Measurement of the quadrupole moment of the 5^- level in ^{206}Hg

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The quadrupole coupling constant of the $\pi(h_{11/2}^{-1}\, s_{1/2}^{-1})_{5^-}$ isomer in polycrystalline ^{206}Hg at 225 K has been measured by means of the perturbed angular distribution method. The result is $e^2Qq/h = 136.5 \pm 3$ MHz, from which we deduce $eQ = 65 \pm 13\ e\,\mathrm{fm}^2$ and an effective charge $e_{\mathrm{eff}}(\pi h_{11/2}^{-1}) = (2.2 \pm 0.5)e$.

INTRODUCTION

Studies of nuclei around doubly magic ^{208}Pb have contributed much to our understanding of single-particle structure and its interplay with collective excitations in nuclei. In particular, the electromagnetic properties of single-particle states often deviate strongly from the predictions of the shell model and they frequently reveal collective admixtures. Very little is known about the electromagnetic properties of the proton-hole orbitals, however, since only a few possibilities exist for investigating neutron-rich or proton-deficient nuclei in this region.

The use of in-beam γ-ray spectroscopy together with reactions induced by tritons has allowed us to study proton-deficient nuclei close to doubly magic ^{208}Pb. In particular, new information has been obtained on the electromagnetic properties of the $h_{11/2}$ proton hole that is often a constituent of isomeric states. The magnetic moment of the $h_{11/2}$ proton orbital has been deduced from a measurement[1] of the g factor of the $\frac{25}{2}^+$ isomer in ^{205}Tl, which has the configuration $(\nu p_{1/2}^{-1} i_{13/2}^{-1} \pi h_{11/2}^{-1})$. In the two-proton-hole nucleus ^{206}Hg, we found an $(h_{11/2}^{-1} s_{1/2}^{-1})_{5^-}$ isomer[2] (Fig. 1). This isomer opens the possibility of measuring a quadrupole moment by time differential observation of the perturbed angular distribution of the γ-ray decay. From the measured quadrupole moment we can determine the $E2$ matrix element $\langle h_{11/2}^{-1} || M(E2) || h_{11/2}^{-1} \rangle$. In this paper we describe the measurement and discuss the results.

EXPERIMENT AND RESULTS

The 5^- isomer in ^{206}Hg was produced by the ^{204}Hg(t,p) reaction at the tandem Van de Graaff accelerator facility of the Los Alamos National Laboratory. A target of polycrystalline Hg, enriched to $>98\%$ ^{204}Hg, was bombarded with a pulsed beam of 16-MeV tritons. The beam pulses were 1 ns wide and separated by 12.8 μs. The target, which was thick enough to stop the beam, was cooled to 225 ± 5 K, 10 deg below the melting point of Hg. Gamma rays from the target were measured by two Ge(Li) detectors, one placed at 0 and the other at 90 deg with respect to the direction of the incident beam. For each detector, the pulse heights and the time delays with respect to the beam pulses were recorded event-by-event on magnetic tape. The data were subsequently sorted off-line to obtain the perturbed angular-distribution patterns. The time scale was calibrated with an accuracy of $\pm 2\%$ by means of cables. Otherwise, the experimental techniques were as described in Ref. 2. Energy spectra of delayed γ rays from the reactions of 16-MeV tritons with ^{204}Hg are presented in Ref. 3.

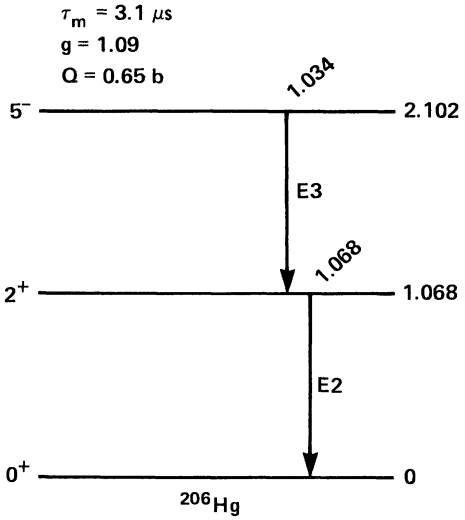

FIG. 1. The $J^\pi = 5^-$ isomer and its decay in ^{206}Hg. Energies are shown in MeV.

Figure 2 shows the perturbed-angular-distribution pattern $R(t)$ of the 1034-keV $E3$ transition:

$$R(t)=\frac{2}{3}\left[\frac{I(0°)-B(0°)}{I(90°)-B(90°)}-1\right]\approx A_{22}G_{22}(t)\ ,$$

where $I(\theta)$ represents the count rates at 0° and 90° and $B(\theta)$ a long lived background. The curve fitted to the data is the theoretical time distribution $G_{22}(t)$ for spin $I=5$ and a polycrystalline target in which the electric field gradient q has axial symmetry. This fit gives the following results:

$$\nu_0=2.275\pm0.05\ \text{MHz}\ ,$$

$$\Delta\nu/\nu=0.11\pm0.04\ ,$$

$$A_{22}=0.32\pm0.04\ .$$

Here ν_0 is the repetition frequency of the whole pattern, or $1/\nu_0$ is the time interval between the major peaks in Fig. 2.

The measured pattern shows a damping of the amplitude with time. The fitted curve reproduces the damping by using a Lorentzian distribution of frequencies rather than a single frequency; $\Delta\nu/\nu$ is the width (full width at half maximum) of this distribution that results in the best fit to the data. A_{22} describes the angular distribution at time zero as $W(\theta)=1+A_{22}P_2(\cos\theta)$; higher terms are neglected.

The quadrupole coupling constant e^2Qq/h is obtained from ν_0 by applying the appropriate spin factor:

$$e^2Qq/h=\nu_0\times4I(2I-1)/3$$

$$=136.5\pm3\ \text{MHz for}\ I=5\ .$$

The quadrupole moment eQ can be derived from this result if the electric-field gradient eq for a mercury nucleus in mercury metal is known. We obtain the electric-field gradient from the work of Haas and Shirley,[4] who measured the coupling constant for the $\frac{5}{2}^-$ level of ^{199}Hg in metallic mercury. They found $e^2qQ/h(^{199}\text{Hg},\ \frac{5}{2}^-)=210\pm20$ MHz at liquid-nitrogen temperature. The quadrupole moment Q of this state is 83 ± 9 fm^2 (Refs. 4 and 5), and therefore the electric-field gradient can be obtained.

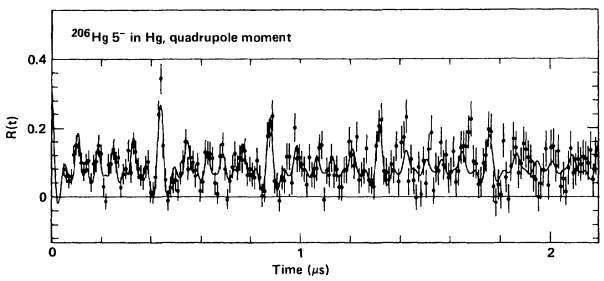

FIG. 2. The perturbed angular distribution pattern of the 1034-keV $E3$ γ ray in the decay of the 5^- isomer in ^{206}Hg.

Since our measurement was made at 225 K, a correction has to be applied for the change of the field gradient with temperature before we can use this result. The temperature dependence of the field gradient for a cadmium impurity in mercury has been measured by Mahnke *et al.*,[6] and the authors give convincing evidence from similar systems that mercury in mercury should have very nearly the same temperature behavior. The result is $q(225\ \text{K})=0.83q(77\ \text{K})$, and the error of this 17% correction can be neglected. Finally,

$$Q(^{206}\text{Hg},5^-)=Q(^{199}\text{Hg},\tfrac{5}{2}^-)\frac{136.5}{0.83\times210}$$

$$=65\pm13\ \text{fm}^2\ .$$

The error of 20% is a conservative estimate of the combined uncertainties in the calibration.

Our measurement is insensitive to the sign of Q; theory predicts $Q>0$. The present result, $A_{22}=0.32\pm0.04$, agrees with $A_{22}=0.38\pm0.05$ found in the g-factor measurement,[2] which was done under similar experimental circumstances except that the target was liquid mercury at room temperature. The pattern for $I=5$ shown in Fig. 2 reproduces nearly all of the details of the measured distribution well, although there are some indications that the target is not a perfect polycrystal. A fit for $I=4$, the spin proposed in Ref. 7 for the 2.102-MeV state, deviates visibly and has $\chi^2(I=4)=1.38$. By comparison, the fit for $I=5$ has $\chi^2(I=5)=1.11$. The confidence range for χ^2 is 1 ± 0.1. Therefore this experiment provides independent evidence of $I=5$ for the ^{206}Hg isomer. We did not try a fit for $I=6$, which is excluded by the γ-ray half-life. It might be noted that the shape of the time distribution depends only on the crystalline structure and on the spin of the isomeric state. The quadrupole interaction frequency gives a scale factor in time, and the A_{22} coefficient determines the overall amplitude. The 1068-keV $E2$ transition ($2^+\rightarrow0^+$) that also occurs in the decay of the isomer has only half the anisotropy of the $E3$ transition; therefore its time distribution was not useful for obtaining the quadrupole interaction. In this experiment the γ-ray lines from the decay of the $\frac{25}{2}^+$ isomer in ^{205}Tl were also observed, but we could not discern any time-perturbed angular distribution pattern for them.

DISCUSSION

The quadrupole moment gives directly the reduced $E2$ matrix element through the relation

$$\langle I||M(E2)||I\rangle$$

$$=\left[\frac{5}{16\pi}\right]^{1/2}\left[\frac{(2I+3)(2I+2)(2I+1)}{2I(2I-1)}\right]^{1/2}Q\ ,$$

which gives

$$\langle5^-||M(E2)||5^-\rangle=1.38eQ(5^-)=90\pm18\ e\ \text{fm}^2\ .$$

From this we can deduce the matrix element $\langle h_{11/2}^{-1}||M(E2||h_{11/2}^{-1}\rangle$, as presented in Table I. For a pure $(h_{11/2}^{-1}s_{1/2}^{-1})_{5^-}$ configuration, only angular-momentum

TABLE I. The reduced $E2$ matrix element $\langle h_{11/2}^{-1}||M(E2)||h_{11/2}^{-1}\rangle$ (e fm^2) for different 5^- wave functions, and comparison with theory (see the text).

	Evaluation of experiment						Theory[d]	
Pure	KH1[a]	KH2[a]	KH3[a]	MT[b]	Adopted	$h_{9/2}$[c]	Shell	FFS
96±19	89	92	92	92	92±20	−51	42	75

[a]References 8 and 9.
[b]Reference 10.
[c]Reference 11.
[d]Reference 12.

algebra is required since the $s_{1/2}$ particle does not contribute to the matrix element. In this case the result is 96 ± 19 e fm^2, as seen in the first column of Table I.

Wave functions of the 5^- level that include configuration mixing have been calculated by Herling and Kuo[8,9] in three different approximations, and by Ma and True.[10] We use these wave functions with the single-particle $E2$ matrix elements of Donahue *et al.*[11] to calculate the contributions of the small components to $Q(5^-)$. In each of the four wave functions, the amplitude of the main $h_{11/2}^{-1}s_{1/2}^{-1}$ component is greater than 0.95, and the contributions to the quadrupole moment due to the small admixtures partially cancel. Therefore the impurities in the 5^- wave function have only a small effect on the corrected values (Table I, second through sixth columns) for the reduced $E2$ matrix element of the $h_{11/2}$ proton hole. The error on the adopted value in the sixth column includes the uncertainty in these corrections. Other corrections, due to complex configurations based on, for example, the ^{208}Pb(3^-) state, were not carried out explicitly, but we estimate that they would not be significant within the accuracy of the present experiment.

In a first approximation, we can compare the $h_{11/2}$ matrix element with the measured value for $\langle h_{9/2}||M(E2)||h_{9/2}\rangle$ as given in the seventh column of Table I. Both matrix elements should be equal in magnitude (and of opposite sign) if the radial wave functions are identical. This comparison shows agreement within a factor of 2, but the difference exceeds the experimental errors.

Ring, Bauer, and Speth[12] used the shell model, and also the theory of finite Fermi systems, to calculate electric-quadrupole moments for nuclei around ^{208}Pb. The latter approach explicitly considers excitations of the ^{208}Pb core that result from the interaction between the quasiparticle states and the core. In the shell model, these excitations are taken into account by using an effective charge $e_{\rm eff}=e+e_{\rm pol}$. Systematic studies[13] in the lead region have shown that $e_{\rm pol}\approx0.5$ for protons and $e_{\rm pol}\approx1$ for neutrons.

The value of the $E2$ matrix element for the $h_{11/2}$ proton orbital, calculated[12] using the theory of finite Fermi systems, is 75 e fm^2 (Table I, ninth column). This agrees reasonably well with our measurement. The shell-model calculation[12] gives $\langle h_{11/2}||M(E2)||h_{11/2}\rangle=42$ e fm^2. The ratio of our measured matrix element to this value gives the effective charge $e_{\rm eff}(h_{11/2}^{-1})=2.2\pm0.5e$, or a polarization charge $e_{\rm pol}=e_{\rm eff}-e=1.2\pm0.5e$. This result can be compared with data from Astner *et al.*,[13] who surveyed polarization charges in the lead region for good shell-model states. They found $e_{\rm pol}(\pi h_{9/2})=0.53\pm0.04e$ and $e_{\rm pol}(\nu i_{13/2}^{-1})=0.96\pm0.04e$. Our measured value of $e_{\rm pol}=1.2\pm0.5e$ is larger, but the error extends close to the $\pi h_{9/2}$ value.

The experimental evidence for the $E2$ polarization of the ^{208}Pb core has previously been based mainly on neutron-hole and proton-particle states. For proton holes, the only measured case has been $B(E2,d_{3/2}^{-1}\rightarrow s_{1/2}^{-1})=280\pm40$ e^2fm^4 in ^{207}Tl.[11] The calculations for this transition give $B(E2)=199$ e^2fm^4 in the theory of finite Fermi systems and $B(E2)=57$ e^2fm^4 in the shell model. Comparing the measurement with the shell-model result gives

$$\frac{e_{\rm eff}}{e}=\left[\frac{280\pm40}{57}\right]^{1/2}=2.2\pm0.2\ ,$$

or

$$e_{\rm pol}=1.2\pm0.2\ ,$$

in agreement with our result for the $\pi h_{11/2}^{-1}$ orbital. Thus, both measurements for proton-hole states give quadrupole matrix elements that are larger by about the same factor than the expected values based on the systematics of other states.

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