

PHYSICAL REVIEW C

NUCLEAR PHYSICS

THIRD SERIES, VOLUME 30, NUMBER 6

DECEMBER 1984

Electric quadrupole photoneutron reactions in ^{16}O

P. C-K. Kuo* and J. W. Jury

Department of Physics, Trent University, Peterborough, Ontario, Canada K9J 7B8

N. K. Sherman and W. F. Davidson

Division of Physics, National Research Council of Canada, Ottawa, Ontario, Canada K1A 0R6

(Received 6 February 1984)

Photoneutron energy spectra from the $^{16}\text{O}(\gamma, n_0)^{15}\text{O}$ reaction were measured as functions of laboratory angle over a range of excitation energies from 30 to 35 MeV. Angular distribution Legendre-polynomial coefficients were extracted up to third order as functions of excitation energy. Nonzero values of the coefficients a_1 ($+0.25 \pm 0.02$) and a_3 (-0.2 ± 0.02) were observed over the energy region explored, indicating interference between states of opposite parity. These values can be accounted for in a simple model incorporating electric quadrupole absorption strength decaying via the ground-state photoneutron channel. When combined with the previously-determined amplitude ratio for the $E1 p \rightarrow s$ and $p \rightarrow d$ single-particle transitions, the present results suggest that about 4% of the isovector energy-weighted sum rule is found in the (γ, n_0) channel in the energy range studied. The value of the (γ, n_0) cross section was found to vary from 1.5 ± 0.1 mb at $E_x = 30$ MeV to 0.8 ± 0.1 mb at 35 MeV. The average magnitude of the $E2$ contribution to this cross section was estimated to be 0.05 ± 0.02 mb. This is in reasonable agreement with a recent continuum random-phase approximation shell model calculation, but is in disagreement with a previous measurement.

I. INTRODUCTION

Among the light nuclei, ^{16}O has been the object of intensive investigation. From the theoretical point of view it is doubly magic and is readily amenable to different forms of shell model calculations (see, for example, Refs. 1–3). Experimental investigation has established the properties of the giant dipole resonance (GDR) in ^{16}O through photoneuclear and radiative-capture angular distribution measurements (see, for example, Refs. 4–6). Within the last few years a number of experiments were performed in search of giant resonances other than the GDR in ^{16}O .

Phillips and Johnson⁷ have measured the angular distributions of photoneutrons from the ground-state reaction $^{16}\text{O}(\gamma, n_0)^{15}\text{O}$. In the energy region of 25–45 MeV, nonzero a_1 , a_3 , and a_4 values (normalized coefficients of a Legendre polynomial fit to the angular distribution) were observed, indicating the presence of electric quadrupole strength in the neutron channel. The $E2$ cross section was extracted as a function of photon energy by making assumptions about the phase differences and transition amplitude ratios extrapolated from previous polarization data from (\vec{p}, γ_0) experiments (e.g., Hanna *et al.*⁶).

The $E2$ cross section deduced by Phillips and Johnson⁷

for the neutron channel was of the same order of magnitude as that for the proton channel. For example, at 35 MeV, the neutron $E2$ cross section was reported to be 0.21 ± 0.01 mb. Measurements⁹ of the proton channel indicate an $E2$ cross section of about 0.22 ± 0.01 mb at this energy. These findings have led to three different interpretations which are of major consequence to the understanding of the physics of this nuclear reaction:

(a) The large photoneutron $E2$ cross section (reported in Ref. 7) might have arisen from an erroneously large a_4 value (approximately $+0.25$ from 30 to 45 MeV) resulting from a possible overfitting of the Legendre series to the experimental data (this was suggested in Ref. 10).

(b) It is possible that no, or very little *direct* absorption is present in the neutron channel in the energy region considered here; all the $E2$ cross section observed might be of *resonant* nature. If this is the case, the results of Ref. 7 constitute dramatic evidence for a giant electric quadrupole resonance (GQR).

(c) There is also the possibility that both direct and resonant absorption strengths are present (perhaps in a 50-50 mixture) as predicted by some direct-semidirect (DSD) models (for example, Ref. 11), and the simple concept of recoil quadrupole effective charge^{12,13} is inaccurate. The effective quadrupole charge of a neutron (in

^{16}O) should be 30 times weaker than that of a proton ($q_n=0.03e$, $q_p=0.91e$). Therefore, direct absorption strength in the neutron channel should be much weaker (900 times) than that in the proton channel. The DSD model predicts 50% of the $E2$ cross section should be attributed to the direct strength. This strength is expected to concentrate in the proton channel. As for the resonant portion, the strength should be divided equally between the neutron and proton channels. This means only 25% of the total $E2$ cross section should be found in the neutron channel. Assuming the DSD model is applicable to the ^{16}O nucleus, one would expect the neutron $E2$ cross section to be about one-third of the proton $E2$ cross section. However, this is contrary to the measurement reported in Ref. 7.

In order to differentiate among these interpretations and to test the validity of the experimental results, a similar experiment to that of Ref. 7 was carried out and is reported in this paper. Although the same experimental approach was employed, many details in the experimental design and procedures, data reduction and analysis, and interpretation of the angular distribution coefficients were significantly different.

II. EXPERIMENTAL DETAILS AND DATA ANALYSIS

The angular distribution measurements for the $^{16}\text{O}(\gamma, n_0)^{15}\text{O}$ reaction were performed at the electron linear accelerator laboratory of the National Research Council of Canada, using the bremsstrahlung-induced photoneutron time-of-flight technique. Since this facility has been described in some detail already by Jury *et al.*¹⁴ and Watson *et al.*,¹⁵ only some of the noteworthy features will be highlighted here.

The photoneutron time-of-flight spectra were recorded at eight angles simultaneously over 10 m flight paths. Each of the eight detectors produced a "stop" signal for its own time-to-digital converter, which was previously started by a signal generated by the photon beam striking a detector placed near the sample. A computer-controlled sample changer cycled the samples into the photon beam for time periods which were very short compared to the duration of a run. This helped to ensure that the photoneutron samples received the same amount of photon flux. The highest 5.2 MeV region of the bremsstrahlung tip corresponds unambiguously to ground-state transitions (the first excited state in ^{15}O is at 5.2 MeV).

Photoneutron data from a total of three samples were recorded. These were D_2O (to monitor photon distribution), H_2O (the ^{16}O sample), and a "blank" (an empty container to measure the backgrounds contributed by the sample containers). In order to reduce the neutron self-scattering within the samples, a hollow right-cylindrical design for the sample containers was employed.¹⁵ Containers of different radii were used for different runs to ensure the angular distributions observed were not due to self-scattering effects in the sample.

The time calibration and determination of the relative efficiencies of the detection systems were obtained in the same manner as previously reported.^{14,15} The relative ef-

iciencies were obtained using a thin (250 μm) titanium radiator, assuming the distribution of the bremsstrahlung produced is that of Schiff,¹⁶ and by using the calculated deuteron photodisintegration cross section.^{17,18}

The reduction of data from the series of photoneutron time-of-flight spectra to yield a total ground-state cross section along with anisotropy coefficients was performed by means of the following steps.

Initial corrections for dead time and natural and container backgrounds were performed on the time-of-flight spectra before they were converted into neutron energy spectra in the center-of-mass system. The neutron energy spectra were corrected for detector efficiency and converted to the excitation energy scale.

Third order Legendre polynomials were fitted to the energy spectra and the coefficients a_i ($i=1$ to 3) in the following expression:

$$\text{yield}(E_x) = \sum_{i=0}^3 A_i P_i(\cos\theta), \quad (1)$$

and $a_i = A_i / A_0$ were extracted as functions of excitation energy using a weighted least-squares technique.¹⁹ The total cross section was obtained by dividing $4\pi A_0$ by the bremsstrahlung distribution scaled to the relative number of nuclei of ^{16}O and ^2H exposed to the photon beam. The scale of the cross section was determined relative to the deuterium cross section, as calculated by Arenhövel *et al.*¹⁷ for E_γ less than 10 MeV and by Partovi¹⁸ for E_γ greater than 10 MeV.

III. RESULTS

The total ground-state cross section is shown in Fig. 1 together with the normalized angular distribution coefficients. The present results were obtained from two independent measurements using an end-point energy of 36 MeV, taken 16 months apart. Results are shown for Legendre fits up to third order. An attempt was made to fit the experimental data to a fourth order Legendre polynomial, but for several energies the cross section extrapolated to 0° became negative. Also, no improvement in the reduced- χ^2 values was observed when the order of the fit was extended to include a_4 . It was concluded that, although the data had very little statistical error (typically about 60000 counts were collected for each data point), there could have been small unobserved systematic errors contributing to the (sensitive) a_4 coefficient. The a_i ($i=1$ to 4) coefficients measured in Ref. 7, on the other hand, do not produce negative cross sections at any energies.

In general, the angular distribution coefficients a_i vary smoothly with energy. The value of a_1 rises from close to zero at 30 MeV to about $+0.50 \pm 0.02$ at 35 MeV; the a_2 values averaged to be about -0.65 ± 0.02 in this energy region; and over the same energy region, the value of a_3 remains approximately constant at -0.20 ± 0.02 . The nonzero values of a_1 and a_3 between 30 and 35 MeV are indicative of multipole absorption other than $E1$. Because $M1-E1$ interference cannot generate a finite a_3 ratio, the observed nonzero value of a_3 is an indication of the presence of $E2$ (or higher) multipole absorption.

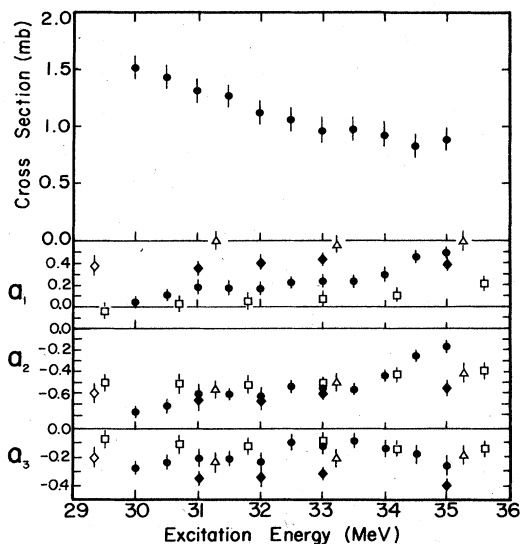


FIG. 1. The angle-integrated cross section of the $^{16}\text{O}(\gamma, n_0)^{16}\text{O}$ reaction is shown along with the normalized angular distribution coefficients a_i ($i=1,2,3$) as functions of excitation energy (solid circles). The scale of the cross section was determined relative to that of the deuteron. Also shown are the angular distribution coefficients obtained in the photoneutron measurement of Ref. 7 (open squares) and those from the photoproton work of Ref. 8 (open diamonds), Ref. 9 (open triangles), and Ref. 20 (solid diamonds).

IV. DISCUSSION

Excellent agreement is found in the total ground-state photoneutron cross section between the present results and those of Ref. 7. They are shown in Fig. 2 together with the ground-state photoproton data of O'Connell and Hanna,⁸ Paul *et al.*,⁹ and the recent work of Anghinolfi *et al.*²⁰ The photoproton measurements of Refs. 9 and 20 are seen to be within equally good agreement with one another. On the average, the photoproton cross section appears to be about one-third lower in magnitude than the pho-

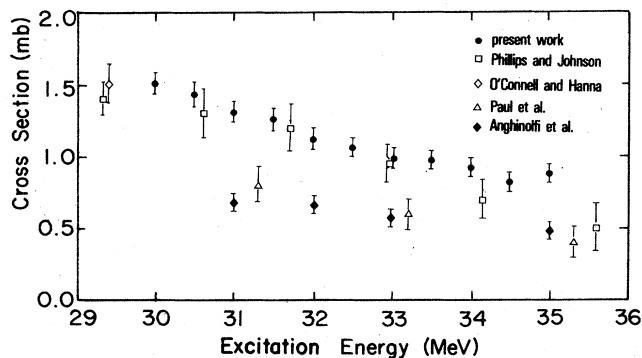


FIG. 2. The present ground-state cross section (solid circles) is compared with the ground-state photoneutron work of Phillips and Johnson (open squares; Ref. 7), and ground-state photoproton cross section deduced from the work of O'Connell and Hanna (open diamond; Ref. 8), Paul *et al.* (open triangles; Ref. 9), and Anghinolfi *et al.* (solid diamonds; Ref. 20).

toneutron cross section, but it exhibits a similar variation with energy.

Figure 1 presents a comparison of the present measurements for the Legendre coefficients a_i ($i=1,2,3$) with the coefficients obtained in Refs. 7–9 and 20. For the a_1 values, the present data lie between the low values ($a_1 \approx 0.1$) of the photoneutron work⁷ and the higher values (0.4 to 0.6) of the photoproton measurements.^{8,9,20} The a_2 values for all sets of results are in reasonable agreement. The present a_3 values agree reasonably well with the previous photoneutron results⁷ but disagree with the photoproton results^{8,9,20} at several energies. This comparison is not totally valid because the present work and the results of Refs. 8 and 9 have employed third order Legendre fits (assuming $a_4=0$), whereas the results of Refs. 7 and 20 have come about using a fourth order fit.

In analyzing the angular distributions of the present experimental results, several assumptions similar to those of Ref. 7 were used. Since no neutron polarization data are available between 30 and 35 MeV, assumptions based on existing neutron polarization and proton polarization measurements at lower energies^{21,22} were made for the transition amplitude ratio of s - to d -wave neutrons (s/d ratio) and (at several energies) the relative phase difference between the s - and d -wave neutrons.

The s/d ratio as obtained in Ref. 22 (solution I) was extrapolated from 27 MeV and assumed to be constant at 0.75 ± 0.05 in the energy region of 30–35 MeV. The neutron polarization measurement of Hanser²¹ found that the phase difference, at lower energies, between the s - and d -wave neutrons is $(-118 \pm 5)^\circ$. The proton polarization work of Ref. 22 yielded a phase difference of $(-120 \pm 5)^\circ$ in the same energy region. We have therefore assumed that the phase difference is $(-120 \pm 5)^\circ$ and that it is approximately constant in the energy region of 30 to 32 MeV. Above 32 MeV, it was possible to extract from the measured a_1 , a_2 , and a_3 coefficients values for this relative phase difference as described in the following. The fact that values of about -120° were obtained (for example at 33 MeV) suggests the validity of this extrapolation.

The Legendre coefficients can be expressed in terms of single-particle matrix elements and relative phase differences. This formulation is readily available in the literature²³ and is often reduced to expressions involving only $E1$ and $E2$ amplitudes for this range of excitation energies (30–35 MeV) and momentum transfer ($q \approx 0$). For the ground state photoneutron reaction in ^{16}O , the $E1$ matrix elements can be represented by s and d and the $E2$ matrix elements by p and f . Relative phases can be represented by δ_{sd} , δ_{pf} , etc.

Table I presents a model-dependent description of the A_i ($i=0$ to 3) coefficients under the assumption that f , one of the $E2$ matrix elements, is small with respect to the other $E2$ matrix element. Because the a_4 coefficient depends only on terms of first or higher order in f , this assumption implies a value of $a_4 \approx 0$. This is reasonable in consideration of the low nuclear penetrability of f -wave neutrons relative to p -wave neutrons. The f -wave "centrifugal barrier" in this nucleus is about 19 MeV. The maximum neutron energy in this experiment was about 18 MeV. The analysis model we are employing allows $E2$

TABLE I. Simplified expression for the angular distribution coefficients for ^{16}O after setting the f -wave amplitude to zero. The matrix elements are represented by the symbols s , p , and d for simplicity, and for example, the relative phase shift between emitted d -wave neutrons (following $E1$ transition) and p -wave neutrons ($E2$ transitions) is represented by δ_{dp} .

$A_0 = 3s^2 + 3d^2 + 5p^2$
$A_1 = -1.34dp \cos\delta_{dp}$ $+ 9.49sp \cos\delta_{sp}$
$A_2 = -1.5d^2 + 2.5p^2$ $+ 4.24sd \cos\delta_{sd}$
$A_3 = +8.05dp \cos\delta_{dp}$

absorption to proceed primarily via the emission of p -wave neutrons. This implies that neutron penetrabilities act to reduce the $E2$ matrix element for the single-particle $p \rightarrow f$ transitions, even though in the absence of the centrifugal barrier these transitions might play a significant role in the reaction. This model is supported by the results of the recent $^{15}\text{N}(p, \gamma_0)^{16}\text{O}$ measurement of Anghinolfi *et al.*²⁰ They found the a_4 values for the (p, γ_0) channel to be zero in this energy region.

Given these assumptions and expressing the total ground-state cross section as

$$\sigma(\text{g.s.})_{\text{tot}} = \sigma_s + \sigma_d + \sigma_p, \quad (2)$$

and the ratio of $E2$ to $E1$ absorption as

$$\frac{\sigma(E2)}{\sigma(E1)} \simeq (p/d)^2, \quad (3)$$

we can derive²⁴ an expression for the $E2$ (p -wave) cross section in terms of the (p/d) ratio. The $E2$ (p -wave) cross section is approximated by

$$\sigma(E2)_p = \frac{\sigma_0(p/d)^2}{0.94 + (p/d)^2}, \quad (4)$$

where σ_0 is the total ground-state cross section and the transition amplitude ratio (p/d) and relative phases δ_{sd} and δ_{pd} can be obtained from the experimental a_1 , a_2 , and a_3 coefficients (refer to Table I). Table II shows the extracted values of p/d , the relative phases δ_{sd} and δ_{pd} , and the $E2$ cross section for the energy region 30 to 35 MeV. For energies up to 32 MeV, the δ_{sd} relative phase was assumed to be -120° , which is an *interpolation* from the measured values of Refs. 21 and 22 at 27 MeV (-120°) and the averaged value of δ_{sd} (-121°) found in this experiment at energies of 32.5 to 33.5 MeV.

To explore the validity of the assumption ($f=0$) used to obtain these $E2$ values, a calculation was carried out under a second simplifying assumption. We assumed that the $E2$ f amplitudes were comparable to the p amplitudes and repeated the extraction of p/d ratios, relative phases, and $E2$ cross section based on measured a_1 , a_2 , and a_3 values. The $E2$ cross section values extracted using this model were seen, at all energies, to be somewhat lower than the results for the model with $f=0$. This suggests that if small $E2$ f -wave transitions were present, no enhancement of the $E2$ cross section would result from our analysis of the measured a_1 , a_2 , and a_3 coefficients.

A recent theoretical calculation was performed using a continuum random phase approximation theory with a Skyrme III force by Cavinato *et al.*¹⁰ These authors have calculated the $E2$ ground-state cross section for the ^{16}O photoneutron reaction. The present $E2$ results are shown in Fig. 3 together with this calculation and the measurements reported in Ref. 7, which are significantly larger than the present values at all energies. The calculation appears to be in much better agreement with the present results. It also reproduces approximately the same magnitude and trend in the $E2$ cross section as are observed in the present experiment; that is, an increase in the cross section near 33.5 MeV.

Assuming that the effective charge concept of Bethe¹² (subsequently elaborated by others, see Ref. 13) is correct, then the amount of direct $E2$ strength in the neutron channel should be negligible.²⁵ Therefore, within this

TABLE II. Values of the electric quadrupole cross section, p/d amplitude ratios, and the relative phase differences δ_{sd} and δ_{pd} extracted from the measured anisotropy coefficients. The phase differences have an estimated uncertainty of $\pm 5^\circ$. Integrated cross section: 0.24 ± 0.09 mb MeV; energy-weighted integrated cross section: 0.216 ± 0.09 $\mu\text{b MeV}^{-1}$; percentage of the EWSR₁ isovector sum rule (5.596 $\mu\text{b MeV}^{-1}$): $3.9 \pm 1.6\%$.

Excitation energy (MeV)	p/d amplitude ratio	δ_{sd} (deg)	δ_{pd} (deg)	$\sigma(E2)$ (mb)
30.0	0.0	-120		0.0 ± 0.02
30.5	0.0	-120		0.0 ± 0.02
31.0	0.231	-120	124	0.071 ± 0.02
31.5	0.194	-120	131	0.048 ± 0.02
32.0	0.162	-120	148	0.030 ± 0.03
32.5	0.156	-120	114	0.026 ± 0.01
33.0	0.154	-122	118	0.023 ± 0.01
33.5	0.160	-122	111	0.025 ± 0.01
34.0	0.210	-109	117	0.041 ± 0.01
34.5	0.326	-92	111	0.088 ± 0.02
35.0	0.380	-84	117	0.127 ± 0.02

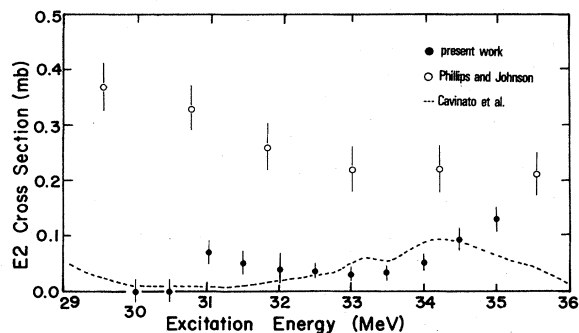


FIG. 3. The present results for the photoneutron $E2$ cross section (solid circles) have been extracted from the measured a_1 , a_2 , and a_3 values in a model where electric quadrupole f amplitudes are assumed to be zero. Also shown are the data of Ref. 7 (open circles) and the RPA-SK III calculation of Cavinato *et al.* (broken line; Ref. 10).

model any $E2$ strength observed is resonant in nature. It can be concluded that there exists an electric quadrupole resonance in the neutron channel even though the present $E2$ cross section results are substantially lower than those previously reported.⁷

The present $E2$ neutron data suggest that the ratio between the direct strength and the resonant strength lies somewhere between a 60% (direct)-40% (resonant) and 70% (direct)-30% (resonant) split. This assumes that all the direct and one-half of the resonant strengths are concentrated in the proton channel, and the other one-half of the resonant strength is found in the neutron channel. The latter ratio of the direct strength to resonant strength will yield a neutron $E2$ cross section of about one fifth of the magnitude of the proton $E2$ cross section which is the case when the results of this experiment are compared to photoproton measurements.^{8,9,20}

From the present experimental results, the energy-weighted integration of the $E2$ cross section between 30 and 35 MeV gives $0.22 \pm 0.09 \mu\text{b MeV}^{-1}$ for the $E2$ ground-state photoneutron channel. This is about 4% of the EWSR_1 (one EWSR_1 for ^{16}O is $5.60 \mu\text{b MeV}^{-1}$ using the EWSR_1 of Ref. 26).

V. CONCLUSIONS

Excellent agreement was found between the present ground-state cross section and a previous (γ, n_0) measurement,⁷ and (γ, p_0) measurements^{8,9,20} were seen to be about one-third lower in magnitude in the energy region studied (30–35 MeV). The angular distributions reported here display nonzero values of the a_1 and a_3 coefficients, indicating the presence of either $E2$ or $M1$ absorption strength (or both) interfering with the $E1$ strength in the neutron channel.

The $E2$ cross section was obtained from the present a_1 , a_2 , and a_3 values [using a model where there is negligible $E2$ (f -wave) neutron emission] and information from previous polarization experiments. In the energy region studied, the present $E2$ results are significantly lower in magnitude than a previous measurement.⁷ We believe that this difference arises in part from fitting the differential cross sections with a different Legendre polynomial order and, in part, from a different interpretation of the measured angular distributions and the single-particle matrix elements describing them.

The recent calculation by Cavinato *et al.*¹⁰ of the $E2$ cross section is in good agreement with the present results over the energy region studied.

The energy-weighted integration of the $E2$ cross section between 30 and 35 MeV gives $0.22 \pm 0.09 \mu\text{b MeV}^{-1}$ for the ground-state neutron channel, exhausting about 4% of the isovector $E2$ energy-weighted sum rule.

To obtain a more accurate and model-independent estimate of the $E2$ photoneutron cross section, polarization measurements are required in energy regions higher than 30 MeV.

ACKNOWLEDGMENTS

The assistance of Mr. Alex Nowak and the staff of the electron linear accelerator laboratory of the National Research Council of Canada was essential during the data-acquisition phase of this experiment and is gratefully acknowledged. This work was supported in part by the Natural Sciences and Engineering Research Council of Canada.

*Present address: Department of Physics, University of Toronto, Toronto, Ontario, Canada M5S 1A7.

¹V. Gillet and H. Vinh Mau, Nucl. Phys. **54**, 321 (1964).

²F. W. K. Firk, Annu. Rev. Nucl. Sci. **20**, 39 (1970).

³Photonuclear Reactions, edited by E. G. Fuller and E. Hayward (Dowden, Hutchinson, and Ross, Stroudsburg, 1967), Vol. 2.

⁴P. C. Nagle, E. J. Winhold, P. E. Yergin, P. Stoler, and S. G. Mecca, Nucl. Phys. **A127**, 669 (1969).

⁵J. W. Jury, J. S. Hewitt, and K. G. McNeill, Can. J. Phys. **48**, 1635 (1970).

⁶S. S. Hanna, H. E. Glavish, R. Avida, J. R. Calarco, E. Kuhlmann, and R. LaCanna, Phys. Rev. Lett. **32**, 114 (1974).

⁷T. W. Phillips and R. G. Johnson, Phys. Rev. C **20**, 1689 (1979).

⁸W. J. O'Connell and S. S. Hanna, Phys. Rev. C **17**, 892 (1978).

⁹J. W. N. Paul, K. A. Snover, M. Suffert, E. K. Warburton, and H. M. Kuan, in *Proceedings of the International Symposium on Highly Excited States in Nuclei, Jülich, 1975*, edited by A. Faessler, C. Mayer-Böricke, and P. Turek (Kernforschungsanlage, Jülich, 1975), Vol. 1, p. 2.

¹⁰M. Cavinato, M. Marangoni, P. L. Ottaviani, and A. M. Saruis, Nucl. Phys. **A373**, 445 (1982).

¹¹M. Potokar, Phys. Lett. **B46**, 346 (1973).

¹²H. A. Bethe, Rev. Mod. Phys. **9**, 69 (1937).

¹³E. Hayward, *Nuclear Structure and Electromagnetic Interactions*, Scottish Universities Summer School 1964, edited by N. MacDonald (Oliver and Boyd, Edinburgh, 1965), pp. 143–145.

¹⁴J. W. Jury, C. K. Ross, and N. K. Sherman, Nucl. Phys.

- A337, 523 (1980).
- ¹⁵J. D. Watson, J. W. Jury, P. C-K. Kuo, N. K. Sherman, W. F. Davidson, and K. G. McNeill, *Phys. Rev. C* **27**, 506 (1983); J. D. Watson, M. S. thesis, Trent University, 1983.
- ¹⁶L. I. Schiff, *Phys. Rev.* **83**, 252 (1951).
- ¹⁷H. Arenhövel, W. Fabian, and H. G. Miller, *Phys. Lett.* **52B**, 303 (1974).
- ¹⁸F. Partovi, *Ann. Phys. (N. Y.)* **27**, 29 (1964).
- ¹⁹P. R. Bevington, *Data Reduction and Error Analysis for the Physical Sciences* (McGraw-Hill, New York, 1969), pp. 148–160.
- ²⁰M. Anghinolfi, P. Corvisiero, M. Taiuti, and A. Zucchiatti, *Phys. Rev. C* **28**, 1005 (1983).
- ²¹F. A. Hanser, Ph.D. thesis, Massachusetts Institute of Technology, 1967.
- ²²S. S. Hanna and W. E. Meyerhof, Progress Report, Stanford University, 1975.
- ²³R. W. Carr and J. E. E. Baglin, *Nucl. Data Tables* **10**, 143 (1971).
- ²⁴P. C-K. Kuo, M. S. thesis, Trent University, 1983.
- ²⁵E. D. Arthur, D. M. Drake, and I. Halpern, *Phys. Rev. Lett.* **35**, 914 (1974).
- ²⁶S. S. Hanna, in *International Conference on Nuclear Physics with Electromagnetic Interactions*, 1975, edited by H. Arenhövel and D. Drechsel (Springer, Berlin, 1975), p. 288.