^{24}Mg yrast sequence up to 30 MeV excitation

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Highly excited states observed in the ^{12}C(^{16}O,α)^{24}Mg reaction are analyzed within the statistical model. Evidence that the $E^* = 28.21$, 33.34, and 38.22 MeV states correspond to high spin states is found. The ^{24}Mg yrast line can be identified up to the 12^+ state located at 28.21 MeV excitation energy.

I. INTRODUCTION

The determination of the ^{24}Mg yrast line has been the subject of great interest and intense investigation[1-6] in recent years for several reasons:

(1). ^{24}Mg is one of the heaviest nuclei where complete shell model calculations are still possible and where pure microscopic and macroscopic methods can both be applied to the same phenomena (i.e., backbending, etc.) and thus be compared.[4-6]

(2). ^{24}Mg is one of the nuclei whose yrast line is best known when compared to the liquid drop limit.

(3). The ^{24}Mg yrast line is particularly interesting since the process ^{12}C + ^{12}C→^{24}Mg* is one of the best candidates for a yrast limitation of the heavy ion fusion cross section.[7]

(4). *Some* of the gross structures observed in the α spectra from the ^{12}C(^{16}O,α) reaction, up to high bombarding energies [40 MeV $< E(^{16}$O$) < 150$ MeV], can be related to statistical decay of the compound nucleus to clusters of states located near the yrast line ("yrast clusters").[8]

Very little information exists on the yrast lines of medium-light nuclei at high excitation energies. This is mainly the result of the low threshold for particle emission ($\sim 10-14$ MeV) that limits the applicability of γ-γ and particle-γ correlation studies. Therefore, particle spectroscopy, and mainly the use of high spin selective heavy-ion reactions, has been one of the most common spectroscopic tools. This method has already been used in the suggestion of a state at $E^*(^{24}$Mg$)=20.26$ MeV as a possible 10^+ yrast member[2] in connection with the compound nucleus description of the reaction.

A variety of theoretical calculations is available in the literature on ^{24}Mg up to very highly excited high spin states. Shell model calculations, performed by Watt et al.,[4] based on the Preedom and Wildenthal (PW) interaction, predict the ^{24}Mg yrast line up to the $I = 12$ state and indicate that it displays an effective rigid rotor behavior up to this state, even though they found that the band structure does not persist beyond $I = 8$. Mülhans et al.[9] studied the pair field of rotating ^{24}Mg, adopting

for the generating two body force the surface delta interaction. In this case, the equilibrium deformation was taken for each angular momentum from a Strutinsky calculation without pairing and an upbending up the yrast line results for $I = 10-16$. However, when the deformation was kept constant, the upbend was suppressed.

When the Hartree-Fock approximation, with the Skyrme force, is applied to describe the ^{24}Mg ground state band, projecting states of good angular momentum,[10] a clear backbending is observed at $I = 10-12$.

These predictions make the region above $E^* \simeq 20$ MeV of special interest. Low lying states in ^{24}Mg have been systematically studied using the ^{12}C(^{16}O,$\alpha\alpha$) correlation.[11,12] However, the branching ratio for the decay of highly excited ^{24}Mg states $E^*(^{24}$Mg$) > 20$ MeV to the ^{20}Ne (g.s.) are negligible, and the spins of such states measured by means of triple correlation methods are ambiguous.[12] Therefore, in these cases a careful statistical analysis of the reaction mechanism and transition probabilities has to be performed using the Hauser-Feshbach formalism.

In this paper we present results on the investigation of highly excited states observed in the ^{12}C(^{16}O,α)^{24}Mg reaction at $E(^{16}$O$) = 97$ MeV. These transitions are associated with ^{24}Mg high spin states on the basis of compound nucleus model fits of the angular distributions and absolute cross sections taking into account *all* the statistical characteristics of the process.

II. EXPERIMENT

Measurements were made on the University of Rochester MP tandem accelerator. A 94 and 97 MeV ^{16}O beam was used with a natural C target of ~ 40 μg/cm^2. Alpha spectra were recorded using the split-pole magnetic spectrograph and a (ΔE-E) position sensitive proportional counter at the focal plane.

Typical α evaporation spectra measured at $\theta_{\text{lab}} = 13°$, 23°, 33°, and 43° at $E(^{16}$O$) = 97$ MeV and at 3° at $E(^{16}$O$) = 94$ MeV are shown in Fig. 1. In order to enhance the visibility of the discrete transitions to ^{24}Mg
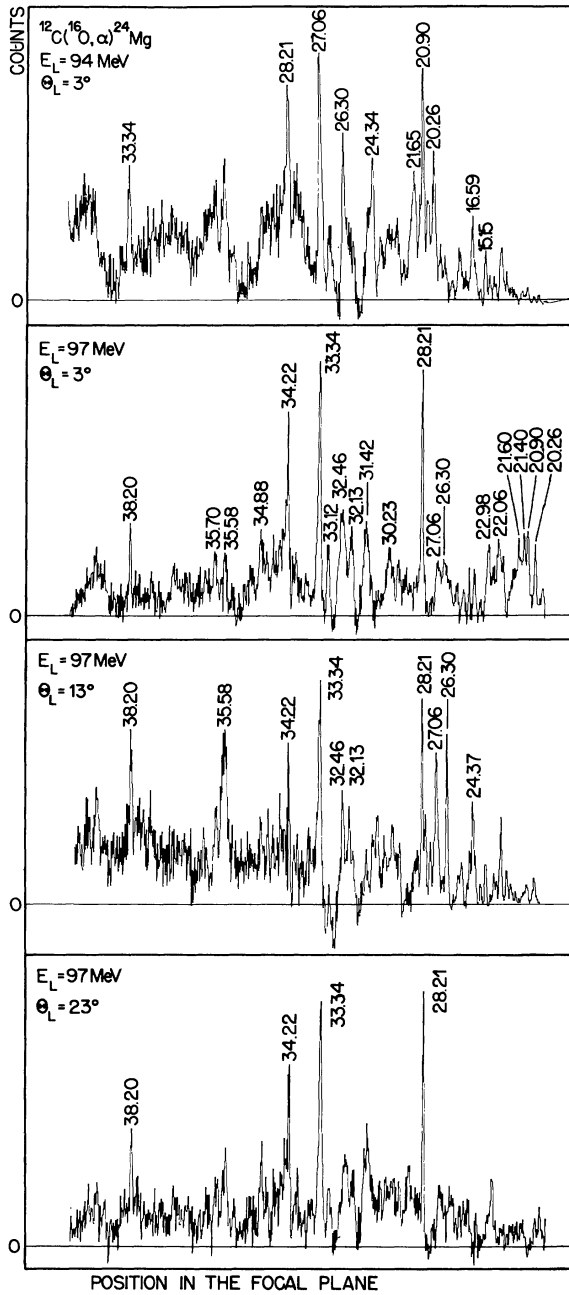
FIG. 1. Background subtracted spectra from the ^{12}C(^{16}O,α)^{24}Mg reaction at $E(^{16}$O)$=94$ and 97 MeV.

final states, the smooth evaporation background has been subtracted (see Fig. 1). The background in these spectra was χ^2 fitted with a fifth order polynomial and has been studied in some detail in Ref. 8.

The overall energy resolution was of the order of 130 keV. The focal plane calibration was performed using the same ^{12}C(^{16}O,α)^{24}Mg reaction at lower bombarding energies $E(^{16}$O)$\simeq$50—65 MeV looking at the well-known low lying transitions. A third order polynomial was used to calibrate the focal plane, and the uncertainties in the excitation functions are estimated to be smaller than 100 keV.

Transitions leading to ^{24}Mg states from $E^*(^{24}$Mg)$=20$ MeV to 38 MeV were then observed in the focal plane.

Differential cross sections of the selectively populated highly excited states were calculated. The absolute cross section scale is accurate to $\sim 20\%$, and possible uncertainties from the background subtractions were also taken into account.

III. RESULTS AND ANALYSIS

In the present paper we focus our attention on the $E^*(^{24}$Mg)$=28.21$, 33.34, 34.22, and 38.20 MeV states which correspond to the transitions which are still selectively populated at backward angles and therefore are yrast state candidates[2] (see Fig. 1).

The background that we have subtracted in these spectra has been studied in considerable detail in Refs. 8 and 13 and is very well described in both magnitude and shape by statistical model calculations for all bombarding energies up to $E(^{16}$O)~ 150 MeV. In particular, at the energies reported in this work, the background is completely consistent with compound nucleus decay, and no significant contributions of other processes such as projectile breakup can be reported.

The absolute differential cross sections obtained in the present work (see Fig. 2) were analyzed on the basis of the Hauser-Feshbach statistical model. Calculations were performed using the code STATIS.[14] Knowing, however, that statistical model predictions are notoriously sensitive to the model parameters, we calibrated them against a very broad range of bombarding energies, angular momentum of final states, and excitation energies.
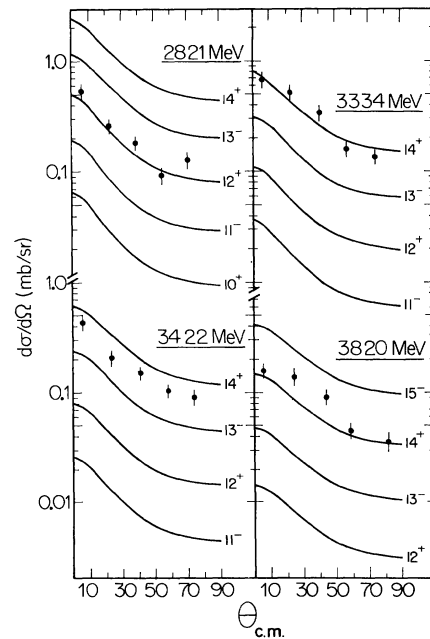

FIG. 2. Experimental angular distributions compared to the calculated differential cross sections (full lines) for different final state angular momenta.

This has been done using the data reported in Ref. 8, consisting of the excitation functions of the $E^*(^{24}\text{Mg})$ =0.0 MeV ($J^\pi=0^+$), 13.21 MeV ($J^\pi=0^+$), 13.21 MeV ($J^\pi=8^+$), 20.26 MeV [$J^\pi=(10^+)$], and 20.9 MeV [$J^\pi=(10^+)$] states from $E(^{16}\text{O})=40$ to 150 MeV. These data provide very severe constraints on the model parameters, guaranteeing that we are sensitive to the entire range of entrance channel partial waves. In order to eliminate the influence of model parameters on the results, we fit the observed cross sections and high spin selectivity at low bombarding energies, where it is accepted that the compound nucleus mechanism dominates the reaction cross section, and then kept them constant for all energies. It can be seen in Ref. 8 that the energy dependences of these selected states are very well described up to $E(^{16}\text{O})=150$ MeV, with reasonable compound nucleus critical angular momenta.[15]

The energy dependences of the exit channel transmission coefficients have a very strong effect on the energy dependence of the high spin selectivity. These parameters were determined by fitting the oscillatory angular distributions of the ground state transition over a broad energy range. These fits have already been presented in Ref. 16, where a complete discussion of this procedure is given. The level density parameters were taken from Ref. 2, where their influence on the statistical model calculation for ^{24}Mg was studied in detail. In particular, the value of $a=A/7.4$ and a spin cutoff parameter appropriate to a rigid ^{24}Mg rotor with $\hbar^2/2\theta=185$ keV were used.

The fact that the fitted data set is so extensive, and the quality of the fit is satisfactory, allows us to compare with some confidence the Hauser-Feshbach calculations to the data obtained in the present work.

It is known that the angular distributions of transitions to lower spin final states in heavy ion compound reactions are very anisotropic (i.e., $1/\sin\theta$) due to the strong alignment of the compound nucleus angular momentum J_{cn} with the orbital angular momentum l_α of the emitted particle. It has also been shown[2] that when higher spin states are populated in the final nucleus, this alignment is lost, giving rise to more isotopic angular distributions. The most isotropic angular distribution is observed when the final state angular momentum (I_f) is comparable to the critical angular momentum (J_c) for the compound nucleus formation. The anisotropies $A=d\sigma(0°)/d\sigma(90°)$ can easily vary from values $A\sim10$ (when $I_f\sim0$) to $A\sim1$ ($I_f\sim J_c$).

The data presented in Fig. 2 display an anisotropy small enough for the states at $E^*(^{24}\text{Mg})=29.21$, 33.34, 34.22, and 38.20 MeV to be considered high spin states. The high selectivity of these states indicates that they are located near the ^{24}Mg yrast line.

X^2 fits of the absolute differential cross section to statistical model calculations, as a function of the spin of the final state (assuming singlets), were performed and are shown in Fig. 3. The 28.21 MeV state overcomes the 95% confidence level for an $I=12^+$ assignment.

In order to estimate the influence of the statistical fluctuations on the interpretation of the data, the effective number of channels N_{eff} as a function of the center of mass angle for differential cross sections and for total
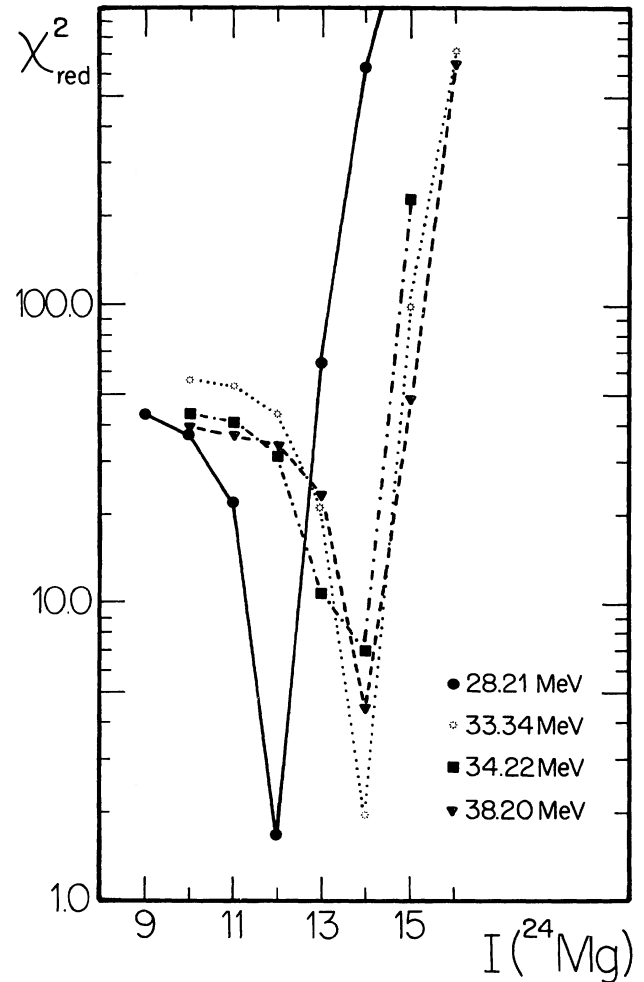

FIG. 3. X^2 fits of the absolute differential cross sections to statistical model calculations as a function of the spin of the ^{24}Mg final state.

cross sections, were evaluated for all the considered transitions using the code STATIS.[14] A value $N_{eff}(\theta_{c.m.}=90°)=5-6$ has been obtained. These numbers are substantially smaller than the maximum value possible, i.e.,

$$N_{max}=\tfrac{1}{2}(2I_A+1)(2I_a+1)(2I_B+1)(2I_b+1)$$

for the reaction $A(a,b)B$, due to the truncation of the entrance channel partial waves at the critical angular momentum which limits fusion cross sections. The fluctuation analysis of the angular distributions, when compared to other states and to the background,[8] indicates that the damping factor for the statistical fluctuations (Ericson fluctuations) is large enough to indicate that the 28.21, 33.34, 34.22, and 38.20 MeV are high spin states and that the spin values suggested are confident within one unit of angular momentum. In particular, the inclusion of the 28.21 MeV excitation energy state in the ^{24}Mg yrast sequence as the 12^+ yrast state is supported by the fluctuation analysis of the angular distribution with 75% probability[17] and at the 95% level according to the X^2 fit.
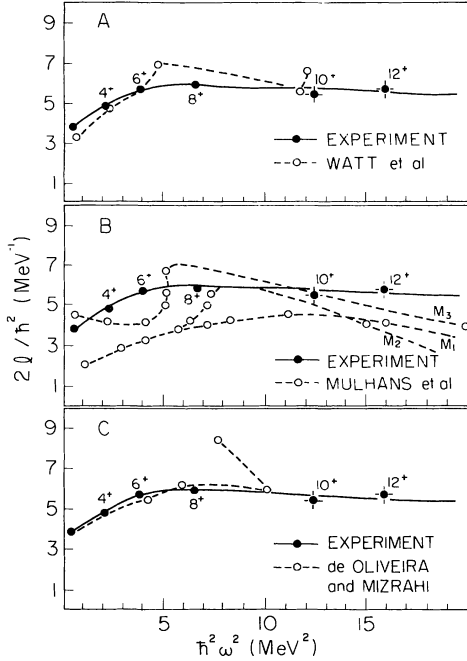
FIG. 4. Effective moment of inertia $2\mathscr{I}/\hbar^2$ as a function of the rotational frequency squared $\hbar^2\omega^2$. The large solid points 4^+, 6^+, 8^+, 10^+, and 12^+ and the full line represent the experimental results. (a) The dashed line (W) represents the predictions from Ref. 4. (b) The dashed lines ($M1$, $M2$, and $M3$) are from Ref. 9: $M1$: $(\beta,\gamma)=$const$=(0.35,-10°)$, $G_0=0.62$; $M2$: (β,γ) varies, $G_0=0.62$; $M3$: (β,γ) varies, $G_0=0$. (c) The dashed line represents the predicted curve from Ref. 10.

It is important to notice that the χ^2 fit to the magnitude of the $E^*=33.34$ MeV state converges to the value 14^+. However, this particular transition displays a broader peak than can be accounted for by the system resolution. In the case of corresponding to a singlet, the final state has a natural width $\Gamma_{33.34}\simeq230$ keV. Further discussion of this state should await higher resolution measurements.

Based on this new proposed yrast sequence, it is possible to calculate the effective moment of inertia as a function of the rotational frequency squared $(2\theta/\hbar^2$ vs $\hbar^2\omega^2)$. A comparison to the theoretical predictions is shown in Fig. 4.

The experimental data (full lines) indicate that ^{24}Mg tends toward an effective rigid rotorlike behavior with constant effective moment of inertia equivalent to an $A=24$ sphere $(2\theta/\hbar^2=5.85$ MeV$^{-1})$. The calculations performed by Mülhans et al. (Ref. 9) in Fig. 4(b) are able to reproduce this tendency only when the deformation of the system is maintained constant $(\beta,\gamma=0.35\ ;-10°)$; otherwise a clear upbend appears already at the $I=8$ state. Even with this constant deformation, together with the interaction strength used in their calculations (i.e., $G_0=0.62$), the moment of inertia turns out to be smaller than the experimental one. The constant deformation adopted in Ref. 9 $(\beta,\gamma=0.35;-10°)$ corresponds to the ground state. However, when the s-d shell band cutoff is reached, ^{24}Mg becomes oblate, and a transition to $(\beta,\gamma)=(0.30;-60°)$ takes place at $I=10$. The predicted upbend is associated with the occupation of the rotationally aligned $1f_{7/2}$ orbital by one neutron and one proton. On the other hand, this model is in accordance with the shell model calculations performed by Watt et al.,[4] who found that the yrast $I=8$ state is formed essentially by rotational alignment of one neutron pair and one proton pair in the $1d_{5/2}$ shell. The predicted yrast sequence of this shell model calculation is shown in Fig. 4(a) (dashed lines). According to these calculations, the lowest predicted 8^+ state (at $E^*=12.53$ MeV) does not belong to the ground state band $K^\pi=0^+$. The 8^+ ($K^\pi=0^+$) is predicted at 13.63 MeV. The calculated yrast 10^+ state decays as strongly to the $I=8$, $K^\pi=0^+$ state as to the $I=9$, $K^\pi=2^+$ state, and the lowest 12^+ decays very weakly to the first $I=10$ state. This leads to the conclusion that the band structure starts to disappear around $I=8-10$.

Calculations using the Hartree-Fock approximation[10] with the Skyrme force [Fig. 4(c)] predict an upbending around spin 12. They also make clear that at these high spin values the s-d shell is no longer sufficient to provide states of the correct J with the necessary configurational structure to continue the band. The coefficients of the s-d shell orbital expansion C_J are $(E^*;\ I,\ C_J^2)=(0.0;\ 0^+,\ 0.78)$, $(1.34$ MeV, 2^+, $0.415)$, $(4.26$ MeV, 4^+, $0.324)$, $(8.33$ MeV, 6^+, $0.12)$, $(13.18$ MeV, 8^+, $0.0211)$, $(19.54$ MeV, 10^+, $0.0017)$, and $(25.00$ MeV, 12^+, $5\times10^{-5})$.

Data from the present work suggest that the 12^+ yrast candidate found at 28.21 MeV determines a constant effective moment of inertia for the ^{24}Mg up to this state. However, since band structure is not expected to survive up to these high spin states, it appears that a constant effective moment of inertia results in spite of the alignment of nucleons in the $f_{7/2}$ shell.

The states observed at 33.34 and 34.22 MeV excitation energy may correspond to $I=13^-$ or 14^+ states. In this case, the predicted upbending in the yrast sequence would occur. If it turns out, however, that the lowest $I=14$ state is the 38.20 MeV, the yrast line would still display a constant moment of inertia. Further experiments will be required to settle this question.

If this latter case is confirmed, and if the spin sequence proposed by Cormier et al. for the ^{12}C + ^{12}C molecular resonances is confirmed,[18] the expected backbending in ^{24}Mg must occur at the lowest at $I=14$ due to the crossing molecular band. This final moment of inertia would not be in conflict with the extreme deformation expected by the liquid drop model.

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[1]A. H. Lumpkin, G. R. Morgan, J. D. Fox, and K. W. Kemper, Phys. Rev. Lett. **40**, 104 (1978).

[2]A. Szanto de Toledo, M. Schrader, E. M. Szanto, G. Rosner, and H. V. Klapdor, Nucl. Phys. **A315**, 500 (1979); Phys. Rev. C **13**, 555 (1979).

[3]A. J. Lazzarini, E. R. Cosman, A. Sperduto, S. G. Steadman, W. Thoms, and G. R. Young, Phys. Rev. Lett. **40**, 1426 (1978).

[4]A. Watt, D. Kelvin, and R. R. Whitehead, Phys. Lett. **63B**, 385 (1976); **63B**, 388 (1976); D. Kelvin, A. Watt, and R. R. Whitehead, J. Phys. G **3**, 1533 (1977).

[5]B. M. Preedom and B. H. Wildenthal, Phys. Rev. C **6**, 1633 (1972).

[6]H. Chanda and U. Mosel, Phys. Lett. **64B**, 373 (1976); Nucl. Phys. **A298**, 151 (1978).

[7]U. Mosel, Comments Nucl. Part. Phys. **9**, 213 (1981).

[8]A. Szanto de Toledo, M. M. Coimbra, N. Carlin Filho, T. M. Cormier, and P. M. Stwertka, Phys. Rev. Lett. **47**, 632 (1981); T. M. Cormier, A. Szanto de Toledo, M. M. Coimbra, N. Carlin Filho, P. M. Stwertka, M. Herman, and N. Nicolis, Phys. Lett. **B118**, 303 (1982); P. M. Stwertka *et al.*, Phys. Rev. Lett. **49**, 640 (1982).

[9]K. Mülhans, E. M. Müller, K. Neergard, and U. Mosel, Phys. Lett. **105B**, 32G (1981).

[10]D. R. de Oliveira and S. S. Mizarahi, Rev. Bras. Fis. **7**, 591 (1977).

[11]L. K. Fifield, R. W. Zurmuhle, and D. P. Balamuth, Phys. Rev. C **8**, 2217 (1983).

[12]M. J. Levine, in Proceedings of the Bormio Conference, 1981, p. 144.

[13]Y.-W. Lui, Y. Mihara, T. Murakami, K. Nagatani, R. Neese, N. Takahasi, D. M. Tanner, R. E. Tribble, and E. Ungrich, Progress in Research, Cyclotron Institute, Texas A&M University, 1982.

[14]R. G. Stokstad, Wright Nuclear Structure Laboratory, Yale University, International Report, 1972, Vol. 52 (unpublished).

[15]The critical angular momentum for compound nuclear formation (l_c) was taken to obey the Glas and Mosel form

$$\sigma_{\text{fus}}^{\text{exp}} = \pi R_c^2 \left[1 - \frac{V_c}{E} \right] = \pi \lambdabar^2 (l_c + 1)^2 ,$$

obtaining $R_c = 1.0 \, (A_1^{1/3} + A_2^{1/2})$ and $V_c = -7.75$ MeV.

[16]A. Szanto de Toledo, T. M. Cormier, M. Herman, B. Lin, P. M. Stwertka, M. M. Coimbra, and N. Carlin Filho, Phys. Rev. Lett. **47**, 1881 (1981).

[17]The probability $P(\Delta, N_{\text{eff}})$ of measuring a differential cross section σ_E so that $1 - \Delta \leq \sigma / \langle \sigma \rangle_{\text{HF}} \leq 1 + \Delta$ is given by the relation

$$P(\Delta, N_{\text{eff}}) = \int_{1-\Delta}^{1+\Delta} P(N_{\text{eff}}, y) \, dy .$$

[18]T. M. Cormier, C. M. Jachcinski, G. Berkowitz, P. Braun-Munzinger, P. M. Cormier, M. Gai, J. W. Harris, J. Barrette, and H. E. Wegner, Phys. Rev. Lett. **40**, 924 (1978).