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Electrodisintegration and electrocapture in primordial nucleosynthesis

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Standard big-bang nucleosynthesis calculations do not take into account nuclear electrodisintegration and its inverse, electrocapture. This note reports a calculation of the rates of the most important of these processes, $e^- + {}^2H \rightleftharpoons e^- + n + p$. At a temperature of 10^9 K, characteristic of the most rapid nucleosynthesis, these electronuclear reactions proceed at about 10^{-5} the rate of the corresponding photonuclear reactions, $\gamma + {}^2H \rightleftharpoons n + p$.

The light-element abundances predicted by big-bang nucleosynthesis calculations provide significant constraints on models in both cosmology and particle physics.[1] Accordingly, the reaction rates on which these predictions are based deserve close scrutiny. Within the past two years, there have been at least three studies of finite-temperature radiative corrections to the most important weak-interaction rates.[2] However, I have not found any mention in the literature of the effect on primordial nucleosynthesis of two mundane electromagnetic processes which are omitted from standard calculations: electrodisintegration and its inverse, electrocapture. In this Brief Report, I show that, for the conditions prevailing during the nucleosynthesis era of the big bang, corrections arising from inclusion of these processes are negligibly small, even compared to the subtle details considered in Ref. 2.

While there is an electronuclear process corresponding to each photonuclear process included in the standard primordial nucleosynthesis reaction network, I consider only the electrodisintegration of deuterium ($e^- + {}^2H \rightarrow e^- + n + p$) and its inverse, neutron-proton electrocapture ($e^- + n + p \rightarrow e^- + {}^2H$). The corresponding photonuclear reactions ($\gamma + {}^2H \rightleftharpoons n + p$) are by far the most important, as they control the "deuterium bottleneck." Moreover, the results presented below show that it is unnecessary to pursue corrections due to electrodisintegration and electrocapture any further.

To begin we need the total cross section for electrodisintegration of deuterium as a function of incident electron energy. This has been calculated by Skopik, Murphy, and Shin,[3] who checked that their results agreed with cross sections extracted from experiment. I found that the cross section shown in Fig. 1 of Ref. 3 could be parametrized as

$$\sigma_D(E) = \sigma_0[(E - E_0)/E_1]^{1/2} \ , \tag{1}$$

where E is the incident electron energy, $E_0 = 2.225$ MeV is the threshold energy, $E_1 = 17.775$ MeV, and $\sigma_0 = 56\ \mu$b. Over the energy range considered in Ref. 3, threshold to 120 MeV, this parametrization deviates by no more than 3% from the curve to which it was fitted.

The inverse mean lifetime of a deuteron against electrodisintegration is, in the notation of Wagoner, Fowler, and Hoyle,[4]

$$\lambda_e(D) = n_{e^-} \langle \sigma_D v \rangle \ , \tag{2}$$

where n_{e^-} is the number density of electrons, v is the electron velocity,[5] and the angular brackets denote a thermal average. Practically all primordial nucleonsynthesis occurs within the temperature range $0.3 < T_9 < 3$, where T_9 is the temperature in units of 10^9 K.[6] At these temperatures, the electrons are relativistic, but—for the accepted range of matter densities in the standard cosmological model—they are not degenerate.[4] The thermal average in the right-hand side (RHS) of Eq. (2) is thus

$$\langle \sigma_D v \rangle = \frac{\mathcal{N}}{c^2} \int_{E_0 + mc^2}^{\infty} dE \sigma_D(E)(E^2 - m^2c^4)\exp(-\beta E) \ , \tag{3}$$

where[7]

$$\mathcal{N}^{-1} = \frac{1}{c^3} \int_{mc^2}^{\infty} dE\, E(E^2 - m^2c^4)^{1/2} \exp(-\beta E) = (m^2c/\beta)K_2(\beta mc^2) \ . \tag{4}$$

In these expressions, $\beta = 1/kT$ with k Boltzmann's constant; m is the electron mass, and $K_2(z)$ is a modified Bessel function. For the cross section given by Eq. (1), the RHS of Eq. (3) may be evaluated analytically, with the result

$$\langle \sigma_D v \rangle = \left[\frac{\sigma_0 \exp[-\beta(E_0 + mc^2)]}{[2m^2c^3K_2(\beta mc^2)]} \right] (\pi/\beta E_1)^{1/2} \times [3.75\beta^{-2} + 3(E_0 + mc^2)\beta^{-1} + E_0(E_0 + 2mc^2)] \ . \tag{5}$$

Next we evaluate the electron number density n_{e^-}. This may be written as

$$n_{e^-} = n_e^{(0)} + n_{\text{pair}} \ , \tag{6}$$

where $n_e^{(0)}$ is the number density of "background" electrons, one per proton, and n_{pair} is the number density of electrons from electron-positron pairs. For the range of temperatures characteristic of primordial nucleosynthesis, Ref. 4 gives

$$n_e^{(0)} \approx 6.02 \times 10^{23} h T_9^3 \sum_i \frac{X_i}{A_i} Z_i \text{ cm}^{-3} \ , \tag{7}$$

and Fowler and Hoyle[8] give

$$n_{\text{pair}} = (1/\pi^2)(mc/\hbar)^3(\beta mc^2)^{-1} K_2(\beta mc^2) \ . \tag{8}$$

In Eq. (7), h is the baryon density parameter;[9] X_i, A_i, Z_i are the mass fraction, mass number, and atomic number of the ith nucleus. For the accepted range of values of the current mean mass density of the universe,[9] h lies in the range $10^{-6} < h < 10^{-3}$, and the sum on the RHS of Eq. (7) is never very different from unity. During the nucleosynthesis era, when $\beta mc^2 < 1$, we may approximate the RHS of Eq. (8) by[8]

$$n_{\rm pair} \approx 1.5\times 10^{29} T_9^{3/2} \exp(-5.93/T_9)\ {\rm cm}^{-3}\ ,$$

so that

$$n_{\rm e}^{(0)}/n_{\rm pair} \approx 4\times 10^{-6} h T_9^{3/2} \exp(5.93/T_9)\ .$$

Taking as typical values $h=10^{-5}$ and $T_9=1$, we have $n_{\rm e}^{(0)}/n_{\rm pair}\approx 10^{-8}$, ample grounds for neglecting $n_{\rm e}^{(0)}$ in the following.[10] Inserting Eqs. (3), (4), and (8) in Eq. (2), we then have

$$\lambda_{\rm e}(D) = \left[\frac{\sigma_0}{2\hbar^3 c^2 (\pi^3 E_1)^{1/2}}\right](kT)^{3/2} \exp\left[\frac{-(E_0+mc^2)}{kT}\right] \times [E_0(E_0+2mc^2)+3(E_0+mc^2)kT+3.75(kT)^2]\ .$$

In a form convenient for numerical evaluation, this reads

$$\lambda_{\rm E}(D) = 8.51\times 10^{11} T_9^{3/2} \exp(-31.75/T_9) \times [1+0.098T_9+0.004T_9^2]\ {\rm sec}^{-1}\ . \quad (9)$$

By comparison, Wagoner[11] uses an inverse mean lifetime for photodisintegration given by

$$\lambda_\gamma(D) = 2.07\times 10^{14} T_9^{3/2} \exp(-25.82/T_9) \times [1-0.860T_9^{1/2}+0.429T_9]\ {\rm sec}^{-1},\quad 0 < T_9 < 5\ .$$

Thus

$$\lambda_{\rm e}(D)/\lambda_\gamma(D) \approx 4.1\times 10^{-3} \exp(-5.93/T_9) \times \left[\frac{1+0.1T_9+0.004T_9^2}{1-0.9T_9^{1/2}+0.4T_9}\right]\ . \quad (10)$$

At $T_9=1$, when nucleosynthesis is proceeding most rapidly,[6] this gives

$$\lambda_{\rm e}(D)/\lambda_\gamma(D) \approx 2\times 10^{-5}\ . \quad (11)$$

Clearly, electrodisintegration is completely negligible, even compared to the finite-temperature radiative corrections evaluated in Ref. 2, which give cumulative corrections of the order of 1%.

By virtue of detailed balance, no further analysis is needed to assess the relative magnitude of the electrocapture rate: When statistical equilibrium prevails, the ratio of rates of each reaction and its inverse must be the same. It follows immediately, then, that[12]

$$[{\rm pn}]_{\rm e}/[{\rm pn}]_\gamma = \lambda_{\rm e}(D)/\lambda_\gamma(D)\ , \quad (12)$$

where $[{\rm pn}]_{\rm e}$ and $[{\rm pn}]_\gamma$ are the inverse mean lifetimes of neutrons against electrocapture and against radiative capture, respectively.[13]

A simple qualitative argument gives much the same result as Eq. (11): if we write

$$\lambda_{\rm e}(D)/\lambda_\gamma(D) \approx (\sigma_{\rm e}/\sigma_\gamma)(v_{\rm e}/c)(n_{\rm e}/n_\gamma)\ ,$$

and use $\sigma_{\rm e}/\sigma_\gamma \approx \alpha \approx \frac{1}{137}$ (because of the extra electromagnetic vertex involved in electrodisintegration),[14] $v_{\rm e}/c \approx 2kT/mc^2 = 2T_9/5.93$, and

$$n_{\rm e}/n_\gamma \approx n_{\rm pair}/n_\gamma \approx \exp(-5.93/T_9)$$

(two photons per pair), we find, at $T_9=1$,

$$\lambda_{\rm e}(D)/\lambda_\gamma(D) \approx (\tfrac{1}{137})(\tfrac{2}{6})(3\times 10^{-3}) \approx 10^{-5}\ .$$

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[5]Here and elsewhere, I neglect the velocities of nucleons and nuclei compared to those of electrons at the same temperature.

[6]See, e.g., Fig. 3 of Ref. 4.

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[10]By the time T_9 has dropped to 0.3, $n_{\rm e}^{(0)}$ is not negligible compared to $n_{\rm pair}$ for the upper range of values of h. However all densities are so low by this point that nucleosynthesis has essentially ceased.

[11]R. V. Wagoner, Astrophys. J. **18**, Suppl. No. 162, 247 (1969).

[12]Strictly speaking, the radiative capture rate used in Eq. (12) should include the effect of stimulated radiative capture, which is omitted from standard nucleosynthesis calculations. At the temperatures in question, this correction is negligible.

[13]The notation is that of W. A. Fowler, G. R. Caughlan, and B. A. Zimmerman, Annu. Rev. Astron. Astrophys. **5**, 525 (1967).

[14]The qualitative argument underestimates the electrodisintegration cross section by an order of magnitude or more, *except* just above threshold. But that is the most important energy range for this calculation.