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Comparison of different collective models describing the low spin structure of ^{168}Er

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The ability of several collective models to reproduce the low-energy spectrum of ^{168}Er is investigated. It is found that the interacting boson approximation, the general collective model, and the Gneuss-Greiner model all describe the data with comparable quality, while the rotation-vibration model appears to be too restrictive.

The low-energy spectrum of ^{168}Er has recently been the object of intensive discussions,[1-3] because the geometrical collective models[4-7] appeared to give different results from the interacting-boson approximation (IBA).[8] On the other hand, it was shown by several authors that the IBA and the geometrical models can be related mathematically.[9-11] The IBA Hamiltonian can be expressed in terms of the Bohr and Mottelson variables,[12] although the IBA still has the additional condition of maximal number of d bosons. For low spins this should not play an important role as long as the total number of s and d bosons is sufficiently large. For high-spin states the pure s-d-boson description fails[13] and other effects, such as g bosons, have to be taken into account. In the case of ^{168}Er the number of bosons in the IBA is 16, quite a large number, so that the low-spin states should be reproducible with both the IBA and the geometrical models with about equal quality (even in the Gneuss-Greiner model,[6] usually only about 30 bosons are used). In that sense the results of Refs. 2 and 3 are surprising.

In the present work we investigated ^{168}Er in the framework of the general collective model (GCM),[7] the Gneuss-Greiner model (GGM),[6] the rotation-vibration model (RVM),[5] and the IBA.[3,8] The more flexible geometric models like the GCM and GGM contain a number of specialized models as limiting cases, e.g., the five-dimensional harmonic oscillator, the RVM, and the triaxial rotor.[14] The experimental data are taken from the impressive work of Casten and co-workers,[2,3] giving complete information about the low-lying spin structure. The error bars in the $B(E2)$ transitions are obtainable from Ref. 1. In Ref. 3 it was shown that the IBA can reproduce the data excellently, while specialized geometric models[4,15] did not fare well. We want to confront the results of different geometric models with the IBA and with experiment, showing that there is no practical difference between the models, although their mathematical foundation is somewhat different and the IBA turned out to be easier to use in some applications. Principal differences between the IBA and a geometrical model as found in Ref. 3 seem to be caused by the choice of a strongly restricted parametrization of the geometrical model.

For the geometric models[6,7] the parameters were determined by least-squares fitting, i.e., such that the weighted sum of squares of the difference between theoretical and experimental values was minimized. One noticeable drawback of this method occurs in the fitting of rotational bands: if the band head does not occur at the right position, the fit will stretch or shrink the band in order to reproduce the other members of the band more closely, instead of reproducing the correct level spacing. For ^{168}Er the effect was not too strong, however.

The data utilized in the fit of the RVM were the excitation energies of the first and second 2^+ states, the first excited 0^+ state, and the $0_g^+ \rightarrow 2_g^+ B(E2)$ value. For the GGM the energy levels of the ground state band (up to 6^+), $2\gamma^+$ band (up to 6^+), 0_β^+ band (up to 4^+), and $0_{\beta'}^+$ band (up to 2^+) as well as the $B(E2)$ ratios of Table I were used; for the GCM, the energy of the 2_γ^+ state was added, but only the $B(E2)$ values for the $0_g^+ \rightarrow 2_g^+$ and $0_g^+ \rightarrow 2_\gamma^+$ were fitted. Note that in the geometric models the $B(E2)$ operator contains no additional parameters, so that the $B(E2)$ values had to be adjusted purely via the Hamiltonian. The optimum fit reached in a least-squares procedure depends, of course, on the weights assigned to the different pieces of data (this problem is discussed in Ref. 16), and careful judgment is necessary for selecting the fit that appears best to represent the data. The calculations in the GCM and GGM were done in a basis of up to 30 phonon states of the five-dimensional harmonic oscillator.[17] Owing to code limitations, only five states could be calculated for each angular momentum in the GCM. The RVM was diagonalized in a basis of 13 states, as described in Ref. 5. The several parameters used in this paper are listed in Table II.

In Fig. 1 the experimental and theoretical spectra are plotted. In spite of the slight stretching caused by the fitting method for the GGM and GCM (as discussed above), the overall fit quality is as good as in the IBA. There appears an additional 4^+ band (in parentheses) not observed in experiment, but also obtained in the IBA (this band was omitted from the figures of Ref. 3—as is mentioned in that reference). The band head is at 1619 keV in the IBA and at 1380, 1464, and 1647 keV in the GGM, GCM, and RVM, respectively, so that it appears in the same energy region in all models. The $K = 0_3^+$ band is interpreted as a two-phonon γ band in these models and is quite well reproduced in energy. Only the RVM yields too high an energy of

TABLE I. $B(E2)$ ratios in ^{168}Er. Theory (Refs. 5–8), compared to experiment. RVM = rotation-vibration model (Ref. 5); GG = Gneuss-Greiner model (Ref. 6); GCM = general collective model (Ref. 7); IBA = interacting-boson approximation (Ref. 3); Expt. = experiment; δ is the average deviation.

Transition ratio	Expt.	IBA (cons. Q) (Ref. 2)	IBA	GG	GCM	RVM
$\frac{2_\gamma^+ \to 0_g^+}{2_\gamma^+ \to 2_g^+}$	0.54	0.54	0.66	0.265	0.209	0.322
$\frac{2_\gamma^+ \to 4_g^+}{2_\gamma^+ \to 2_g^+}$	0.068	0.076	0.06	0.054	0.023	0.072
$\frac{3_\gamma^+ \to 2_g^+}{3_\gamma^+ \to 2_\gamma^+}$	0.026	0.026	0.027	0.039	0.039	0.033
$\frac{4_\gamma^+ \to 2_g^+}{4_\gamma^+ \to 2_\gamma^+}$	0.016	0.017	0.025	0.005	0.003	0.010
$\frac{5_\gamma^+ \to 4_g^+}{5_\gamma^+ \to 3_\gamma^+}$	0.029	0.035	0.043	0.031	0.029	0.034
$\frac{0_\beta^+ \to 2_g^+}{0_\beta^+ \to 2_\gamma^+}$	≥0.196		0.055	0.289	0.399	55.941
$\frac{4_\beta^+ \to 2_g^+}{4_\beta^+ \to 2_\beta^+}$	0.0002		0.0009	0.009	0.013	0.021
$\frac{4_\beta^+ \to 5_\gamma^+}{4_\beta^+ \to 2_\beta^+}$	0.0019		0.028	0.199	0.239	0.004
$\frac{6_\beta^+ \to 4_\gamma^+}{6_\beta^+ \to 4_\beta^+}$	0.0011		0.0009	0.003	0.003	0.002
$\frac{2_{\beta'}^+ \to 0_g^+}{2_{\beta'}^+ \to 0_{\beta'}^+}$	0.0009		0.000 06	0.011	0.007	0.000 07
$\frac{2_{\beta'}^+ \to 4_g^+}{2_{\beta'}^+ \to 0_{\beta'}^+}$	0.01		0.0002	0.056	0.041	0.0002
δ			0.3719	0.6511	0.7771	63.007

1.586 MeV, which indicates that the restriction to a harmonic potential in γ makes the γ vibrations too stiff.

Table I lists some important $B(E2)$ ratios in comparison to experiment. Two main properties were stressed by Warner *et al.*,[3] namely, that the $\beta\to\gamma$ transitions dominate over the $\beta\to g$ ones, and that the $\gamma\to g$ transitions are stronger than the $\beta\to g$ ones. This was seen as a confirmation of the IBA model, because the geometrical models did not appear to reproduce this transition structure. Examining the results in Table I, we see that the geometrical models are able to reproduce the data with about equal average quality as the IBA. For example, the ratio of theoretical

TABLE II. Number and values of parameters. All possible additional parameters are zero and are not included in the fit.

Five parameters of IBA	(see Ref. 3)	12 parameters of GCM	$V_{20}=-41.82$ MeV $V_{30}=1.33$ MeV $V_{40}=129.96$ Mev $V_{60}=61.06$ MeV $V_{21}=-1.147$ MeV $V_{51}=127.06$ MeV $V_{22}=2.890$ MeV $V_{32}=3.331$ MeV $V_{62}=-5.822$ MeV $V_{23}=-8.731$ MeV $B_2=46.19\times10^{-42}$ MeV sec^2 $P_3=0.0119\times10^{+42}$ MeV^{-1} sec^{-2}
Four parameters of RVM	$E_{\rm rot}=3\epsilon=0.068$ MeV $E_\beta=1.217$ MeV $E_\gamma=0.793$ MeV $\beta_0=0.337$		
Eight parameters of GG	$B_2=93.52\times10^{-42}$ MeV sec^2 $P_3=-0.021\,28\times10^{42}$ MeV^{-1} sec^{-2} $C_2=-171.98$ MeV $C_3=390.27$ MeV $C_4=2167.45$ MeV $C_s=-5067.32$ MeV $C_6=-577.64$ MeV $D_6=-6.775$ MeV		

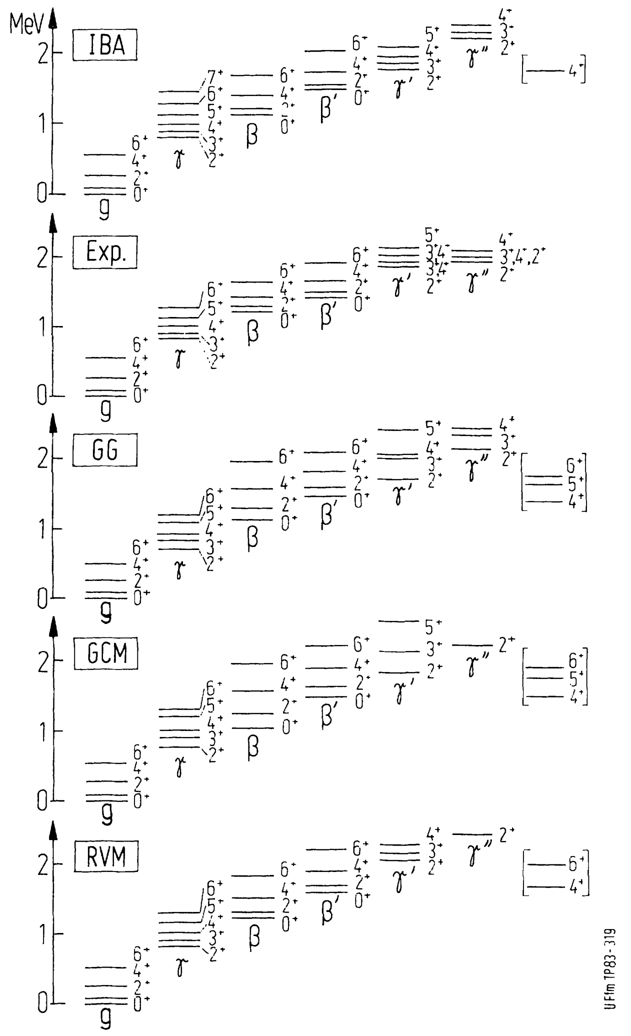

FIG. 1. Comparison of low energy spectra of ^{168}Er with experiment, calculated in terms of the interacting-boson approximation (IBA), the Gneuss-Greiner model (GG), the general collective model (GCM), and the rotation-vibration model (RVM).

and experimental values of some $B(E2)$ ratios are given for the IBA, GG, GCM, and RVM, respectively, as

$$\left(\frac{0_\beta^+ \rightarrow 2_g^+}{0_\beta^+ \rightarrow 2_\gamma^+}\right)_{\text{theor}} \bigg/ \left(\frac{0_\beta^+ \rightarrow 2_g^+}{0_\beta^+ \rightarrow 2_\gamma^+}\right)_{\text{expt}} = 0.28, 1.47, 2.04, 285.41 \ ,$$

$$\left(\frac{4_\beta^+ \rightarrow 5_\gamma^+}{4_\beta^+ \rightarrow 2_\gamma^+}\right)_{\text{theor}} \bigg/ \left(\frac{4_\beta^+ \rightarrow 5_\gamma^+}{4_\beta^+ \rightarrow 2_\gamma^+}\right)_{\text{expt}} = 14.7, 105, 126, 2.105,$$

$$\left(\frac{4_\beta^+ \rightarrow 2_g^+}{4_\beta^+ \rightarrow 2_\beta^+}\right)_{\text{theor}} \bigg/ \left(\frac{4_\beta^+ \rightarrow 2_g^+}{4_\beta^+ \rightarrow 2_\beta^+}\right)_{\text{expt}} = 4.5, 45, 65, 105 \ ,$$

$$\left(\frac{2_\beta^+ \rightarrow 0_\beta^+}{2_{\beta'}^+ \rightarrow 4_g^+}\right)_{\text{theor}} \bigg/ \left(\frac{2_{\beta'}^+ \rightarrow 0_\beta^+}{2_{\beta'}^+ \rightarrow 4_g^+}\right)_{\text{expt}} = 50, 0.18, 0.24, 50 \ .$$

This demonstrates that for some data the IBA agrees better with experiment than the geometrical models and vice versa. The deviations between IBA and each of the geometrical models are not decisively bigger than those between the geometrical models themselves. A suitable measure of the quality may be the average deviation δ

$$\delta = \frac{\sum_i |A_{\text{th}} - A_{\text{exp}\,i}|}{\sum_i A_{\text{exp}\,i}} \ ,$$

given in Table I.

The values of the branching ratio $2_\gamma^+ \rightarrow 2_g^+/2_\beta^+ \rightarrow 2_g^+$ of 7.261, 5.553, 2.064 for the GG, GCM, and RVM, respectively, indicate the dominance of the $\gamma \rightarrow g$ transitions. The notable exception is the RVM, which does not yield the dominance of $\beta \rightarrow g$, indicating that this model is too restricted in the parametrization of the potential-energy surface.

As we have emphasized in the introduction, the connec-

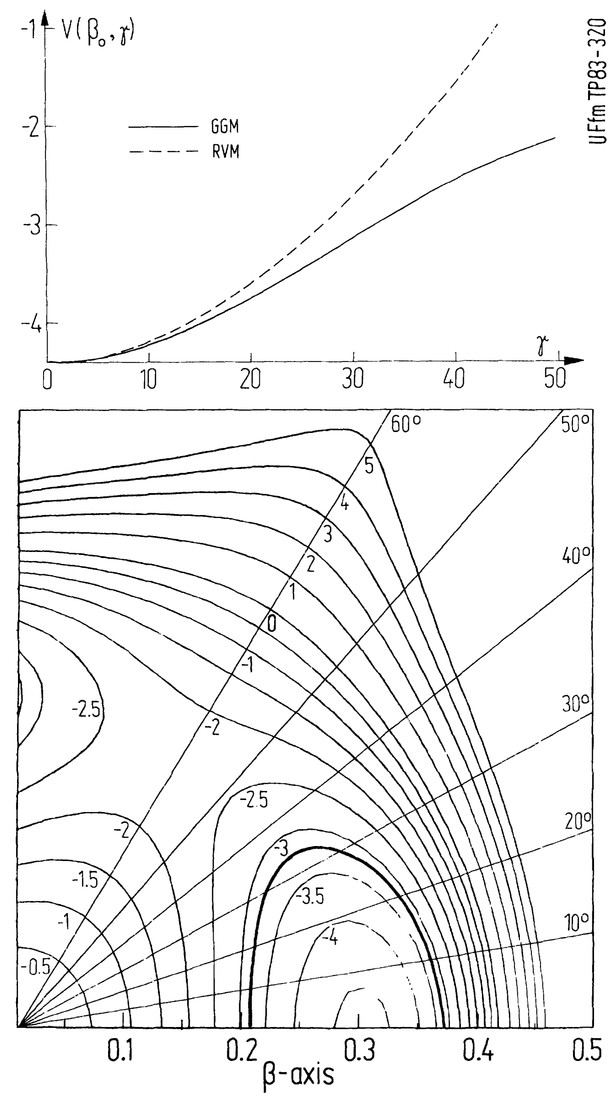

FIG. 2. Potential energy surface of ^{168}Er resulting from the Gneuss-Greiner model (GGM). In the upper part the dependence on γ is shown and compared to the rotation-vibration model (RVM). The lower part shows the potential as a contour plot dependent on β and γ.

tion between the IBA and the geometric models established in Refs. 9 and 10 should imply that both types of models can describe low-spin states equally well, as long as the total number of bosons allowed in the IBA is not too small. Since in practice the GCM and GGM Hamiltonians are also diagonalized in a space of finite boson number, it is easy to test this dependence and it was found[6,7] that except for very strongly deformed nuclei the low-spin states did not change appreciably for boson numbers changing from 15 to 30.

In this Rapid Communication it was shown that indeed the general collective description[6,7] of collective excitations gives similar results of the IBA model.[8] Even discrepancies from experiment are similar, most notably the additional 4^+ band not observed experimentally. It is only the restriction to quite specialized models such as the RVM, which is really applicable only to relatively stiff rotational nuclei, that produces serious discrepancies. For ^{168}Er this is clearly visible in Fig. 2, where the potential energy surface in the GGM (the GCM gives a very similar behavior) is plotted as a function of γ at equilibrium deformation. In the RVM the potential is purely harmonic and, as indicated in the figure, differs drastically from the softer γ dependence obtained in the GGM.

It is interesting to see that the RVM as a representation of a collective model with strongly limited parametrization of the potential-energy surface fails in the branching ratio $0_\beta^+ \rightarrow 2_g^+/0_\beta^+ \rightarrow 2_\gamma^+$, whereas all other branching ratios are in reasonable agreement with experiment, the IBA, and more refined collective models. Notable is the fact that the GCM yields less accurate agreement with experiment than the GGM model, although the GCM has the more flexible parametrization containing the GM as a special case. This apparent contradiction is caused by problems to fit the many model parameters.[16] Finding a set of parameters yielding reasonable agreement with the experimental data is the more difficult and time consuming, the more parameters are to be determined. Practically the quality of the fits decreases with the number of parameters. This problem is quite general and occurs in all models dealing with many adjustable parameters. Moreover, the $K=2_1^+$ band is interpreted in all of these models as a collective band because of the measured strong $B(E2)$ transition $0_1^+ \rightarrow 2_2^+$ of $0.13e^2\text{b}^2$.

Regarding the $K=0_2^+$ and 0_3^+ bands, Bohr and Mottelson[4] doubted their collectivity because of the weak transitions of these bands to the g band of only a fraction of a Weisskopf unit. But this does not necessarily imply that such bands must be of microscopic origin. The interference of the basis wave functions can result in very small collective transition probabilities also. We find a $B(E2;0_1^+ \rightarrow 2_3^+)$ of $0.0435e^2\text{b}^2$, $0.111e^2\text{b}^2$, a $B(E2;0_1^+ \rightarrow 2_4^+)$ of $0.0323e^2\text{b}^2$, $0.035e^2\text{b}^2$, for the GGM and GCM, respectively, which gives the right trend. The RVM with $B(E2;0_1^+ \rightarrow 2_3^+)=0.208e^2\text{b}^2$ and $B(E2);0_1^+ \rightarrow 2_4^+)=0.0005e^2\text{b}^2$ deviates from these more refined collective models.

However, as discussed previously, the numerical application of the more general collective models is quite complicated, and for cases such as ^{168}Er and IBA provides a useful and easy tool for interpretation of the spectra. Although there is no direct geometric interpretation of the results, the corresponding geometric picture can be obtained by a translation as given in Refs. 9–11.

We hope that this work clarifies the discussion on the IBA and geometric models by showing that in practical applications the flexibility of the specific parametrization and not the type of model used determines the quality of the agreement with experiment.

[1] W. F. Davidson, D. D. Warner, R. F. Casten, K. Schreckenbach, H. G. Börner, J. Simić, M. Stojanović, M. Bogdanović, S. Koicki, W. Gelletly, G. B. Orr, and M. L. Stelts; J. Phys. G **7**, 455 (1981); **7**, 843 (1981).

[2] R. F. Casten, in *Proceedings of the 1982 Institute for Nuclear Study International Symposium on Dynamics of Nuclear Collective Motion* (Institute for Nuclear Study, Tokyo 1982), p. 431.

[3] D. D. Warner, R. F. Casten, and W. F. Davidson, Phys. Rev. C **24**, 1713 (1981).

[4] A. Bohr and B. R. Mottelson, Phys. Scr. **25**, 28 (1982).

[5] A. Faessler and W. Greiner, Z. Phys. **168**, 425 (1962); **170**, 105 (1962); **170**, 105 (1962); **177**, 190 (1964); A. Faessler, W. Greiner, and R. K. Sheline, Nucl. Phys. **80**, 417 (1965).

[6] G. Gneuss, Ph.D. thesis, University Frankfurt/Main, West Germany, 1970; G. Gneuss and W. Greiner, Nucl. Phys. **A171**, 449 (1971).

[7] P. O. Hess, Ph.D. thesis, University Frankfurt/Main, West Germany, 1980; P. O. Hess, M. Seiwert, J. A. Maruhn, and W. Greiner, Z. Phys. A **296**, 147 (1980); P. O. Hess, J. A. Maruhn, and W. Greiner, J. Phys. G **7**, 737 (1981).

[8] A. Arima and F. Iachello, Phys. Rev. Lett. **35**, 1069 (1975); Ann. Phys. **99**, 253 (1976); **111**, 201 (1978).

[9] M. Moshinsky, Nucl. Phys. **A338**, 156 (1980).

[10] J. N. Ginocchio and W. Kirson, Phys. Rev. Lett. **44**, 1744 (1980); Nucl. Phys. **A350**, 31 (1980).

[11] O. Castaños, A. Frank, P. O. Hess, and M. Moshinsky, Phys. Rev. C **24**, 1367 (1981).

[12] A. Bohr and B. R. Mottelson, K. Dan. Vidensk. Selsk. Mat. Fys. Medd. **27** (1953); *Nuclear Structure* (Benjamin, London-Amsterdam, 1969, 1975), Vols. I and II.

[13] H. Ower, Ph.D. thesis, University of Frankfurt, West Germany, 1980; H. Ower, Th. W. Elze, J. Idzko, K. Stelzer, E. Grosse, H. Emling, P. Fuchs, D. Schwalm, and H. J. Wollersheim, Nucl. Phys. **A388**, 421 (1982).

[14] A. S. Davidov and B. F. Filippov, Nucl. Phys. **8**, 237 (1958). A. S. Davidov and V. S. Rostovsky, *ibid.* **12**, 58 (1959); A. S. Davidov and A. A. Chabon; *ibid.* **20**, 499 (1960).

[15] L. L. Riedinger, Ph.D. thesis, Vanderbilt University, 1969; L. L. Riedinger, N. R. Johnson, and J. H. Hamilton, Phys. Rev. **179**, 1214 (1969).

[16] P. O. Hess, J. A. Maruhn, and W. Greiner, in *Future Directions in Studies of Nuclei Far from Stability,* edited by J. H. Hamilton *et al.* (North-Holland, Amsterdam, 1980), p. 151.

[17] E. Chacón, M. Moshinsky, and R. T. Sharp, J. Math. Phys. **17**, 668 (1976); E. Chacon and M. Moshinsky, *ibid.* **18**, 870 (1977).