar{\psi}^{(-)}_{\vec{k}}(\vec{q})a_{\vec{Q}-\vec{q}/2}a_{\vec{Q}+\vec{q}/2}\,, \tag{5.2}$$

or, by introducing the rescattering function f through the equation

$$\bar{\psi}^{(-)}_{\vec{k}}(\vec{p})=(2\pi)^3\delta(\vec{k}-\vec{p})+f(\vec{k},\vec{p})\,, \tag{5.3}$$

we find

$$\hat{j}_{\vec{k}\vec{k}}=\left[\frac{\hbar}{2Mi}\vec{\nabla}_{\vec{R}}+\text{H.c.}\right]\int\frac{d^3P\,d^3p\,d^3Q\,d^3q}{(2\pi)^6}\delta(\vec{K}-\vec{P})[(2\pi)^3\delta(\vec{k}-\vec{p})+f^*(\vec{k},\vec{p})]$$

$$\otimes a^\dagger_{\vec{P}+\vec{p}/2}a^\dagger_{\vec{P}-\vec{p}/2}\delta(\vec{K}-\vec{Q})[(2\pi)^3\delta(\vec{k}-\vec{q})+f(\vec{k},\vec{q})]a_{\vec{Q}-\vec{q}/2}a_{\vec{Q}+\vec{q}/2}\,. \tag{5.4}$$

Note that for $f=0$ this current operator will give us back the two-particle cross section obtained without final-state interactions. For the calculation of the rescattering terms, we use the local momentum approximation

$$f(\vec{k},\vec{p})=\left[\frac{1}{f_J(-|\vec{k}|)}-1\right](2\pi)^3\delta(\vec{k}-\vec{p})\,, \tag{5.5}$$

where f_J is the N-N Jost function.[41] Using this equation and taking the expectation value of the current, we get (going back to original coordinates)

$$\langle\Phi(\infty)|J_{\vec{k}\vec{q}}|\Phi(\infty)\rangle=\frac{d^6\sigma_f^{(2)}}{dk^3dq^3}=\frac{1}{\left|f_J\left(-\left|\frac{\vec{k}-\vec{q}}{2}\right|\right)\right|^2}\frac{d^6\sigma^{(2)}}{dk^3dq^3}\,. \tag{5.6}$$

Here $\sigma^{(2)}$ denotes the inclusive cross section of Eq. (3.27). We see that the effect of the final-state interaction is to enhance the two-particle cross section by a factor $1/|f_J|^2$. For energies less than 50 MeV the correlated wave function $\bar{\psi}_k^{(-)}$ will be dominated by the contribution from the deuteron pole. As our residual interaction, we assume a separable Yukawa potential of the form[42]

$$V(\vec{k},\vec{k}')=\frac{\lambda_0}{4\pi}g(\vec{k})g(\vec{k}')$$

with

$$g(\vec{k})=\frac{1}{k^2+\beta^2}\,.$$

The strength λ_0 is adjusted to produce the deuteron binding energy, and we obtain

$$\frac{1}{f(-k)}=\cos\delta e^{i\delta}\left[1-\frac{\pi}{2}\frac{\lambda_0\beta^4}{(k^2+\beta^2)^2-\lambda_0\frac{\beta^2}{2}(k^2-\beta^2)}\right]\,.$$

We use the effective range expansion for the phase δ,

$$\tan\delta=\frac{\lambda_0\beta^3}{(k^2+\beta^3)^2-\lambda_0\frac{\beta^2}{2}(k^2-\beta^3)}\,.$$

The values of λ_0 and β obtained are $\beta=285$ MeV, $\lambda_0=-2.19$.

We have computed the inclusive two-particle cross section without and with final-state interactions, Eqs. (3.27) and (5.6), respectively. Using the Galilean invariance of these cross sections, we transform them into the relative and center-of-mass coordinates of the two emitted particles and define a cross section $\bar{\sigma}$ which is only a function of the relative momentum by

$$\bar{\sigma}(p)=\int d^3P\frac{d^6\sigma^{(2)}}{dk^3dq^3}\,,$$

$$\vec{P}=\vec{k}+\vec{q},\quad \vec{p}=\frac{\vec{k}-\vec{q}}{2}\,.$$

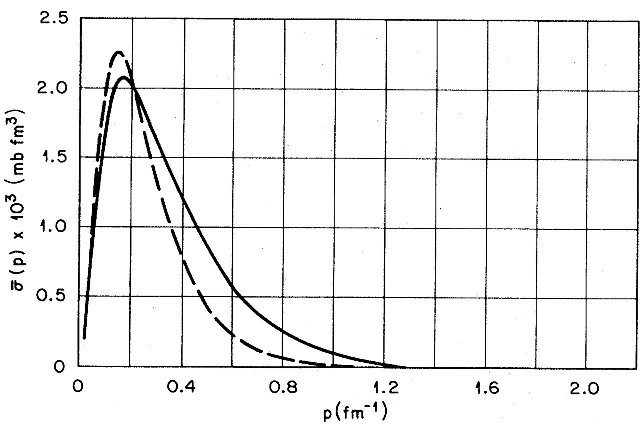

FIG. 18. Two-particle cross section $\bar{\sigma}(p)$ (mb fm^3) without final-state interactions (solid curve) is plotted as a function of the relative momenta (fm^{-1}). The dashed curve shows the same quantity with the inclusion of final-state interaction.

The functions $\bar{\sigma}$, with and without the final-state interaction, are shown in Fig. 18. We observe that the addition of the final-state interaction enhances the two-particle cross section by a factor of ~ 100 but leaves the general form the same. A naive application of the uncertainty principle to the half-width of the two-particle cross section of Fig. 18 gives a space localization of $\Delta r \cong 2.5$ fm, about the radius of a ^{16}O nucleus.

VI. CONCLUSIONS

In summary, we have developed a time-dependent mean-field model for nucleon emission in heavy-ion reactions. In the model the coupling between the nucleons in the target with the projectile is through a one-body field having a time dependence arising from the motion of the ions on classical trajectories. We have found that the time dependence of the field induced by the motion of the ions is strong and must be solved exactly. Inclusive cross sections are obtained by summation over the appropriate final states. The model is easily calculable by introducing a phenomenological form for the mean field. The neutron emission from the Nb-like fragment is neglected in the present work but could be treated assuming the emission from the target and projectile along the same impact parameter to be independent processes. The results of our calculation are quite encouraging and successfully reproduce the general features of the spectra obtained by phenomenological and statistical models discussed earlier. The exponential dependence of the spectra on energy emerges naturally, and although there are significant numbers of nucleons which are emitted with velocities greater than the incident beam velocity, the mean velocity of these particles is found to be related to the change of the relative velocity of the two centers. It is also possible to extract a so-called "temperature" parameter by making a functional fit to the energy spectrum.

Specifically, we have calculated the inclusive neutron cross sections from the deep-inelastic and fusion branches of the ^{16}O+^{93}Nb reaction of $E_{\text{lab}}=204$ MeV. The results for energy-angle double-differential cross sections are in good agreement with the data with the noted exception at angles greater than 50° in the laboratory. For this system, we have also calculated the two-particle cross sections as a function of the relative momentum of the two correlated nucleons. Although there is no experimental measurement of these correlations for this system, our results show the general features of the two-particle correlations obtained for other systems.[40,11] A naive use of the uncertainty principle indicates a localized source for the particle emission which is about the size of the ^{16}O nucleus.

There now exists several models for the emission of fast particles in heavy-ion collisions. Each makes different assumptions concerning the underlying physics. A systematic set of data which clarifies the systematics of the emission will be necessary to discern between these models. The dependence of the cross section on beam energy, ion masses, and the dependence on projectile-target mass asymmetry are some of the features that need to be studied. A further refinement of the models to reduce the number of free parameters and their application to a large body of data will clarify the underlying physics which governs the early stages of ion-ion reactions.

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APPENDIX

Here we will derive a closed form expression for the time evolution of Eqs. (4.10) over an infinitesimal time step Δt. The discretization of the momentum integrals in Eqs. (4.10), and the choice of evaluating the C_k's (for notational simplicity we have dropped the λ and l indices) on the same mesh[37] allows us to write the matrix equation,

$$\dot{X}(t)=\frac{4\pi\Lambda_0}{i\hbar}A'X(t)\ . \tag{A1}$$

The elements of matrix A' are the elements of matrix A, given by Eq. (4.8), multiplied by $k_i/(2\pi)^{3/2}(dk)^{1/2}$, where k_i is the ith mesh point of the momentum discretization. We also make the transformation

$$C'_{k_i}=\frac{k_i}{(2\pi)^{3/2}}(dk)^{1/2}C_{k_i}\ , \tag{A2}$$

which makes A' an Hermitian matrix. In terms of these quantities, vector $X(t)$ is defined as

$$X^T(t)\rightarrow[C_\alpha(t),C'_{k_1}(t),\ldots,C'_{k_N}(t)]\ . \tag{A3}$$

In this form the matrix A' can be written as a product of two vectors, with elements

$$A'_{ij} = a_i a_j^* \;, \tag{A4}$$

where

$$a_\alpha = \left\{ \int_0^\infty dq\, q^2 \chi_{\alpha l}^*(q) F_l[qa(t)] \right\} e^{-i\epsilon_\alpha t/\hbar}$$

and

$$a_{k_i} = \frac{k_i}{(2\pi)^{3/2}} (dk)^{1/2} \times \left\{ \int_0^\infty dq\, q^2 \chi_{kl}^{+}(q) F_l[qa(t)] \right\} e^{-i\epsilon_{k_i} t/\hbar} \;.$$

The general solution of Eq. (A1) over a small time step Δt can be written, in component form, as

$$X_i(t_0+\Delta t) = \left\{ \exp\left[\frac{4\pi\Lambda_0}{i\hbar} A'(t_0)\Delta t \right] \right\}_{ij} X_j(t_0) \;.$$

By noting the identity

$$(A'^2)_{ij} = N A'_{ij}; \quad N = \sum_i |a_i|^2 \;,$$

we can write all of the terms in the expansion of the exponential as an expression which is linear in matrix A',

$$\exp\left[\frac{4\pi\Lambda_0}{i\hbar} A'(t_0)\Delta t \right] = 1 + \frac{1}{N} \left[\exp\left[\frac{4\pi\Lambda_0}{i\hbar} \Delta t \right] - 1 \right] A' \;, \tag{A5}$$

which simplifies our calculations.

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