Microscopic relativistic nucleon-nucleus inelastic scattering

J. R. Shepard and E. Rost
Department of Physics, University of Colorado, Boulder, Colorado 80309

J. Piekarewicz
Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania 19104
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We formulate a microscopic relativistic treatment of nucleon-nucleus inelastic scattering. Both the continuum and bound state single nucleon wave functions are obtained by solving the Dirac equation with scalar and timelike vector potentials. The interaction which drives the transition is expressed in terms of invariant combinations of Dirac matrices. The structure of the resulting amplitudes is presented, and certain of their features, such as selection rules, are discussed.

I. INTRODUCTION

Recent years have seen the development of relativistic models of the nucleon-nucleus interaction. These provide descriptions of infinite nuclear matter[1,2] and ground state structure of finite nuclei[3-5] which are appealing in many ways. In addition, phenomenological relativistic descriptions of proton-nucleus elastic scattering are qualitatively consistent with the relativistic models and have been shown[6,7] to be superior in many ways to the standard nonrelativistic phenomenology. More recently, it has been demonstrated[8,9] that an essentially parameter-free model of proton-nucleus elastic scattering based on the relativistic impulse approximation[10] is in excellent agreement with data and is superior to comparable nonrelativistic results at all bombarding energies above $\simeq$100 MeV.[11]

These successes suggest extensions of the relativistic approaches to include other processes which might provide more stringent tests of the models. For example, Miller[12] and Serot[13] have examined the influence of relativistic nuclear dynamics on electron-nucleus elastic scattering. Shepard *et al.*[14] have formulated a relativistic microscopic treatment of electron inelastic scattering. Several workers[15] have looked at the implications of relativistic nuclear dynamics for (e,e′p) processes. There has also been a preliminary attempt[16] to assess the influence of nuclear relativity on weak interaction amplitudes. None of these models has yet confronted experiment in such a way that firm conclusions about the appropriateness of the relativistic description of the nucleon-nucleus interactions can be drawn.

Intermediate energy nuclear physics facilities, over the past several years, have generated a great deal of high quality nucleon-nucleus inelastic scattering data. At present, an impressive capability for the measurement of exotic spin observables is being developed. In intermediate energy elastic scattering, the spin dependent observables provided the clearest signature of nuclear relativity to date. This strongly suggests that inelastic scattering, where the selectivity of various nuclear transitions can be exploited to emphasize specific elements of the effective NN interaction (many of which *cannot* participate in elastic scattering), will provide crucial tests of relativistic models of nuclear structure and scattering.

Macroscopic (or collective) relativistic models of nucleon-nucleus inelastic scattering have already been developed.[17,18] While such models have many important applications, their utility is severely limited in that they contain only the dynamics already present in elastic scattering. In the present paper, we present a fully microscopic formulation of nucleon-nucleus inelastic scattering which apart from the explicit treatment of exchange, encompasses the full range of dynamics for single-step inelastic processes.

II. THE TRANSITION AMPLITUDE

We wish to calculate the transition amplitude for nucleon-nucleus inelastic scattering in the framework of a relativistic distorted wave impulse approximation (DWIA). We note that in the present work we do not include *explicit* treatment of exchange processes. However, there are exchangelike effects which are implicit in a relativistic treatment. We consider a process in which a nucleus is excited from an initial state, $\Psi_{J_iM_i}$, to a final state $\Psi_{J_fM_f}$. We then take the transition amplitude to be

$$T_{fi}=\sum_{n=1}^{A}\psi^{\dagger(-)}_{\vec{k}'s'}\Psi^{\dagger}_{J_fM_f}\gamma^0(0)\gamma^0(n)\hat{t}(0,n)\psi^{(+)}_{\vec{k}s}\Psi_{J_iM_i}\ , \qquad (1)$$

where integration over the A target nucleons and the projectile (0) is implied. The projectile wave functions, $\psi_{ks}(\mp)$, have boundary conditions specified by $(-)$ or $(+)$ and asymptotic momentum and spin projection indicated by $\vec{k}$ and s, respectively; the nuclear wave functions Ψ_{JM} are functions of the coordinates of all A constituent nucleons. In Eq. (1), γ^0 is the usual timelike vector Dirac matrix and $\hat{t}$ is the relativistic nucleon-nucleon interaction which drives the transition. We assume that the relativistic wave functions are solutions to a fixed energy Dirac equation containing as yet unspecified relativistic potentials, i.e.,

$$(\vec{\alpha}\cdot\vec{\mathrm{p}}+\beta m+V_0)\psi_{\vec{\mathrm{k}},s}=E_k\psi_{\vec{\mathrm{k}},s} \tag{2a}$$

for the projectile (0) and

$$\left[\sum_{n=1}^{A}(\vec{\alpha}_n\cdot\vec{\mathrm{p}}_n+\beta_n m)+V_t\right]\Psi_{JM}=E\Psi_{JM} \tag{2b}$$

for the target (t). In specific applications, these potentials will usually consist of strong scalar and timelike vector interactions which characterize current relativistic models of nuclear dynamics. The detailed nature of these potentials, as well as the techniques used to solve Eq. (2), will be presented in a future publication in which quantitative results using the formulation presented in this paper will be compared with data and with comparable nonrelativistic calculations.

We now introduce a complete set of momentum eigenstates, Fourier transform the configuration space projectile and target wave functions, and write

$$\begin{aligned}T_{fi}=\sum_{n=1}^{A}\int &\frac{d^4p_0'd^4p_0d^4p_n'd^4p_n}{(2\pi)^4(2\pi)^4(2\pi)^4(2\pi)^4}d^4x'\,d^4y'\,d^4x\,d^4y\,\psi_{k's'}^{\dagger(-)}(x')\\ &\times\Psi_{J_fM_f}^{\dagger}(y_1,\ldots,y_n',\ldots,y_A)\gamma^0(0)\gamma^0(n)e^{-ip_0'\cdot x'}e^{-ip_n'\cdot y'}\langle p_0'p_n'\,|\,\hat{t}\,|\,p_0p_n\rangle\\ &\times e^{ip_0\cdot x}e^{ip_n\cdot y}\psi_{ks}^{(+)}(x)\Psi_{J_iM_i}(y_1,\ldots,y_n,\ldots,y_A)\;.\end{aligned} \tag{3}$$

We make the impulse approximation and assume that the (off-shell) NN interaction, $\hat{t}$, is given by the form of the free (on-shell) NN amplitude, t^{NN}. In analogy with Eq. (1), we can write the elementary free NN amplitude as

$$\begin{aligned}t_{fi}^{\mathrm{NN}}&=\psi_{k_1's_1'}^{\dagger}\psi_{k_2's_2'}^{\dagger}\gamma^0(1)\gamma^0(2)t_{\mathrm{NN}}\psi_{k_1s_1}\psi_{k_2s_2}\\ &=\int\frac{d^4p_1'd^4p_2'd^4p_1d^4p_2}{(2\pi)^4(2\pi)^4(2\pi)^4(2\pi)^4}d^4x_1'\,d^4x_2'\,d^4x_1\,d^4x_2\psi_{k_1's_1'}^{\dagger}(x_1')\\ &\quad\times\Psi_{k_2's_2'}^{\dagger}(x_2')\gamma^0(1)\gamma^0(2)e^{-ip_1'\cdot x_1'}e^{-ip_2'\cdot x_2'}\langle p_1'p_2'\,|\,\hat{t}_{\mathrm{NN}}\,|\,p_1p_2\rangle e^{ip_1\cdot x_1}e^{ip_2\cdot x_2}\psi_{k_1s_1}(x_1)\psi_{k_2s_2}(x_2)\;.\end{aligned} \tag{4}$$

The nucleon spinors now satisfy the free Dirac equation, i.e.,

$$\begin{aligned}\psi_{ks}(x)&=e^{-ik\cdot x}u(k,s)\;,\\ u(k,s)&=\left[\frac{E+m}{2m}\right]^{1/2}\begin{bmatrix}1\\ \dfrac{\vec{\sigma}\cdot\vec{\mathrm{k}}}{E+m}\end{bmatrix}\chi_s\;,\end{aligned} \tag{5}$$

where χ_s is the Pauli spinor for spin projection s. The integrations indicated in Eq. (4) can now be performed trivially giving

$$t_{fi}^{\mathrm{NN}}=\bar{u}(k_1',s_1')\bar{u}(k_2',s_2')\hat{t}_{\mathrm{NN}}u(k_1,s_1)u(k_2,s_2) \tag{6a}$$

$$=\frac{-8\pi i p_{\mathrm{NN}}}{E_{\mathrm{NN}}}\bar{u}(k_1',s_1')\bar{u}(k_2',s_2')F_{\mathrm{NN}}u(k_1,s_1)u(k_2,s_2)\;, \tag{6b}$$

where F_{NN} are the familiar[20] set of relativistic invariants used to describe the NN amplitudes and where p_{NN} and E_{NN} are the nucleon momentum and total energy, respectively, in the NN center-of-momentum frame. In the present context, the impulse approximation consists specifically of making the following "operator" identification for the nucleon-nucleon interaction $\hat{t}$ in Eqs. (1) and (3):

$$\hat{t}\rightarrow\frac{-8\pi i p_{\mathrm{NN}}}{E_{\mathrm{NN}}}\hat{F}_{\mathrm{NN}}\;, \tag{7}$$

where the caret on $\hat{F}_{\mathrm{NN}}$ now indicates that it is an operator in the space of four-component spinors of two nucleons. Note that this implies a specific off-shell extrapolation[21] since off-shell (*and* negative energy) spinors *are implicitly* present in the Fourier transforms of Eq. (3). In contrast, those of Eq. (4) which define t_{NN} contain contributions from on-shell, positive-energy spinors only.

We now exploit the harmonic time dependence of solutions of fixed energy Dirac equations to perform the indicated time and energy integrations. Then, explicitly imposing conservation of three-momentum, we have

$$T_{fi}=\frac{-8\pi i p_{\rm NN}}{E_{\rm NN}}\sum_{n=1}^{A}\int\frac{d\vec{p}_0}{(2\pi)^3}\frac{d\vec{p}_n}{(2\pi)^3}\frac{d\vec{q}}{(2\pi)^3}d\vec{x}\,'d\vec{y}\,'_n d\vec{x}\,d\vec{y}_n$$
$$\times\psi^{\dagger(-)}_{\vec{k}',s'}(\vec{x}\,')\Psi^{\dagger}_{J_fM_f}(\vec{y}_1,\ldots,\vec{y}\,'_n,\ldots,\vec{y}_A)e^{i(\vec{p}_0-\vec{q})\cdot\vec{x}\,'}e^{i(\vec{p}_n+\vec{q})\cdot\vec{y}\,'_n}\hat{F}_{\rm NN}(p_0',p_n';p_0,p_n)$$
$$\times e^{-i\vec{p}_0\cdot\vec{x}}e^{-i\vec{p}_n\cdot\vec{y}_n}\psi^{(+)}_{\vec{k},s}(\vec{x})\Psi_{J_iM_i}(\vec{y}_1,\ldots,\vec{y}_n,\ldots,\vec{y}_A)\,, \qquad (8)$$

where $p_0=[E(p),\vec{p}_0]$, $p_n=[E_0(n),\vec{p}_n]$, $p_0'=[E'(p),\vec{p}\,'_0]$, $p_n'=[E'(n),\vec{p}\,'_n]$, with E (E') being the total energy of the indicated particle in the initial (final) state.

We know that $\hat{F}_{\rm NN}$ must be a function only of the relativistic kinematic invariants

$$s=(k_1+k_2)^2=(k_1'+k_2')^2$$

and

$$t=(k_1-k_1')^2=(k_2'-k_2)^2\,.$$

In order to simplify Eq. (8), we assume that the explicit dependence of $\hat{F}_{\rm NN}$ on s can be ignored and take $\hat{F}_{\rm NN}$ at a fixed value, s_0, evaluated from the asymptotic four-momentum in some appropriate frame such as the nucleon-nucleus center-of-momentum frame or the Breit frame. We also make the approximation that $t^2\simeq-\vec{q}\cdot\vec{q}$, where $\vec{q}$ is the local three-momentum transfer in our chosen reference frame. These assumptions result in a local form for $\hat{F}_{\rm NN}$ and we perform several of the remaining integrations in Eq. (8), obtaining

$$T_{fi}=\frac{-8\pi i p_{\rm NN}}{E_{\rm NN}}\sum_{n=1}^{A}\int d\vec{x}\,d\vec{y}_n\psi^{\dagger(-)}_{\vec{k}'s'}(\vec{x})\Psi^{\dagger}_{J_fM_f}(\vec{y}_1,\ldots,\vec{y}_A)\gamma^0(0)\gamma^0(n)\hat{F}_{\rm NN}(s_0,|\vec{x}-\vec{y}|)\psi^{(+)}_{\vec{k}s}(\vec{x})\Psi_{J_iM_i}(\vec{y}_1,\ldots,\vec{y}_A)\,, \qquad (9)$$

where

$$\hat{F}_{\rm NN}(s_0,|\vec{x}-\vec{y}|)=\frac{1}{(2\pi)^3}\int d\vec{q}\,e^{-i\vec{q}\cdot(\vec{x}-\vec{y})}\hat{F}_{\rm NN}(s_0,\vec{q}^{\,2})\,.$$

This is the specific form of transition amplitude which we employ throughout the remainder of the present work.

III. EVALUATION OF THE TRANSITION AMPLITUDE

In order to make progress in evaluating the amplitude of Eq. (9), it is useful to examine the specific structure of $\hat{F}_{\rm NN}$. As already stated, this object has a well known[20] (but *not* unique[21]) structure, namely,

$$\hat{F}_{\rm NN}=\hat{F}_S+\gamma(1)\cdot\gamma(2)\hat{F}_V+\gamma^5(1)\gamma^5(2)\hat{F}_P+\gamma^5(1)\gamma^5(2)\gamma(1)\cdot\gamma(2)\hat{F}_A+\sigma^{\mu\nu}(1)\sigma_{\mu\nu}(2)\hat{F}_T \qquad (10)$$

where S, V, P, A, and T refer to scalar, vector, pseudoscalar, axial vector, and antisymmetric tensor, respectively, with complex amplitudes $\hat{F}$. We use the Bjorken and Drell[22] convention for the form of the Dirac matrices. Note that throughout this work, and in Eq. (10) in particular, isospin indices are suppressed for clarity. In order to calculate amplitudes for transitions between discrete nuclear levels, it is convenient[18] to rewrite the expression in Eq. (10) in a form which explicitly shows the spin dependence and the implied combinations of upper and lower components of the relativistic wave functions. We can write

$$\begin{aligned}
&\gamma(1)\cdot\gamma(2)=\Gamma_2(1)\Gamma_2(2)-\Gamma_4(1)\Gamma_4(2)\vec{\sigma}(1)\cdot\vec{\sigma}(2)\,,\\
&\gamma^5(1)\gamma^5(2)=\Gamma_3(1)\Gamma_3(2)\,,\\
&\gamma^5(1)\gamma^5(2)\gamma(1)\cdot\gamma(2)=\Gamma_4(1)\Gamma_4(2)-\Gamma_2(1)\Gamma_2(2)\vec{\sigma}(1)\cdot\vec{\sigma}(2)\,,\\
&\sigma^{\mu\nu}(1)\sigma_{\mu\nu}(2)=2[\Gamma_1(1)\Gamma_1(2)+\Gamma_3(1)\Gamma_3(2)]\vec{\sigma}(1)\cdot\vec{\sigma}(2)\,,
\end{aligned} \qquad (11)$$

where the structure matrices, Γ_ν, are defined by

$$\Gamma_1=1=\begin{bmatrix}1&0\\0&1\end{bmatrix},\quad \Gamma_2=\gamma^0=\begin{bmatrix}1&0\\0&-1\end{bmatrix},$$
$$\Gamma_3=\gamma^5=\begin{bmatrix}0&1\\1&0\end{bmatrix},\quad \Gamma_4=\gamma^0\gamma^5=\begin{bmatrix}0&1\\-1&0\end{bmatrix}. \qquad (12)$$

It should be noted that these Γ_ν matrices have no other content than the specification of the particular combinations of upper and lower components. The interactions of Eq. (10) then take the compact form

$$\gamma^0(1)\gamma^0(2)\hat{F}_{NN}(|\vec{x}-\vec{y}|)=\sum_{\nu=1}^{4}[f^{\nu}(|\vec{x}-\vec{y}|)+g^{\nu}(|\vec{x}-\vec{y}|)\vec{\sigma}(1)\cdot\vec{\sigma}(2)]\Gamma_\nu(1)\Gamma_\nu(2)\ , \tag{13}$$

where we have defined

$$f^1=\hat{F}_V,\ f^2=\hat{F}_S,\ f^3=\hat{F}_A,\ f^4=\hat{F}_P\ ,$$
$$g^1=-\hat{F}_A,\ g^2=2\hat{F}_T,\ g^3=-\hat{F}_V,\ g^4=2\hat{F}_T\ . \tag{14}$$

Note that, apart from the Γ_ν's, the form of the interaction in Eq. (13) is much simpler than for the corresponding "nonrelativistic" (e.g., Wolfenstein[23]) form of the NN interaction. Specifically, it is entirely local and contains only "central" and "spin-spin" pieces. The additional complexity of the amplitude (such as nonlocalities and spin-orbit or tensor terms) is contained implicitly in the Γ_ν matrices.

From Eqs. (13) and (14) we observe that, for the spin-independent terms, the scalar and vector amplitudes involve only upper-upper and lower-lower coupling while the pseudoscalar and axial vector amplitudes involve only upper-lower combinations. For the spin-dependent terms, the roles of vector and axial vector amplitudes are reversed while the antisymmetric tensor amplitude contributes to both types of coupling.

To proceed further in the calculation we perform a multipole expansion of the invariant interaction operator:

$$\gamma^0(0)\gamma^0(n)\hat{F}_{NN}(|\vec{x}-\vec{y}_n|)=\sum_{\nu=1}^{4}\sum_{LSJM}(-1)^{J+L+S}h^{\nu}_{L,S}(x,y)[Y_L\sigma_S(0)]^*_{JM}[Y_L(\hat{y}_n)\sigma_S(n)]_{JM}\Gamma_\nu(0)\Gamma_\nu(n)\ , \tag{15}$$

where

$$h^{\nu}_{L,S}=f^{\nu}_L\ \text{ for } S=0\ ,$$
$$\quad =g^{\nu}_L\ \text{ for } S=1\ , \tag{16}$$

and where $\sigma_0=1$. This technique enables us to factorize the calculation into target and projectile parts which will now be treated in turn.

A. Target space evaluation

We expand the operators in the target space using second quantization and replace the operator

$$\sum_{n=1}^{A} h^{\nu}_{L,S}(x,y_n)[Y_L(\hat{y}_n)\sigma_S(n)]_{JM}\Gamma_\nu(n)$$

with the equivalent operator

$$\sum_{j_f m_f j_i m_i}\langle\psi_{j_f m_f}(y)\,|\,h^{\nu}_{L,S}(x,y)[Y_L(\hat{y})\sigma_S]_{JM}\Gamma_\nu\,|\,\psi_{j_i m_i}\rangle a^{\dagger}_{j_f m_f}a_{j_i m_i}$$
$$=-\sum_{j_i j_f}\left[\frac{(2j_f+1)}{(2J+1)}\right]^{1/2}\langle\psi_{j_f}||h^{\nu}_{L,S}(x,y)[Y_L(\hat{y})\sigma_S]_J\Gamma_\nu||\psi_{j_i}\rangle[a^{\dagger}_{j_f}\tilde{a}_{j_i}]_{JM}\ ,$$

where the "bra" is defined by $\langle\psi|=\psi^{\dagger}=\bar{\psi}\gamma^0$. Here, $a^{\dagger}_{jm}$ and a_{jm} are the single nucleon creation and annihilation operators, respectively, in the target space and the ψ_{jm} are single particle orbitals to which the creation and annihilation operators refer. For our purposes they are four-component eigenfunctions of a Dirac equation like Eq. (2a). The hole creation operator is related to the particle annihilation operator by

$$\tilde{a}_{jm}=(-1)^{j-m}a_{j,-m}\ . \tag{17}$$

Since $[a^{\dagger}_{j_f}\tilde{a}_{j_i}]_{JM}$ is now the only operator active in the target space, we can take matrix elements between the initial and final nuclear states obtaining

$$\langle\Psi_{J_f M_f}\,|\,[a^{\dagger}_{j_f}\tilde{a}_{j_i}]_{JM}\,|\,\Psi_{J_i M_i}\rangle=(J_i J M_i M\,|\,J_f M_f)\mathscr{A}^{J_f J_i}_{J(j_f j_i)}\ , \tag{18}$$

in terms of conventional[24] nuclear structure amplitudes $\mathscr{A}$. The reduced matrix elements follow the convention of Brink and Satchler.[25]

We are now able to write the transition form factor (an operator in the projectile space) as

$$\langle J_f M_f\,|\,\gamma^0\hat{F}_{NN};\vec{x}\,|\,J_i M_i\rangle=\sum_{\nu LSJ}G^{J_f J_i}_{LSJ\nu}(x)(J_i J\,M_i M\,|\,J_f M_f)[Y_L(\hat{x})\sigma_S]^*_{JM}\Gamma_\nu\ , \tag{19}$$

where

$$G^{J_fJ_i}_{LSJ\nu}(x)=-\sum_{j_fj_i}(-1)^{J+L+S}\left[\frac{2j_f+1}{2J+1}\right]^{1/2}\mathscr{A}^{J_fJ_i}_{J(j_fj_i)}\langle\psi_{j_f}(\vec{y})||h^{\nu}_{LS}(x,y)[Y_L(\hat{y})\sigma_S]_J\Gamma_\nu||\psi_{j_i}(\vec{y})\rangle\ . \tag{20}$$

The angular momentum selection rules are determined by the reduced matrix element in Eq. (20) which is

$$\langle\psi_{j_f}||\cdots||\psi_{j_i}\rangle=[2(2L+1)(2S+1)(2J+1)(2\lambda_i+1)(2j_i+1)]^{1/2}$$
$$\times(4\pi)^{-1/2}(L\,\lambda_i\,0\,0\,|\,\lambda_f\,0)\begin{Bmatrix}\frac{1}{2}&\lambda_i&j_i\\\frac{1}{2}&\lambda_f&j_f\\S&L&J\end{Bmatrix}\times\text{ radial integral.} \tag{21}$$

The single particle bound state wave functions are written as[14,22]

$$\psi_{ljm}(\vec{y})=\begin{bmatrix}u_{lj}(y)\mathscr{Y}_{ljm}(\hat{y})\\iw_{lj}(y)\mathscr{Y}_{l'jm}(\hat{y})\end{bmatrix}, \tag{22}$$

where l' is the "other" l giving the same j and u_{lj} and w_{lj} are real. The λ values appearing in Eq. (21) are either l or l' as determined by the component combinations implied by Γ_ν. The specific selection rules are fixed by the parity Clebsch-Gordan coefficient and the 9-j symbol in Eq. (21).

In order to make Eq. (20) more explicit, we consider the specific case of a 0^+ initial target state. Then, for a final state with spin-parity J^π, we have

$$G^{J0}_{LSJ\nu}(x)=\sum_{j_fj_i}(-1)^{J+L+S}\left[\frac{2j_f+1}{2J+1}\right]^{1/2}\mathscr{A}^{J0}_{J(j_fj_i)}H^{\nu}_{LSJ}(x)\ ,$$

where

$$\begin{aligned}H^1_{LSJ}(x)&=\int_0^\infty y^2dy(u_fh^1_{LS}u_iB^{LSJ}_{l_fl_i}+w_fh^1_{LS}w_iB^{LSJ}_{l'_fl'_i})\ ,\\H^2_{LSJ}(x)&=\int_0^\infty y^2dy(u_fh^2_{LS}u_iB^{LSJ}_{l_fl_i}-w_fh^2_{LS}w_iB^{LSJ}_{l'_fl'_i})\ ,\\H^3_{LSJ}(x)&=i\int_0^\infty y^2dy(u_fh^3_{LS}w_iB^{LSJ}_{l_fl'_i}-w_fh^3_{LS}u_iB^{LSJ}_{l'_fl_i})\ ,\\H^4_{LSJ}(x)&=i\int_0^\infty y^2dy(u_fh^4_{LS}w_iB^{LSJ}_{l_fl'_i}+w_fh^4_{LS}u_iB^{LSJ}_{l'_fl_i})\ ,\end{aligned} \tag{23}$$

and where

$$B^{LSJ}_{\lambda_f\lambda_i}=\langle[\lambda_f\tfrac{1}{2}]_{j_f}||[Y_L\sigma_S]_J||[\lambda_i\tfrac{1}{2}]_{j_i}\rangle\ . \tag{24}$$

We complete the evaluation of the full transition amplitude by combining Eqs. (19) and (9), obtaining, in general,

$$T_{fi}=\frac{-8\pi ip_{\rm NN}}{E_{\rm NN}}\sum_{LSJ\nu}(J_i\,J\,M_i\,M\,|\,J_fM_f)\int d\vec{x}\,\psi^{\dagger(-)}_{\vec{k}'s'}(\vec{x})G^{j_fj_i}_{LSJ\nu}(x)[Y_L(\hat{x})\sigma_S]^*_{JM}\Gamma_\nu\psi^{(+)}_{\vec{k}s}(\vec{x})\ . \tag{25}$$

The remaining integration may be evaluated by two methods, one involving a partial-wave expansion of the distorted wave and the other a simpler eikonal semianalytical treatment.

B. Projectile space evaluation: partial waves

In partial wave expansion, the distorted waves can be written as[18]

$$\psi^{(+)}_{\vec{k}s}(\vec{x})=\left[\frac{4\pi}{kx}\right]^{1/2}\left[\frac{E+m}{2m}\right]^{1/2}\sum_{J_aL_a}i^{L_a}e^{i\delta^C_{L_aJ_a}}\sqrt{2L_a+1}(L_a\,\tfrac{1}{2}\,0\,s\,|\,J_a\,s)\begin{bmatrix}g_{J_a}(x)\mathscr{Y}_{L_aJ_as}(\hat{x})\\if_{J_a}(x)\mathscr{Y}_{L'_aJ_as}(\hat{x})\end{bmatrix}, \tag{26}$$

$$\psi^{\dagger(-)}_{\vec{k}'s'}(\vec{x})=\frac{4\pi}{k'x}\left[\frac{E'+m}{2m}\right]^{1/2}\sum_{J_bL_bM_b}i^{-L_b}e^{i\delta^C_{L_bJ_b}}(L_b\,\tfrac{1}{2}\,M_b\,s'\,|\,J_b\,s'+M_b)$$
$$\times[g_{J_b}(x)\mathscr{Y}_{L_bJ_bs'+M_b}(\hat{x}),-if_{J_b}(x)\mathscr{Y}_{L'_bJ_bs'+M_b}(\hat{x})]Y^*_{L_bM_b}(\hat{k}')\ , \tag{27}$$

where δ^C are Coulomb phase shifts and $g_J(x)$ and $f_J(x)$ are the upper and lower component radial solutions to the Dirac equation which are specified in detail in Ref. 18.

Substitution of Eqs. (26) and (27) into Eq. (25) allows us to evaluate the angular and radial integrals and to obtain, after some manipulation, for a definite J transfer,

$$T_{fi}^{ss'M}=\frac{-8\pi i p_{\rm NN}}{E_{\rm NN}}\frac{4\pi}{kk'}\sum_{L_b}\beta_{L_b}^{ss'M}Y_{L_b,s-s'-M}(\theta,0)\ , \tag{28}$$

where

$$\beta_{L_b}^{ss'M}=\sum_{LSJ_aL_aJ_b\nu} i^{L_a-L_b}(-1)^{J+L}[2(2J+1)(2L+1)(2S+1)(2L_a+1)(2J_b+1)(2\lambda_b+1)]^{1/2}$$
$$\times(\lambda_b\,L\,0\,0\,|\,\lambda_a\,0)(J_b\,J\,s-M\,M\,|\,J_a\,s)\begin{Bmatrix}\lambda_b & \frac{1}{2} & J_b\\ L & S & J\\ \lambda_a & \frac{1}{2} & J_a\end{Bmatrix}$$
$$\times(L_a\,\tfrac{1}{2}\,0\,s\,|\,J_a\,s)(L_b\,\tfrac{1}{2}\,s-s'-M\,s'\,|\,J_b\,s-M)I_{J_a\lambda_aJ_b\lambda_b}^{LSJ\nu} \tag{29}$$

and θ is the scattering angle. In Eq. (29), $\lambda=L$ (L') for an upper (lower) component and the radial integrals are

$$I_{J_a\lambda_aJ_b\lambda_b}^{LSJ\nu}=e^{i(\delta_{L_aJ_a}^C+\delta_{L_bJ_b}^C)}\int dx\, h_{J_b\lambda_b}^{\nu}(x)G_{LSJ_\nu}^{J_fJ_i}(x)h_{J_a\lambda_a}^{\nu}(x)\ , \tag{30}$$

where $h_{J\lambda}^{\nu}(x)$ denotes $g_{J\lambda}(x)$ or $\pm i f_{J\lambda}(x)$ according to how the upper-lower component combinations are specified by the index ν. The $+i$ factor for $f_{j\lambda}$ is used for initial projectile states with $-i$ used for final projectile states as required by the Hermitian conjugation operation in Eq. (25).

C. Projectile space evaluation: Eikonal approximation

An alternative approach for calculating the relativistic distorted waves which is suitable at medium energies uses the Dirac eikonal approximation.[19] In this framework the distorted waves have a simple analytic form given by

$$\psi_{\vec{k}s}^{(\pm)}(\vec{x})=\left[\frac{E+m}{2m}\right]^{1/2}\begin{bmatrix}1\\ \dfrac{1}{E+m+V_S-V_V}\ \vec{\sigma}\cdot\vec{p}\end{bmatrix}e^{i\vec{k}\cdot\vec{x}}e^{iS^{(\pm)}(\vec{x})}\chi_s\ , \tag{31}$$

where V_S and V_V are the Dirac scalar and vector potentials and where the eikonal phase $S(\vec{x})$ is written in terms of the effective central and spin-orbit distortions [functions of V_S and V_V, see Ref. (19)] as

$$S^{(\pm)}(\vec{x})=-\frac{m}{k}\int_{\mp\infty}^{z}dz'\{V_C(\vec{b},z')+V_{SO}(\vec{b},z')[\vec{\sigma}\cdot(\vec{b}\times\vec{k})-ikz']\}\ . \tag{32}$$

and $\vec{x}=(\vec{b},z)$. Note that $S(\vec{x})$ is still an operator in spin space.

The transition amplitude is evaluated by substituting Eq. (31) into Eq. (25) to obtain

$$T_{fi}=\frac{-8\pi i p_{\rm NN}}{E_{\rm NN}}\frac{(E+m)}{2m}\sum_{LSJ\nu}(J_iJM_iM\,|\,J_fM_f)$$
$$\times\int e^{-i\vec{k}_f\cdot\vec{x}}e^{-iS^{\dagger(-)}(\vec{x})}\left[1,\frac{\vec{\sigma}\cdot\vec{p}}{E+m+V_S-V_V}\right]G_{LSJ\nu}^{J_fJ_i}(x)$$
$$\times[Y_L(\hat{x})\sigma_S]_{JM}^{*}\Gamma_\nu\begin{bmatrix}1\\ \dfrac{1}{E+m+V_S-V_V}\ \vec{\sigma}\cdot\vec{p}\end{bmatrix}e^{i\vec{k}_i\cdot\vec{x}}e^{iS^{(+)}}(\vec{x})\ . \tag{33}$$

Working to leading order in the eikonal expansion, we can evaluate the above expression as

$$T_{fi}=\frac{-8\pi i p_{\rm NN}}{E_{\rm NN}}\sum_{LSJ\nu}(J_i J M_i M\,|\,J_f M_f)\int d\vec{x}\,e^{i\vec{q}\cdot\vec{x}}e^{-\chi(\vec{b})}\left[\frac{E+m+V_S-V_V}{2m}\right]$$
$$\times U^{\dagger}(\vec{\Phi})\left\{\left[1,\frac{\vec{\sigma}\cdot\vec{k}_f}{E+m+V_S-V_V}\right]\right.$$
$$\left.\times G_{LSJ\nu}^{J_fJ_i}(x)[Y_L(\hat{x})\sigma_S]^{*}_{JM}\Gamma_\nu\begin{bmatrix}1\\ \frac{1}{E+m+V_S-V_V}\,\vec{\sigma}\cdot\vec{k}_i\end{bmatrix}\right\}U(\vec{\Phi})\,, \qquad (34)$$

where $U(\vec{\Phi})=e^{-(1/2)\vec{\sigma}\cdot\vec{\Phi}}$ is a spin rotation operator by an angle

$$\vec{\Phi}=\left[2mb\int_{\infty}^{z}V_{\rm SO}(\vec{b},z')dz'\right](\hat{b}\times\hat{k})$$

and the phase $\chi(\vec{b})$ is given by

$$\chi(\vec{b})=-\frac{m}{k}\int_{-\infty}^{\infty}dz'[V_C+V_{\rm SO}\vec{\sigma}\cdot(\hat{b}\times\hat{k})]\,. \qquad (35)$$

We note that the presence of distortion will favor small values of z in which case the rotation angle takes the simple form $\vec{\Phi}\simeq=-\chi_{\rm SO}(\vec{b})\hat{k}\times\hat{b}$ and $\chi_{\rm SO}(\vec{b})$ involves only the spin-orbit (SO) part of Eq. (35).

IV. DISCUSSION

The general structure of the various terms comprising the full transition amplitude, T_{fi}, is indicated schematically along with the related selection rules in Table I. Note that in the table, the operations implied by the Γ_ν matrices in Eq. (20) and (25) (i.e., the connection of upper and lower components in various combinations) have been explicitly carried out.

The origin and meaning of the selection rules are most easily understood by consideration of specific examples. Let us consider idealized transitions in ^{12}C. We assume the 0^+ ground state to consist of a closed $p_{3/2}$ shell and all transitions to consist of the promotion of a $p_{3/2}$ nucleon to the $p_{1/2}$ shell. Thus we have

$$|0^+_{\rm g.s.}\rangle=|0\rangle,\quad |J^+\rangle=[p_{1/2}p_{3/2}^{-1}]_J\,|0\rangle,\quad J=1,2\,.$$

Consider the excitation of a natural parity 2^+ level via the scalar interaction, which clearly involves

$$\Gamma_1=1=\begin{bmatrix}1&0\\0&1\end{bmatrix}$$

in Eqs. (20) (for target) and Eq. (25) (for projectile). In this case, only upper-to-upper and lower-to-lower component combinations are possible and no explicit spin-spin operators are present. These facts are reflected by the form of the amplitudes containing the scalar interaction in Table I. The selection rules come from the angular momentum and parity restrictions associated with the nuclear matrix elements of Eq. (20). In the present case, we can only connect upper-to-upper and lower-to-lower components of $p_{3/2}$ and $p_{1/2}$ wave functions. The upper-to-upper connection is identical to that encountered in the normal nonrelativistic treatment where the parity Clebsch-Gordan coefficients [Eq. (21)] ensure that only even L transfers ($L=2$ in this case) contribute. In examining the lower-to-lower combination, however, we must recall that the lower component of the bound nucleon wave function [Eq. (22)] has the same j as but opposite parity from the upper component. We therefore observe that the lower component of the $p_{3/2}$ wave function looks like a $d_{3/2}$ wave function while the lower component of the $p_{1/2}$ wave function looks like an $s_{1/2}$ wave function. Consequently, in the lower-to-lower amplitude, only an $L=2$ transfer is allowed, as before. Quite generally, the selection rules for all component combinations associated with a given operator [Eq. (20)]

TABLE I. Relativistic (p,p′) amplitudes and selection rules. Interactions: S (scalar), V (vector), P (pseudoscalar), A (axial vector), and T (antisymmetric tensor). Spinors:

$$|ij\rangle=|i\rangle_{\rm projectile}\times|j\rangle_{\rm target}\,,$$

$$|1\rangle\rightarrow \text{upper component},\quad |2\rangle\rightarrow \text{lower component}\,.$$

Amplitudes	Natural parity	Unnatural parity
$(11\,\|\,(V+S)\,\|\,11)$	$L=J$	forbidden
$(12\,\|\,(V-S)\,\|\,12)$	$L=J$	forbidden
$(21\,\|\,(V-S)\,\|\,21)$	$L=J$	forbidden
$(22\,\|\,(V+S)\,\|\,22)$	$L=J$	forbidden
$(11\,\|\,(A+P)\,\|\,22)$	forbidden	$L=J$
$(12\,\|\,(A-P)\,\|\,21)$	forbidden	$L=J$
$(21\,\|\,(A-P)\,\|\,12)$	forbidden	$L=J$
$(22\,\|\,(A+P)\,\|\,11)$	forbidden	$L=J$
$(11\,\|\,(2T-A)\sigma_p\cdot\sigma_t\,\|\,11)$	$L=J$	$L=J\pm1$
$(12\,\|\,(-2T-A)\sigma_p\cdot\sigma_t\,\|\,12)$	$L=J$	$L=J\pm1$
$(21\,\|\,(-2T-A)\sigma_p\cdot\sigma_t\,\|\,21)$	$L=J$	$L=J\pm1$
$(22\,\|\,(2T-A)\sigma_p\cdot\sigma_t\,\|\,22)$	$L=J$	$L=J\pm1$
$(11\,\|\,(2T-V)\sigma_p\cdot\sigma_t\,\|\,22)$	$L=J\pm1$	$L=J$
$(12\,\|\,(-2T-V)\sigma_p\cdot\sigma_t\,\|\,21)$	$L=J\pm1$	$L=J$
$(21\,\|\,(-2T-V)\sigma_p\cdot\sigma_t\,\|\,12)$	$L=J\pm1$	$L=J$
$(22\,\|\,(2T-V)\sigma_p\cdot\sigma_t\,\|\,11)$	$L=J\pm1$	$L=J$

$$[Y_L(\hat{y})\sigma_S]_{JM}\Gamma_\nu \quad (36)$$

are identical.

We now consider our unnatural parity example, the 1^+ excitation. For amplitudes involving the scalar interaction, the preceding discussion showed that only even-L transfers are possible. Since no spin operator is present, we have $L=J$ and such an unnatural parity transition is forbidden as indicated in Table I. However, for amplitudes involving the pseudoscalar interaction, the operator Γ_3 is present in Eqs. (20) and (25) and upper-lower and lower-upper combinations are implied. We therefore connect $p_{3/2}$ to $s_{1/2}$ and $d_{3/2}$ to $p_{1/2}$ with the spin-angle function $[Y_L\sigma_S]_{JM}$. The parity Clebsch-Gordan coefficient then requires odd-L transfers ($L=1$ in this case) implying that this transition is allowed. Note that there is no overall parity change in the transition because upper and lower components themselves have opposite parity and are connected by an odd-L transfer. All of the selection rules in Table I follow from similar considerations.

The transition amplitude T_{fi} forms a matrix in the spin spaces of the projectile and target and all observables are constructed from it by taking the usual traces. The overall normalization of the cross section is straightforwardly determined by kinematical considerations. Wallace,[26] for example, defines an invariant nucleon-nucleon amplitude as $(2ip)^{-1}f_{\rm NN}$. Then the elastic NN amplitude in the nucleon-nucleus center-of-momentum frame is given by

$$f_{\rm NA}=\frac{p_{\rm NA}}{p_{\rm NN}}f_{\rm NN}=p_{\rm NA}\left[\frac{-E_{\rm NN}}{4\pi p_{\rm NN}}\right]t_{\rm NN} ,$$

where $p_{\rm NN}$ is the projectile momentum in the nucleon-nucleus frame. We can then write the nucleon-nucleus frame NN elastic cross section as

$$\frac{1}{4}\sum_{\substack{\text{all}\\ \text{spins}}}|f_{\rm NA}|^2=\frac{1}{4}\left[\frac{m}{2\pi}\right]^2\left[\frac{p_{\rm NA}E_{\rm NN}}{2p_{\rm NN}m}\right]^2\sum_{\substack{\text{all}\\ \text{spins}}}|t_{\rm NN}|^2 .$$

We then have, for nucleon-nucleus inelastic scattering,

$$\frac{d\sigma}{d\Omega}(\text{p,p}')\,|_{\rm NA}=\frac{1}{2(2J_i+1)}\left[\frac{m}{2\pi}\right]^2\eta(E_L,M)\sum_{ss'M_t}|T_{fi}^{ss'M_t}|^2 , \quad (37)$$

where

$$\eta(E_L,M)=\left[\frac{p_{\rm NA}E_{\rm NN}}{2p_{\rm NN}m}\right]^2=\frac{[(E_L+m)/(2m)]^2}{1+2E_L/M+(m/M)^2} . \quad (38)$$

In Eq. (38), E_L is the total energy of the projectile in the laboratory frame and m and M are the projectile and target rest masses, respectively. This normalization is consistent with that implied by Eq. (I.25) of the original Kerman, McManus, and Thaler (KMT) paper.[27]

The primary difference between our work and standard nonrelativistic treatments is that several nonlocalities are included via the lower component wave functions which have no direct counterparts in the nonrelativistic theories. In the latter approaches, these kinds of nonlocalities enter in practice only through the explicit treatment of the knock-on exchange process. Detailed comparison of the present formulation with nonrelativistic treatments and with experimental data will be made in future publications. Preliminary comparisons with data are highly encouraging.

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